Analysis of Construction Dynamic Plan Using Fuzzy Critical Path Method

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Critical Path Method (CPM) technique has become widely recognized as valuable tool for the planning and scheduling large construction projects. The aim of this paper is to present an analytical method for finding the Critical Path in the precedence network diagram where the duration of each activity is represented by a trapezoidal fuzzy number. This Fuzzy Critical Path Method (FCPM) uses a defuzzification formula for trapezoidal fuzzy number and applies it on the total float (slack time) for each activity in the fuzzy precedence network to find the critical path. The method presented in this paper is very effective in determining the critical activities and finding the critical paths.

Key words: *critical path method, trapezoidal fuzzy number, network diagram, defuzzification*

1. INTRODUCTION

The Critical Path Method (CPM) is a vital tool for the planning and control of complex projects. The successful implementation of CPM requires the availability of a clearly defined duration for each activity. However, in practical situations there are many cases where the activity duration can not be presented in a precise manner.

For construction projects, on account of long duration of the construction and risks that accompany this process, it is often very difficult or almost impossible to accurately predict the duration of an activity, and consequently to take it for granted that the given activity will be finished on the very same day that is given in the dynamic plan of construction.

In engineering practice, durations of different activities are usually taken from productivities rates for man-hours calculation, which are often too generalized and sometimes obviously not accurate. For example, productivity rates for man-hours calculation for in-situ reinforcement fixing are based only on total amount of the reinforcing steel [1], regardless of the pattern complexity which can greatly affect time needed for proper placing, tying and control. Because of that, patterns consisting of 12Ø16 and 3Ø32 bars, respectively, have exactly the same total amount of steel and consequently the same theoretical number of man-hours needed for placing and fixing, although it is obvious that such result would not be realistic, as was proven in studies [2–4]. Besides that, Proverbs at al. [5] have proven that productivity rates can significantly wary from country to country.

All the above mentioned problems can lead to an unreliable dynamic plan for a given construction project.

Deterministic version of CPM, known in practice for decades, is characterized by the fact that the duration of any activity in the network diagram is known and expressed deterministically (by exactly one number). However, it would be more realistic to have the duration of any construction activity and deadline for its accomplishment in the dynamic plan of construction expressed as an interval of a few days rather than one specific day (date).

The first solution of this problem has emerged in the form of PERT (Program Evaluation and Review Technique) method, which is based on the theory of probability. But there is a problem in the application of this method in practice because there are no norms with defined optimistic, normal and pessimistic durations of the activities.

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A possible solution of this problem would be to use the fuzzy logic to create dynamic construction plans. Implementation of fuzzy logic and fuzzy sets would offer a new alternative – the development of the Fuzzy Critical Path Method (FCPM) for the analysis of time in the network diagram considering uncertain duration of construction activities. In the other words, it would apply the fuzzy theory instead of the probability theory.

In the literature there are many works that deal with the definition of fuzzy critical paths in a network diagram [6, 7]. The basic assumption is that the duration of any given activity can be expressed by a fuzzy number, which requires knowledge of the relevant algebraic operations with fuzzy numbers, ranking of fuzzy numbers and defuzzyfication.

This paper presents an analytical method for finding the critical path in the fuzzy precedence network diagram, which applies the method of defuzzificating total float (slack time) of every activity in the network diagram.

2. FUZZY NUMBERS

In this study, duration of a given activity (marked with DUR) is represented by a trapezoidal fuzzy number $ft_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij},)$, as shown in Figure 1.

