SUDAR SUSJEDNIH NESIMETRIČNIH VIŠESPRATNIH ZGRADA USLED UTICAJA ZEMLJOTRESA

POUNDING OF ADJACENT NON-SYMMETRIC MULTISTORY BUILDINGS DUE TO AN EARTHQUAKE

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1 UVODNE NAPOMENE

Mnogi gradovi u svijetu izgrađeni su po sistemu takozvanih "kontinualnih sistema zgrada" gdje se svaka zgrada obično sa dvije suprotne strane oslanja na susjedne zgrade. U Solunu u Grčkoj postoje čak tri gradska bloka u kojima nema dilatacionih razdjelnica između susjednih zgrada, [3]. Dilatacione razdjelnice između susjednih zgrada su potrebne da bi se: obezbijedio slobodan prostor unutar koga zgrade mogu vibrirati u slučaju pojave zemljotresa i omogućilo nesmpriječeno širenje zgrada usled sezonskih temperaturnih uticaja. Svi savremeni tehnički propisi zahtjevaju dilatacionu razdjelnicu između susjednih zgrada. Međutim, to nije lako primjeniti jer postoji jako protivljenje od strane vlasnika (investitora), izvođača i inženjera, [2]. To je posledica: visoke cijene placeva, težnje da se dobije što veći korisni prostor, a time i što veći profit, teškoće pri izvođenju dilatacionih razdjelnica (izvođačima radova najviše odgovara da postojeći zid susjedne zgrade bude spoljašnja oplata za zid nove zgrade), kao i problemi kasnijeg održavanja razdjelnica.

Da su susjedne zgrade izgrađene u skladu sa postojećim tehničkim propisima njihov sudar ne bi se nikada dogodio, jer one usled zemljotresa načelno vibriraju međusobno nezavisno. Međutim, ako se njihove relevantne dinamičke karakteristike (krutost, masa, prigušenje) bitno razlikuju susjedne zgrade neće vibrirati sinhrono i ako je pri tome dilataciona razdjelnica između

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1 INTRODUCTION

Many towns in the world are built according to the so-called "continuous buildings system" where buildings are virtually connected, usually on both lateral sides, even though the lateral walls for two buildings are not the same. In Thessaloniki, Greece, there are even three city blocks without any seismic separation between buildings, [3]. Dilatation joints between buildings are necessary in order to provide the empty space where building may freely vibrate in the case of an earthquake and also to provide unrestricted thermal dilatation due to seasonal climate conditions. All current building codes require the existence of dilatation joint between neighboring buildings. However, it is not easy to implement sufficient separation distances in evry case, because of a strong opposition of the owners (or investors) and also contractors, [2]. It is due to high prices of construction lots, strong tendency to obtain as much usefull area as possible, and therefore as much profit as possible, and also due to difficulties in practical implementation of dilatation joints (construction engineers much more prefere to use the existing wall of the neighboring building as the outter form for the new wall) and also latter maintenance of the expansion joint.

If the two adjacent building were built strictly according to technical building codes, the pounding between buildings would have never happened: they would vibrate during earthquakes independently from each other. If the

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njih nedovoljnih dimenzija može doći do njihovog sudara. Iskustva stečena poslije mnogih većih zemljotresa pokazuju da se sudar susjednih zgrada izloženih ovoj prirodnoj pojavi zaista događa, [3], [5] i [13]. Poslije rušenja stepenišnog tornja bolnice "Olive View Hospital" u Kaliforniji usled sudara sa glavnom zgradom bolnice za vrijeme zemljotresa "San Fernando" 1971.godine praktično i započinje razmatranje problema sudara zgrada, [5]. Intezivnije se počinje raditi na ovom problemu poslije zemljotresa u Mexico City 1985.godine kada se srušio neuobičajeno veliki broj zgrada, a u nekim slučajevima glavni uzrok rušenja je bio sudar susjednih zgrada, [13]. Od 330 višespratnih zgrada koje su bile znatno oštećene ili se srušile za vrijeme tog zemljotresa, sudar sa susjednim objektima desio se u preko 40% slučajeva, dok je 15% od svih slučajeva doveo do rušenja, [2].

Analiza mogućeg sudara zgrada za vrijeme zemljotresa je složena i nedovoljno ispitana oblast primjenjene mehanike. Numerička i analitička istarživanja tog problema su relativno rijetka. U osnovi postoje dva pristupa u analizi. Prvi se zasniva na uvođenju posebnih linearno viskozno - elastičnih "udarnih" elemenata između dvije susjedne zgrade koji se aktiviraju tek po ostvarivanju kontakta između dvije vibrirajuće mase, [1], [4] i [15]. Pri tome se krutosti ovakvih udarnih elemenata usvajaju u relativno velikom iznosu (znatno većem od krutosti zgrada), čime se simuliraju udarne sile, dok je konstanta viskoznog prigušenja određena prema procjeni disipacije energije tokom sudara, dovođenjem u korelaciju sa koeficijentom sudara. U radu [15] zgrade su tretirane kao ekvivalentni sistemi sa po jednim stepenom slobode, dok je u radovima [1] i [4] dat napredniji pristup gdje su zgrade tretirane kao sistemi sa više stepeni slobode. U drugom pristupu uslovi ostvarivanja sudara između pojedinih dijelova zgrada se nameću kao ograničenja jednačinama kretanja zgrada primjenom metode Lagranžeovih multiplikatora veza, [14]. U svim ovim radovima zgrade su tretirane kao simetrični sistemi, gdje svaka tavanica vrši samo translatorno kretanje sa jednim stepenom slobode.

U ovom radu prikazana je analiza mogućeg sudara višespratnih nesimetričnih zgrada usled dejstva zemljotresa. Zgrade su tretirane kao trodimenzionalni sistemi, gdje svaka tavanice vrši ravno kretanje u svojoj horizontalnoj ravni sa po tri stepena slobode (translacije u i v i rotacija j), kao što je prikazano u radu [12]. Dakle, zgrada sa N spratova ima 3N stepeni slobode kretanja. Korišćen je pristup koji je u osnovi radova [6] i [7], gdje je analiziran mogući sudar jednospratnih nesimetričnih susjednih zgrada. Mogući sudar zgrada analiziran je kombinacijom direktne numeričke integracije u vremenskom domenu korak po korak i klasične analize sudara dva kruta tijela pri ravnom kretanju.

relevant building's dynamic properties (stiffness, mass and damping) are substantially mutually different, then adjacent buildings will not vibrate in phase (in sinhronous fashion), and if the separation distance is insufficient, then the pounding may occur. Experiences from many major earthquakes show that pounding of neighboring buildings is really happening, see [3], [5] or [13]. After the collapse of the staircase tower of the "Olive's View Hospital", due to collisions of the tower with the main building of the hospital during the San Fernando earthquake in 1971, the analysis of the pounding of buildings, as a subset of earthquake engineering, practically started, [5]. More intensive analysis of pounding started after the Mexico City earthquake in 1985, where the rather substanial number of buildings was heavily damaged or crushed down and in some cases the main reason for demolition was attributed to pounding, [13]. Out of about 330 relatively multi-story buildings that were heavily damaged or demolished during that earthquake, pounding with neighboring buildings occured in more then 40% of cases, while in 15% of demolition the pounding was direct cause, [2].

Analysis of the possible pounding of buildings during earthquake is a rather complicated and still unexplored field of the applied mechanics. Numerical and analytical investigations of that problem are relatively rare. In the essencee there are two main approaches to that problem. One is based on the introduction of the special linear visco-elastic impact elements between two adjacent buildings, that are being activated after the contact of the two vibrating masses, [1], [4], [15]. The stiffness of such impact elements are assumed as relatively high (much higher then the stiffness of the buildings), so the impact forces are being simulated, while the viscous damping is estimated according to evaluation of dissipation of energy during collision, by assuming some correlation with the impact coefficient. In [15] buildings are treated as the equivalent single-degree-of-freedom systems, while in [1] and [4] the buildings are considered as multy-degree-of-freedom systems. In the other approach the conditions of the contact-impact problem between certain parts of neighboring buildings are imposed as restrictions of the differential equations of motion by the Lagrange multiplier method, [14]. In all of this papers buildings are treated as symmetric systems, where each slab is performing only a translation with one degree of freedom.

This paper is presenting the analysis of the possible pounding of multi-story non-symmetric buildings in the event of an earthquake. Buildings are treated as trodimensional systems, where each floor slab is performing the planar motion in its horizontal plane, with three degrees of freedom each (translations u and v and rotation φ), as shown in [12]. Therefore, a building with N stories has 3N degrees of freedom. The approach used here is based upon the approach presented in [6] and [7], where the possible pounding of single-story non-symmetric buildings was analyzed. The possible pounding is analyzed by the combination of direct numerical integration in the time domain step-by-step and the classical impact analysis of the two rigid bodies in planar motion.

2 MOGUĆI SUDAR TAVANICA ISTOG NIVOA SUSJEDNIH ZGRADA

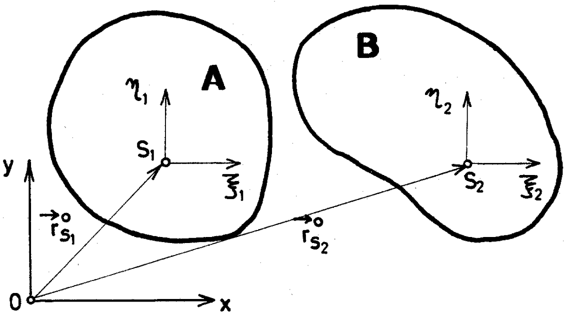
Usled uticaja zemljotresa susjedne zgrade istih spratnih visina, izgrađene u skladu sa postojećim tehničkim propisima, načelno vibriraju nezavisno. Međutim, ako se njihove relevantne dinamičke karakteristike bitno razlikuju i ako je dilataciona razdjelnica između njih nedovoljnih dimenzija može da dođe sudara tavanica susjednih zgrada na istom visinskom nivou.

Na slici 1 prikazane su tavanice A i B istog nivoa susjednih nesimetričnih zgrada. Za opisivanje položaja tavanica tokom njihovog ravnog kretanja usvojeni su referentni (globalni) koordinatni sistem Oxy i materijalni (lokalni) koordinatni sistemi $S_1\xi_1\eta_1$ i $S_2\xi_2\eta_2$, postavljeni u centru masa posmatranih tavanica. U početnoj konfiguraciji (prije pojave zemljotresa) lokalni sistemi su paralelni sa globalnim sistemom.

