FUZZIFICATION OF THE SAATY’S SCALE AND A PRESENTATION OF THE HYBRID FUZZY AHP-TOPSIS MODEL: AN EXAMPLE OF THE SELECTION OF A BRIGADE ARTILLERY GROUP FIRING POSITION IN A DEFENSIVE OPERATION

Dragan S. Pamučar\textsuperscript{a}, Darko I. Božanić\textsuperscript{b}, Dejan V. Kurtov\textsuperscript{c}

University of Defence in Belgrade, Military Academy, Belgrade, Republic of Serbia
\textsuperscript{a} e-mail: dpamucar@gmail.com, ORCID ID: http://orcid.org/0000-0001-8522-1942
\textsuperscript{b} e-mail: dbozanic@yahoo.com, ORCID ID: http://orcid.org/0000-0002-9657-0889
\textsuperscript{c} e-mail: dejankurtov@gmail.com, ORCID ID: http://orcid.org/0000-0002-7403-5914

DOI: 10.5937/vojtehg64-9262

FIELD: Mathematics, Operational Research
ARTICLE TYPE: Original Scientific Paper
ARTICLE LANGUAGE: English

Summary:

This paper presents the new way of fuzzification of the Saaty’s scale. In this model of fuzzification, the confidence interval of fuzzy numbers that describes the comparison in pairs’ degree is not determined before the comparison. It is defined (calculated) during and after the comparison, based on the degree of certainty of decision-makers/experts. Thus, the confidence interval can vary depending on the comparison, no matter if it refers to the same degree of comparison. Also, in group deciding, confidence intervals differ depending on the decision-maker/expert’s opinion. Such explanation is supported by the new fuzzified Saaty’s scale. The application of the new scale is shown in the hybrid fuzzy AHP-TOPSIS model when choosing firing positions of an artillery brigade group in a defensive operation. Finally, the impact of the degree of certainty on the final decision is analyzed, where it is demonstrated that this element affects output results. The impact is reflected in the change of the size of weight vectors, i.e. output results (the distance from the ideal alternative), as well as in the change of the rank of alternatives, but only at limit values.

Key words: Fuzzy logic, Firing position, Hybrid model, TOPSIS, AHP.
Introduction

Decision making is one of the most important management elements. It has been empirically proven that decision making makes even 92% of a manager’s job structure (Ćupić, Suknović, 2010, p.xxiv). Considerable attention is paid to the decision-making process in military organizations. The reason for this can be found in the fact that a man is in the center of every decision, and it is not expected that all people respond equally to situations in which they find themselves, as expressed particularly in combat operations, where the consequences of wrong decisions can often be irreparable (Pamučar, et al., 2011a, p.3). In the Army of Serbia many decisions are made in the processes of planning, organization and preparation for the execution of missions and tasks. The useful tools which support the decision-making process are the methods of multi criteria decision making.

This paper presents a hybrid model, using the fuzzificated Saaty’s scale and the TOPSIS method (Technique for Order Preference by Similarity to Ideal Solution). The paper is focused on the demonstration of a new way of fuzzification of the Saaty’s scale used for comparison in pairs with varying a confidence interval depending on the comparison. The scale is used for obtaining criteria weight coefficients, while the TOPSIS method is used for the final ranking. This model is illustrated by an example of decision making during the selection of a brigade artillery group (BrAG) firing position area in a defensive operation. The example presents only one segment from a series of decisions that decision makers face in the preparation and execution of (military) operations.

Fuzzy logic and fuzzy sets

In conventional logic, belonging of an element to the given set is strictly defined, i.e., an element can belong or not belong to the set. In fuzzy logic, belonging of an element to the specific set is not precisely defined - the element can be more or less part of the set; therefore, it is closer to human perception than conventional logic (Pamučar, et al., 2011b, p.594). Fuzzy logic allows quantification of seemingly imprecise information, which is a very common situation when describing social phenomena.

The first step in designing fuzzy sets is defining the degree of the membership of an element x (x ∈ X) to the set A. This is described with the membership function μ_A(x), which in the classic theory has a value of 0 (does not belong) or 1 (belongs), while in a fuzzy set the membership...
function can have any value between 0 and 1. So, it can be said that the closer the \( \mu_A(x) \) is to 1, the belonging of the \( x \) to \( A \) is greater, and vice versa. Every fuzzy set is completely and uniquely defined by its membership function (Zadeh, 1965). A fuzzy set is defined as a set of ordered pairs

\[
A\{(x, \mu_A(x)) | x \in X, 0 \leq \mu_A(x) \leq 1 \}.
\]

where:

– \( X \) is a universal set or a set of considerations based on which the fuzzy set \( A \) is defined;

– \( \mu_A(x) \) is a membership function of the element \( x \) to the set \( A \).