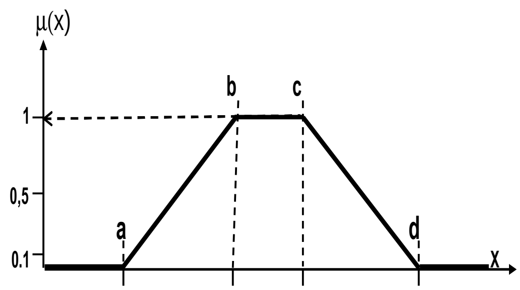

Figure 1 - Graphical presentation of a trapezoidal fuzzy number

Fuzzy number $ft_{ij} = (a_{ij}, b_{ij}, c_{ij}, d_{ij},)$ is a trapezoidal fuzzy number if its membership function can be expressed by the following equation:

$$\mu(x) = \begin{vmatrix} \dfrac{x-a}{b-a}; & a \le x \le b \\ 1; & b \le x \le c \\ \dfrac{d-x}{d-c}/; & c \le x \le d \end{vmatrix} \qquad (1)$$

The formulation of fuzzy numbers that would define the durations of construction activities can be achieved by the expert methods, or by asking four experienced professionals (experts) for the duration of activities.

2.1 Algebraic operations with fuzzy numbers

In order to carry out necessary steps in creating and analysis of the network diagram, one has to know elementary algebraic operations with fuzzy numbers.

Addition of fuzzy numbers is conducted as follows [8]:

$$\begin{aligned} FN_1 + FN_2 &= (a_1,b_1,c_1,d_1) + (a_2,b_2,c_2,d_2) = \\ &= (a_1+a_2, b_1+b_2, +c_1+c_2, d_1+d_2) \end{aligned} \qquad (2)$$

while the subtraction is conducted as [8]:

$$\begin{aligned} FN_1 - FN_2 &= (a_1,b_1,c_1,d_1) - (a_2,b_2,c_2,d_2) \\ &= (a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2) \end{aligned} \qquad (3)$$

2.2 Defuzzification

Comparison of fuzzy numbers is not always easy, especially if they are partly overlapping each other. To make it possible, it is necessary to conduct defuzzification, i.e. to translate them into real numbers.

In the literature one can find more than forty different methods for comparison and ranking fuzzy numbers. In this study, the following formula was used for defuzzification of a given fuzzy number FN (a, b, c, d) and translating it into real number [9]:

$$D = \frac{(c^2+d^2+cd)-(a^2+b^2+ab)}{3[(c+d)-(b+a)]} \qquad (4)$$

This defuzzification formula can not be applied to the trapezoidal fuzzy number with equal elements because that would be a crisp number [8].

3. CPM IN FUZZY NETWORK DIAGRAMS

CPM is an analysis technique with three main purposes:

- To calculate the project finish date;
- To identify to what extent each activity in the schedule can slip (float) without delaying the project;
- To identify the activities with the highest risk, i.e. the ones that cannot slip without changing project finish date.

The process of defining fuzzy critical path (FCPM) is the same as in the deterministic critical path method (CPM), but with using fuzzy numbers and the relevant algebraic operations [7].

Precedence diagramming places the activities on the nodes and uses arrows between the nodes to show the sequence between each activity. Such node is represented by a square, normally divided into 7–9 boxes. Each box contains information such as activity name, its duration (DUR), early start and finish (ES, EF), late start and finish (LS, LF) and floats like free float and total float (TF) [4]. Since there are no standards for organization of these boxes, an adequate legend has to be provided with a precedence diagram to show a meaning of each box, as shown in Figure 2.

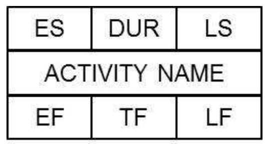

Figure 2 - The legend of activities in a precedence network diagram

Application of the fuzzy critical path method will be illustrated by the example presented in Figure 3.

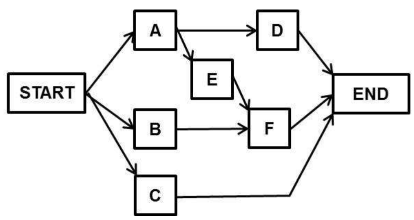

Figure 3 - Precedence network diagram numbers

Durations for given activities were obtained from four different sources (experts) and their values are presented in Table 1.

