RECENT ADVANCES IN UNDERSTANDING DEFORMATION AND FLOW OF GRANULAR MATTER

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Summary: *By means of graph theory, we analyze the changes in topology of a granular assembly during deformation. The elementary mechanism of diffuse deformation consists of intermittent flips. We show that dilatancy is the direct result of: an increasing number of flips, and, elastic relaxation of particles upon flips. Both are dependent on particles' elastic potential energy prior to flip and after the flip. The latter is the result of nonuniform distribution of interparticle forces in force chains. Next, we consider shear bands in granular materials. Formation of shear bands is accompanied by accompanied by massive rolling of particle. Since rolling is constrained by neighbors, a characteristic rolling correlation length appears. The transmission of rotations in a particular direction depends on the strength of the force chain branches in the direction of propagation and across. The maximum propagation distance is comparable to observed widths of shear bands. Finally, we turn to the question of vortex formation within shear bands and argue that vortex pattern minimizes the dissipation/resistance in granular fluid.*

Keywords: *Dilatancy, shear bands, vortex flow, length scale*

1. INTRODUCTION

Dilatancy in granular materials has been known since XIX century [1]. Despite of wealth of experimental observations [2-4] and phenomenological macroscopic models, the current understanding of dilatancy is purely empirical. The original Reynolds' rational for dilatancy, that nearly rigid particles must climb over each other to accommodate shear [5], only brings about other questions. Why materials particulate on the atomic level, such as crystals and simple fluids, don't dilate, as the rigid sphere model of densely packed atoms would predict? Moreover, the boundary between dilation and compaction behaviour, the *critical state*, depends on both porosity and pressure. Yet, the only existing rational for dilatancy, excludes pressure dependence.
Depending on the state of a particle assembly and boundary conditions, deformation may localize into a persistent shear band. Numerous experimental observations of shear bands of width 10-20 particle diameters [6-10] indicate that this width may be a universal length scale. The formation of a shear band is accompanied by massive rolling of particles within the band [11, 12]. Sliding engages frictional dissipation and thus

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requires more work for the same displacement than rolling, so that the latter is always the energetically preferred form of transverse relative motion. However, in a densely packed assembly, pure rolling motion is impossible, owing to the constraint posed by neighboring particles, as illustrated in Figure 1. Since perfect rolling is impossible, sliding dissipates the energy so that only a part of the angular velocity of a rotating particle is transmitted to its neighbors. Therefore, the information about rotation of a particle diminishes with distance from the particle through successive interparticle contacts. This implies the existence of a length scale, the *rotation transmission distance*, associated with the deformation process, which represents the distance from a particle beyond which the information about particle's rotation is not transmitted. For spherical particles, this length will depend on friction, pressure, particle size distribution, and the state of the assembly (e.g., the level of compaction). For non-spherical particles, additional parameters, such as the aspect ratio, will play a role.

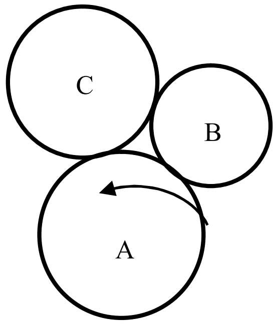

Figure 1. Necessity of sliding in a cluster of particles. The centers of three particles A, B and C are fixed and the particles are in contact with each other. Impose the counterclockwise rotation on particle A, as shown. Relative sliding is defined by non-zero relative velocities of the contact points. To avoid sliding with respect to A, both B and C must both rotate in the clockwise direction. But this will result in sliding on the B-C contact.

The rotation transmission is the result of frictional constraint and must depend on normal forces which are locally directionally dependent, following the pattern of force chains [13-17]. Within shear bands the flow pattern is not laminar, but develops vortices [18]. Such flow pattern is undoubtedly related to the charctristic lengths. In this work we provide a tentaive explanation of why such pattern develops. The localization mechanism based on buckling of force chains [19-23], although based on some phenomenological elements (such as confining forces and contact moments), does predict a characteristic buckling length. Particle level kinematics has, until recently, received less attention than force chains statics [24-29].