The membership function forms and the width of the confidence interval are usually selected on the basis of subjective assessment or experience, so that they best describe the phenomenon they represent. In practice, a variety of membership functions is used: triangular, trapezoidal, Gaussian, etc.

In this paper, triangular fuzzy numbers will be used. They will be presented in the form \( T = (t_1, t_2, t_3) \), where (Figure 1):

– \( t_2 \) is where the membership function of a fuzzy number has a value of 1;

– \( t_1 \) is the left distribution of the confidence interval of the fuzzy number \( T \), and

– \( t_3 \) is the right distribution of the confidence interval of the fuzzy number \( T \) (Pamučar, 2011, p.45).

![Triangular fuzzy number T](image)
The membership function of the fuzzy number $T$ is defined in the following way:

$$
\mu_T(x) = \begin{cases} 
0, & x < t_1 \\
\frac{x-t_1}{t_2-t_1}, & t_1 \leq x \leq t_2 \\
\frac{t_3-x}{t_3-t_2}, & t_2 \leq x \leq t_3 \\
0, & x > t_3 
\end{cases} 
$$

(2)

For its final purpose, the fuzzy number $T = (t_1, t_2, t_3)$ is converted into a real number. Different methods are used for this procedure (Herrera, Martínez, 2000).

**Fuzzification of the Saaty's scale**

The Analytical Hierarchy Process method belongs to the group of light optimization methods. It is based on the interpretation of complex problems in a hierarchy, with an aim at the top and criteria, sub-criteria and alternatives at the levels and sub-levels of the hierarchy (Saaty, 1980). One of the key phases in the application of this method is the development of the comparison matrix by pairs, corresponding to every level of the hierarchy. A pairwise comparison is performed according to the data collected and by measuring them, as well as based on the beliefs, estimates or experiences of those who carry out the assessment (Čupić, Suknović, 2010). The main problem in the pairwise comparison is to quantify linguistically formulated selections or phrases (Kujačić, 2001). Different evaluation scales are developed for this purpose. Basic approaches in its development are: linear (Ma, Zheng, 1990) and exponential (Lootsma, 1988). Previous analyzes have shown that there is no ideal scale, but their quality varies from case to case (Triantaphyllou, et al., 1988). The standard for the AHP method presents the Saaty's scale, Table 1 (Saaty, 1980).

**Table 1 – Saaty’s scale for comparison in pairs**

<table>
<thead>
<tr>
<th>Definition</th>
<th>Standard values</th>
<th>Inverse values</th>
</tr>
</thead>
<tbody>
<tr>
<td>The same importance</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Low dominance</td>
<td>3</td>
<td>1/3</td>
</tr>
<tr>
<td>High dominance</td>
<td>5</td>
<td>1/5</td>
</tr>
<tr>
<td>Very high dominance</td>
<td>7</td>
<td>1/7</td>
</tr>
<tr>
<td>Absolute dominance</td>
<td>9</td>
<td>1/9</td>
</tr>
<tr>
<td>Intermediate values</td>
<td>2, 4, 6, 8</td>
<td>1/2, 1/4, 1/6, 1/8</td>
</tr>
</tbody>
</table>
The Saaty’s scale is applied by decision-makers or analysts performing comparisons in pairs on the basis of semantic preferences from the left column of the Saaty’s scale, or by direct association. Number values in columns two or three of Table 1, which correspond to the semantic preferences in the left column, are entered in the square comparison matrix.

\[ A = \begin{bmatrix} C_1 & C_2 & \cdots & C_n \\ C_1 & a_{11} & a_{12} & \cdots & a_{1n} \\ C_2 & a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ C_n & a_{n1} & a_{n2} & \cdots & a_{nn} \end{bmatrix} \] (3)

Since it is true that \( a_{ji} = 1 / a_{ij} \) and \( a_{ii} = 1 \) for every \( i, j = 1, 2, \ldots, n \), the matrix \( A \) is positive, symmetrical and reciprocal. The essential information on elements of preferences is located only in the upper triangle of the matrix, but all methods for its further analysis use the reciprocal value from the lower triangle (Bozanić, et al., 2013).