Table 1. Activities' durations obtained by experts

	Expert			
Activity	Exp1	Exp2	Exp3	Exp4
A	4	6	6	8
B	9	6	12	10
C	2	4	7	5
D	3	5	6	7
E	1	6	3	9
F	9	6	12	6

Therefore, their fuzzy annotations are as follows: A(4, 6, 6, 8), B(6, 9, 10, 12), C(2, 4, 5, 7), D(3, 5, 6, 7), E(1, 3, 6, 9) and F(6, 9, 9, 12).

Activities Start and End are so-called milestone activities, which means that they just represent significant events so they have no duration, i.e. their values are Start (0, 0, 0, 0) and End (0, 0, 0, 0).

4. CRITICAL PATH ANALYSIS

CPM uses activity durations and relationships between activities to calculate schedule dates. This calculation is done in three passes through the activities in a given project:

- forward pass calculation;
- backward pass calculation;
- total float.

4.1 Forward pass

The forward pass is the first part of the CPM calculation procedure. The purpose of the forward pass is to determine the earliest moment in which the activities can finish or start without impacting the overall project.

The procedure starts with the first activity in the schedule and placing the project start date in the early start time box (ES) of milestone activity "Start", where the term "milestone activity" denotes activity with zero duration that typically represents a significant event, usually the beginning and/or the end of the project.

Many projects are scheduled according to work days and therefore, if the weather impact is not considered or if it is not otherwise instructed, a project may be assumed to start on a day zero.

Consequently, the early start time of the activity "Start" is ES_{start} = (0, 0, 0, 0).

If we denote the duration of any activity as DUR, its early finish time (EF) can be calculated as:

$$EF = ES + DUR \tag{5}$$

Early finish time (EF) of the activity "Start" will be:

$$EF_{start} = ES_{start} + DUR_{start} = (0,0,0,0) + (0,0,0,0) \tag{6}$$

The early finish time of any given activity becomes the early start time of its subsequent activities (Figure 4).

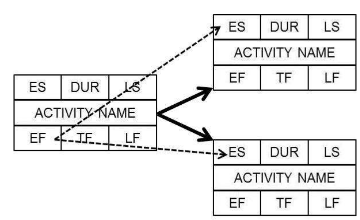

Figure 4 - Correlation between early finish and early start of the subsequent activities

Therefore, for example presented in Fig. 2, it will be:

$$ES_A = ES_B = ES_c = EF_{start} = (0,0,0,0) \tag{7}$$

For activities with multiple priors, the ES for any activity is the greatest (i.e. the latest) EF of all preceding activities, based upon network logic (Fig. 5).

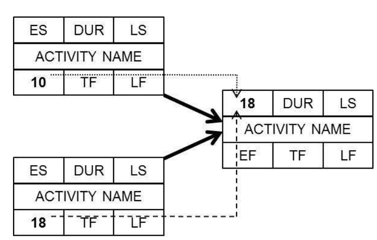

Figure 5 - Early start in case of multiple preceding activities

In example presented in Fig.3, activity F has two preceding activities (E and B) so it will be:

$$ES(F) = \max[EF(B); EF(E)] =$$
$$= \max\left[\langle ES(B) + DUR(B)\rangle; \langle ES(E) + DUR(E)\rangle\right] =$$
$$= \max\left[\langle (0,0,0,0) + (6,9,10,12)\rangle \langle (4,6,6,8) + (1,3,6,9)\rangle\right] =$$
$$= \max\left[(6,9,10,12); (5,9,12,17)\right]$$

In order to determin which of the obtained values is greater, it is necessary to apply the defuzzification formula (4) to calculate real values of given fuzzy numbers.

