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A DEMAND INDEPENDENT INVENTORY MODEL

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Abstract: This paper is an extension of Deng et al. (2007) that was published in the European Journal of Operational Research. We have generalized their model from ramp type demand to arbitrary positive demand while theoretically discovering an important phenomenon: the optimal solution is actually independent of the demand as pointed out by Wou (2010), Hung (2011) and Lin (2011). We extend their inventory models in which the deteriorated rate is any non-negative function and backlog rate is inversely linearly related to the waiting time. Our findings will provide a new inventory system to help decision makers decide the optimal ordering quantity and replenishment policy.

Keywords: Inventory model, Deteriorating item, Partially backlogged.

MSC: 90B05.

1. INTRODUCTION

This paper investigates an inventory model with stock at the beginning and shortages allowed, but then partially backlogged. In the entire time horizon, there is only one order placed at the beginning and backorders are filled at the end. This kind of inventory model was first proposed by Hill [4], and then further investigated by Mandal and Pal [11], Wu and Ouyang [15], and Deng et al. [3], Skouri et al. [10], in which the demand is ramp type demand. Moreover, we also improve inventory models of Wou [12], Yang et al. [19], Hung [5], and Lin [8] with a more general deterioration rate, backordered rate, and set up cost. Deng [2] improved Mandal and Pal [11] and Wu et al.

[13] to provide a more complete solution procedure. According to Mandal and Pal [11], Wu and Ouyang [15], and Deng et al. [3], Wou [12] studied the inventory models from a ramp type demand to any nonnegative function. He provided an explanation, from managerial point of view, of deriving the optimal solution without constructing the objective function, which dramatically simplifies the solution structure. Based on the inventory model of Mandal and Pal [11], Wu and Ouyang [15] and Wu [14], Yang et al. [19] developed an inventory model in which the demand is extended from a ramp type demand to positive demand, showing that the optimal solution is independent of the demand type. Owing to Mandal and Pal [11], Wu and Ouyang [15], Deng et al. [3], and Skouri et al. [10], Hung [5] revised their models from special ramp type demand to generalized nonnegative function, where the deteriorated rate is any nonnegative function and the backlogged rate needs an extra condition to guarantee the existence of the optimal solution. Lin [8] amended Deng et al. [3] and Cheng and Wang [1] to construct a multi-period inventory model from a trapezoidal type demand to any positive function with constant deterioration and complete backorder. The set up costs are included in the inventory models of Cheng and Wang [1] and Lin [8]. Recently, Lin et al. [7] used smoothly connected property to simplify the solution procedure of Wu et al. [16] and then applied their approach to Deng et al. [3]. Hung [6] published a paper to compare the interior local minimum and the boundary local minimum. Roy and Chaudhuri [9] developed an EPLS model with a variable production rate and demand depending on price. Wu et al. [17] applied the Newton method to locate the optimal replenishment policy for EPQ model with present value. Yadav et al. [18] studied an inventory model of deteriorating items with two warehouse and stock dependent demand.

2. ASSUMPTIONS AND NOTATION

We have generalized the inventory model of Mandal and Pal [11], Wu and Ouyang [15], Deng et al. [3], Wou [12], Hung [5] and Lin [8] with the following assumptions and notations for the deterministic inventory replenishment policy with a general demand:

1. The replenishment rate is infinite; thus, replenishments are instantaneous.
2. The lead time is zero.
3. T is the finite time horizon under consideration. We follow the assumption of Deng et al. [3] and set $T=1$ so that the length of the inventory model equals to the unit time.
4. A is the set up cost that was proposed by Lin [8].
5. C_h is the inventory holding cost per unit per unit of time.
6. C_s is the shortage cost per unit per unit of time.
7. C_d is the cost of each deteriorated item.
8. C_l is the opportunity cost due to lose sale per unit
9. The deterioration is generalized from a constant θ to a nonnegative function $\theta(t)$.
10. $I(t)$ is the on-hand inventory level at time t over the ordering cycle $[0,T]$.