2 POSSIBLE IMPACT OF SLABS AT THE SAME LEVEL OD ADJACENT BUILDINGS

In the case of an earthquake, neighboring buildings with the same story heights, built in accordance to technical building codes, will vibrate independently. However, if their relevant dynamic characteristics are substantiallly different and if the separation distance between them is unsufficient, then it is quite possible that pounding between slabs at the same level will occur.

Fig. 1 is presenting two arbitrary slabs A and B at the same level of the two neighboring non-symmetric buildings. In order to describe slab positions during their planar motion the common global (inertial) coordinate system Oxy is adopted, as well as the two material (or local) coordinate systems $S_1\xi_1\eta_1$ and $S_2\xi_2\eta_2$, assumed in the center of mass of each slab. In the initial configuration (prior to the earthquake), local systems are parallel with the global one.



Slika 1. Tavanice istog nivoa susjednih nesimetričnih zgrada u početnoj konfiguraciji
Figure 1. Slabs at the same level of adjacent non-symmetric buildings in the initial configuration

Tavanice A i B, kao posledica uticaja zemljotresa, usled nastalog ravanskog kretanja svake tavanice, u nekom trenutku vremena mogu da zauzmu jedan od sledećih međusobnih položaja: da nisu u kontaktu, da se dodiruju u jednoj tački i da se preklapaju. U prvom slučaju nema sudara tavanica. Ako je ostvaren kontakt između tavanica u jednoj tački to ne znači da je došlo do njihovog sudara, jer je ostvarivanje kontakta u jednoj tački samo potreban ali ne i dovoljan uslov sudara tavanica. Moguće je da su tavanice tokom kretanja ostvarile kontakt u jednoj tački, ali tako da su pri tome brzine tačaka dodira jedne i druge tavanice jednake nuli ili su takvih smjerova koji ukazuju na međusobno razdvajanje tavanica u sledećem trenutku, pa zbog toga nema sudara tavanica, već je samo ostvaren kontakt. Ukoliko je došlo do preklapanja tavanica onda to znači da se sudar dogodio u nekom ranijem trenutku vremena koji treba odrediti.

2.1 Uslovi sudara tavanica

Potrebni i dovoljni uslovi sudara tavanica u nekoj tački Q tokom njihovog ravnog kretanja formulisani su kao:

- uslov sudara po položaju: ostvaren kontakt tavanica u jednoj tački,

As a consequence of an earthquake, due to sudden beginning of the planar motion of slabs, at any instant of time the slabs A and B may occupy any of the following mutual positions: to be without any contact, to have a contact at a single point or to overlap with certain areas. In the first case there is obviously no impact between slabs. If the contact at a single point of the two slabs is established, it does not necessarily mean the the impact has occured, because a contact at a point is just the necessary, but not the sufficient condition of an impact. Namely, it is possible that the contact at a point has occured, but in such a way that the velocities of that common point of both slabs are either equal to zero, or with such senses as to indicate the separation of slabs at the next instance, so there is no impact, only connection. If the slabe are overlapping, it means that the impact has already occured in some previous instant of time that has to be determined.

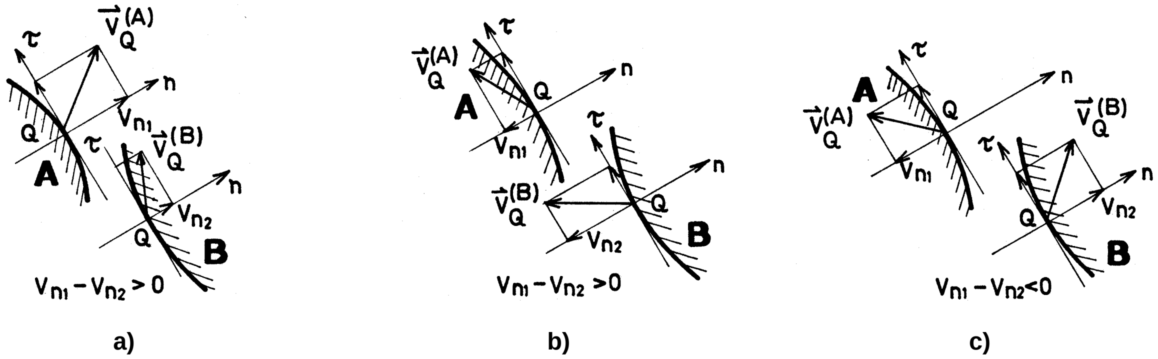
2.1 The conditions of impact of two slabs

The necessary and sufficient conditions of impact of the two slabs at some point Q, during their planar motion are formulated as:

- the position condition of impact: the contact of slabs is established at a single point,

- uslov sudara po brzinama: razlika projekcija brzina jedne i druge tavanice u tački ostvarenog kontakta u pravcu normale na konturu u tački dodira treba da ukazuje na tendenciju preklapanja tavanica u sledećem trenutku vremena, slika 2 (a i b - tendencija ka preklapanju tavanica, c - tendencija ka razdvajanju tavanica u sledećem trenutku vremena).

- the velocity condition of impact: the difference of projections of velocities of both slabs at the point of contact must be such to indicate the tendency of overlapping of slabs at the next instant of time, Fig. 2 (a,b - tendency to overlapping of slabs, c - tendency of separation of slabs at the next instant of time).



a) **b)** **c)**

Slika 2. Uslov sudara izražen po brzinama
Figure 2. The velocity condition of impact

Ako se sa $\vec{r}_Q^{(A)}$ i $\vec{r}_Q^{(B)}$, odnosno sa $\vec{v}_Q^{(A)}$ i $\vec{v}_Q^{(B)}$, označe vektori položaja i brzine tačaka Q tavanica A i B, koji se posmatraju u odnosu na isti prostorni koordinatni sistem i ako se sa $\vec{n}$ označi ort spoljašne normale na konturu jedne od tavanica onda se uslovi sudara tavanica, u vektorskom obliku, mogu prikazati kao:

If the vectors $\vec{r}_Q^{(A)}$ and $\vec{r}_Q^{(B)}$, and also $\vec{v}_Q^{(A)}$ and $\vec{v}_Q^{(B)}$, denote the position vectors and the velocity vectors of the point Q of both slabs A and B, expressed with respect to the same inertial (global) coordinate system and if $\vec{n}$ denotes the unit vector of the outward normal with reference to the contour of one of the slabs, then the conditions of impact of slabs may be expressed in the vector form as:

$$\vec{r}_Q^{(A)} - \vec{r}_Q^{(B)} = 0 \tag{1}$$

$$(\vec{v}_Q^{(A)} - \vec{v}_Q^{(B)})\vec{n} > 0 \tag{2}$$

odnosno, u skalarnom obliku kao:

or, in the scalar form as:

$$x_Q^{(A)} - x_Q^{(B)} = 0 \qquad y_Q^{(A)} - y_Q^{(B)} = 0 \tag{3}$$

$$v_{n1} - v_{n2} > 0 \tag{4}$$

Ako je $\vec{n}$ ort spoljašnje normale na konturu tavanice A u tački Q onda je:

If $\vec{n}$ is the ort of the outward normal with respect to the contour of slab A at the point Q, then

$$v_{n1} = \vec{v}_Q^{(A)} \cdot \vec{n} \quad \text{i} \quad v_{n2} = \vec{v}_Q^{(B)} \cdot \vec{n} \tag{5}$$

a ako je ort $\vec{n}$ definisan kao spoljašnja normala za konturu tavanice B u tački Q, onda se v_{n1} odnosi na tavanicu B, a v_{n2} na tavanicu A:

and also, if $\vec{n}$ is defined as the outer normal for the contour of the slab B at the point Q, then v_{n1} is referring to slab B and v_{n2} to slab A:

$$v_{n1} = \vec{v}_Q^{(B)} \cdot \vec{n} \quad \text{i} \quad v_{n2} = \vec{v}_Q^{(A)} \cdot \vec{n} \tag{6}$$

Vektor brzine neke tačke P tavanice proizvoljnog oblika, čije su materijalne koordinate date sa P(ξ_P,η_P), po definiciji je:

The velocity vector of some point P of an arbitrary slab, whose material coordinates are given by P(ξ_P,η_P), is given as:

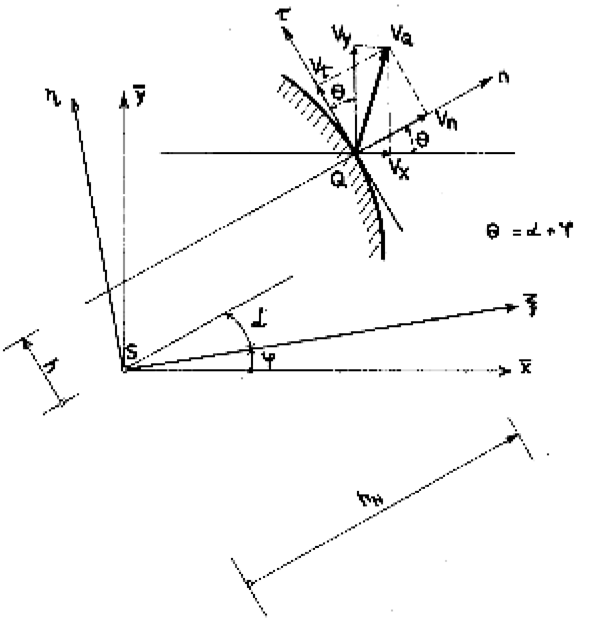
$$\vec{v}_P=\dot{\vec{r}}_P=\begin{Bmatrix}v_x\\ v_y\end{Bmatrix}=\begin{Bmatrix}\dot{u}\\ \dot{v}\end{Bmatrix}+\dot{\varphi}\begin{bmatrix}-\varphi & -1\\ -1 & \varphi\end{bmatrix}\begin{Bmatrix}\xi_P\\ \eta_P\end{Bmatrix} \tag{7}$$

pri čemu su uzete u obzir sledeće aproksimacije: $\cos\varphi\approx 1$ i $\sin\varphi\approx\varphi$, imajući u vidu da su pomjeranja *u* i *v* i obrtanje tavanice φ male vrijednosti, dok je $\vec{r}_P$ vektor položaja tačke P prikazan u radu [12].

Kako je tačka kontakta Q tačka na konturi tavanice, potrebno je odrediti brzine ove tačke u pravcima normale i tangente na konturnu liniju tavanice, kao što je prikazano na slici 3.

