

# An Advanced Model of Partial Discharge in Electrical Insulation

Nenad Kartalović, Dragan Kovačević, and Srđan Milosavljević

**Abstract:** The paper presents an advanced concept for a computer model of partial discharge (PD) in insulation. The advanced model concept is based on a well-known model, using condensers for modeling of electrical network and the cavity where discharge occurs. The fundamental advancement is the modeling of initial conditions of each discharge event by introducing a controlled voltage source and modeling its control, the modeling of parameters of PD current pulse, and advanced modeling of all circuit elements. The paper presents basic analysis of PD processes and the analysis of basic shortcomings of the well-known model. The elements of the advanced model are established, and the way to implement it. It is shown that commercial software packages for electric circuits analysis are indeed suitable for PD modeling. Physical and electrical parameters of PD pulse are established, as well as the response of observed object's electric network to the ensuing excitation. The results of the model show the possibility of modeling of PD current pulse in a wide range of parameters.

**Keywords:** Cavities, detectors, dielectric materials, dielectric breakdown, gas insulation, insulation testing, modeling, partial discharges.

## 1 Introduction

Partial discharge (PD) occurs if dielectric strength within a certain region of the insulation of an electric device is locally exceeded, but the surrounding insulation is strong enough to prevent total breakdown. In order to understand the occurrence of PD and the response of electrical network to the excitation by discharge pulses, numerous hypotheses and models have been developed. There is no need to prove that electrical processes of PD have an electrical model; the only question

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Manuscript received on October 1, 2010

The authors are with Electrical Engineering Institute Nikola Tesla, Koste Glavinića 8, 11 000 Belgrade, Serbia (e-mails [nened.kartalovic, dragan.kovacevic, srdjan.milosavljevic]@ieent.org).

is what level of approximation i.e. reduction of system of equations is considered satisfying. In order to form an electrical model of PD, electrical and physically meaningful PD phenomena in insulation sample are described. PD site such as a cavity in insulation may be modeled with a simple three-capacitance model shown in Fig. 1.

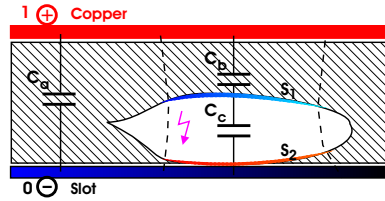


Fig. 1. Gaseous inclusion in an electrical insulation system.  $S_1$ ,  $S_2$  active surfaces (metaled and carbonized surfaces, surface charges)

Figure 2 presents the most frequently used model of electrical insulation with PD in a sinusoidal field [1]. In this model, a gaseous inclusion in electrical insulation system is represented by the capacitance  $C_c$ , which has a short-circuit in the moment of discharge. The electrical insulation capacitance in series with the gaseous inclusion is denoted  $C_b$ . The remainder of electrical insulation is represented by the capacitance  $C_a$ . The electric spark discharge resistance is represented by  $R_{sp}(t)$ .

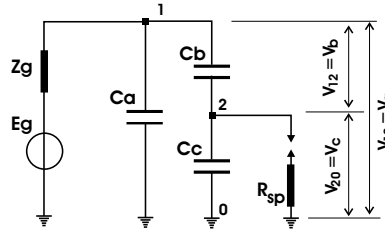


Fig. 2. Electrical model of gaseous inclusion defect in an insulation system in which PD takes place.

According to this model, the usual description of PD process is as follows: if the insulation system is subjected to alternating voltage  $V_a(t)$ , an electric field  $E_a(t)$  appears in the insulation system and an electric field  $E_c(t)$  in the cavity, Figures 2 and 3. Then all the insulation system capacitances get charged to the corresponding voltages  $V_a$ ,  $V_b$  and  $V_c$ . At the moment the cavity voltage  $V_c$  reaches the critical value  $U_p$  (during positive half-period it has subscript  $p$ , while during negative its subscript is  $n$ ), i.e. when the critical value of electric field  $E_{bd}$  is reached, there occurs a cavity discharge (positive breakdown) and a rapid fall in the voltage across it, in cca. 100ns. If it were not for the cavity discharge, its voltage would have