2. INTERMITTENT FLIPS AND DILATANCY

The mathematical description, endowed with information about connectivity of particles, is given by the Delaunay graph of the assembly for spheres [30], or its generalization for an assembly of convex particles, the *space cells* [31]. Delaunay or space cell graph provides a direct transition from discrete kinematics to the equivalent continuum. It allows unambiguous definition of strain, rotation, and their rates, at the level of individual cell. It also provides a mathematical distinction between two types of deformation of dense granular material. The *isotopologic* deformation is characterized by the deformed space cell graph which is topologically equivalent to the reference one,

i.e., each particle must have the same nearest neighbors in reference and deformed configuration. The strains produced by such mechanism are of the order 10^{-4} [29], much smaller than the strains of interest here ($10^{-2} - 10^{-1}$). The *heterotopologic* deformation includes topological changes in the space cell graph. Since these are stochastic, the probability of any such change reversing itself upon unloading is vanishingly small.
The changes in topology of the space cell graph, characterizing the range of plastic deformation of interest, can only occur by a few generic mechanisms – *flips* [32]. Only one generic flip exists in 2D – the 2-2 flip, as illustrated in Figure 2. In 3D, with tetrahedra as cells, there are two such mechanisms – the 2-3 and 3-2 flips. (The numbers indicate the numbers of original and final cells in the generic flip.)
We performed a series of simulations on assemblies of cohesionless elastic particles with friction. Computational details are given in [33], while the details of the analysis are given in [34]. The results demonstrate that the inelastic deformation of dense granular matter occurs by *intermittent flips*. At each deformation increment, only a small fraction of cells is undergoing a flip, and a completely different set of cells is flipping in the next increment. Such *diffuse deformation* mechanism governs the deformation regime where the dilatancy persists and the critical state is reached.

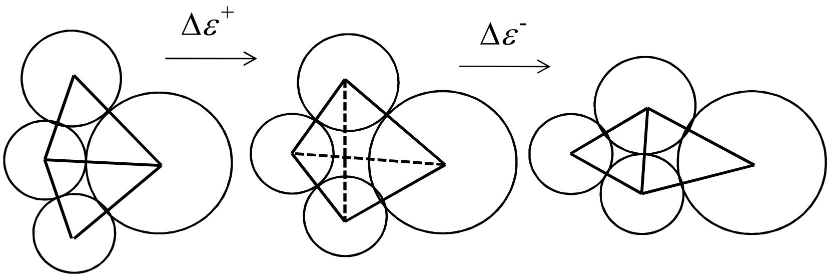

Figure 2. The generic 2-2 flip in 2D, after [32]. Left to right. As the cluster is compressed laterally, the nearest neighbors change. In the Delaunay graph, one diagonal is replaced by the other. The total volume (area in 2D) first increases by $\Delta\varepsilon^+$, then decreases by $\Delta\varepsilon^-$. Note that, in a large assembly of particles, a flip in opposite direction is equally probable.

We find that about ¾ of the total volume change correspond to the increasing flipping fraction, while the remaining ¼ is the result of elastic relaxation of particles forming the flipping cells. This release of elastic energy is possible only because of the specific nonuniform organization of contact forces in force chains of different strength, which enables particles to flip from a strong force chain to a weaker one. Moreover, the nonuniform distribution of elastic energy is the underlying driving force for the increasing fraction of flipping cells. The role of elastic potential in inelastic deformation has been proposed on a phenomenological level [35, 36], but the results in [34] summarized here demonstrate its existence and give it a clear micromechanical meaning. The elastic potential energy of a particle prior to a flip depends on its history, particularly on maximum past pressure. After the flip, the particle has relaxed and its elastic energy corresponds to the current pressure. The flipping potential of a particle is the difference between the two elastic energies. Thus, the rate of dilation will depend on the current pressure.

