

A New Averaged Switch Model Including Conduction Losses for PWM Converters Operating in Discontinuous Inductor Current Mode

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Abstract: In this paper a new PWM switch model including conduction losses, for Discontinuous Inductor Current Mode (DICM) dc-dc converters is developed. Compared to other models for DICM, the proposed model is more accurate, its derivation considering the correct, exponential waveforms for the currents and voltages. The proposed model is verified through PSpice simulation. A possibility of simplifying the model is also presented and verified through PSpice simulation.

Keywords: Conduction losses, discontinuous inductor current mode, averaged switch models

1 Introduction

Averaged switch models are powerful tools for studying the behaviour of PWM dc-dc converters.

In [1] an averaged model for Continuous Conduction Mode (CCM) dc-dc converters, including switch conduction losses and an averaged model for lossless DICM dc-dc converters are described.

A PWM switch model for PWM converters operating in DICM, including conduction losses is described in [2].

In this paper a new averaged model for DICM converters, including conduction losses is developed. The real diode is modeled, using the classical approach, as an ideal diode in series with a constant voltage source V_D and a linear resistor R_D . The transistor is modeled as an ideal switch in series with a resistor R_{on} . The real inductor is modeled as an ideal inductor in series with a resistor R_L .

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The DICM averaged switch model including switch conduction losses is derived in Section 2. A possibility of simplifying the model is presented in Section 3. In Section 4 the models are verified through PSpice simulation. Some conclusions are presented in Section 5. The PSpice netlists of the exact model and the simplified one are presented in the Appendix.

2 DICM Averaged Switch Model Including Conduction Losses

Let us examine DICM operation of the Boost converter.

As it is known, for the capacitor voltage v_C the negligible ripple assumption is still valid in DICM and therefore it can be admitted constant during one switching cycle, T_s .

Let us follow the averaged switch modeling approach [1] to derive an equivalent circuit that models the averaged (over one switching period, T_s) terminal variables of the switch network. The terminal variables of the switch network, $v_1(t)$, $i_1(t)$, $v_2(t)$ and $i_2(t)$ are defined in Fig. 1.

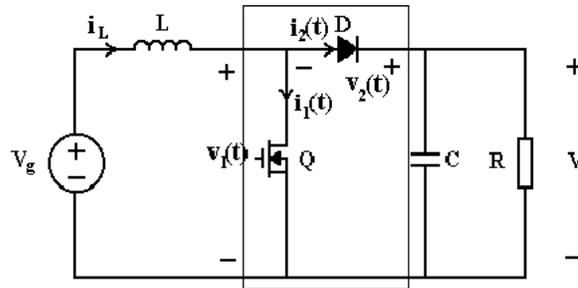


Fig. 1. The Boost converter, with switch network identified.

The equivalent circuits of the ĆUK converter in the three topological states of DCVM operation are shown in Fig. 2.

Due to the non-ideal components, the inductor current waveform is not triangular, as in case the components were ideal. In fact, the inductor current rises and falls exponentially, and its peak value is smaller, as shown in Fig. 3.

During the transistor on-time ($0 - dT_s$), the switch network variables are given by:

$$i_1(t) = i_L = \frac{V_g}{(R_{on} + R_L)} \left(1 - e^{-\frac{(R_{on} + R_L)t}{L}} \right) \quad (1)$$