11. Shortage is partially backlogged with backordered rate $B(t)=\dfrac{1}{1+a(T-t)}$ with $a\geq 0$. For the special case with $a=0$, $B(t)\equiv 1$ that is the fully backlogged case of Deng et al. [3].
12. The demand rate $R(t)$ is assumed to be any positive function with $R(t)>0$ for $t>0$.
13. t_1 is the time taken for the inventory level to reach zero.
14. t_1^* is the optimal solution for t_1.
15. $f(t_1)$ is an auxiliary function defined as $f(t_1)=C_h\int_0^{t_1} e^{\int_t^{t_1}\theta(y)dy}dt+C_d\left(e^{\int_0^{t_1}\theta(y)dy}-1\right)-(C_s+aC_l)\dfrac{T-t_1}{1+a(T-t_1)}$.
16. $C(t_1)$ is the total cost that consists of set up cost, holding cost, deterioration cost, opportunity cost and shortage cost.

3. OUR PROPOSED INVENTORY MODEL

Replenishment occurs at time $t=0$ when the inventory level attains its maximum inventory level. From $t=0$ to t_1, the inventory level reduces due to both demand, $R(t)$ and deterioration. At t_1, the inventory level achieves zero, after which shortages are allowed during the time interval (t_1,T), and all of the demand during the shortage period (t_1,T) are partially backlogged. The inventory levels of the model are described by the following equations:

$$\frac{d}{dt}I(t)+\theta(t)\,I(t)=-R(t),\ \ 0<t<t_1 \tag{1}$$

and

$$\frac{d}{dt}I(t)=-\frac{R(t)}{1+a(T-t)},\ \ t_1<t<T \tag{2}$$

We directly solve Equations (1) and (2) to get

$$I(t)=e^{-\int_0^t\theta(y)dy}\int_{t_1}^{t}(-1)R(x)e^{\int_0^x\theta(y)dy}dx\text{, for }0\leq t\leq t_1 \tag{3}$$

and

$$I(t) = \int_{t_1}^{t} (-1)\frac{R(x)}{1+a(T-x)}dx \text{ , for } t_1 \le t \le T \tag{4}$$

The amount of deteriorated items during $[0,t_1]$ is evaluated

$$I(0) - \int_0^{t_1} R(x)dx = \int_0^{t_1} R(x)\left(e^{\int_0^x \theta(y)dy} - 1 \right)dx \tag{5}$$

Using integration by part, the holding cost during $[0,t_1]$ is evaluated

$$C_h \int_0^{t_1} I(t)dt = C_h \int_{x=0}^{t_1} \int_{t=0}^{x} e^{-\int_0^t \theta(y)dy} R(x) e^{\int_0^x \theta(y)dy} dtdx \tag{6}$$

The shortage cost during $[t_1,T]$ is evaluated through integration by part

$$C_s \int_{t_1}^{T} (-1)I(t)dt = C_s \int_{x=t_1}^{T} \frac{R(x)}{1+a(T-x)}(T-x)dx \tag{7}$$

The opportunity cost for lost sale is evaluated as

$$C_l \int_{t_1}^{T} \frac{a(T-x)}{1+a(T-x)} R(x)dx \tag{8}$$

Therefore, the total cost is the sum of inventory holding cost, deterioration cost, shortage cost, opportunity cost and set up cost

$$\begin{aligned} C(t_1) &= C_h \int_{x=0}^{t_1} \int_{t=0}^{x} e^{-\int_0^t \theta(y)dy} R(x) e^{\int_0^x \theta(y)dy} dtdx + C_d \int_0^{t_1} R(x)\left(e^{\int_0^x \theta(y)dy} - 1 \right)dx \\ &+ C_s \int_{x=t_1}^{T} \frac{R(x)}{1+a(T-x)}(T-x)dx + C_l \int_{t_1}^{T} \frac{a(T-x)}{1+a(T-x)} R(x)dx + 2A \end{aligned} \tag{9}$$