According to (4), fuzzy numbers (6, 9, 10, 12) and (5, 9, 12, 17) can be translated into real numbers as follows:

$$D(6,9,10,12) = \frac{(10^2 + 12^2 + 10 \cdot 12) - (6^2 + 9^2 + 6 \cdot 9)}{3[(10+12) - (9+6)]}$$

$$D(6,9,10,12) = 9.19$$

$$D(5,9,12,17) = \frac{(12^2 + 17^2 + 12 \cdot 17) - (5^2 + 9^2 + 5 \cdot 9)}{3[(12+7) - (9+5)]}$$

$$D(5,9,12,17) = 10.8$$

Since 10.8 is greater than 9.19, the number (5, 9, 12, 17) is greater than (6, 9,10,12) and therefore ES(F) = (5, 9, 12, 17).

All calculated values of ES and EF for activities given in a network diagram in Fig. 3 are presented in Tab 2.

Table 2. The values of ES and EF for activities in a network diagram in Fig. 3

Activity	ES	DUR	EF
START	(0,0,0,0)	(0,0,0,0)	(0,0,0,0)
A	(0,0,0,0)	(4,6,6,8)	(4,6,6,8)
B	(0,0,0,0)	(6,9,10,12)	(6,9,10,12)
C	(0,0,0,0)	(2,4,5,7)	(2,4,5,7)
D	(4,6,6,8)	(3,5,6,7)	(7,11,12,15)
E	(4,6,6,8)	(1,3,6,9)	(5,9,12,17)
F	(5,9,12,17)	(6,9,9,12)	(11,18,21,29)
END	(11,18,21,29)	(0,0,0,0)	(11,18,21,29)

4.2 Backward Pass

The backward pass is the second step of the CPM calculation procedure and its purpose is to determine the latest moment that an activity can finish or start without impacting the overall project.

Backward pass calculation should be startedwith the last activity in the network and performed towards the first activity in the network.

The first step in the backward pass is to take the early finish time of the last activity in the schedule as the late finish time (LF) of that activity, i.e. EF_{END} = LF_{END} (Figure 6).

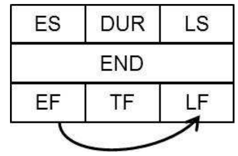

Figure 6 - Early and late finish of the last activity

Once the last activity's late finish time has been set, one can calculate the activity's late start time (LS) by subtracting the activity duration (DUR) from the late finish time:

$$LS = LF - DUR \tag{8}$$

The late start time of any given activity becomes the late finish time for its preceding activity or activities (Fig. 7).

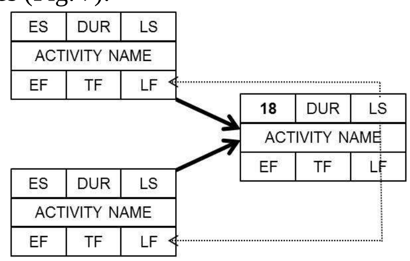

Figure 7 - Correlation between late start and late finish of the preceding activities

For example presented in Fig. 3, it will be:

$$LF_C = LF_D = LF_F = LS_{END} = (11,18,21,29) \tag{9}$$

In case of activities with multiple successors, the late finish time of a given activity is the earliest of the successors' late start times (Fig 8).

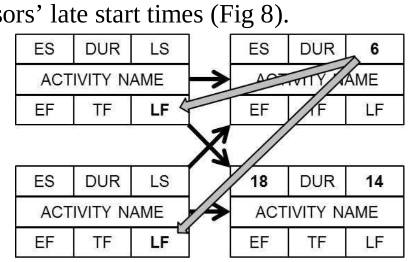

Figure 8 - Late finish in case of multiple succeeding activities

For example, in the network diagram presented in Fig. 3, activity A has two succeding activities (D and E), so it will be:

$$LF(A) = \min[LS(D); LS(E)]$$
$$= \min\left[\langle LF(D) - DUR(D)\rangle; \langle LF(E) - UR(E)\rangle\right]$$
$$= \min\left[\langle (11,18,21,29) - (3,5,6,7)\rangle \langle (-1,9,12,23) - (1,3,6,9)\rangle\right]$$
$$= \min\left[4,12,16,26); (-10,3,9,22)\right]$$