In Eq.(7) the approximations $\cos\varphi\approx 1$ and $\sin\varphi\approx\varphi$ are taken into account, having in mind that displacements *u* and *v* and slab rotation φ are small values, while $\vec{r}_P$ is the position vector of point P shown in [12].

Since the point of contact Q is located on the contour of both slabs, it is necessary to determine the velocity components of that point with respect to the normal and tangent unit vectors defined for the contour of the slab, as presented in Fig. 3.



Slika 3. Vektor brzine u tački kontakta Q
Figure 3. Velocity vector of the point of contact Q

Ako je θ ugao između normale n i prostorne ose x onda su komponente brzine u odnosu na pravce normale i tangente na konturu tavanice u tački Q date sa:

If θ is the angle between the normal n and the global axis x, then the components of velocity vector with respect to normal and the tangent to the contour of the slab at the point Q are given by:

$$\vec{v}_Q=\begin{Bmatrix}v_n\\ v_\tau\end{Bmatrix}=\begin{bmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{bmatrix}\begin{Bmatrix}v_x\\ v_y\end{Bmatrix} \tag{8}$$

Kao što se vidi na slici 3, ugao θ jednak je zbiru uglova α i φ, pri čemu je α ugao između materijalne ose ξ i normale n i on je konstantan za datu konturnu tačku, dok je φ ugao između referentne ose x i materijalne ose ξ, tj. ugao rotacije tavanice.

Kada se u relaciju (8) unesu poznate komponente brzina tačke Q u odnosu na prostorni sistem date sa relacijom (7) dobija se:

As may be seen in Fig. 3, the angle θ is equal to the sum of angles α and φ, where α is the angle between the local (material) axis ξ and the normal n and for the given point Q it has a constant value, while φ is the angle between the global axis x and the local (material) axis ξ, i.e. it represents the angle of rotation of the slab.

When one inserts the known components of velocity of the point Q, expressed with reference to the global system and given by Eq.(7), into relation (8), one obtains:

$$\vec{v}_Q = \begin{Bmatrix} v_n \\ v_\tau \end{Bmatrix} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \dot{y} \end{Bmatrix} + \dot{\varphi} \begin{Bmatrix} -h \\ h_N \end{Bmatrix} \tag{9}$$

gdje je:

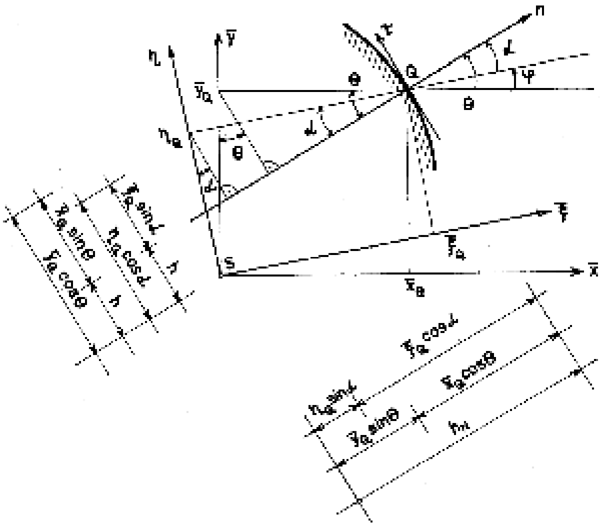
where:

$$h = -\xi_Q \sin\alpha + \eta_Q \cos\alpha \tag{10}$$

$$h_N = \xi_Q \cos\alpha + \eta_Q \sin\alpha \tag{11}$$

Na slici 4 prikazane su veličine h i h_N koje predstavljaju projekcije duži SQ na ose τ i n.

Fig. 4 presents the quantities h and h_N which represent the projections of the segment SQ on the axes τ and n.



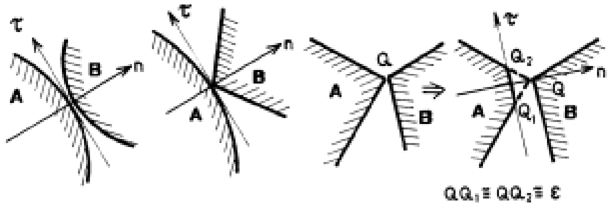
Slika 4. Geometrija konture tačke Q
Figure 4. Geometry of the contour point Q

U tački kontakta mora biti definisan, barem za jednu od tavanica, pravac normale na konturu da bi se mogle odrediti komponente brzine tačke kontakta u pravcu normale na konturu tavanice. Postoje razne situacije vezane za određivanje pravca normale na konturu u tački kontakta i prikazane su na slici 5. Tavanice mogu biti definisane kao glatke zatvorene konture ili kao poligonalne oblasti. Ako su obje tavanice sa glatkim konturama onda su tangenta i normala jednoznačno definisane za obje konture, slika 5a. Za tavanice poligonalnog oblika su moguće dvije situacije. Prva situacija je da tjeme jedne od tavanica bude u kontaktu sa glatkim dijelom konture druge tavanice, slika 5b. U ovom slučaju tangenta i normala su definisane u tački kontakta Q u odnosu na tavanicu sa glatkom konturom. Druga situacija je slučaj kontakta dva tjemena poligonalnih kontura tavanica, slika 5c. U ovom slučaju u tački kontakta nisu definisane tangenta i normala ni za jednu od tavanica. Tada se, kao inženjersko rješenje, tačka Q jedne od tavanica zamijeni sa dvije bliske tačke Q_1 i Q_2 koje su u maloj, odnosno ε okolini tačke Q na susjednim stranicama konture tavanice tj. $\overline{QQ_1} \cong \overline{QQ_2} \cong \varepsilon$, slika 5d. U ovom slučaju pravac tangente određen je pravcem Q_1Q_2, pri čemu se usvaja smjer tangente u obilasku oko tavanice u smjeru

At the point of contact, at least for one of the slabs, direction of the outward normal n with respect ot the contour must be defined in order to be able to determine the velocity components for the point of contact. There might be various situations related to determination of the direction of the outward normal, as presented in Fig. 5. The slabs may be difined as the smooth closed curves, or as the closed polygonal areas. If both slabs are with smooth contours, then the tangent and normal lines are uniquely defined for both contours, Fig. 5a. For the polygonal slabs two situations are possible. The first one is when the corner point of one slab is in a contact with the smooth part of the other slab, Fig. 5b. In this case the tangent and tne normal are defined in the point of contact with reference to the smooth surface. The second situation is the contact of two corner points of polygonal slabs Fig. 5c. In this case, the normal and the tangent lines at the contact point are not defined for neither of the slabs. As the engineering approach, the corner point Q for one of the slabs is substituted with two close points Q_1 and Q_2 which are in some small, or ε surrounding of the point Q on neighboring sides of the contour, i.e. $\overline{QQ_1} \cong \overline{QQ_2} \cong e$, Fig. 5d. In this case, the tangent is defined by direction Q_1Q_2, with the sense adopted according to the counter-clock-wise direction.

suprotnom od kazaljke na satu. Ovo “odbacivanje” malog dijela tavanice moglo bi da se shvati kao deformisanje tjemena jedne od tavanica usled sudara.

This "cutting-off" of the small part of the slab might be considered as the deformation of the corner point of one slab due to an impact.



a) b) c) d)

Slika 5. Tangenta i normala na konturu u tački sudara
Figure 5. Tangent and normal lines defined for the contour of the slab at the point of impact

2.2 Analiza sudara dvije tavanice

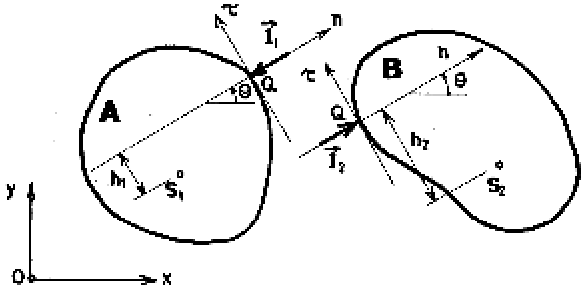
Tokom sudara, koji se ostvaruje unutar beskonačno kratkog intervala vremena, smatra se, a u skladu sa pristupom u klasičnoj mehanici, da nema pomjeranja ploča koje se nalaze u sudaru već samo da dolazi do nagle promjene njihovih brzina. U tački sudara javljaju se kao unutrašnje sile veze između ploča odgovarajuće udarne sile, koje su beskonačno velikog inteziteta i traju beskonačno kratko, jer one postoje samo tokom sudara, pa je samo njihov impuls konačna veličina. Za razliku od toga, impulsi svih ostalih “neudarnih sila” su beskonačno male veličine zbog njihovih konačnih intenziteta, a beskonačno kratkog vremenskog intervala. Eventualno trenje tokom sudara se zanemaruje, pa zbog toga udarne sile između ploča imaju pravac koji se poklapa sa normalom na konturu ploča, a suprotnog su smjera i istih intenziteta u skladu sa aksiomom akcije i reakcije.

Na slici 6 prikazane su razdvojene tavanice masa m_1 i m_2 tokom sudara i njihovi unutrašnji udarni impulsi $\vec{I}_1$ i $\vec{I}_2$ koji djeluju u pravcu normale $\vec{n}$, definisane u odnosu na jednu od tavanica, u tački sudara Q. Udarni impulsi su usmjereni uvijek tako da to odgovara pritisku na ploče, jer su u pitanju reakcije jednostranih (nezadržavajućih) veza, pa je $\vec{I}_1 = -I\vec{n}$, a $\vec{I}_2 = I\vec{n}$.

2.2 Impact analysis of two slabs

Duration of the impact is assumed to be infinitely small, according to the classical rigid body approach, and also, the main assumption is that there is no displacement during impact, only the sudden change in the velocity field. At the point of impact there are two internal impact forces between slabs, whose intensities are infinitely large, but their duration is infinitely small, because impact forces exist only during the impact, so only their impuls has the finite value. As opposed to that, impulses of all other "non-impact" forces are infinitelly small, due to their finite intensities and infinitelly small time interval. The possible friction forces during the impact are neglected, so the impact forces between slabs have the known direction coinciding with the normal to the contour line of the slabs and they are of the opposite senses and equal intensities due to the Law of Action and Reaction.