3. ROTATION LENGTH SCALE

To analyze the transmission of particle rotations through the contact network of an assembly, we perform numerical experiments on a 2D assembly of particles using the discrete element method [33]. A roughly circular assembly of particles (Figure 3) is subjected to confining pressure. Then, a disk on a strong force chain is forced to rotate with a prescribed angular velocity. Details are given in [37].

The non-dimensional parameters considered include: the friction coefficient μ, the quasistatic coefficient [33], the non-dimensional standard deviation of the particle size distribution σ/R, the non-dimensional pressure coefficient ε_0, and the solid volume fraction. For a specified initial compaction procedure and a sufficiently small quasistatic coefficient, the rotation transmission length depnds on three parameters: σ/R, ε_0 and μ. We have covered this parameter space in [37]. Typical results are shown in Figure 4, where the magnitudes of angular velocities of particles, normalized with the prescribed angular velocity of the forced particle ω_0, are plotted as functions of the distance from the forced particle for a given particle size distributions, pressure and the coefficient of friction.

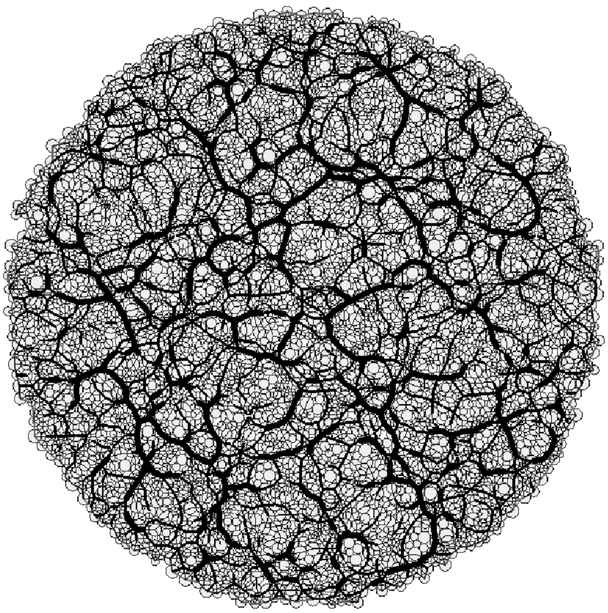

Figure 3. Force nework of a pressurized assembly of particles.

The angular velocities diminish with distance from the forced particle. The distance from the forced particle at which the angular velocities decay to the level of noise associated with the simulation is the rotation transmission distance. This length scale is the fundamental length scale exhibited by granular materials and should correlate with the shear band widths observed in experiments. The rotation transmission distance increases with increasing width of the particle size distribution. This corresponds well qualitatively with observations in numerical experiments [38], where the shear band width is lowest for the nearly monodisperse assembly. As the size distribution of particles narrows and approaches the monodisperse assembly, the kinematic constraints for particle rearrangement become more severe, yielding a shorter rotation transmission distance and thus – a narrower shear band.

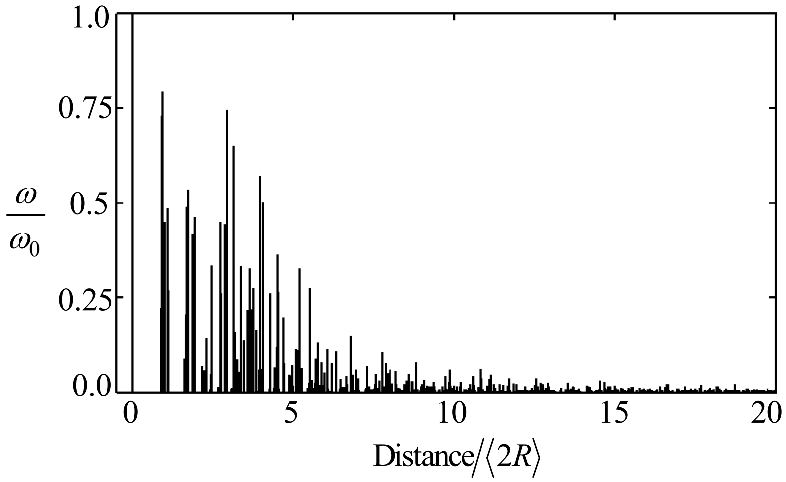

Figure 4. Evolution of angular velocity with distance for particle assemblies with $\varepsilon_n = 6.647 \times 10^{-4}$. $u = 0.5$ *and* $\sigma/R = 0.1925$.