Comparison of obtained fuzzy numbers will be possible after defuzzification in accordance with (4).

$$D(4,12,16,26) = \frac{(16^2 + 26^2 + 16 \times 26) - (4^2 + 12^2 + 4 \times 16)}{3[(16+26) - (4+12)}$$
$$= 14,6$$

This means that fuzzy number (4,12,16,26) is equivalent to the real number 14.6.

$$D(-10,3,9,22) = \frac{(9^2 + 22^2 + 9\times 22) - ((-10)^2 + 3^2 + (-10)\times 3)}{3[(9+22) - ((-10)+3)}$$
$$= 6$$

Fuzzy number $(-10,3,9,22)$ is equivalent to the real number 6.

Therefore:

$$LS(D) > LS(E) \qquad (10)$$

Consequently:

$$LF(A) = \min[LS(D), LS(E)] = LS(E) \qquad (11)$$

Therefore:

$$LF(A) = (-10,3,9,22) \qquad (12)$$

The calculated values of LF and LS of all activities in a network diagram are presented in Tab. 3

Table 3. The values of LF and LS for activities in a network diagram in Fig. 3

Activity	LF	DUR	LS
START	(0,0,0,0)	(0,0,0,0)	(-18,-3,3,18)
A	(-10,3,9,22)	(4,6,6,8)	(-18,-3,3,18)
B	(-1,9,12,23)	(6,9,10,12)	(-13,-1,3,17)
C	(11,18,21,29)	(2,4,5,7)	(4,13,17,27)
D	(11,18,21,29)	(3,5,6,7)	(4,12,16,26)
E	(-1,9,12,23)	(1,3,6,9)	(-10,3,9,22)
F	(11,18,21,29)	(6,9,9,12)	(-1,9,12,23)
END	(11,18,21,29)	(0,0,0,0)	(11,18,21,29)

4.3. Total Float

The third part of schedule calculation using CPM is the calculation of the total float or slack time.

Total float (TF) is the amount of time by which an activity (or a whole chain of activities) can be delayed from its early start without delaying the contract completion date. The total float for any activity can be calculated as:

$$TF = LF - EF \qquad (13)$$

Calculated values of total float for activities in a network diagram in Figure 3 are presented in Table 3.

Although total float has to be calculated and reported for every activity in a network, it is an attribute of a network path and therefore not associated with any specific activity along the path.

For example, total float of the activity A, defined as TF_A = (–18, –3, 3, 18), after defuzzification in accordance with (4), will be:

$$D(A) = \frac{(3^2 + 18^2 + 3\times 18) - ((3)^2 + (-18)^2 + (-3)\times(-18)}{3[(3+18) - ((-3)+(-18))]} = 0$$

which means that fuzzy number (-18,-3,3,18) is equivalent to the real number 0 (zero).

Table 4. The values of total float (TF) of activities in a network diagram in Fig. 3

Activity	LF	EF	TF
START	(0,0,0,0)	(0,0,0,0)	(-18,-3,3,18)
A	(-10,3,9,22)	(4,6,6,8)	(-18,-3,3,18)
B	(-1,9,12,23)	(6,9,10,12)	(-13,-1,3,17)
C	(11,18,21,29)	(2,4,5,7)	(4,13,17,27)
D	(11,18,21,29)	(7,11,12,15)	(-4,6,10,22)
E	(-1,9,12,23)	(5,9,12,17)	(-18,-3,3,18)
F	(11,18,21,29)	(11,18,21,29)	(-18,-3,3,18)
END	(11,18,21,29)	(11,18,21,29)	(-18,-3,3,18)

In network diagrams, there are many possible paths, but the critical path is the one that extends from the start of the project to its end on which all total floats have the zero value.