Fig. 6 presents two separated slabs, with masses m_1 and m_2, during the impact and the corresponding internal impact impulses $\vec{I}_1$ and $\vec{I}_2$ whose line of action is the normal $\vec{n}$ defined with reference to one of the slabs at the point of impact Q. Impact impulses are oriented in such a way that it corresponds to the pressure on the slabs, because impact forces are reaction forces of one-sided restrains, so $\vec{I}_1 = -I\vec{n}$ and $\vec{I}_2 = I\vec{n}$.



Slika 6. Razdvojene tavanice u toku sudara
Figure 6. Separated slabs during the impact

Sam proces sudara opisuje se pomoću zakona o promjeni količine kretanja i o promjeni momenta količine kretanja u konačnom obliku, koji su napisani za svaku razdvojenu tavanicu posebno (i=1, 2), neposredno poslije i neposredno prije sudara:

The process of impact is described by the Law of momentum and the Law of the moment of momentum in the finite forms, written for each separated slab (i=1,2), immediatelly after and immediately before the impact:

$$\vec{K}_i'' - \vec{K}_i' = \vec{I}_i \qquad (12)$$

$$\vec{D}_i'' - \vec{D}_i' = \vec{H}_i = \vec{r}_i \times \bar{I}_i \qquad \Rightarrow \qquad D_i'' - D_i' = H_i \qquad (13)$$

gdje su:

– $\vec{K}_i = m_i \cdot \vec{v}_i$ vektori količine kretanja tavanice (m$_i$ je masa, a $\vec{v}_i$ brzina centra mase tavanice i),

– $D_i = J_{\zeta_i} \cdot \dot{\varphi}_i$ vektori momenta količine kretanja tavanice (komponente upravne na ravan tavanice), pri čemu je J_{ζ_i} centralni momenat inercije mase za osu upravno na tavanicu, a $\dot{\varphi}_i$ ugaona brzina rotacije tavanice oko vertikalne ose,

– $\vec{I}_i$ udarni impulsi između tavanica,

– H$_i$ impulsni momenti u odnosu na centre masa tavanica, tj. u odnosu na centralne ose upravne na tavanice ζ_i.

U relacijama (12) i (13) sa $(...)'$ su označene sve veličine neposredno prije sudara, a sa $(...)''$ sve veličine neposredno poslije sudara.

Ako se navedeni zakoni (12) i (13) napišu za svaku od razdvojenih tavanica u skalarnom obliku u odnosu na inercijalni sistem Oxy, dobija se:

where:

– $\vec{K}_i = m_i \cdot \vec{v}_i$ are the vectors of the momentum of the slab i (m$_i$ is the mass, and $\vec{v}_i$ is the velocity of the center of mass of the slab i),

– $D_i = J_{\zeta_i} \cdot \dot{\varphi}_i$ are the vectors of the moment of momentum of the slab i (components that are perpendicular to slabs), while J_{ζ_i} is the central mass moment of inertia for the axis perpendicular to slab, and $\dot{\varphi}_i$ is the angular velocity of rotation of the slab around the vertical axis,

– $\vec{I}_i$ are the impact impulses between slabs,

– H$_i$ are the impuls moments with respect to the centers of mass of slabs, i.e. with respect to the central axes perpendicular to slabs ζ_i.

In relations (12) and (13) the single prime $(...)'$ denotes a quantity immediately before the impact, while the double prime $(...)''$ denoter a quantity immediately after the impact.

If the Laws (12) and (13) are written for each of the separated slabs in the scalar form, with respect to the global system Oxy, one obtains:

Tavanica (A):

Slab (A):

$$m_1\dot{x}_1'' - m_1\dot{x}_1' = -I\cos\theta \qquad (14)$$

$$m_1\dot{y}_1'' - m_1\dot{y}_1' = -I\sin\theta \qquad (15)$$

$$J_{\zeta 1}\dot{\varphi}_1'' - J_{\zeta 1}\dot{\varphi}_1' = I\,h_1 \qquad (16)$$

Tavanica (B):

Slab (B):

$$m_2\dot{x}_2'' - m_2\dot{x}_2' = I\cos\theta \qquad (17)$$

$$m_2\dot{y}_2'' - m_2\dot{y}_2' = I\sin\theta \qquad (18)$$

$$J_{\zeta 2}\dot{\varphi}_2'' - J_{\zeta 2}\dot{\varphi}_2' = -I\,h_2 \qquad (19)$$

Sa h$_i$ (i=1, 2) označeno je rastojanje pravca normale u tački Q do odgovarajućeg centra mase koje je dato relacijom (10).

U ovih šest jednačina (14)-(19), osim šest nepoznatih generalisanih brzina neposredno poslije sudara, postoji i sedma nepoznata veličina, a to je unutrašnji udarni impuls I. Da bi se riješio ovaj sistem jednačina, uvodi se koeficijent sudara (ili koeficijent restitucije) k definisan kao:

Symbols h$_i$ (i=1, 2) denote the distance of the normal n in the point Q to the corresponding center of mass, as given by the expression (10).

Presented six equations (14)-(19), besides the six unknown generalized velocities immediately after the impact, contain also the seventh unknown quantity, which is the internal impact impulse I. In order to solve this system of equations, the coefficient of impact (or the coeffcient of restitution) k is introduced as:

$$k = \frac{\left| v''_{n2} - v''_{n1} \right|}{\left| v'_{n1} - v'_{n2} \right|} \qquad k \in [0,1] \tag{20}$$

gdje su v'_{ni} i v''_{ni} (i=1, 2) komponente brzina tačke sudara Q u pravcu normale jedne i druge tavanice neposredno prije odnosno neposredno poslije sudara, date sa relacijom (9) kao:

where v'_{ni} and v''_{ni} (i=1, 2) are the velocity components of the point of impact Q of both slabs in direction of the normal, immediately before and immediately after the impact, given by (9) as:

$$v'_{ni} = \dot{u}'_i \cos\theta + \dot{v}'_i \sin\theta - \dot{\varphi}'_i h_i \qquad (i = 1, 2) \tag{21}$$

$$v''_{ni} = \dot{u}''_i \cos\theta + \dot{v}''_i \sin\theta - \dot{\varphi}''_i h \qquad (i = 1, 2) \tag{22}$$

Koeficijent sudara je realan broj u intervalu [0, 1]. Ukoliko je k=1, onda je u pitanju idealno elastičan sudar tokom kojeg nema gubitka u ukupnoj kinetičkoj energiji. Slučaj k=0 predstavlja idealno plastičan sudar kod koga je ostvaren najveći gubitak u ukupnoj kinetičkoj energiji koji je utrošen na plastično deformisanje materijala obje tavanice.

Ako se u relaciju (20) unesu izrazi (21) i (22), a zatim se nepoznate generalisane brzine tavanica neposredno poslije sudara izraze preko nepoznatog impulsa iz relacija (14)-(19), dobija se sledeća jednačina u kojoj figuriše samo nepoznati udarni impuls:

The coefficient of impact is the real number in the interval [0,1]. The case of k=1 represents the ideally elastic impact where there is no global loss of kinetic energy. The case k=0 represents the ideally plastic impact with the largest loss in the total kinetic energy which is spent on the plastic deformation of the material of both slabs.

If the expressions (21) and (22) are introduced into relation (20), and if the unknown generalized velocities immediately after the impact are expressed through the unkonown impuls, using equations (14)-(19), one obtains the following equation with the impact impuls as the only unknown:

$$\left| aI - b \right| = k \left| b \right| \tag{23}$$

gdje su:

where the coefficients a and b are given by:

$$a = \sum_{i=1}^{2} \left(\frac{1}{m_i} + \frac{h_i^2}{J_{\zeta i}} \right) \tag{24}$$

$$b = v'_{n1} - v'_{n2} \tag{25}$$

Može da se konstatuje da su koeficijenti a i b dati sa (24) i (25) pozitivni realni brojevi. To je očigledno za koeficijent a koji predstavlja zbir pozitivnih brojeva, a koeficijent b je pozitivan jer predstavlja uslov sudara tavanica po brzinama, dat sa (4).

Nepoznati unutrašnji impuls I, kao rješenje jednačine (23), može se prikazati u obliku:

It could be established that the coefficients a and b, given by (24) and (25), are positive real numbers. It is quite obvious for the coefficient a which is the sum of positive numbers, and the coefficient b is positive because it represents the velocity condition of impact, given by (4).

The unknown internal impuls I, as the solution of equation (23), may be presented in the form:

$$I = (1 + k) \frac{b}{a} \tag{26}$$

Unošenjem izraza (26) u izraze (14)-(19) dobijaju se generalisane brzine za obje tavanice neposredno poslije sudara.

Formalno bi postojalo i rješenje jednačine (23) u obliku :

Introducing the expression (26) into (14)-(19) one obtains the generalized velocities for both slabs immediately after the impact.

The formal solution of Eq. (23) may be also presented in the form:

$$I = (1 - k) \frac{b}{a} \tag{27}$$

Međutim, u ovom slučaju za idealno elastičan sudar (k=1) imuls bi bio jednak nuli, pa bi tada generalisane brzine za obje tavanice neposredno poslije sudara ostale iste kao i neposredno prije sudara, što nema fizičkog smisla.

However, in this case for the ideally elastic impact (k=1) the impuls would be equal to zero, so in that case the generalized velocities of both slabs immediately after the impact would be the same as immediately before the impace, which does not have the physical sense.

3 ANALIZA MOGUĆEG SUDARA SUSJEDNIH ZGRADA USLED ZEMLJOTRESA

Iako, u toku zemljotresa dolazi do naglog, potpuno nepravilnog oscilovanja površinskih slojeva zemljine kore, njegov uticaj na zgradu, zbog relativno malih dimenzija osnove zgrade u odnosu na talasnu dužinu seizmičkih talasa u tlu, se posmatra kao prinudno kretanje temelja zgrade u horizontalnoj ravni. Metode analize zgrada usled dejstva zemljotresa, zavise od vrste primjenjene analize (statičke ili dinamičke) i usvojenog modela konstrukcije (linearnog ili nelinearnog), prikazane su u radovima [8] i [10]. Primjenom nelinearne dinamičke analize, koja je suviše komplikovana za praktičnu primjenu, dobijaju se najtačniji rezultati. U radu [9], primjenom nelinearne dinamičke analize, je analizirano ponašanje konstrukcije na dejstvo standardnih i impulsnih zemljotresa. Za razliku od standardnih (tipičnih) zemljotresa gdje dolazi do kretanja tla koje je slično oscilacijama, odnosno do oscilacija zgrade, kod impulsnih zemljotresa dolazi do iznenadnog i veoma velikog unosa energije u konstrukciju zbog naglog trzaja podloge.