The propagation of rotation depnds on the direction. The primary direction of transmission of rotations is along strong force chains. In [37], we have proposed a quantitative description, analogous to the fabric tensor and analyzed the spatial distribution of rotations in some detail. When the forced particle is crossed only by weak force chains, rotations propagate radially only up to the first transverse strong force chain. Such a force chain prevents propagation of rotations in the radial direction. Thus, strong force chains have a dual role in propagation of rotations: they improve the rotation propagation along the chain and impede the propagation across the chain.

4. VORTICES IN SHEAR BANDS

Granular material at critical state behaves like a viscous fluid. It flows at a constant stress. The flow is localized in a shear band and the observed nearly constant width of the shear band (in units of particle size) indicates the internal length scale. The flow inside the shear band exhibits a pattern of vortices, not expected for a Newtonian fluid at such low Reynolds numbers. The critical questions are: (1) Why do vortices form? (2) What kind of compressible fluid model is appropriate for granular fluid, or, at least, which aspects of the constitutive model can predict the vortex pattern? (3) What are the origins of the characteristic length scale?

To answer the first question, we consider that the formation of a shear band as given. The fluid like granular material shears flows in fixed channel between nearly rigid solids, as illustrated in Figure 1. A Newtonian incompressible fluid flows with a linear distribution of velocities across the height of the channel (Figure 5(a)). However, if the viscosity is dependent on density, and the material can redistribute density to minimize the shear resistance *T*, the idealized pattern is illustrated in Figure 5(b). Concentration of material into nearly rigid (high viscosity) rotating discs, with vanishing viscosity on the remainder of the volume, produces vanishing shear resistance.

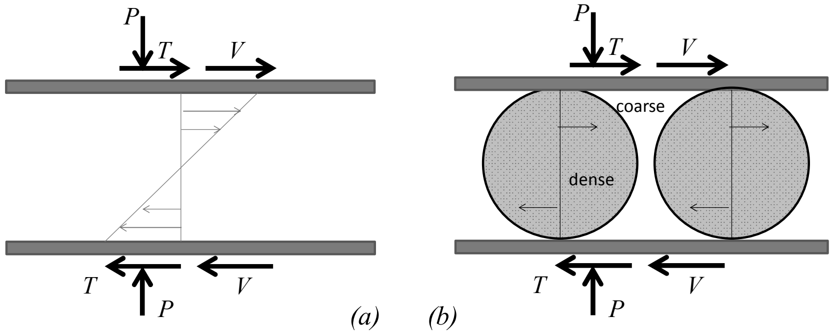

Figure 5. (a) Shear flow of a Newtonian fluid. (b) Idealized shear flow pattern of a material with density dependent viscosity, which is allowed to arbitrarily redistribute its mass to minimize shear resistance.

5. SUMMARY AND DISCUSSION

Consider a sample of high porosity, subjected to a constant pressure *p* and continuous vibrations. The sample will compact until the porosity reaches its minimum value. For a given size distribution of particles and their shape, this minimum porosity will depend on the applied pressure. This state is called random dense packing (RDP) [39], originally an empirical concept that has only recently been subjected to a more rigorous rational treatment [40]. Numerical simulations of random packings of spheres under varying simulated gravity [41] (equivalent to varying pressure) confirm the pressure dependence of the RDP porosity.