followed the graph  $[V_c]$ , Figure 3. Cavity voltage  $V_c$  falls to the value of "positive" spark extinction value  $V_p$ . At the same time, the voltage  $V_b$  across capacitance  $C_b$  rapidly rises. (Voltage  $V_b$  is not shown, but the relation:  $V_b = V_a - V_c$  holds, Figure 3). In the external circuit one can register a pulse of current, i.e. a PD pulse, recharging capacitance  $C_b$  from capacitance  $C_a$  and the supply circuit. Due to further rise of external voltage  $V_a$ , the condition for recharging (adding charge to) the condenser  $C_c$  is fulfilled. So the voltage  $V_c$  reaches the value  $U_p$ , when partial discharge takes place again. Let us notice that values  $U_p$  and  $V_p$  are by their nature stochastic, Figure 3.

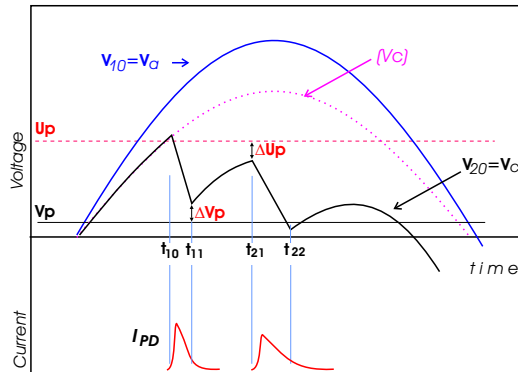


Fig. 3. Voltage and current diagrams (PD), where:  $V_{10} = V_a$  - voltage across object,  $[V_c]$  - cavity voltage when there is no PD,  $V_{20} = V_c$  - cavity voltage when there is PD,  $I_{pd}$  - PD current pulses.  $\Delta U_p$ ,  $\Delta V_p$  - Stochastic deviation of the values of  $U_p$  respectively  $V_p$

One should underline that the values of the capacitances are estimated on the basis of geometrical and physical parameters. Special attention is paid to the estimation of the values of  $C_b$  and  $C_c$  based on their interaction during the voltage and charge distribution process. Their distribution without the presence of spark is relatively slow and can encompass substantially greater surfaces of the cavity. During spark (quick changes), distribution depends on the free charge on surfaces, and surface conductivities  $\gamma_{1,2}$  and  $\gamma_c$ , Figure 1 and 4a. For instance, if the insulation is delaminated, one can have a situation where only a minor part of cavity surface takes part in a spark, due to small discharge surface (spark channel width of the order of magnitude of a few microns), and insufficient surface conductivity of the surface, Figure 4a. In some cases, it is possible to obtain an analytic expression for the voltage and charge distribution, respectively quantities  $C_b$  and  $C_c$ , [2–5]. They researched the influence of variable applied frequency on the physics of discharge process, on voltage and charge distribution, as well as on the stochasticity of the process. It is possible to experimentally ascertain necessary parameters in order for the cavity discharge model to be incorporated in this integral model [6, 7].

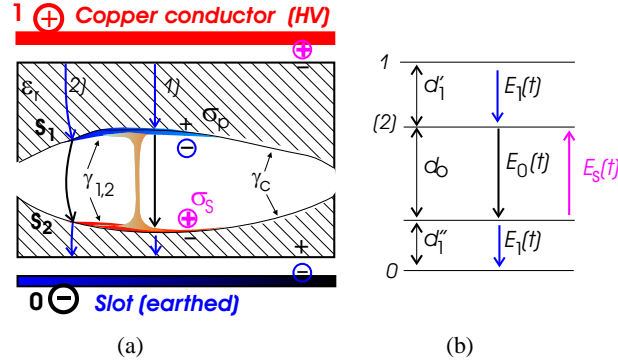


Fig. 4. A simplified sample of an insulation system with a cavity. (a) distribution of free ( $\delta_s$ ) and polarization charges ( $\delta_p$ ); (b) electric fields within insulation and cavity ( $E_C = E_0 - E_S$ );  $\gamma_{1,2}$  surface conductivity with free charges;  $\gamma_c$  surface conductivity without free charges; ( $\gamma_{1,2} \gg \gamma_c$ ).