Table 5. The values of Total Float (TF) of activities in a network diagram in Fig.3

Activity	TF (fuzzy number)	TF (real number)
START	(-18,-3,3,18)	**0**
A	(-18,-3,3,18)	**0**
B	(-13,-1,3,17)	1.62
C	(4,13,17,27)	11.97
D	(-4,6,10,22)	8.08
E	(-18,-3,3,18)	**0**
F	(-18,-3,3,18)	**0**
END	(-18,-3,3,18)	**0**

This means that the critical path for the given problem consists of activities Start, A, E, F and End (Figure 9).

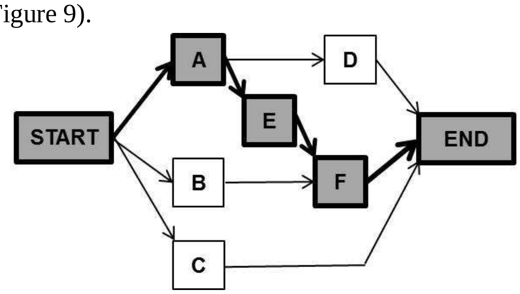

Figure 9 - Fuzzy critical path

5. DISCUSSION

If the same problem was solved using deterministic version of CPM, there would be four different solutions, each one based on different set of data obtained from a different source (Figure 10).

Experts 1 and 2 would prognose the same total duration of the given project (18 days), but with significantly different critical paths and important data about starts and finishes of activities.

Experts 3 and 4 would prognose much longer duration of project (24 and 23 days, respectively), and they would also provide different critical paths.

The only solution consistent with the one obtained by the fuzzy critical path method is the one based on data provided by Expert 2, but there would also be some discrepancies considering starts, finishes and total floats of critical activities.

Of course, this does not mean that experts 1, 3 and 4 are not reliable, but that their opinions are based on different experiences and attitudes. If we presume that all data presented in Table 1 are relevant in a sufficient degree, it can be concluded that the solution obtained by the fuzzy critical path method is the most probable one because it includes remarkably different informations from four different but reliable sources and gives time limits not defined deterministically but probabilistically, i.e. as the time intervals.

It should be noted here that activities' durations in presented example are purposefully taken to be remarkably different in order to use small set of activities for illustrating how different opinions and sources can lead to significantly different conclusions and decisions. For example, in case of the activity E, it is highly unlikely that for the same activity one expert would prognose that it would last one day, while the other one would say it would be nine days. On the other hand, presented example has only six activities, while the real construction projects consist of several dozens or even hundreds of activities and therefore even small differences of a day or two can build up to a significant discrepancy in the total project duration or the critical path structure.

In figure 10, it can be seen that results significantly vary considering the both most important aspects of the problem, i.e. the total duration of the project and the structure of the critical path.