The second stage of the flip mechanism in Figure 2 is dynamic; it sends waves through the assembly. If the sample is loose, flips act as a disturbance, similar to externally applied vibrations, causing compaction of the assembly. However, owing to a small number of flips and fast attenuation of waves in granular assembly with interparticle friction, the effect of flip disturbance is localized and small compared to the effect of externally applied vibrations, even if the vibrations are achieved by gentle tapping on the sides of the container. Therefore, we expect that the critical state porosity is larger than the RDP porosity. Indeed, the experiments on identical steel beads (with friction) [42] indicate that the critical porosity is higher than the RDP value: 0.4 and 0.36 respectively, at zero pressure. In the absence of friction, the critical state could be as dense as the RDP, as simulations with frictionless spheres indicate [43]. The above observations on the intermittent flips mechanism enable us to formulate a micromechanical definition of the critical state as the state at which the compaction rate [caused by disturbances following flips] and dilation rate [caused elastic relaxation in flips] balance out, resulting in zero net volume change. For a given particle assembly, the critical state is a function of both pressure and porosity. The line in the pressure-porosity space delineating dilating and compacting states is the critical state line. Low pressure and low porosity indicate a state with net dilation, while high pressure and porosity indicate a state with net compaction. Note that both, local dilation and compaction of flipping cell clusters occur on both sides of the critical state line, but their net sum is different.

As the mechanism of transverse relative motion of particles in a dense granular assembly, rolling is energetically preferred to frictional sliding. Owing to the geometrical constraints, pure rolling is impossible, so that dissipative frictional sliding is engaged. Consequently, the information about rotation of a particle diminishes with distance from the particle, resulting in an intrinsic length scale – rotation transmission distance. Our numerical simulations indicate that this distance increases with increasing width of particle size distribution, with increasing friction, and (weakly) with decreasing pressure.

Numerical simulations reveal that the structure of force chains greatly affects the transmission of rotations in a densely packed granular material. Rotations propagate easily along strong force chains but not across strong force chains. The rotation transmission through a particle is governed by the kinematic constraints imposed by the surrounding force network which consists of both favorably and unfavorably aligned force chains. The nonlocal force chain fabric tensor has been defined which describes directional distribution of contact force strengths in a neighborhood of a particle.

Our results of rotation transmission distance [37] correlate with existing observations of shear band widths in experiments [6-10] and numerical simulations [38], as well as with the buckling force chain model [22]. The magnitude of angular velocity of particles decreases with distance from the center of the shear band [20, 24]. Interestingly, the nature of this distribution is preserved during the continuing deformation concentrated in the shear band [44]. This indicates that our rotation transmission distance corresponds to roughly half of the shear band width.

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СКОРАШЊА ДОСТИГНУЋА У РАЗУМЕВАЊУ ДЕФОРМАЦИЈЕ И ТЕЧЕЊА ЗРНАСТИХ МАТЕРИЈАЛА

Резиме: *Користећи теорију графова, анализирали смо тополошке промене у скупу зрна током деформације. Елементарни механизам разуђене (нелокализоване) деформације састоји се од спорадичних тополошких обрта. Показали смо да је дилатација директан резултат: пораста фреквенције тополошких обрта и еластичне релаксације која прати сваки обрт. Оба механизма зависе од еластичног потенцијала зрна пре и после обрта, а други је резултат неуједначене* *дистрибуције међузрнских сила организованих у ланце сила. Затим, посматрали смо траке смицања у зрнастим материјалима. Настанак трака смицања је праћен великим порастом ротације (котрљања) зрна. Пошто је котрљање осујећено суседним зрнима, појављује се карактеристична дужина. Пренос ротације у датом правцу зависи од интензитета ланца сила у том правцу и нормално на њега. Максимална удаљеност преноса ротације је упоредива са измереним ширинама трака смицања. Коначно, разматрамо питање стварања вртлога унутар траке смицања и показујемо да вртложно течење минимизира дисипацију/отпор смицању у зрнастом флуиду.*

Кључне речи: *Дилатација, трака смицања, вртложно течење, дужина скале*