$$i_2(t) = 0 \quad (2)$$

$$v_1(t) = R_{on} i_1 \quad (3)$$

$$v_2(t) = V_0 - R_{on} i_1 \quad (4)$$

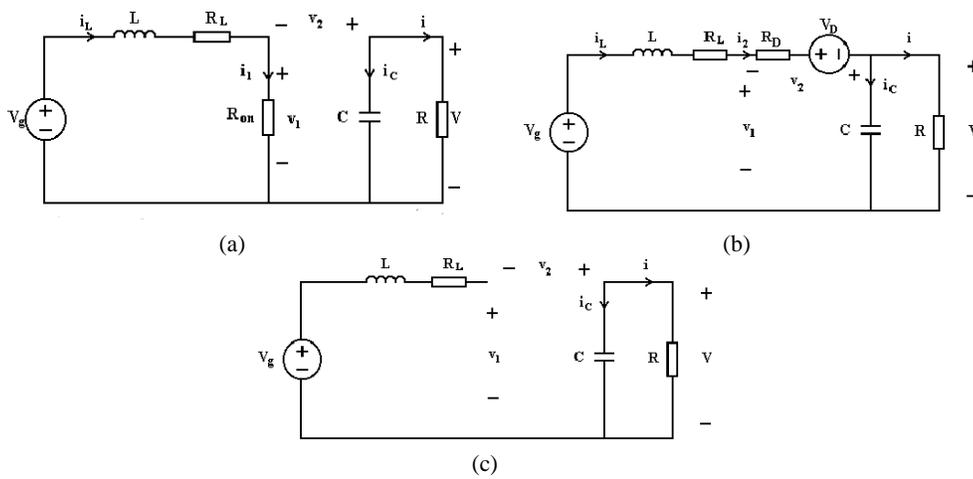


Fig. 2. The three topological states of DICM Boost converter.

During the transistor on-time the inductor current, i_L is equal to the switch network input current, i_1 . Therefore, from Eqn. (1) the peak value of the inductor current, I_{Lmax} can be obtained as:

$$I_{Lmax} = i_1(dT_s) = \frac{V_g}{(R_{on} + R_L)} \left(1 - e^{-\frac{(R_{on} + R_L)}{L f_s} d} \right) \quad (5)$$

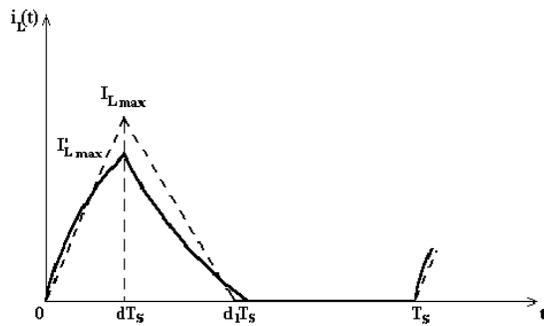


Fig. 3. Inductor current with ideal switches (dashed line) and considering conduction losses (solid line).

The voltage across the inductor is given by:

$$v_L = V_g - R_{on}i_1 - R_Li_1 \quad (6)$$

During the diode on-time ($dT_s - d_1T_s$), the switch network variables are given by:

$$i_1 = 0 \quad (7)$$

$$i_2 = i_L = \frac{V_g - V_0 - V_D}{R_D + R_L} + \left(I_{Lmax} - \frac{V_g - V_0 - V_D}{R_D + R_L} \right) e^{-\frac{R_D + R_L}{L f_s} (\frac{t}{T_s} - d)} \quad (8)$$

$$v_1 = V_0 + V_D + R_D i_2 \quad (9)$$

$$v_2 = -V_D - R_D i_2 \quad (10)$$

The voltage across the inductor is:

$$v_L = V_g - V_0 - V_D - R_D i_2 - R_L i_2 \quad (11)$$

During the third topological state of DICM operation, the switch network variables are given by:

$$i_1 = 0 \quad (12)$$

$$i_2 = 0 \quad (13)$$

$$v_1 = V_g \quad (14)$$

$$v_2 = V_0 - V_g \quad (15)$$

and v_L is:

$$v_L = 0 \quad (16)$$

As in the second and third topological states i_1 equals 0, from Eqn. (1) the average value over one switching cycle of i_1 can be obtained as:

$$\bar{i}_1 = \frac{1}{T_s} \int_0^{T_s} i_1(t) dt = \frac{V_g d}{R_{on} + R_L} - \frac{V_g L f_s}{(R_{on} + R_L)^2} \left(1 - e^{-\frac{(R_{on} + R_L)}{L f_s} d} \right) = \frac{V_g}{R_e} \quad (17)$$

where R_e is given by:

$$R_e = \frac{1}{\frac{d}{R_{on} + R_L} - \frac{L f_s}{(R_{on} + R_L)^2} \left(1 - e^{-\frac{(R_{on} + R_L)}{L f_s} d} \right)} \quad (18)$$