U ovom radu kao osnova, za analizu mogućeg sudara višespratnih nesimetričnih zgrada u uslovima zemljotresa, poslužio je matematički model uticaja zemljotresa na jednu višespratnu nesimetričnu zgradu (preko zadatog akcelerograma standardnog zemljotresa) prikazan u radu [12].

Sada se posmatraju dvije susjedne višespratne nesimetrične zgrade sa različitim brojem spratova, N_1 odnosno N_2. Zgrade su istih spratnih visina i izložene su istoj seizmičkoj pobudi koja je definisana sa akcelerogramom $\ddot{u}_g(t)$ i sa dominantnim pravcem pod uglom β, koji se mjeri u horizontalnoj ravni u odnosu na osu x usvojenog globalnog koordinatnog sistema.

Diferencijalne jednačine kretanja zgrada date su kao:

3 ANALYSIS OF THE POSSIBLE IMPACT OF NEIGHBORING SLABS DURING EARTHQUAKE

Even though during earthquakes a sudden and completely nonregular vibrations of the surface layers of the Earth's crust occure, its influence upon buildings, due to relatively small dimensions in building's plan with respect to the characterisctic wave length of seismic waves in soil, is considered as the imposed motion of building's foundation in the horizontal plane. The methods of analysis of buildings during earthquakes depend upon the applied approach (static or dynamic) and adopted numerical model of a building (linear or non-linear), are presented in papers [8] and [10]. Applying the non-linear dynamic analysis, which is too complicated for practical everyday's engineering use, one obtains the most accurate results. Applying the non-linear dynamic analysis, paper [9] is considering behaviour of structures under the influence of the standard and impulsive earthquakes. As opposed to the standard (typical) earthquakes, where the soil motion dominantly looks like vibration, so the vibrations of buildings are happening as a consequence, during impulsive earthquakes, due to the sudden jerk or impulse at the supporting soil, the sudden and large input of energy into the structure occurs.

The present analysis of the possible pounding of multistory non-symmetric buildings during earthquakes is based upon the numerical model of dynamic analysis of the multistory non-symmetric building exposed to an earthquake, defined by the given accelerogram of the standard earthquake, as presented in [12].

Now, two neighboring multistory non-symmetric buildings, with different numbers of stories N_1 and N_2 are considered. It is assumed that story heights of both buildings are the same and that both buildings are exposed to the same earthquake excitation, defined by the accelerogram $\ddot{u}_g(t)$ and the dominant direction defined by the angle β measured in the horizontal plane with respect of the axis x of the global coordinate system.

Differential equations of motion of both buildings are given by:

$$\mathbf{M}_1\ddot{\boldsymbol{\delta}}_1+\mathbf{C}_1\dot{\boldsymbol{\delta}}_1+\mathbf{K}_1\boldsymbol{\delta}_1=-\mathbf{M}_1\mathbf{b}_1\ddot{u}_g(t)=\mathbf{g}_1(t) \tag{28}$$

$$\mathbf{M}_2\ddot{\boldsymbol{\delta}}_2+\mathbf{C}_2\dot{\boldsymbol{\delta}}_2+\mathbf{K}_2\boldsymbol{\delta}_2=-\mathbf{M}_2\mathbf{b}_2\ddot{u}_g(t)=\mathbf{g}_2(t) \tag{29}$$

gdje su $\mathbf{M}_i$, $\mathbf{K}_i$ i $\mathbf{C}_i$ matrice masa, krutosti i prigušenja, dok d_i i $\mathbf{g}_i$ predstavljaju vektor generalisanih pomjeranja i vektor opterećenja za pojedine zgrade. Ove jednačine su detaljnije prikazane u radu [12].

Proces analize mogućeg sudara zgrada započinje se simultanim rješavanjem jednačina kretanja za obje zgrade koristeći α postupak direktne numeričke integracije. Znači, u svakom intervalu vremena Δt rješava se prvo za jednu, a zatim za drugu zgradu ekvivalentni “statički” problem:

where $\mathbf{M}_i$, $\mathbf{K}_i$ and $\mathbf{C}_i$ represent the matrices of mass, stiffness and damping, while d_i and $\mathbf{g}_i$ represent the vector of generalized displacements and the loading vector for each building. This equations are presented in more details in [12].

The process of analysis of the possible pounding of buildings starts by the simultaneous solution of the equations of motion of both buildings using the α method of direct numerical time integration. It means that within the each time interval Δt the equivalent "static" problem is solved first for one, and then for the other building:

$$\mathbf{K}_i^*\boldsymbol{\delta}_{i,n+1}=\mathbf{g}_{i,n+\alpha}^* \qquad (i=1,2;\ n=1,2,\ldots,n_{t-1}) \tag{30}$$

gdje su $\mathbf{K}_i^*$ efektivne matrice krutosti, a $\mathbf{g}_{i,n+\alpha}^*$ efektivno opterećenje zgrada. Rješavanjem jednačina (30) dobijaju se vektori generalisanih pomjeranja obje zgrade na kraju posmatranog intervala vremena. Sa određenim vektorima generalisanih pomjeranja određuju se vektori brzine i ubrzanja za obje zgrade na kraju posmatranog intervala vremena prema relacijama:

where $\mathbf{K}_i^*$ are the effective stiffness matrices and $\mathbf{g}_{i,n+\alpha}^*$ are the effective loading of buildings. By solving the linear algebraic equations (30) one obtains the vectors of the generalized displacements of both buildings at the end of the considered time interval. With obtained vectors of generalized displacemenst the vectors of generalized velocities and accelerations for both buildings, at the end of considered time interval, are calculated as:

$$\dot{\boldsymbol{\delta}}_{n+1} = \frac{\gamma}{\beta\Delta t}\boldsymbol{\delta}_{n+1} - \frac{\gamma}{\beta\Delta t}\boldsymbol{\delta}_{n} - \left(\frac{\gamma}{\beta} - 1\right)\dot{\boldsymbol{\delta}}_{n} - \left(\frac{1}{2\beta} - 1\right)\ddot{\boldsymbol{\delta}}_{n} \tag{31}$$

$$\ddot{\boldsymbol{\delta}}_{n+1} = \frac{1}{\beta\Delta t^2}\boldsymbol{\delta}_{n+1} - \frac{1}{\beta\Delta t^2}\boldsymbol{\delta}_{n} - \frac{1}{\beta\Delta t}\dot{\boldsymbol{\delta}}_{n} - \left(\frac{1}{2\beta} - 1\right)\ddot{\boldsymbol{\delta}}_{n} \tag{32}$$

Zatim se, od poslednjeg sprata niže zgrade pa sve do prvog sprata, ispituje međusobni položaj tavanica susjednih zgrada koje se nalaze na istom visinskom nivou. To znači da se prema dobijenim vektorima pomjeranja na kraju posmatranog intervala vremena za svaku zgradu određuju prostorne koordinate oblasti u kojima se nalaze tavanice istog nivoa, naravno, izraženo u odnosu na zajednički globalni koordinatni sistem.

Ukoliko, na kraju posmatranog intervala vremena, nijedan par tavanica istog nivoa nije u međusobnom kontaktu nastavlja se sa simultanim rješavanjem jednačina kretanja zgrada za sledeći interval vremena. Pri tome se vektori brzine i ubrzanja obje zgrade na kraju posmatranog intervala vremena tretiraju kao početni vektori brzine i ubrzanja za naredni interval vremena.

Ako se pri provjeri međusobnog položaja tavanica konstatuje da je ostvaren njihov kontakt u jednoj tački, tada treba provjeriti provjeriti da li su tavanice u sudaru, tj. da li je ispunjen i uslov sudara po brzinama. Nakon ovoga treba još provjeriti da li je došlo do sudara između ostalih parova tavanica istih nivoa, jer sudar se može istovremeno ostvariti između više parova tavanica. Tada se svim tavanicama koje su se sudarile, umjesto brzina koje su dobijene prema relaciji (31) nametnu brzine koje su određene prema klasičnoj analizi ekscentričnog sudara dvije krute ploče koje vrše ravno kretanje prema relacijama (14)-(19). Dakle, generalisane brzine tavanica, odnosno brzine centara mase i ugaone brzine tavanica koje su dobijene postupkom numeričke integracije prema relaciji (31) se tretiraju kao brzine neposredno prije sudara, dok se brzine neposredno poslije sudara, koje su određene prema postupku analize sudara prikazanom u dijelu 2, tretiraju kao početne brzine za sledeći interval vremena.

Ukoliko je, tokom provjere međusobnih položaja tavanica istih nivoa, došlo do preklapanja tavanica, to znači da se sudar već bio dogodio unutar posmatranog intervala vremena. U tom slučaju ponovo se posmatra trenutak na početku posmatranog intervala vremena i ponavlja se postupak, ali sada sa upola smanjenim intervalom vremena u odnosu na prethodni, sa ciljem da se iterativno odredi baš onaj trenutak kada je došlo do kontakta tavanica. Kada odredimo taj trenutak u kome se na kraju intervala vremena dogodio sudar, onda imamo prethodni slučaj i analiza sudara se vrši na prethodno objašnjeni način. Posle izvršene analize

After that, starting with the top floor of the lower building and down to the first floors, the mutual positions of the corresponding slabs at the same level of both buildings is established. It means that using the obtained generalized displacements at the end of the time interval, spatial coordinates of the slab areas for each building are determined, of course, with respect to the same global coordinate system.

If, at the end of considered time interval, not a single pair of neighboring slabs at the same levels is not in a contact, simultaneous solution of differential equations of motion of both buildngs is continued for the next time step (or time interval). Of course, obtained vectors of generalized velocities and accelerations at the end of the previous time interval are treated as the initial velocities and accelerations at the beginning of the next time interval.

If, while checking the positions of slabs at the same level, one obtains that a contact at a point is established, then one must check if also the velocity condition of impact is satisfied too, which means that the collision of considered slabs has occured at the end of considered time interval. Of course, it is also necessary to check if the impact has occured between any other pair of slabs at the same level, because it is quite possible that impact happens between several pairs of slabs at the same time. In such a case, to all pair of slabs that are in the condition of impact, instead of velocities obtained according to relation (31), velocoties that are calculated according to the classical collision of two rigid plates in planar motion, as presented by (14)-(19), are imposed. Therefore, the generalized velocities of slabs, i.e. velocities of the centers of mass and the angular velocities of slabs, obtained by numerical integration according to relation (31), are treated as velocities immediatelly before the impact, while velocities immediately after the impact, which are calculated according to analysis given in section 2, are considered as the initial velocities at the beginning of the next time interval.