When applying the classic Saaty’s scale, relations in the pairwise comparison are strictly defined. However, very often, when defining these values, one cannot be completely sure of the relations of the pairs compared. Therefore, in the literature there is an increasing number of papers which approach the fuzzification of the Saaty’s scale in various ways ((Ray, Triantaphyllou, 1999), (Zhu, et al., 1999), (Chen, 2007), (Srđević, et al., 2008), (Gardašević-Filipović, Šaletić, 2010), (Janacković, et al., 2013), (Rezaei, et al. 2014), (Janjić, et al., 2014) and others). Most authors use a predetermined interval of a fuzzy number in the fuzzification, i.e., preset the left and the right distribution of the most commonly used triangular fuzzy number \( T = (t_1, t_2, t_3) \). Some authors have recognized the necessity to leave the possibility of some uncertainty, such as in (Božanić, et al., 2011), (Božanić, et al., 2013), (Pamučar, et al., 2011c) (Pamučar, et al., 2012), (Pamučar, 2013) (Pamučar, et al., 2015), (Božanić, et al., 2015b), where the level of uncertainty for the whole scale is pre-defined, based on which the left and right distribution of the fuzzy number \( T \) are calculated. This level of uncertainty, i.e., the confidence interval, changes depending on the case or depending on the decision-maker.

On the basis of the Saaty’s scale and the idea that the confidence interval of the fuzzy number does not always have to be identical, as shown in (Božanić, et al., 2011), (Božanić, et al., 2013), (Pamučar, et al., 2011c) (Pamučar, et al., 2012), (Pamučar, 2013) (Pamučar, et al., 2015), (Božanić, et al., 2015b), a new scale is defined, Table 2 (Božanić, Pamučar), (Božanić, et al., 2015a), (Božanić, et al., 2016). Defining this new fuzzyficated Saaty’s scale has started from the assumption that decision
makers and analysts have a different degree of certainty \( \gamma \) concerning the accuracy of comparisons in pairs. This degree of certainty differs from one comparative pair to the other. The value of the degree of certainty belongs to the interval \( \gamma \in [0, 1] \). In the cases when \( \gamma = 0 \), it is considered that the decision-maker/analyst has no data about this relationship, so it should not be used in the decision-making process, because it points to the absolute ignorance of the decision-making subject. The value of the degree of certainty where \( \gamma = 1 \) describes the absolute certainty of decision-makers and analysts in the defined comparison. The lower the certainty in the performed comparison is, the lower the element \( \gamma \).

Table 2 – Fuzzified Saaty’s scale for comparison in pairs (Božanić, Pamučar), (Božanić et al., 2015a), (Božanić et al., 2016)

<table>
<thead>
<tr>
<th>Definition</th>
<th>Standard values</th>
<th>Fuzzy number</th>
<th>Inverse values of the fuzzy number</th>
</tr>
</thead>
<tbody>
<tr>
<td>The same importance</td>
<td>1</td>
<td>(1, 1, 1)</td>
<td>(1, 1, 1)</td>
</tr>
<tr>
<td>Low dominance</td>
<td>3</td>
<td>((3\gamma, 3(2 - \gamma), 3))</td>
<td>((1/(2 - \gamma), 3, 3/3\gamma))</td>
</tr>
<tr>
<td>High dominance</td>
<td>5</td>
<td>((5\gamma, 5(2 - \gamma), 5))</td>
<td>((1/(2 - \gamma), 5, 5/5\gamma))</td>
</tr>
<tr>
<td>Very high dominance</td>
<td>7</td>
<td>((7\gamma, 7(2 - \gamma), 7))</td>
<td>((1/(2 - \gamma), 7, 7/7\gamma))</td>
</tr>
<tr>
<td>Absolute dominance</td>
<td>9</td>
<td>((9\gamma, 9(2 - \gamma), 9))</td>
<td>((1/(2 - \gamma), 9, 9/9\gamma))</td>
</tr>
<tr>
<td>Intermediate values</td>
<td>2, 4, 6, 8</td>
<td>((x\gamma, x, (2 - \gamma), x))</td>
<td>((1/(2 - \gamma), x, 1/x, 1/x\gamma))</td>
</tr>
</tbody>
</table>

By defining different values of the parameter \( \gamma \), the left and the right distribution of fuzzy numbers change from comparison to comparison, according to the expression:

\[
T = (t_1, t_2, t_3) = \begin{cases} 
  t_i = \psi t_2, & t_i \leq t_2, \quad t_i, t_2 \in [1/9, 9] \\
  t_i = t_2, & t_2 \in [1/9, 9] \\
  t_i = (2 - \gamma)t_2, & t_i \leq t_2, \quad t_i, t_2 \in [1/9, 9] 
\end{cases}
\]

(4)

the value of \( t_2 \) represents the value of linguistic expressions from the classic Saaty’s scale, which in a fuzzy number has a maximum membership \( t_2 = 1 \).
The fuzzy number \( T = (t_1, t_2, t_3) = (x\gamma, x(2-\gamma)x), \ x\in[1,9] \) is defined by the expressions:

\[
t_1 = x\gamma, \quad \forall \ 1 \leq x\gamma \leq x
\]

\[
t_2 = x, \quad \forall \ x\in[1,9]
\]

\[
t_3 = (2-\gamma)x, \ \forall \ x\in[1,9]
\]