It has to be stressed that the expression of R_e , obtained for Boost converter, is the same for Buck and Buck-Boost converters.

In DICM, the diode switches off when the inductor current reaches zero. Hence, $i_L(0) = i_L(T_s) = 0$, and consequently, $\bar{v}_L = 0$. The average value of the switch network input voltage, v_1 can be written, for Boost converter, as:

$$\bar{v}_1 = V_g - (\bar{i}_1 + \bar{i}_2) R_L - \bar{v}_L = V_g - (\bar{i}_1 + \bar{i}_2) R_L \quad (19)$$

From Eqns. (17) and (19) a general expression for \bar{i}_1 can be obtained in the form:

$$\bar{i}_1 = \frac{\bar{v}_1 + \bar{i}_2 R_L}{R_e - R_L} \quad (20)$$

From Eqns. (6), (11) and (16), the average value of the voltage across the inductor can be obtained as:

$$\bar{v}_L = dV_g - (R_{on} + R_L)\bar{i}_1 + (d_1 - d)(V_g - V_0 - V_D) - (R_D + R_L)\bar{i}_2 = 0 \quad (21)$$

On the other side, from Eqns. (1)-(4), (7)-(10), (12)-(15), \bar{v}_1 and \bar{v}_2 are given by:

$$\bar{v}_1 = R_{on}\bar{i}_1 + R_D\bar{i}_2 + (d_1 - d)(V_D - V) + (1 - d_1)V_g \quad (22)$$

$$\bar{v}_2 = dV - R_{on}\bar{i}_1 - R_D\bar{i}_2 - (d_1 - d)V_D + (1 - d_1)(V - V_g) \quad (23)$$

Replacing V from Eqn. (21), V_g from Eqn. (19) and \bar{v}_1 from Eqn. (22) into Eqn. (23), the average value of the switch network output voltage can be written as:

$$\bar{v}_2 = \frac{d}{d_1 - d}\bar{v}_1 - V_D - \frac{R_{on}\bar{i}_1}{d_1 - d} - \frac{R_D\bar{i}_2}{d_1 - d} - \frac{R_L(\bar{i}_1 + \bar{i}_2)(1 - d_1)}{d_1 - d} \quad (24)$$

Defining an effective switch conversion ratio, μ ,

$$\mu = \frac{\bar{i}_1}{\bar{i}_1 + \bar{i}_2} \quad (25)$$

and using Eqn. (24), it can be finally concluded that the model is described by the following equations:

$$\bar{i}_1 = \frac{\mu}{1 - \mu}\bar{i}_2 \quad (26)$$

$$\bar{v}_2 = \frac{\mu_1}{1 - \mu_1}\bar{v}_1 - V_D - (R_{on1}(d_1) + R_{D1}(d_1) + R_{L1}(d_1))\bar{i}_2 \quad (27)$$

where:

$$\left\{ \begin{array}{l} \frac{\mu_1}{1 - \mu_1} = \frac{d}{d_1 - d} \\ R_{on1}(d_1) = R_{on} \frac{1}{d_1 - d} \frac{\mu}{1 - \mu} \\ R_{D1}(d_1) = R_D \frac{1}{d_1 - d} \\ R_{L1}(d_1) = R_L \frac{1 - d_1}{d_1 - d} \frac{1}{1 - \mu} \end{array} \right. \quad (28)$$

The length of diode on-time, $(d_1 - d)T_s$ can be obtained from the condition that i_L becomes zero at the end of the diode on-time. The difference $d_1 - d$ is given by:

$$d_1 - d = \frac{L f_s}{R_D + R_L} \ln \left[1 - \frac{I_{Lmax}(R_D + R_L)}{R_L(\bar{i}_1 + \bar{i}_2) - v_2 - V_D} \right] \quad (29)$$

Although the equations that describe the model are complicated, the model is general, so it can be used in many different converters, without deriving a different averaged switch network model for each converter configuration. This is a very useful feature for constructing averaged circuit models for simulation.