If, during the process of checking the positions of pairs of slabs of the same level, at the end of the time interval, one obtains the situation of interlapping of slab areas, it means that the impact has already occured sometimes within the considered time interval. In that case, the beginning of previously considered time interval is considered again and the time stepping procedure is done again, but now with the time interval sudara za posmatrani interval vremena ponovo se vrši simultano rješavanje jednačina (30) sa prvobitnim, a ne sa smanjenim intervalom vremena koji je dobijen u slučaju preklapanja tavanica.

4 NUMERIČKI PRIMJER

U cilju numeričke realizacije prikazane analize mogućeg sudara nesimetričnih višespratnih zgrada razvijen je odgovarajući kompjuterski program Sudar_3D, [11]. Program, osim što daje vremenske odgovore višespratnih nesimetričnih zgrada usled zadatog akcelerograma, može da ima još i praktičnu primjenu pri određivanju potrebnih širina dilatacionih razdjelnica između susjednih zgrada.

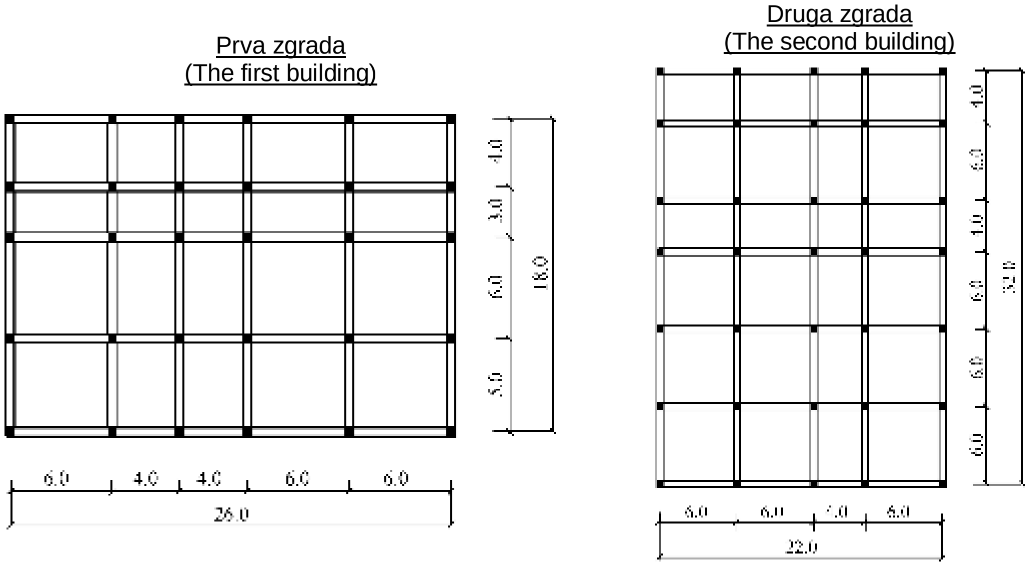
Kao primjer razmatrane su dvije susjedne višespratne nesimetrične zgrade istih spratnih visina, sa sedam odnosno deset spratova. Osnove zgrada, sa neophodnim geometrijskim podacima i podacima o spratnim masama, prikazane su na slici 7, dok su 3D modeli dobijeni primenom komercijalnog programa Tower 6 dati na slici 8. Periodi oscilovanja prvog tona razmatranih susjednih zgrada, dobijeni korišćenjem komercijalnog programa Tower 6, iznose 0.765s, odnosno 1.119s.

reduced by half with respect to the previous time step, in order to capture the moment of impact just at the end of considered time step. If that is achieved, then it is the previously considered situation of the iompact at the end of the time interval, so the impact is considered as previously explained. After the collision analysis for considered (reduced) time interval is performed, the usual simultaneous time integration of both buildings, given by (30), is done again, but with the originally selected time interval and not with the reduced one as obatined in the case of overlapping of slabs.

4 NUMERICAL EXAMPLE

In order to implement presented analysis of the possible pounding of non-symmetric multi-story buildings, the corresponding computer code, called Impact_3D [11], was developed. The code, besides producing the time history response of multi-story non-symmetric buildings due to a given accelerogram, may have also the practical aspect in determination of the necessary separation distance between neighboring buildings.

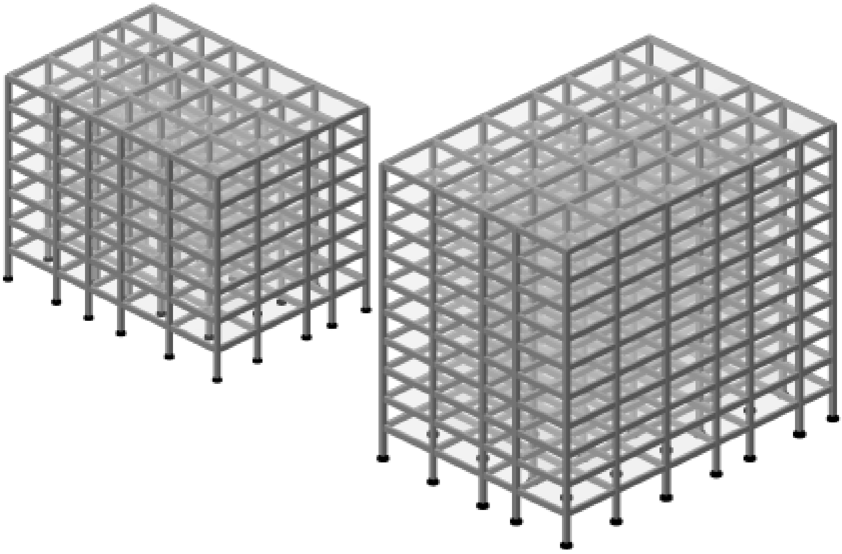
As an illustrative example, two neighboring multi-story non-symmetric buildings with seven and ten floors


Prva zgrada
(The first building)

Druga zgrada
(The second building)

Podaci (Basic data)		**Prva zgrada (First building)**	**Druga zgrada (Second building)**
Broj spratova: N (Number of stories)		7	10
Dimenzije stubova: b/h [cm] (Cross section of columns)		45/45	50/50
Dimenzije rigli: b/h [cm] (Cross section of beams)		35/60	35/60
Debljina ploče: d_{pl} [cm] (Slab thickness)		20	22
Modul elastičnosti: E [kN/m^2] (Modulus of elasticity)		3.15*10^7 (MB 30)	3.15*10^7 (MB 30)
Spratne mase: (Story masses)	m_1=...=m_{N-1} [kNs^2/m]	555	845
	m_N [kNs^2/m]	530	810

Slika 7. Osnove susjednih nesimetričnih zgrada
Figure 7. Plan of the neighboring non-symetric buildings



Slika 8. Modeli nesimetričnih susjednih zgrada (Tower 6)
Figure 8. 3D Models of non-symmetric neighboring buildings (Tower 6)

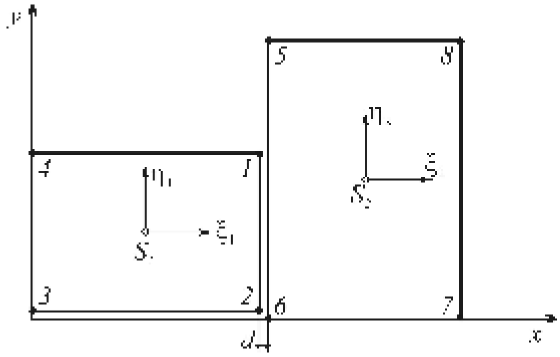
Na slici 9 prikazan je međusobni položaj razmatranih zgrada, sa d je označena dilataciona razdjelnica između njh. Takođe su prikazani usvojeni koordinatni sistemi: referentni Oxy i lokalni $S_1\xi_1\eta_1$ i $S_2\xi_2\eta_2$, postavljeni u centru masa (težištu) tavanica, kao i položaj karakterističnih tačaka kontura razmatranih susjednih zgrada.

Posmatrane zgrade su izložene zemljotresu čiji akcelerogram odgovara frekventnom zapisu zemljotresa El Centro, iz decembra 1940. godine, komponenta NS, sa dominantnim pravcem delovanja u pravcu globalne ose x (β=0°). Akcelerogram je skaliran tako da maksimalno ubrzanje tla iznosi 0.32g ($g=9.81 m/s^2$). Izračunavanje je urađeno dva puta: jednom sa dilatacionom razdjelnicom dovoljnih dimenzija (d=0.5m), tako da zgrade usled zemljotresa osciluju međusobno nezavisno, a drugi put sa dilatacionom razdjelnicom nedovoljnih dimenzija (d=0.2m), tako da dolazi do sudara tavanica tokom vibriranja zgrada. U tom slučaju usvojen je koeficijent sudara u iznosu od k=0.5. Odgovor zgrada je posmatran u ukupnom trajanju akcelerograma zemljotresa El Centro (12.2s) sa usvojenim vremenskim korakom $\Delta t=0.05s$.

and with the same story heights are considered. The plan of both buildings, with the necessary geometric and mass data, are presented in Fig. 7, and isometric views obtained in the commercial computer code Tower 6, are given in Fig. 8. Natural periods of the first mode of free vibrations of considered adjacent buildings, using the commercial code Tower 6, are obtained as 0.765s and 1.119s.

Fig. 9 presents the mutual configuration in plan of considered buildings, with d denoting the separation gap between them. Also, adopted coordinate systems are presented: the global system Oxy and the local ones $S_1\xi_1\eta_1$ and $S_2\xi_2\eta_2$, which are assumed in the center of mass of floor slabs, as well as the characteristic points on the contours of both adjacent buildings.

Considered buildings are exposed to the earthquake whose frequency contents corresponds to the El Centro record, from Dec. 1940, direction N-S, with the dominant direction along the x axis (β=0°). The accelerogram is scaled to the maximum acceleration equal to 0.32g ($g=9.81 m/s^2$). Calculations are performed twice: once with the separation distance of more then sufficient value (d=0.5m), so the buildings are vibrating independently



Slika 9. Položaj karakterističnih tačaka kontura susjednih zgrada
Figure 9. Disposition of the characteristic points of the contours of adjacent buildings

Sudar zgrada, u toku trajanja akcelerograma, se desio 11 puta: osam puta su se sudarile tavanice sedmog, dva puta tavanice šestog i jednom tavanice petog sprata. Kontakt između tavanica sedmog sprata ostvaren je još jednom, ali pri tome nije došlo do sudara jer nije bio ispunjen i uslov sudara po brzinama u tački kontakta dotičnih tavanica. Prvi sudar se desio između tavanica sedmog sprata, što je bilo i očekivati s obzirom da je tavanica sedmog sprata poslednja za prvu (nižu) zgradu, u trenutku t=3.1257s.