PD numerical model suggested in this paper is based on commercially available software packages for electric (electronic) circuits modeling and analysis. There are many reasons for this decision. Developed software packages solve complex systems of integral-differential equations using highly advanced algorithms. In the packages, numerical modeling of nonlinear electronic elements is possible, as well as modeling of randomness of their specific parameters. Analysis is possible in both time and frequency domains. Highly advanced signal processing tools have been developed. Also developed were user-friendly graphical interfaces that enable straightforward modeling of equation systems, as well as intuitive and easy understanding of models by a wide circle of users. Moreover, in specific situations certain relationships between parameter values are easily spotted, which enables efficient simplification of the model (the system of equations) depending on the case at hand.

## 2 Physical Elements of the PD Model

As already stated, it is not possible to model a cavity with discharge by a single condenser  $C_c$  and arrester only, without the so-called memory effect etc. This paper suggests a more complex model. It discusses the importance of process occurring before discharge, of the discharge process itself, and of the interdependence and stochasticity of all processes. With the help of adequate parameters, processes are then integrated in a complex electrical network (electrical models of cavities, objects, supply, measurement circuits, etc.) so as to obtain an entire picture of the PD.

For the opening analysis a simplified sample of insulation with an internal cav-

ity will be taken, Figure 4a. When an external voltage is applied to the copper conductor, it creates certain electric fields in the insulation between the copper and the earthed iron core, Figure 4b.

According to Figures 1 and 2, the dielectric field is  $E_1(t)$  (capacitance  $C_b$ ), and the corresponding (related) cavity field component is  $E_0(t)$  (capacitance  $C_c$ ). The second field component in the cavity,  $E_s(t)$ , stems from the free charge (whose density is  $\sigma_s$ ) of the formed discharge. The resulting cavity field is, Figure 4b:

$$\vec{E}_c(t) = \vec{E}_o(t) - \vec{E}_s(t) \quad (1)$$

When the critical cavity voltage  $U$  is obtained,  $Ec(t_i) \rightarrow E_{bd}(t_i)$ , there occur PDs. The breakdown is preceded by processes of intensive ion and electron generation, which can be incorporated in this model by suitably modeled ion or electron generator parameters. Processes before the breakdown are cumulative and need a certain time to develop. Of special interest for the model is the spark channel forming time, which determines the rise time of the current pulse. For estimation of the characteristic spark channel forming time  $\tau_{sf}$  depends on the average intensity of the electric field in the cavity  $E_{bd}$  and the characteristic Toppler's constant  $k_T$  of the gas in the cavity (Toppler's Law), [8]:

$$\tau_{sf} = 4.4 \frac{k_T}{E_{bd}} \quad (2)$$

For air,  $k_T = (0.5 - 0.6)10^{-4} \text{ V s cm}^{-1}$ . Average electric field in the gap is arrived at on the basis of the estimated voltage  $U_p$  and the shape factor of the gap.

Under the influence of the local electric field, electrons and ions are separated in a relatively short time, forming compound local pulse of current  $i(t)$ . "Characteristic duration of current pulse" parameter  $\tau_i$  is determined by a complex relationship of the energy accumulated in the cavity capacitance, physical processes in the cavity, electric interaction with the circuit etc. In any case, the spark lasts until the cavity voltage drops to spark extinction voltage  $V$ . This  $V$  depends on conditions for the development of the spark and is stochastic by nature. The greater the energy of discharge (that is, the greater the breakdown voltage  $U$ ), the more efficient the process of ionization, so the spark extinction voltage gets lower [9].

Electrode or rather surface effects are determined primarily by stochastic and time-variable processes of electron emission from the surface (collision emission, thermal emission, field emission). So, the electron emissivity is influenced by the kind of material, the condition of the surface, its temperature, intensity of electric field and some other synergistic effects of the system as a whole.

Among the volume effects one can enumerate relevant processes within the volume of gas: excitations of atoms and molecules, changes of charge, ionizations and recombinations, charge diffusion and drift, synergies of multiple effects etc.

Most of the processes are basically quantum mechanical and stochastic. Cross sections of individual reactions and their randomness depend on the intensity of electric field, the kind and condition of the gas and cavity surfaces (pressure, temperature, material, bulk and surface conductivity etc.), electrochemical characteristics etc. To a great extent, randomness depends also on several macro factors of the cavity (shape, size etc.), as well as on the coupling of the discharge and external circuit (local impedances, frequency, generator harmonics, generator impedance, etc.) [9–11].