3 A Simplified Model for DICM Including Conduction Losses

The averaged model for the switch network in DICM presented in Section 2 can be simplified, by making some approximations. Of course, the precision of the model is decreased.

One approximation that can be made is to consider that the waveforms of i_L , i_1 and i_2 are linear. The average value of the inductor current is given by:

$$\bar{i}_L = \frac{1}{2} I_{Lmax} d_1 \quad (30)$$

From Eqn. (30) a simpler expression for d_1 can be obtained as:

$$d_1 = \frac{2\bar{i}_L}{I_{Lmax}} \quad (31)$$

The model can be described, in this approximation, by the following relations:

$$\bar{i}_1 = \frac{d}{d_1 - d} \bar{i}_2 = \frac{\mu}{1 - \mu} \bar{i}_2 \quad (32)$$

$$\bar{v}_2 = \frac{\mu}{1 - \mu} \bar{v}_1 - V_D - \left[\frac{R_{on}}{d} \left(\frac{\mu}{1 - \mu} \right)^2 + \frac{R_D}{d} \frac{\mu}{1 - \mu} + \frac{R_L}{d} \frac{\mu}{(1 - \mu)^2} \left(1 - \frac{d}{\mu} \right) \right] \bar{i}_2 \quad (33)$$

or, considering i_1 and v_2 as the independent variables,

$$\bar{i}_2 = \frac{d_1 - d}{d} \bar{i}_1 = \frac{1 - \mu}{\mu} \bar{i}_1 \quad (34)$$

$$\bar{v}_1 = \frac{1 - \mu}{\mu} (\bar{v}_2 + V_D) + \left[\frac{R_{on}}{d} + \frac{R_D}{d} \frac{1 - \mu}{\mu} + \frac{R_L}{d} \frac{1}{\mu} \left(1 - \frac{d}{\mu} \right) \right] \bar{i}_1 \quad (35)$$

It is important to mention that, if μ is equal to d , as in CCM is, and R_L is zero, the model given by Eqns. (34) and (35) is identical to the model obtained in [1] for CCM including switch conduction losses. If the converter operates in DICM, μ is greater than the duty cycle, d .

Based on the Eqn. 34 and 35, the averaged circuit model of the switch network can be constructed, as in Fig. 4:

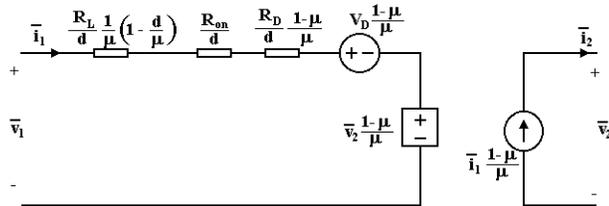


Fig. 4. The large signal simplified averaged switch model for DICM with conduction losses.

4 Results and Discussion

In order to validate the models, a Buck converter and a Boost converter are examined. The component values are the following:

1. Buck converter: $V_g = 24V$, $L = 20\mu H$, $f_s = 50kHz$, $d = 0.25$, $R = 6\Omega$, $C = 47\mu F$.
2. Boost converter: $V_g = 24V$, $L = 10\mu H$, $f_s = 50kHz$, $d = 0.25$, $R = 12\Omega$, $C = 47\mu F$.