Na slikama 10-17, za zadati akcelerogram, prikazani su vremenski odgovori razmatranih susjednih zgrada, odnosno njihovih tavanica sedmog sprata (poslednji sprat za prvu-nižu zgradu). Pri tome je sivom linijom prikazan odgovor u slučaju kada je dilataciona razdjelnica dovoljnih dimenzija, pa ne dolazi do sudara (d=0.5m), a crnom linijom u slučaju kada je dilataciona razdjelnica nedovoljnih dimenzija, pa dolazi do sudara zgrada (d=0.2m).

Vremenska promjena generalisanih pomjeranja u_7, v_7, φ_7 i generalisanih brzina $\dot{u}_7, \dot{v}_7$ i $\dot{\varphi}_7$ centra mase tavanice sedmog sprata za obje zgrade prikazana je na slikama 10-13.

Pri nezavisnom oscilovanju zgrada, tj. u slučaju dilatacione razdjelnica dovoljnih dimenzija (d=0.5m) maksimalna pomjeranja centra mase tavanice sedmog sprata u pravcu x i y ose (u_{7max} i v_{7max}) za prvu zgradu iznose 20.1cm i 2cm (slika 10), a za drugu 16.2cm i 0.9cm (slika 12).

Na slikama 14 i 15 prikazana je vremenska promjena pomjeranja karakterističnih tačaka "1" i "2" tavanice broj 7 prve zgrade $u_7^{"1"}$, $v_7^{"1"}$, $u_7^{"2"}$ i $v_7^{"2"}$, a na slikama 16 i 17 vremenska promjena pomjeranja karakterističnih tačaka "5" i "6" tavanice broj 7 druge zgrade $u_7^{"5"}$, $v_7^{"5"}$, $u_7^{"6"}$ i $v_7^{"6"}$.

during the earthquake, and the second time with dilatation joint of a smaller value (d=0.2m), so the pounding between slabs during earthquake may occur. In this case, the coefficient of impact is assumed with the value of k=0.5. The time response of buildings was calculated during the complete duration of El Centro earhquake (12.2s) and the adopted time step was Δt=0.05s.

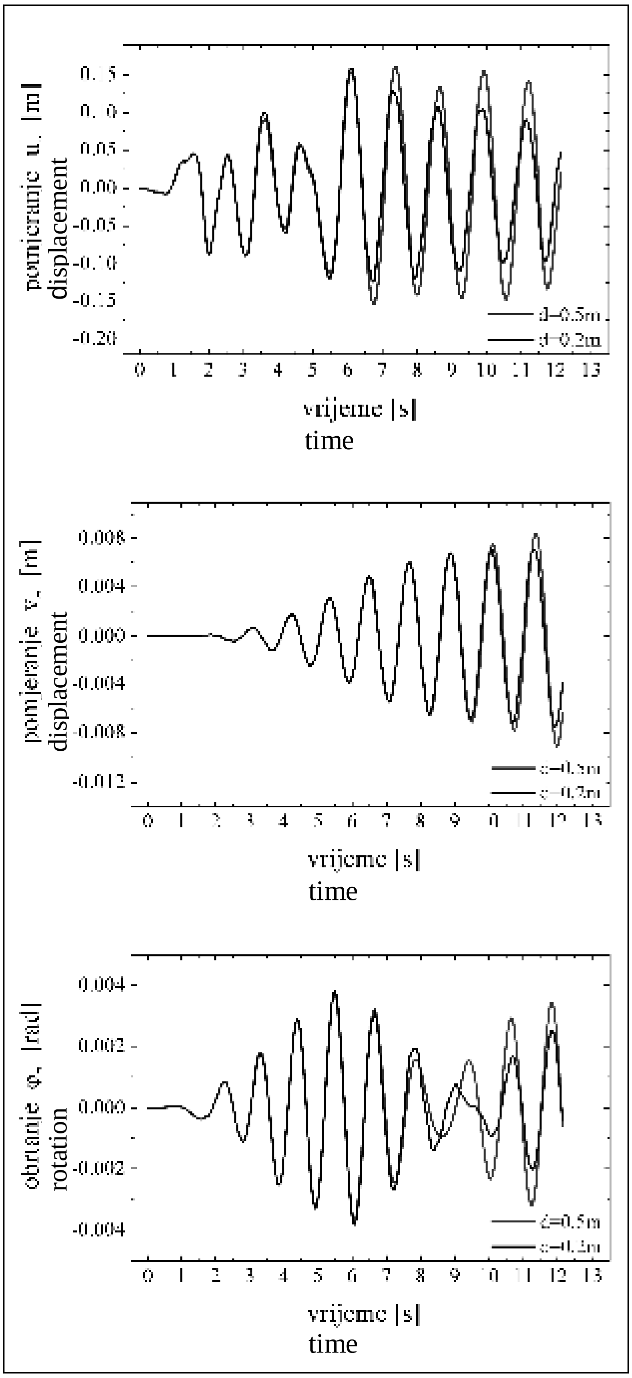
During the earthquake duration pounding has occured 11 times: eight times between slabs at the seventh floor, two times slabs at the sixth floor and once slabs at the fifth floor. Also, the contact of slabs of the seventh floor has occured once, but the pounding did not happen, because the velocity condition at the point of contact was not satisfied at the same time. The first pounding occured, at the time t=3.1257s, between slabs at the seventh floor, which was to be expected, since the seventh floor is the top floor for the first (lower) building.

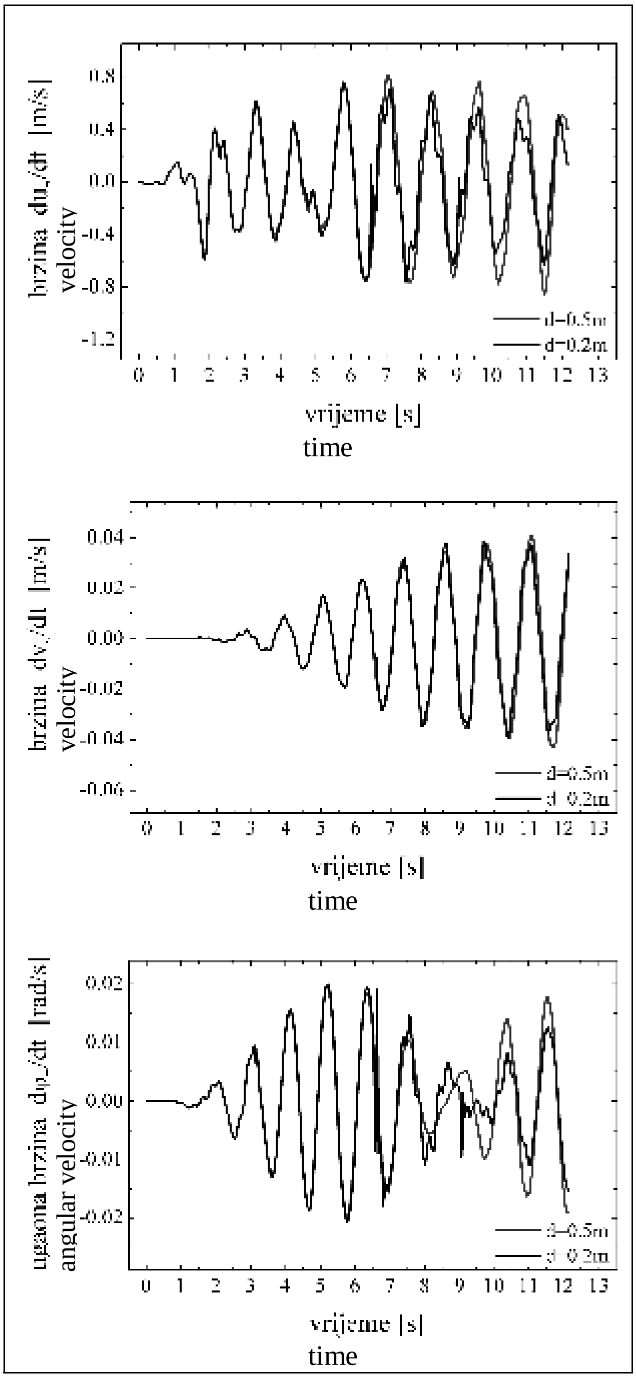
Figs. 10-17 are presenting the time response of both buildings, due to considered earthquake excitation, or rather, the time response of their slabs of the seventh floor (which is the top floor for the lower building). The gray line is used for the time history response for the case when the separation distance is sufficient (d=0.5m), so there is no pounding, while the black line represents the second case when the separation distance is unsufficient (d=0.2m), so the pounding happened.

The time history response of the generalized coordinates u_7, v_7, φ_7 and the generalized velocities $\dot{u}_7, \dot{v}_7$ and $\dot{\varphi}_7$ of the center of mass of the seventh floor for both buildings is presented in Figs. 10-13.

In the case of independent vibrations of buildings, i.e. in the case when the separation joint is sufficient (d=0.5m), the maximum displacements of the center of mass of the slab at the seventh floor with respect to axes x and y (i.e. u_{7max} and v_{7max}) for the first building are equal to 20.1cm and 2cm (Fig. 10), and for the second building 16.2cm and 0.9cm (Fig. 12).

Figs. 14 and 15 are presenting the time history response of displacements of the characteristic points "1" and "2" of the slab 7 of the first building, i.e. $u_7^{"1"}$, $v_7^{"1"}$, $u_7^{"2"}$ and $v_7^{"2"}$, while Figs. 16 and 17 present the time history response of characteristic points "5" and "6" of the slab 7 of the second building, i.e. $u_7^{"5"}$, $v_7^{"5"}$, $u_7^{"6"}$ and $v_7^{"6"}$.