The inverse fuzzy number \( T^{-1} = (1/t_1, 1/t_2, 1/t_3) = \frac{1}{(2-\gamma)x}, \ x\in[1,9] \) is defined as follows:

\[
1/t_1 = \frac{1}{(2-\gamma)x}, \ \forall \ \frac{1}{(2-\gamma)x} < 1
\]

\[
1/t_2 = 1/x, \ \forall \ 1/x\in[1,9]
\]

\[
1/t_3 = \frac{1}{\gamma x}, \ \forall \ 1/x\in[1,9]
\]

The defined scale is further used in standard steps of the AHP method, which is described in a number of papers (Saaty, 1980), (Lootsma, 1988), (Nikolić, Borović, 1996), (Srđević, Srđević, 2004) (Čupić, Suknović, 2010), (Karović, Pušara, 2010), (Devetak, Terzić, 2011), (Indić, et al., 2014) and others.

Based on the pre-defined scale, decision-makers and analysts fill in the new, modified matrix:

\[
A = \begin{bmatrix}
C_1 & C_2 & \cdots & C_n \\
\begin{bmatrix}
a_{11};\gamma_{11} & a_{12};\gamma_{12} & \cdots & a_{1n};\gamma_{1n} \\
\vdots & \vdots & \ddots & \vdots \\
a_{n1};\gamma_{n1} & a_{n2};\gamma_{n2} & \cdots & a_{nn};\gamma_{nn}
\end{bmatrix}
\end{bmatrix}
\]

As it can be seen, the matrix is extended with the degree of certainty in the comparison made, whereby \( \gamma_{ij} = \gamma_j \), where \( \gamma_j \in [0,1] \). After the calculation is finished, the defuzzification can be performed using one of well-known methods. Some of well-known expressions for defuzzification (Seiford, 1996) are the following ones:

\[
A = ((t_3 - t_1) + (t_2 - t_1)) / 3 + t_1
\]

\[
A = \left[ \lambda t_1 + t_2 + (1-\lambda) t_1 \right] / 2
\]
where $\lambda$ represents the degree of optimism (Božanić, et al., 2015b). Also, it is possible to perform further calculations using fuzzy weight coefficients without defuzzification, which would be done at the end of the calculation of the criteria functions of alternatives.

The scale presented can be implemented in the classic application of the AHP method, where weight coefficients are calculated first, and then criteria functions are evaluated for every alternative studied. The scale is also suitable for the evaluation of criteria weights with the aim of later implementation of some other method.

The defined scale is also suitable for group decision making, which has recently become more and more popular. Involving experts greatly improves the quality of decisions made because knowledge and experience are collected and consolidated into a single unit. The most widely used approach in data collecting by experts is the Delphi method (Lootsma, 1988). The scale defined in this paper is applied in group decision making, as well as in the standard AHP method (more information in (Srđević, Zoranović, 2003) (Zoranović, Srđević, 2003)).

The TOPSIS method

The TOPSIS method was developed by Hwang and Joon (1981). This method consists in ranking alternatives by multiple criteria comparisons based on the distance from the ideal solution and the negative ideal solution. The ideal solution minimizes the cost type criteria and maximizes the benefit type criteria, while the negative ideal solution is reverse (Srđević, et al., 2002). The optimum alternative is the one that is the closest in a geometrical sense to the ideal solution, i.e., the farthest from the negative ideal solution (Srđević, et al., 2002).

The starting point of this method is the initial decision-making matrix.

$$
\begin{bmatrix}
C_1 & C_2 & C_3 & \cdots & C_m \\
W_1 & W_2 & W_3 & \cdots & W_m \\
A_1 & r_{11} & r_{12} & \cdots & r_{1m} \\
A_2 & r_{21} & r_{22} & \cdots & r_{2m} \\
A_n & r_{n1} & r_{n2} & \cdots & r_{nm} \\
\end{bmatrix}
$$

$$P = \begin{bmatrix}
\vdots & \vdots & \vdots & \cdots & \vdots \\
A_1 & r_{1g1} & r_{1g2} & \cdots & r_{1gM} \\
A_2 & r_{2g1} & r_{2g2} & \cdots & r_{2gM} \\
A_n & r_{ng1} & r_{ng2} & \cdots & r_{ngM} \\
\end{bmatrix}
$$

(14)

With the decision-making matrix, the $n$ alternatives and the $m$ criteria are defined. The weight of the criteria $w_i$ is joined to each criterion. Criteria weights should meet the following requirement:

$$\sum_{i=1}^{n} w_i = 1$$

(15)
After defining the decision-making matrix, the TOPSIS method can be implemented. The application of the method can be divided into six steps:

- **First step**: normalization of the decision-matrix values:
  \[
  x_{ij} = r_{ij} \left[ \sum_{j=1}^{n} r_{ij}^2 \right]^{-1}
  \]
  After the application of expression (16), a new dimensionless matrix is obtained:
  \[
  \begin{array}{cccc}
  C_1 & C_2 & \cdots & C_n \\
  w_1 & w_2 & \cdots & w_n \\
  \mathbf{A} = \begin{bmatrix}
  x_{11} & x_{12} & \cdots & x_{1n} \\
  x_{21} & x_{22} & \cdots & x_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  x_{m1} & x_{m2} & \cdots & x_{mn}
  \end{bmatrix}
  \end{array}
  \]

- **Second step**: multiplication of the normalized values and the criteria weight coefficients:
  \[
  v_{ij} = x_{ij} w_j; \ j = 1, 2, ..., m
  \]
  After the application of expression (18), a new matrix is obtained:
  \[
  \begin{array}{cccc}
  C_1 & C_2 & \cdots & C_n \\
  w_1 & w_2 & \cdots & w_n \\
  \mathbf{A} = \begin{bmatrix}
  v_{11} & v_{12} & \cdots & v_{1n} \\
  v_{21} & v_{22} & \cdots & v_{2n} \\
  \vdots & \vdots & \ddots & \vdots \\
  v_{m1} & v_{m2} & \cdots & v_{mn}
  \end{bmatrix}
  \end{array}
  \]

- **Third step**: determination of ideal solutions. The ideal and negative ideal solution are obtained using the following expression:
  \[
  A^+ = \left\{ \max_{j \in C} v_{ij} \mid j \in C, i = 1, 2, ..., n \right\} = \left\{ v_{1}^*, v_{2}^*, \ldots, v_{n}^* \right\}
  \]
  \[
  A^- = \left\{ \min_{j \in C} v_{ij} \mid j \in C, i = 1, 2, ..., n \right\} = \left\{ v_{1}^-, v_{2}^-, \ldots, v_{n}^- \right\}
  \]
  where: C is the set of benefit type criteria, and C' is the set of cost type criteria.
Fourth step: determination of the distance of alternatives from the ideal solution.

\[ S_i^+ = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j)^2}, i = 1, 2, ..., n \]  

\[ S_i^- = \sqrt{\sum_{j=1}^{m} (v_{ij} - v_j)^2}, i = 1, 2, ..., n \]  

Fifth step: determination of the relative proximity of alternatives to the ideal solution. Determination of the distance from the ideal alternative is performed by applying the expression:

\[ Q_i = \frac{S_i^-}{S_i^+ + S_i^-}, i = 1, 2, ..., n \]  

Sixth step: ranking alternatives. Alternatives are ranked on the basis of the results obtained by applying expression (24). The best alternative is considered the alternative whose value \( Q_i \) is the highest, and vice versa.

Selection of a BrAG firing position area by using the hybrid FAHP-TOPSIS model

Description of the problem and defining the criteria

A brigade artillery group is a temporary formation of artillery units, formed at the tactical level. It is intended for artillery firing support of one’s own forces in combat operations of a land forces brigade and territorial brigades (Vojni leksikon, 1981, p.72). A BrAG performs its tasks from firing positions (primary, supplementary, alternate, temporary). A firing position (FP) is a part of the land in the area of the operation, prepared and occupied or intended to be occupied by the artillery units for the execution of firing support (Vojni leksikon, 1981, p.658). An FP area of a BrAG is determined depending on its purpose, type of weapons and ammunition, targets of artillery firing support, combat, space, time and other conditions.

There are rules and regulations governing fundamental criteria of grouping artillery and its deployment into action, as well as a part of the conditions an FP area should meet in order to be selected as an alternative. However, in the available literature, this problem has not been fully developed, systematized, nor the method of selection of an FP area has been elaborated (neither criteria are precisely defined, nor their weight values and their relationship, nor the particularities of various
combat operations). Consequently, decision-makers usually have to select an FP of BrAG area relying on the acquired theoretical knowledge, experience and assessment in the specific situation. A number of criteria that influence the ranking and selection of alternatives indicate a possibility of applying multiple criteria methods.

For the purposes of the selection of the best firing position for a BrAG in a defensive operation, the following six criteria are defined (Kurtov, et al., 2014, p.708):

- **C₁** - "distance from the ideal location for action" - the ideal location is generally defined as 1/3 of the rank of artillery weapon from the front end of one's own forces;
- **C₂** - "sheltering height" - represents the height of the obstacles that allow hiding or masking combat effects from the survey instruments, electronic effects and enemy fire;
- **C₃** - "masking conditions" - terrain features that enable successful masking of the BrAG and movement of parts or the whole BrAG;
- **C₄** - "fortification conditions" - terrain features that allow successful fortification of artillery to enhance force protection;
- **C₅** - "conditions for maneuver" – terrain features based on which the assessment of the possibility of fast moving to the following firing position is performed;
- **C₆** - "average height difference between individual instruments" - for successful actions, it is necessary for all individual instruments in BrAG artillery batteries to be located on the height difference less than 20m.