First, the exact model, given by Eqn. (26) and (27) is applied to the two converters to study the steady state performances, compared to the switching circuit. The simulation waveforms of the output voltage, considering small losses ($R_{on} = 0.2\Omega$, $R_D = 0.11\Omega$, $R_L = 0.1\Omega$) are shown in Fig. 5a, b for the Buck and the Boost converter. Then, the model is applied to the two converters, considering large conduction losses ($R_{on} = 0.5\Omega$, $R_D = 0.61\Omega$, $R_L = 0.5\Omega$). The simulation waveforms of the output voltage are shown in Fig. 5c, d.

The results confirm the accuracy of the model.

The results for the output voltage (V_0) and the efficiency (η), for the two converters, considering both models developed in this paper, compared to the values obtained by simulating the switching circuit, and to the values obtained using the lossless model described in [1], are presented in Table 1 for the Buck and the Boost converter. For the output voltage the errors made by each averaged model are determined.

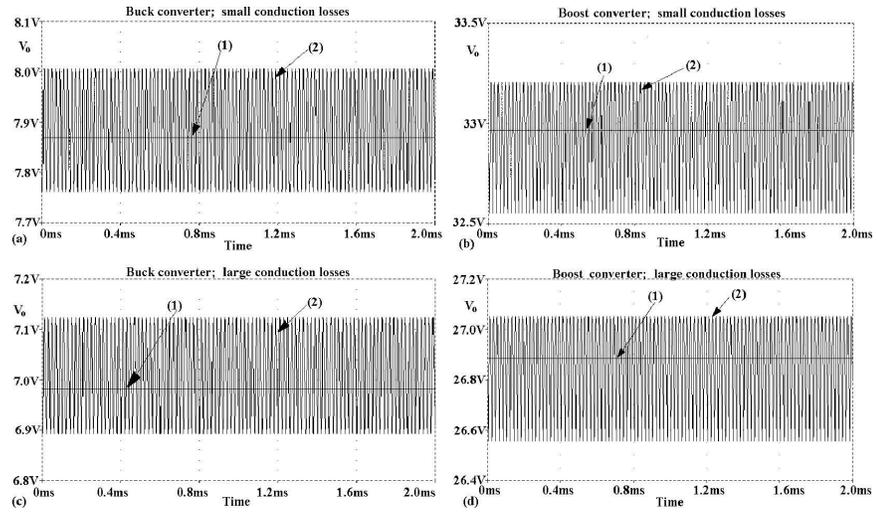


Fig. 5. Simulation results (output voltage) for: (a) the Buck converter with small conduction losses, (b) the Boost converter with small conduction losses, (c) the Buck converter with large conduction losses, (d) the Boost converter with large conduction losses; (1)-simulation using the averaged model, (2)-simulation of the switching converter.

Table 1. The output voltage and the efficiency for the Buck and the Boost converter.

Buck converter				
		V	error	η
small losses	switching circuit	7.89V	-	87.2%
	exact model	7.87V	0.25%	87.1%
	approximate model	7.92V	0.38%	87.4%
	lossless model [1]	8.38V	6.21%	100%
large losses	switching circuit	7.01V	-	69%
	exact model	6.98V	0.42%	70.4%
	approximate model	7.17V	2.28%	74%
	lossless model [1]	8.38V	19.5%	100%
Boost converter				
		V	error	η
small losses	switching circuit	32.95V	-	90.3%
	exact model	32.96V	0.03%	90.2%
	approximate model	33.42V	1.42%	92.05%
	lossless model [1]	36V	9.25%	100%
large losses	switching circuit	26.85V	-	71.2%
	exact model	26.88V	0.11%	71.3%
	approximate model	27.90V	3.91%	75%
	lossless model [1]	36V	34%	100%

Table 1 shows that the exact model can be used even for large losses. The errors given by the approximate model increase when the losses are larger, but for small losses it still can be used.