During the current pulse, there changes surface density of the spark free charge  $\delta_s$  on cavity walls, Figure 4a. That charge creates the opposing field and brings about the extinction of the spark. The free charge accumulated in the cavity changes between two discharges, due to surface conductivity of cavity walls and drift or diffusion of the charge. That phenomenon can be characterized by the corresponding "characteristic time of cavity charge change" parameter  $\tau_e$ , which will be a subject of this model.

### 3 Mathematical Elements of PD Model

Mathematical models of electrical elements, systems of equations and solving algorithms developed in commercial programs for the analysis of electric circuits. Generally, one can state that a circuit element  $D$  (resistor, condenser, inductance, controlled source etc.) is dependent on voltage, current, charge, flux, time etc. respectively:

$$D = d(u, i, q, \phi, t, \dots) \quad (3)$$

Such a concept offers broad possibilities for taking into account changes, interdependencies, stochasticity (as already mentioned for  $C_b$  and  $C_c$ ). For instance, in the case of delamination of generator insulation, the value of condensers can change depending on temperature, time, current, surface conductivity etc. (e.g. strong electromagnetic forces and thermal dilatation can decrease the cavities, according to experience).

As previously mentioned, an insulation sample can be represented by three condensers, Figure 2. The corresponding distribution of charge and electric fields has already been shown in Figure 4. Discussion will start with the zero phase moment of the voltage  $V_a (= V_{10})$  and no previous PD in the cavity. If the amplitude of the external voltage is denoted  $V_{am}$  and it varies sinusoidally over time, the condenser

voltages can be written as, Fig. 2:

$$\begin{aligned} V_a(t) &= V_{10}(t) = V_{am} \sin(\omega t) \\ V_b(t) &= V_{12}(t) = \frac{C_c}{C_b + C_c} V_{am} \sin(\omega t) \\ V_c(t) &= V_{20}(t) = \frac{C_b}{C_b + C_c} V_{am} \sin(\omega t); \quad V_c < U_p \end{aligned} \quad (4)$$

For the moment the first PD pulse is initiated (Figure 3):

$$t_{10} = t[V_{20} - U_p(t_{10})] = [U_p \pm \Delta U_p(t_{10})] \quad (5)$$

where  $U_p(t_{10})$  is the value of the critical cavity voltage at the moment, in accordance with Figure 3.  $\Delta U_p(t_{10})$  points to the stochastic nature, where dependence on the previous values is also possible  $\Delta U_p(t_{ij}) = f(t_{ij}, \Delta_p(t_{i-1, j-1}, \dots))$  with index denoting initiation ( $i$ ) or extinction ( $j$ ) of the spark. In accordance with Figures 2, 4, and 5 one can write:

$$V_{20}(t_{10}) = \int_d \vec{E}_0 \, dl = U_p(t_{10}) \quad (6)$$

$$V_{10}(t_{10}) = \int_d \vec{E}_1 \, dl + \int_d \vec{E}_0 \, dl = \vec{V}_{12}(t_{10}) + \vec{U}_p(t_{10}) \quad (7)$$

where  $\vec{E}_1$  is the field in the dielectric of total width  $d_1$  and  $E_0$  is the field in the cavity  $d_0$  wide, Figure 4. It should be stressed that, the value of the integral is the same for paths 1) and 2), Figure 4a, in the case when the electrical field is perpendicular to the surfaces  $S_1$  and  $S_2$ , according to boundary conditions for the dielectric flux density and electric field strength. This situation is to be found in the case of delamination of insulation and in all cases when surfaces  $S_1$  and  $S_2$  become sufficiently conductive ( $\gamma_{1,2}$ ) with time (because charge and aging).

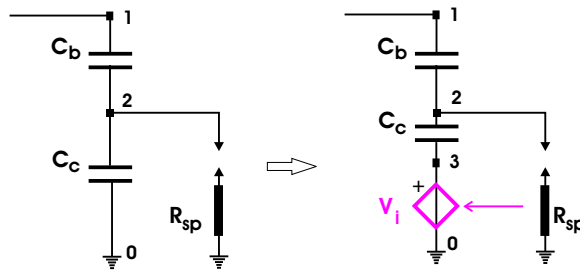


Fig. 5. (a) Model of electric insulation with condensers; (b) Improved model with voltage controlled source  $V_i$ .