The values of the criteria **C₁**, **C₂** and **C₆** are described numerically and the values of the criteria **C₃**, **C₄** and **C₅** are described with fuzzy linguistic descriptors, Figure 2.

![Graphic display of the fuzzy linguistic descriptors](image)
The membership functions of the fuzzy linguistic descriptors are defined through the expressions:

\[
\mu_{Li} = \begin{cases} 
1, & 1 \geq x \\
2 - x, & 1 \leq x \leq 2
\end{cases} \quad (25)
\]

\[
\mu_{Li} = \begin{cases} 
x - 1, & 1 \leq x \leq 2 \\
3 - x, & 2 \leq x \leq 3
\end{cases} \quad (26)
\]

\[
\mu_{Li} = \begin{cases} 
x - 2, & 2 \leq x \leq 3 \\
4 - x, & 3 \leq x \leq 4
\end{cases} \quad (27)
\]

\[
\mu_{Li} = \begin{cases} 
x - 3, & 3 \leq x \leq 4 \\
5 - x, & 4 \leq x \leq 5
\end{cases} \quad (28)
\]

\[
\mu_{Li} = \begin{cases} 
x - 4, & 4 \leq x \leq 5 \\
1, & x \geq 5
\end{cases} \quad (29)
\]

**Calculation of the criteria weight coefficients**

The first step in defining the weight coefficients is to define the square comparison matrix. Two elements of the hierarchy (models) are compared using the Saaty’s classic scale and by defining the degree of certainty of a given claim (according to expression 11). The degree of inconsistency of the given matrix is 0.08.

The values of the matrix A are converted into fuzzy numbers by applying the fuzzyficated Saaty’s scale (Table 2), so a new matrix A’ is obtained.
The weight vector $w$ of every criterion of the matrix $A'$ is the sum of the linguistic expressions that describe the criteria in the same row of the matrix $A'$, which is divided by the sum of all linguistic expressions that describe the criteria of the matrix $A'$.

After the calculation is performed, the weight vectors of the criteria are obtained:

$$w = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \\ w_5 \\ w_6 \end{bmatrix} = \begin{bmatrix} 0.282;0.393;0.382 \\ 0.212;0.159;0.162 \\ 0.112;0.093;0.105 \\ 0.089;0.060;0.058 \\ 0.272;0.269;0.254 \\ 0.033;0.026;0.039 \end{bmatrix}$$

**Ranking alternatives**

The model is applied using the illustrative values of seven alternatives shown in the initial decision matrix and taken from (Kurtov, et al., 2014, p.709).

$$\begin{array}{ccccccc} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ w_1 & 1100 & 15 & 1 & 4 & 3 & 3 \\ w_2 & 2000 & 23 & 4 & 3 & 6 \\ w_3 & 3800 & 27 & 3 & 4 & 2 \\ w_4 & 500 & 5 & 2 & 4 & 2 & 1 \\ w_5 & 1850 & 32 & 5 & 1 & 3 & 4 \\ w_6 & 3200 & 10 & 5 & 1 & 3 & 9 \\ w_7 & 900 & 20 & 3 & 3 & 5 & 5 \end{array}$$

The six-step TOPSIS method application results in the values of the distance of alternatives from the ideal alternative, based on which they are ranked:

$$\begin{array}{c} \bar{q}_1 = 0.578;0.664;0.658 \\ \bar{q}_2 = 0.597;0.573;0.574 \\ \bar{q}_3 = 0.410;0.293;0.300 \\ \bar{q}_4 = 0.538;0.659;0.647 \\ \bar{q}_5 = 0.611;0.591;0.598 \\ \bar{q}_6 = 0.276;0.242;0.252 \\ \bar{q}_7 = 0.748;0.812;0.802 \end{array}$$
Finally, defuzzification is performed by applying expression 12, and the final values of the distance of alternatives from the ideal alternative are obtained:

\[
\begin{align*}
Q'_1 &= 0.633 \\
Q'_2 &= 0.581 \\
Q'_3 &= 0.334 \\
Q'_4 &= 0.612 \\
Q'_5 &= 0.600 \\
Q'_6 &= 0.257 \\
Q'_7 &= 0.787 
\end{align*}
\]

Based on the obtained distances from the ideal alternative, it can be concluded that alternative seven (A7) is the most appropriate alternative, i.e., alternative six (A6) is the least favorable one.