To show the separate effect of R_{on}/R , R_D/R and R_L/R on the dc conversion ratio, $M = V/V_g$, and on the efficiency, η , the model is used to generate dc conversion ratio and efficiency dependencies on the duty cycle for the Boost converter. The effects of R_{on}/R , R_D/R and R_L/R are shown in Fig. 6.

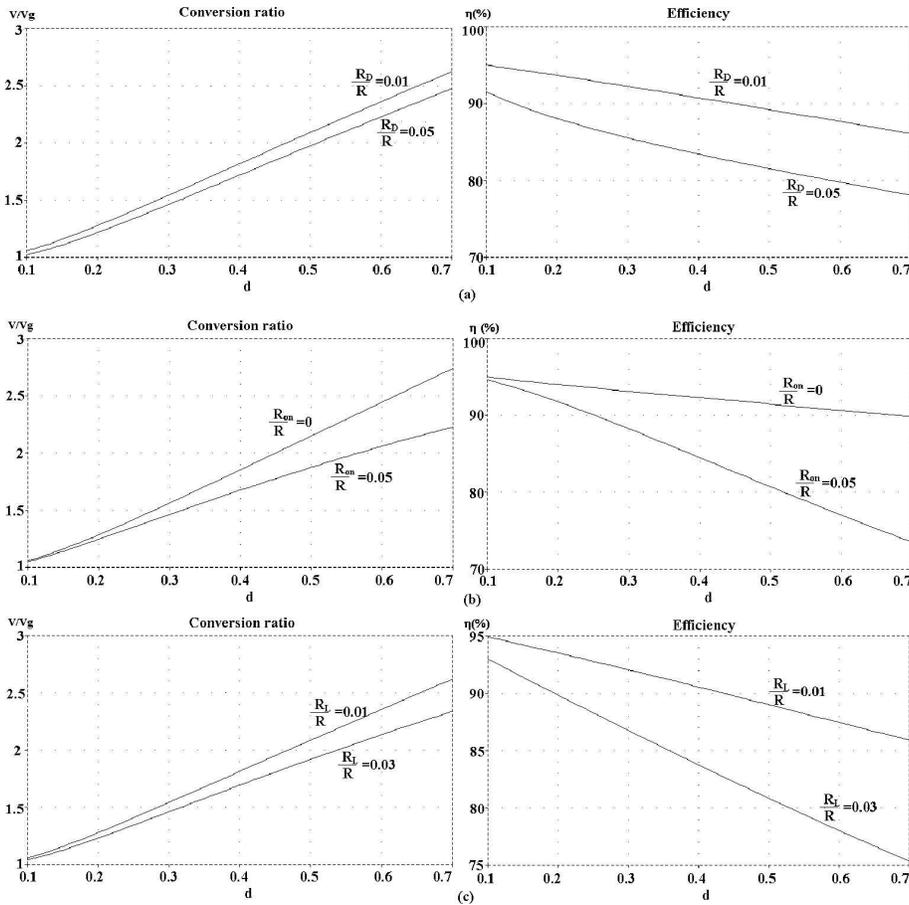


Fig. 6. Boost converter: dc conversion ratio and efficiency, for different (a) R_D/R values, (b) R_{on}/R values, (c) R_L/R values.

Remark: The range of duty cycles from 0.7 to 1 is not covered because in this range the converter operates in CCM. At high duty cycles, the conduction losses increase, so the efficiency decreases. Based on the simulation results, the circuit components can be selected to obtain a desired output voltage and efficiency.

The model can also be used for studying the effect of conduction losses on the dynamic behaviour of PWM converters. Fig. 7 shows the simulated (using PSpice) control-to-output transfer function for the Boost converter. The only effect of conduction losses on the dynamic response is that the open loop gain of the power stage is smaller.