During PD pulse, voltage  $V_{10}(t)$  rises relatively slowly (Figure 3), while voltage  $V_{12}(t)$  rises and voltage  $V_{20}(t)$  falls very quickly, until the first PD pulse is

extinguished at the moment  $t_{11}$  (at spark voltage  $V_p$ ):

$$t_{11} = t[V_{20} - V_p(t_{11})] \quad (8)$$

$$V_{10}(t_{11}) = \int_{d_1} E_1(t_{11}) dl + \int_{d_0} E_0(t_{11}) dl + \int_{d_0} E_s dl \quad (9)$$

where  $E_s$  is the field made by the free charge on cavity walls, Figure 4.

Let it be noted that fields in the dielectric  $E_1$  and in the cavity  $E_0$  have changed (risen instantaneously) due to a new redistribution of the charge in the condenser  $C_b$  (while the voltage applied  $V_{10}$  is roughly the same). One observes that another member has appeared in equation (9), which can be written down as:

$$- \int_{d_0} E_s dl = V_{30} = V_i \quad (10)$$

This member will be modeled by the controlled voltage source  $V_i$ , which takes into account the influence of the free charge formed in the cavity, Figure 5b.

Voltage across cavity is now composed of voltages on condenser  $C_c$  and source  $V_i$  at any given moment  $t$ , i.e. at the moment of spark initiation  $t_{10}$ :

$$V_{20} = V_{23}(t) + V_i(t) \quad (11)$$

$$V_{20}(t_{10}) = U_p(t_{10}) = V_{23}(t_{10}) + V_i(t_{10}) \quad (12)$$

Discharge will be extinguished when cavity voltage  $V_{20}$  falls to the value  $V_p$ , while the external voltage remains the same:

$$V_{20}(t_{11}) = V_p \pm \Delta V_p(t_{11}) = V_{23}(t_{11}) + V_i(t_{11}) \quad (13)$$

$$V_i(t_{11}) = V_p \pm \Delta V_p(t_{11}) - V_{23}(t_{11}) \quad (14)$$

When, at the moment  $t_{21}$ , cavity voltage  $V_{20}(t_{21})$  reaches critical value  $U_p$  again, there occurs a new discharge extinguished at the moment  $t_{22}$ , when one has (Figure 3):

$$V_{20}(t_{21}) = U_p \pm \Delta U_p(t_{21}) = V_{23}(t_{21}) + V_i(t_{21}) \quad (15)$$

$$V_{20}(t_{22}) = V_p \pm \Delta V_p(t_{22}) = V_{23}(t_{22}) + V_i(t_{22}) \quad (16)$$

As outputs of the numerical model one can obtain electric charge of PD and average values of the electric field in the cavity. The quantity of induced charge  $q_b$  in condenser  $C_b$  are:

$$q_b(t) = C_b(t)[U(t) - V(t)] \quad (17)$$



### 4 Graphical Representation of PD Model

Models described can be used to form a graphical model of PD, using some of the software packages (Matlab Simulink, Multisim, Electronics Workbench, Spice AS, etc...) for analysis of electric circuits.

As mentioned before, in the improved model, a sample of the insulation is modeled by condensers  $C_a$  and  $C_b$ . Cavity is modeled by the condenser  $C_c$  and the controlled source  $V_i$ . Generally, corresponding impedances  $Z_a, Z_b, Z_c$ , etc. can be added (Figure 6, block "sample").

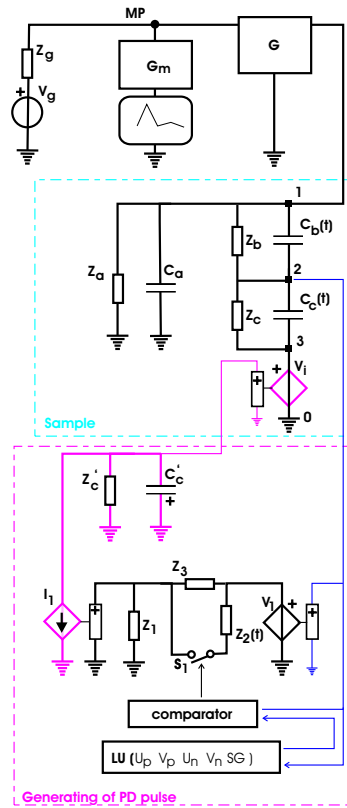


Fig. 6. Numerical model of partial discharges represented by symbols of electronic elements.  $C_a, C_b$  and  $C_c$  are corresponding capacitances of the sample.