Discussion and Conclusions

A practical example of the new scale demonstrated a possibility of using the hybrid fuzzy AHP-TOPSIS model, i.e., its performance in ranking the offered alternatives. In order to determine the differences between the application of the classic Saaty’s scale and the scale demonstrated in this paper, a comparative overview of the output results is presented (Table 3).

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Classic scale</th>
<th>Saaty’s scale</th>
<th>Fuzzyficated Saaty’s scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1</td>
<td>0.664</td>
<td>2</td>
<td>0.633</td>
</tr>
<tr>
<td>A2</td>
<td>0.573</td>
<td>5</td>
<td>0.581</td>
</tr>
<tr>
<td>A3</td>
<td>0.293</td>
<td>6</td>
<td>0.334</td>
</tr>
<tr>
<td>A4</td>
<td>0.650</td>
<td>3</td>
<td>0.612</td>
</tr>
<tr>
<td>A5</td>
<td>0.591</td>
<td>4</td>
<td>0.600</td>
</tr>
<tr>
<td>A6</td>
<td>0.242</td>
<td>7</td>
<td>0.257</td>
</tr>
<tr>
<td>A7</td>
<td>0.812</td>
<td>1</td>
<td>0.787</td>
</tr>
</tbody>
</table>
Analyzing the output results, it can be observed that the rank of alternatives has not changed. However, the values obtained by applying the classic Saaty’s scale and the fuzzyficated Saaty’s scale are different. The differences between the alternatives A_1, A_4 and A_5 are significantly reduced, and the difference between the alternatives A_1 and A_5 is reduced more than twice. It is also evident that the difference between the alternatives A_3 and A_6 increased.

Although the differences shown are significant, there is no change in the rank of alternatives. However, it would be important to determine whether changes in the elements of the initial decision matrix may result in a change of the rank of alternatives when using the classic and fuzzyficated Saaty’s scale. These changes will be presented with two examples.

**Example 1:** Changing the value of the alternative A_1 by the criterion C_2 in the initial decision-making matrix from 15m to 11m would lead to a different way of ranking alternatives (Table 4).

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Classic Saaty’s scale</th>
<th>Fuzzyficated Saaty’s scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>A_1</td>
<td>0.646</td>
<td>0.611</td>
</tr>
<tr>
<td>A_2</td>
<td>0.574</td>
<td>0.582</td>
</tr>
<tr>
<td>A_3</td>
<td>0.295</td>
<td>0.336</td>
</tr>
<tr>
<td>A_4</td>
<td>0.648</td>
<td>0.610</td>
</tr>
<tr>
<td>A_5</td>
<td>0.592</td>
<td>0.601</td>
</tr>
<tr>
<td>A_6</td>
<td>0.242</td>
<td>0.257</td>
</tr>
<tr>
<td>A_7</td>
<td>0.811</td>
<td>0.786</td>
</tr>
</tbody>
</table>

In the mentioned example, it is noted that, when the classic Saaty’s scale is used, the alternative A_1 is ranked as the third. However, when the fuzzyficated scale is used, it is ranked as the second, while the alternative A_4 switched the position with the A_1.

**Example 2:** Changing the value of the alternative A_5 by the criterion C_1 in the initial decision-making matrix from 1850 m to 1500 m would lead to a different way of ranking alternatives (Table 5). In the mentioned example, it is noted that when using the classic Saaty’s scale the alternative A_1 is ranked as the second, while when using the fuzzyficated scale the alternative A_5 is ranked as the second, and vice versa.
Both examples show the importance of the degree of certainty of decision makers when ranking alternatives. Of course, the degree of certainty should not be the deciding factor in ranking, but an additional element that comes to the fore at the limit values. This can be analyzed through the already given examples.

If we consider the calculation with the values from the initial decision matrix, it can be noted that the rank of alternatives is identical when using both classic and fuzzyficated scale. In examples 1 and 2, the rank of alternatives has changed. If we continue and further reduce the value of the alternative A1 by the criterion C2 in the initial decision-making matrix from the original 15 m, and next 11 m, to 10, new solutions would be obtained (Table 6).

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Classic Saaty's scale</th>
<th>Fuzzyficated Saaty's scale</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$Q_i^c$</td>
<td>Rank</td>
</tr>
<tr>
<td>A1</td>
<td>0.667</td>
<td>2</td>
</tr>
<tr>
<td>A2</td>
<td>0.573</td>
<td>5</td>
</tr>
<tr>
<td>A3</td>
<td>0.290</td>
<td>6</td>
</tr>
<tr>
<td>A4</td>
<td>0.654</td>
<td>4</td>
</tr>
<tr>
<td>A5</td>
<td>0.661</td>
<td>3</td>
</tr>
<tr>
<td>A6</td>
<td>0.241</td>
<td>7</td>
</tr>
<tr>
<td>A7</td>
<td>0.813</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 5 – Comparative summary of the solutions with the change of the initial elements