From Equation (9), it follows that

$$\begin{aligned} C'(t_1) &= C_h R(t_1) \int_0^{t_1} e^{\int_t^{t_1} \theta(y)dy} dt + C_d R(t_1)\left(e^{\int_0^{t_1} \theta(y)dy} - 1 \right) \\ &- C_s \frac{R(t_1)}{1+a(T-t_1)}(T-t_1) - C_l R(t_1) \frac{a(T-t_1)}{1+a(T-t_1)} \end{aligned} \tag{10}$$

From Equation (10), we assume an auxiliary function, say $f(t_1)$, as follows

$$f(t_1) = C_h \int_0^{t_1} e^{\int_t^{t_1} \theta(y)dy} dt + C_d \left(e^{\int_0^{t_1} \theta(y)dy} - 1 \right) - (C_s + aC_l) \frac{T - t_1}{1 + a(T - t_1)} \tag{11}$$

By taking the derivative of $f(t_1)$, to derive that

$$f'(t_1) = C_h \left(1 + \int_0^{t_1} e^{\int_t^{t_1} \theta(y)dy} \theta(t_1)dt \right) + C_d e^{\int_t^{t_1} \theta(y)dy} \theta(t_1) + \frac{(C_s + aC_l)}{[1 + a(T - t_1)]^2} > 0 \tag{12}$$

We know that

$$f(0) = -(C_s + aC_l) \frac{T}{1 + aT} < 0 \tag{13}$$

and

$$f(T) = C_h \int_0^{T} e^{\int_t^{T} \theta(y)dy} dt + C_d \left(e^{\int_0^{T} \theta(y)dy} - 1 \right) > 0 \tag{14}$$

We combine the results of equations (12-14) to imply that $f(t)$ is an increasing function from $f(0) < 0$ to $f(T) > 0$ such that there is an unique point, say t_1^*, that satisfies

$$f(t_1^*) = 0 \tag{15}$$

Because $f(t_1) < 0$ for $0 \le t_1 < t_1^*$, therefore, $C'(t_1) < 0$, for $0 < t_1 < t_1^*$. Similarly, because $f(t_1) > 0$ for $t_1^* < t_1 \le T$, therefore, $C'(t_1) > 0$, for $t_1^* < t_1 < T$. Hence, $C(t_1)$ decreases for $0 \le t_1 \le t_1^*$ and $C(t_1)$ increases for $t_1^* \le t_1 \le T$ such that t_1^* is the minimum solution. We summarize our findings in the following theorem.

Independent of Demand Theorem: *For the inventory model beginning with stock, where the demand is a positive function, with a generalized deterioration and the partial backlog inversely proportional to the waiting time, the minimum solution satisfies the condition* $f(t_1^*) = 0$ *and is independent of the demand.*

4. NUMERICAL EXAMPLES

We provide three numerical examples to demonstrate our inventory model and the solution procedures with the following data: the demand $R(t) = 20$ for $0 \leq t \leq 1$; the deterioration rate $\theta(t) = 0.1$ for $0 \leq t \leq 1$; the holding cost $C_h = 1.5$ per item per unit time; the cost $C_d = 2.5$ per item; the shortage cost $C_s = 3.5$ per item per unit time; the opportunity cost $C_l = 4$ per item; the set up cost $A = 20$; and three different values of a, for products with the loyal customers $a = 1$, for items with the ordinary customers $a = 3$, and models with the impatient customers $a = 10$. We list the computation results in the next table 1.

Table 1. The optimal replenishment time and the minimum cost

	a	t_1^*	$C(t_1^*)$
Loyal customers case	1	0.770249	69.095
Ordinary customers case	3	0.855437	70.709
Impatient customers case	10	0.934913	72.085

From our numerical examples, for three different cases, we have provided the optimal replenishment time and minimum cost that will help researchers solve inventory models with deterioration items and partially backlogging.

5. CONCLUSION

We have noticed an important phenomenon for one cycle finite time horizon inventory models where the optimal solution is independent of the demand pattern. Consequently, all the complicated solution procedures in Mandal and Pal [11], Wu and Ouyang [15], Deng et al. [3], and Skouri et al. [10] will be rendered unnecessary, helping researchers reach further insight into these kinds of inventory models.

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