The rest of the observed high-voltage object will be modeled by the corresponding transfer function  $G$ . High-voltage source will be modeled by the source  $V_g$  and impedance  $Z_g$ . Measurement network will be modeled by the transfer function  $G_m$ .

Control of the source  $V_i$  is done by the voltage of the condenser  $C'_c$ , it being the replica of the condenser  $C_c (= C'_c)$ , "generating PD pulse" block in Figure 6.

If in the condenser  $C'_c$  one generates the quantity of charge generated by PD, then the voltage of the condenser is increased. The required compensation in the sample circuit is obtained through the voltage source  $V_i$ . On the other hand, by "dislocating" of the capacitance  $C'_c$ , one makes sure that the redistribution of voltages  $V_{10}$ ,  $V_{12}$  and  $V_{20}$  does not influence directly the change of the cavity free charge in the absence of the spark. However, there are many reasons for the change of charge between two discharges. The free charge can decrease due to diffusion of charge, depending on the impedance  $Z'_c$  and the condenser  $C'_c$ . The decrease is enhanced by the neutralization of charge due to currents through the imperfect insulation, which is indirectly modeled by the values of  $Z_b$  and  $Z_c$  and their ratio.

On the other hand, if the cavity walls have relatively good conductivity, cavity voltage will bring about the redistribution of charge in the direction of the spark conducting channel. Voltage  $V_{20}$  is the cause of charge redistribution and of the discharge in the cavity. For easier modeling of necessary values, that voltage is mirrored by the controlled voltage source  $V_1 (= V_{20})$ . Voltage  $V_1$  and impedance  $Z_3$  ( $|Z_3| \gg 1 \Omega$ ) cause permanent leakage of the current  $I_1$  and increase of the free charge in the cavity. Basically, that will continue to happen until cavity voltage is completely annulled.

When cavity voltage reaches the spark initiation voltage  $V_{20}(t) \rightarrow U \pm \Delta U(t)$ , as determined by the comparator, switch  $S_1$  is turned on. The values of spark channel impedance  $Z_{2t}$  and  $Z_1$ ,  $Z'_c$ ,  $C'_c$  determine the time constant of the increase in current of the controlled current source  $I_1$ , simulating spark pulse. That current source fills the capacitance  $C'_c$  in the direction marked, Figure 6, which causes negative increase of the voltage  $V_i$  i.e. decrease of cavity voltage  $V_{20}$ . When the voltage falls to the value of spark extinction voltage  $V_{20}(t) \rightarrow V \pm \Delta V(t)$  (as determined by the comparator), switch  $S_1$  is turned off. Then the values  $Z_1$ ,  $Z'_c$  and  $C'_c$  model parameters of the transient in the electrical circuit generated because of extinction of the spark.

The values of spark extinction and initiation voltages are determined by the logical unit LU, Figure 6. They are determined on the basis of given values ( $U_p$ ,  $U_n$  and  $V_p$ ,  $V_n$ ) and the random variation generator (module SG) of those values  $[(\pm \Delta U(t), \pm \Delta V(t))]$ , depending on the cavity voltage and given statistical parameters.

## 5 Estimation of Model Parameters

Estimated values of certain parameters of the PD model differ for the positive and the negative half-period, so they will be indexed with p or n respectively: characteristic capacitances ( $C_a$ ,  $C_b$  and  $C_c$ ), critical breakdown voltages ( $U_p$  and  $U_n$ ),

discharge extinction voltages ( $V_p$  and  $V_n$ ), characteristic times of spark channel forming ( $\tau_{sf}$ ), current pulse duration characteristic time ( $\tau_i$ ), characteristic time of cavity charge change ( $\tau_e$ ), randomness parameters of characteristic voltages, capacitances, impedances and times.

PD parameters for different objects can vary in a wide range. In practice, the capacitances of all three condensers differ widely, while the following conditions are mostly fulfilled,  $C_b \ll C_c \ll C_a$ .

Objects' capacitances  $C_a$  are usually of the 1nF order of magnitude, going to 1 $\mu$ F. Cavity  $C_c$  and the associated dielectric  $C_b$  capacitances are usually of the 1pF order of magnitude, going to 1nF. As has been stated previously, in special cases these values can be completely different.