Table 6 – Comparative summary of the solutions with the change of the initial elements
Table 6 shows that, when using different scales, the rank of alternatives is identical again, except that the alternatives A₁ and A₄ switched their positions (which is considered to be expected because the characteristics of the alternative A₁ improved). The situation is similar in example 2. If the values of the alternative A₅ by the criterion C₁ in the initial decision-making matrix are reduced from 1850 m, and 1500 m, to 1400 m, the identical situation occurs as in the previous case (Table 7). The ranks of alternatives are identical applying either scale, except that there is the expected replacement in the rank of alternatives due to improving the alternative value by one criterion.

Table 7 – Comparative summary of the solutions with the change of the initial elements

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Classic scale</th>
<th>Saaty’s scale</th>
<th>Fuzzyficated Saaty’s scale</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Q_i</td>
<td>Qi</td>
<td>Qi</td>
</tr>
<tr>
<td>A₁</td>
<td>0.668</td>
<td>3</td>
<td>0.637</td>
</tr>
<tr>
<td>A₂</td>
<td>0.572</td>
<td>5</td>
<td>0.580</td>
</tr>
<tr>
<td>A₃</td>
<td>0.289</td>
<td>6</td>
<td>0.330</td>
</tr>
<tr>
<td>A₄</td>
<td>0.655</td>
<td>4</td>
<td>0.617</td>
</tr>
<tr>
<td>A₅</td>
<td>0.680</td>
<td>2</td>
<td>0.677</td>
</tr>
<tr>
<td>A₆</td>
<td>0.241</td>
<td>7</td>
<td>0.255</td>
</tr>
<tr>
<td>A₇</td>
<td>0.814</td>
<td>1</td>
<td>0.789</td>
</tr>
</tbody>
</table>

In all cases it can be observed that Q_i changes, i.e., that the differences between the alternatives increase/decrease depending on the degree of certainty of decision makers.

From all of the above mentioned, it can be concluded that the new fuzzyficated Saaty’s scale improves decision making taking into account the degree of certainty of decision makers in the shown pairwise comparison. Considering the degree of certainty of decision makers, a change occurs in ranking alternatives at the limit values, thereby maintaining the decisive role of the comparison itself, which is the essence of the Saaty’s scale.

References

Božanić, D., & Pamučar, D. Modifikacija Saaty-jeve skale primenom fuzzy broja sa promenljivim intervalom poverenja: Primer procene opasnosti od poplava, Analitički hijerarhijski proces - Teorijske osnove i primena u energetici, zaštiti radne i životne sredine i obrazovanju. Tematski zbornik, (accepted for publication/rad prihvaćen za objavljivanje).


985
FAZIFIKACIJA SAATY-JEVE SKALE I PRIKAZ HIBRIDNOG MODELA FUZZY AHP – TOPSIS: PRIMER IZBORA VATRENOG POLOŽAJA BRIGADNE ARTILIJERSKE GRUPE U ODBRAMBENOJ OPERACIJI

Dragan S. Pamučar, Darko I. Božanić, Dejan V. Kurtov
Univerzitet odbrane u Beogradu, Vojna akademija, Beograd, Republika Srbija

OBLAST: matematika, operaciona istraživanja
VRSTA ČLANKA: originalni naučni članak
JEZIK ČLANKA: engleski

Sažetak:

Ključne reči: fuzzy logika, vatreni položaj, hibridni model, TOPSIS, AHP.

Paper received on / Дата получения работы: 12. 10. 2015.
Manuscript corrections submitted on / Дата получения исправленной версии работы: 10. 06. 2016.
Paper accepted for publishing on / Дата окончательного согласования работы: 12. 06. 2016.

© 2016 The Authors. Published by Vojnotehnički glasnik / Military Technical Courier (www.vtg.mod.gov.rs, втг.мо.упр.срб). This article is an open access article distributed under the terms and conditions of the Creative Commons Attribution license (http://creativecommons.org/licenses/by/3.0/rs/).

© 2016 Авторы. Опубликовано в "Военно-технический вестник / Vojnotehnički glasnik / Military Technical Courier" (www.vtg.mod.gov.rs, втг.мо.упр.срб). Данная статья в открытом доступе и распространяется в соответствии с лицензией "Creative Commons" (http://creativecommons.org/licenses/by/3.0/rs/).

© 2016 Autori. Objavio Vojnotehnički glasnik / Military Technical Courier (www.vtg.mod.gov.rs, втг.мо.упр.срб). Ovo je članak otvorenenog pristupa i distribuiru se u skladu sa Creative Commons licencom (http://creativecommons.org/licenses/by/3.0/rs/).