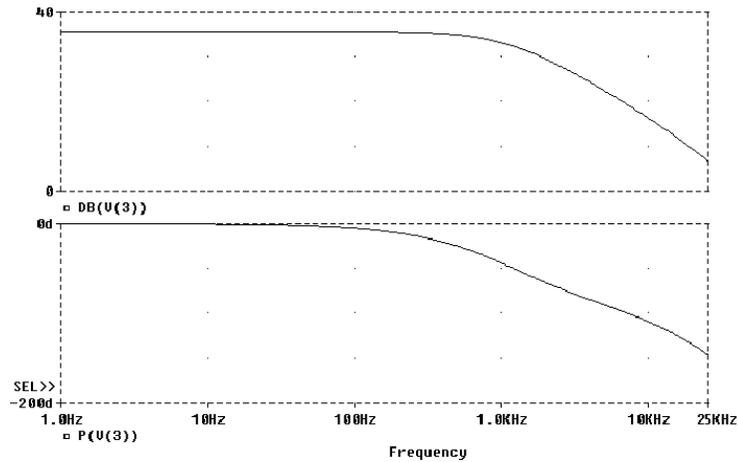


Fig. 7. Control-to-output transfer function of the Boost converter.

5 Conclusions

In this paper a new PWM switch model including conduction losses, for Discontinuous Inductor Current Mode (DICM) dc-dc converters is developed. Compared to other models for DICM, the proposed model is more accurate because the correct exponential waveforms for currents and voltages are considered. A possibility of simplifying the model is also presented. The simulation results confirmed the accuracy of the proposed models.

Appendix

The PSpice netlist of the exact model

```
*****
*MODEL:DCM1
*Application:DICM, including losses
*****
*Parameters:
* L=equivalent inductance for DCM
* fs=switching frequency
```

```

*****
*Nodes:
*1:transistor positive
*2:transistor negative
*3:diode cathode
*4:diode anode
*5:duty cycle control input
*****
.subckt DICM1 1 2 3 4 5
+params:L=100u fs=1E5 Ron=0 VD=0 RD=0 Rl=0
Et 1 1x value={(v(3,4)+VD)*v(d1)/v(5)}
Er 1x 2 value={i(Et)/v(5)*(Ron+RD*(1-v(u))/
+v(u)+Rl/v(u)*(1-v(5)-v(d1)))}
Gd 4 3x value={i(Et)*(1-v(u))/v(u)}
Eg 3x 3 value={0}
Ed d1 0 value={L*fs/(RD+Rl)*log(1-(v(1,2)+Rl*i(Eg)
+/(1-v(u)))/(Ron+Rl))+1-exp(-(Ron+Rl)*v(5)/L/fs))*
+(RD+Rl)/(-VD+Rl*i(Eg)/(1-v(u))-v(3,4))}
Eu u 0 value={(i(Eg)*Rl+v(1,2))/(i(Eg)/(
+v(5)/(Ron+Rl)-L*fs/(Ron+Rl)/(Ron+Rl)
+*(1-exp(-(Ron+Rl)*v(5)/L/fs))+v(1,2))}
.ends
*****
\end{verbatim}

```

The PSpice netlist of the approximate model:\

```

\
\begin{verbatim}
*****
.subckt DICM2 1 2 3 4 5
+params:L=100u fs=1E5 Ron=10 VD=0 RD=0 Rl=0
Er 1 1x value={i(Et)*(Ron+(1-v(u))*RD/v(u)+
+Rl/v(u)*(1-v(5)/v(u)))/d}
Et 1x 2 value={(1-v(u))*(v(3,4)+VD)/v(u)}
Gd 4 3 value={(1-v(u))*i(Et)/v(u)}
Ga 0 0 value={i(Et)}
Va a b
Ra b 0 1k
Eu u 0 value={1/(1+(1/(d/(Ron+Rl)-L*fs/(Ron+Rl)/(
+Ron+Rl)*(1-exp(-(Ron+Rl)*d/L/fs))-Rl)/(v(1,2)*
+v(u)/((1-v(u))*i(Va)+Rl)))}
.ends
*****

```

References

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