Critical breakdown voltages  $U_p$  and  $U_n$  can be estimated on the basis of physical-chemical characteristics of insulation system and its defects. The presence and the condition of metal electrodes (surfaces) is extremely important. Table 1 shows an estimate of the breakdown voltages for an internal cavity  $d = 0.1\text{mm}$  high, filled with air at pressure  $p = 1\text{bar} = 10^5\text{Pa}$ . According to Paschen's Law,  $p_d = 10^5\text{Pa} \times 10^{-4}\text{m} = 0.1\text{Pam}$  and taking account of practical experience, the breakdown voltage is estimated at  $U_p = U_n = 1\text{kV}$ . If one of the electrodes in the cavity is a metal surface, breakdown physics changes significantly, Table 1.

According to experience, rise time of current pulse i.e. spark channel forming time  $\tau_{sf} \sim 100\text{ns}$ . Values are predominantly estimated from the relationship  $\tau_{sf} \rightarrow f(Z_1, Z_2(t))$ , Figure 6.

According to experience, current pulse characteristic duration time  $\tau_i$  is of the order of microseconds. The time observed is determined by many elements, but approximately one can model it as  $\tau_i \rightarrow f(Z_i, Z_2(t), Z'_c, C'_c, \dots)$ .

The characteristic time of cavity charge change  $\tau_e$  is determined by the values of great many elements,  $\tau_e \rightarrow f(Z_i, Z_2(t), Z'_c, C'_c, \dots)$ , Figure 6. This time is usually relatively long and does not influence charge redistribution. In the case of carbonized cavity (aging), it becomes of the same order of magnitude (microseconds) as PD processes, and influences significantly the character and the intensity of PD.

In this model, randomness can be dealt with in many ways. In the first place, comparison thresholds  $U_p$ ,  $U_n$ ,  $V_p$ , and  $V_n$  can be specified as stochastic voltage sources with a chosen distribution. Distribution parameters are estimated on the basis of experience and the theoretical data [4–6]. It should be noted that such an approach to modeling affords many other possibilities of randomness modeling, by modeling of values of the individual model components ( $C(t)$ ,  $Z(t)$  etc.). The basis of that modeling must be formed by the specifics of the physical process of discharge itself.

## 6 Some Results of The Model

On the basis of previous analysis, some average or specific values of all elements in the model shown in Figure 6 can be estimated. Estimated values depend on the object being modeled (rotating machines, transformers, insulation samples ...). In this example, object capacitance can be estimated at  $C_a = 1\text{nF}$ , series capacitance at  $C_b = 0.25\text{pF}$ , and cavity capacitance at  $C_c = 1\text{pF}$  ( $C'_c = 1\text{pF}$ ). Table 1 shows the values of the breakdown voltage thresholds for specific cases of PD.

As previously stated, cavity voltage depends on electric field and shape of the cavity, state of the insulator and the condition of cavity walls (carbonization). According to this model and models discussed in all references, electric field in the cavity is calculated as a superposition of the electric field in the dielectric and the electric field of the free charge in the cavity. Initial electric field stems from the electric source and after some redistribution in the dielectric, it has the form:

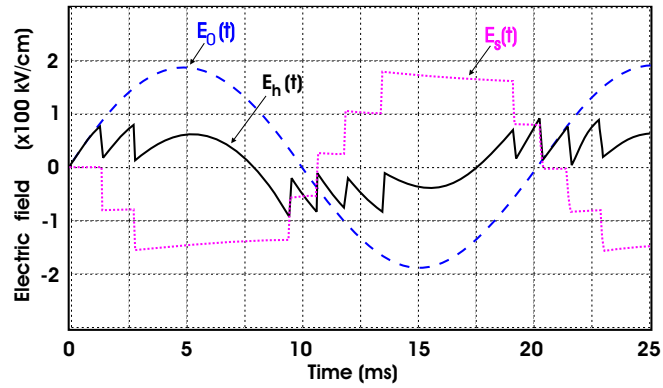
$$E_h(t) = \sin(\omega t) \quad (18)$$

With every PD, quantities of new free ions and electrons are being generated. Their redistribution causes instantaneous change of electric field in the cavity and in the dielectric of the condenser  $C_b$ . The free charge on the walls of the cavity changes between two successive PDs on account of diffusion in the surrounding insulation material, and drift or creeping of the charge on the surface of the cavity. Generated charge in the case of homogenous and slowly changing field can be described by [4]:

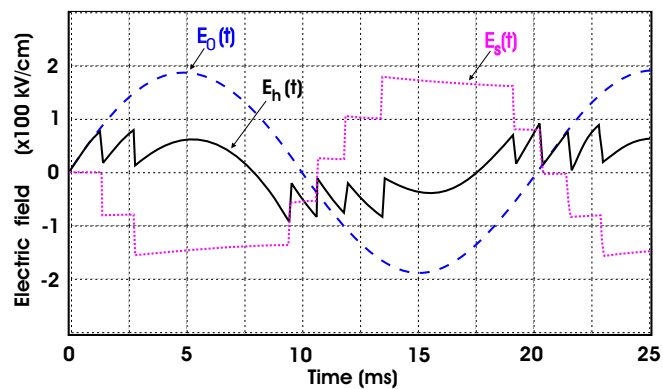
$$q_s = k(1 - e^{-\frac{t}{\tau}}) \quad (19)$$

In the case of relatively high insulation and cavity walls conductivity and quickly changing electric field, situation is more complex. Figure 7 shows a diagram of electric fields in a symmetrical internal cavity, obtained from the model. It is characteristic of it that PD has the same intensities in the negative and in the positive half-period of external voltage. That was modeled by an equal breakdown field and an equal spark extinction field. Figure 7a shows that between two breakdowns, the quantity of free charge changes (falls), as well as the corresponding electric field  $E_s(t)$  ( $\sigma_s \sim E_s(t)$ ). In this case, graph of  $E_s(t)$  can be approximated by the corresponding exponential function. For example, if the field causes a drift of charge, it can also accumulate from the insulator or in some other way, which significantly complicates the previous simplified picture.

After significant carbonization of the cavity (due to aging,  $\gamma_c$  increase, Figure 4) or when drift and creeping currents become significant, the change of free charge in the cavity in the interval between two breakdowns becomes very significant. If carbonization is modeled in the Figure 7a example, one obtains the results shown



(a)



(b)

Fig. 7. Diagram of electric fields in a symmetrical internal cavity. The (resulting) field in the cavity when PDs exist  $E_h(t)$ , the field in the air (vacuum)  $E_0(t)$ , compensating field due to free charge  $E_s(t)$ . (a) normal cavity; (b) carbonized cavity.

in Figure 7b. Namely, the level of partial discharges falls (from 4 to 3 PD pulses per half-period, specifically) when the cavity begins to carbonize. Distribution of free charge has completely different dependence compared to the previous PD, and equation (19) is not applicable.

## 7 Conclusion

The paper presents a concept of an advanced model for analysis of partial discharges in insulation. The model developed by the authors of the papers mentioned has been advanced. All the relevant physical PD processes can be integrated, through adequate parameters of the discharge model, in an integrated model of an

electrical network. The so-called memory effect can be modeled by a controlled voltage source. By choosing specific parameters one can model PD process dynamics, while with transfer functions one can model objects with distributed parameters (such as cables, transformers, generators, measuring equipment, etc).

Since all these phenomena have corresponding electrical parameters, they are easily modeled in today's modern commercial software for analysis of electric circuits. Each circuit element can be modeled as a nonlinear multiparameter element. The accuracy of model's results depends on the knowledge of and the availability of object characteristics, and the desired numerical accuracy.

This model practically gives the opportunity to obtain and compare the discharge pulse shape and its parameters at two very important circuit points: outside, where the pulse can be experimentally measured, and inside the investigated specimen. Moreover, by adequate choice of parameters, the model can integrate results of contemporary experimental and theoretical researches. That way one can model PD cases: Electrodes at floating potential, Voids in cast resin, free particles in gas-insulated switchgear, Turn-to-turn faults in power transformers, Surface discharge, Contamination or foreign particles in the insulation system, Delaminating, Variable applied frequency, Variable current, and many others.

As an example, three cases taken from practice have been discussed and successfully modeled. The case of change of the cavity charge between two PD sparks, the case of asymmetrical discharge when one of the cavity walls is metal held at given potential, and the case of PD voltage pulses observed in practice.

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