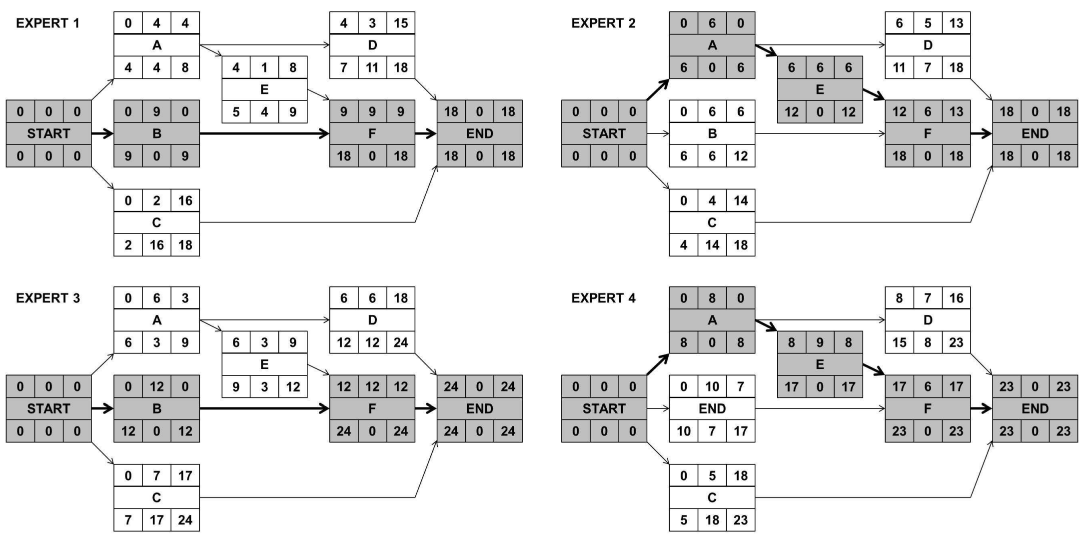

Figure 10 - Results obtained by solving the problem using deterministic critical path method

6. CONCLUSION

In practice, the duration of the construction activities in the dynamic construction plans is usually assessed on the basis of professional experience and a sense of experienced individuals. Therefore, the fuzzy logic and fuzzy numbers can be a very good choice and a suitable mathematical tool to solve the problem of imprecision and to avoid any subjectivity when considering the duration of construction activities.

Analytical method presented in this paper can be very effective tool for solving the problems of the dynamic construction plans.

REFERENCES

[1] Mijatović, R.; Norms and Standards in Civil Engineering, Book 2 (in Serbian), Građevinska knjiga, Belgrade, 2008.

[2] Milajić, A., Beljaković, D., Pejičić, G.: Optimal reinforced concrete beams design using hybrid GA-TABU algorithm. Technics Technologies Education Management, 8 (2), 2013, pp. 533–540.

[3] Milajić, A. (2009) Linear programming in optimal design of reinforced concrete structures. u: Mladenović N., Urošević D. (ur.) SYM-OP-IS, Ivanjica, Serbia, proceedings, str. 165-168, in Serbian.

[4] Milajić, A. (2007) Primena Simpleks metode u projektovanju armiranobetonskih konstrukcija. Beograd, magistarska teza.

[5] Proverbs, D.; Holt, G. D.; Olomolaiye, P. A comparative evaluation of planning engineers' formwork productivity rates in European construction. Building and Environment, 33 (4), 1998,. pp. 181–187.

[6] Dutina, V., Marković, Lj., Kovačević, M.: Application of possibilitistic procedure when planning the time for completion of construction projects, Materijali i konstrukcije u građevinarstvu, 4, 2011, pp. 25–40.

[7] Kurij, K. and Jovanović, S.: Application of fuzzy theory in the development of dynamic plans of building the critical path method, The scientific journal „Tehnička dijagnostika – Građevinarstvo“, 1 (2011), pp.22–27.

[8] Ravi Shankar, N., Sireesha, V. and Phani Bushan Rao, P.: An Analytical Method for Finding Critical Path in a Fuzzy Project Network, Int. J. Contemp. Math. Sciences, 5 (20), 2010, pp. 953–962.

[9] Chen, C. T. and Huang, S. F.: Applying fuzzy method for measuring criticality in project network, Information sciences, 177, 2007, pp. 2448–2458.

REZIME

ANALIZA DINAMIČKOG PLANA U GRAĐEVINARSTVU PRIMENOM FAZI METODE KRITIČNOG PUTA

Metoda kritičnog puta postala je široko prihvaćena kao koristan alat za planiranje velikih građevinskih projekata. Cilj ovog rada je da se predstavi analitički metod iznalaženja kritičnog puta u mrežnom dijagramu u kome je trajanje svake aktivnosti predstavljeno trapezoidnim fazi brojem. Fazi metoda kritičnog puta prikazana u ovom radu veoma je delotvorna u određivanju kritičnih aktivnosti i iznalaženju kritičnog puta.

Ključne reči: *metod kritičnog puta, trapezoidni fazi broj, mrežni plan, defazifikacija*