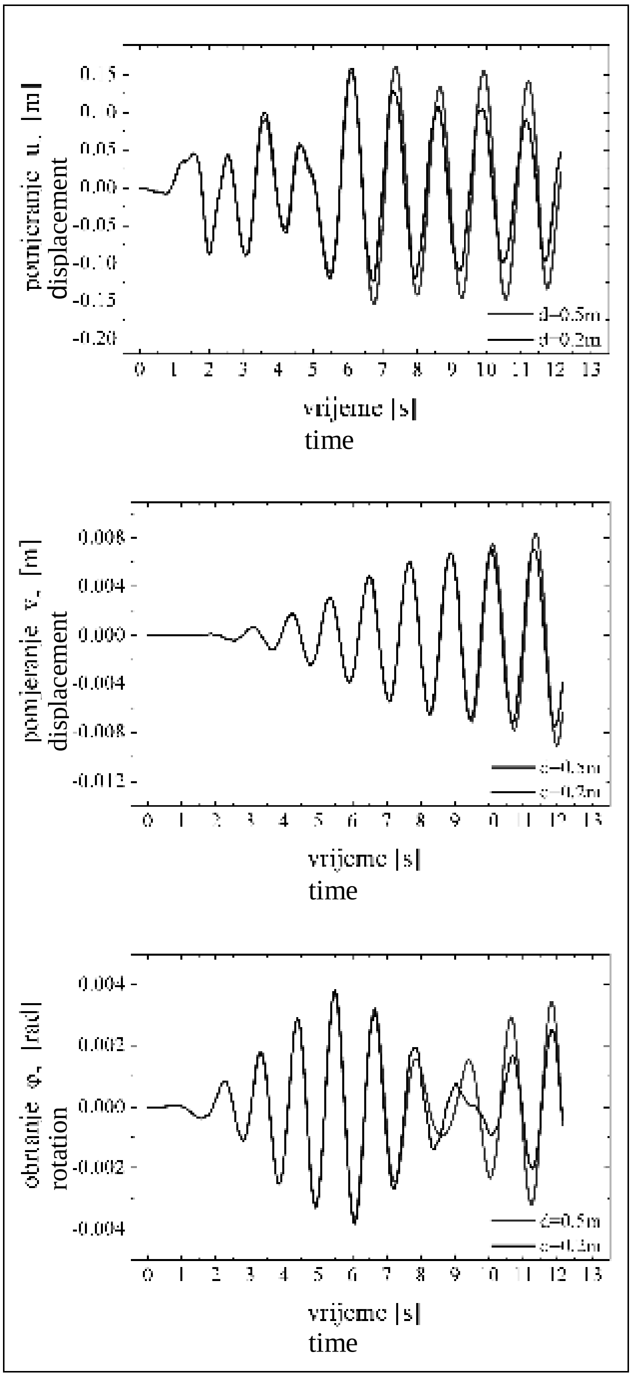


Slika 10. Vremenska promjena generalisanih pomjeranja u_7, v_7 *i* j_7 *prve zgrade*

Figure 10. Time history of generalized displacements u_7, v_7 *and* j_7 *of the first building*

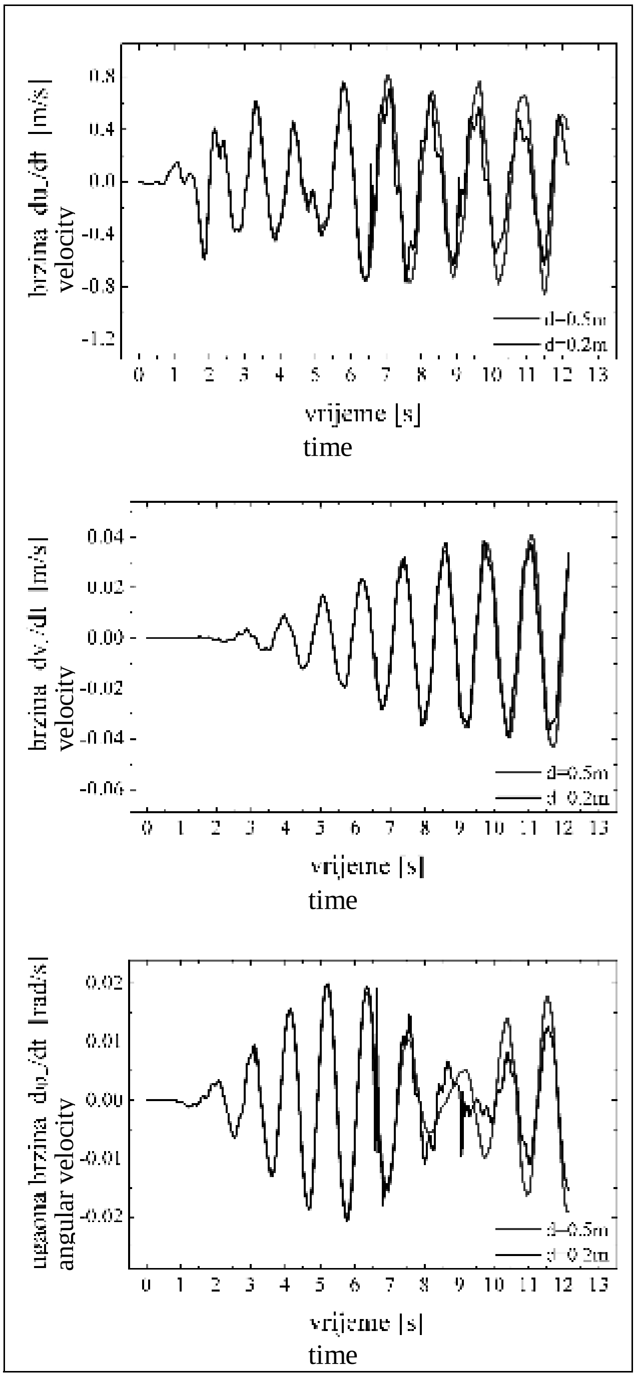
Slika 11. Vremenska promjena generalisanih brzina $\dot{u}_7, \dot{v}_7$ *i* $\dot{\varphi}_7$ *prve zgrade*

Figure 11. Time history of generalized velocities $\dot{u}_7, \dot{v}_7$ *and* $\dot{\varphi}_7$ *of the first building*



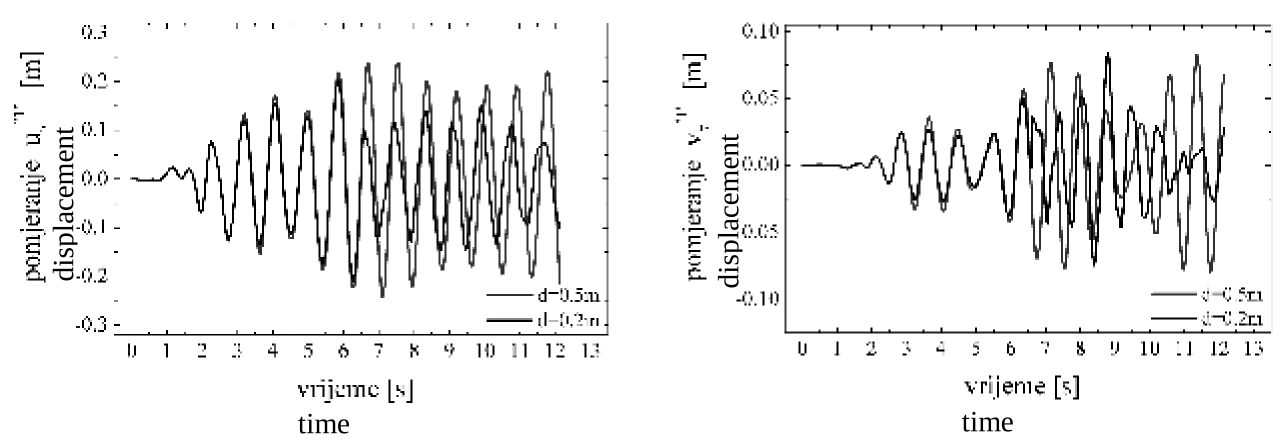
Slika 12. Vremenska promjena generalisanih pomjeranja u_7,v_7 *i* j_7 *druge zgrade*

Figure 12. Time history of generalized displacements u_7,v_7 *and* j_7 *of the second building*

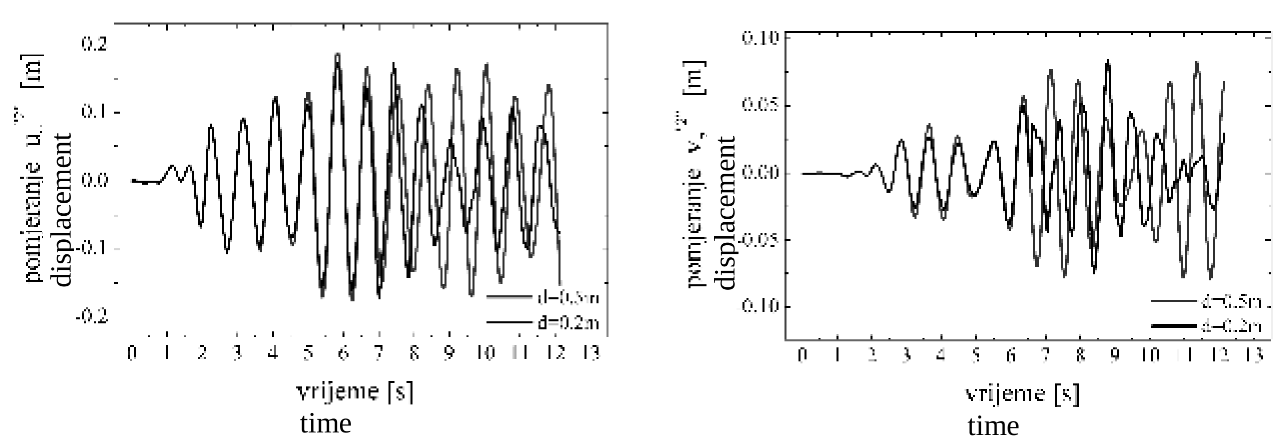


Slika 13. Vremenska promjena generalisanih brzina $\dot{u}_7,\dot{v}_7$ *i* $\dot{\varphi}_7$ *druge zgrade*

Figure 13. Time history of generalized velocities $\dot{u}_7,\dot{v}_7$ *and* $\dot{\varphi}_7$ *of the second building*



Slika 14. Vremenska promjena pomjeranja $u_7^{"1"}$ *i* $v_7^{"1"}$ *(prva zgrada)*
Figure 14. The time history response of displacements $u_7^{"1"}$ *and* $v_7^{"1"}$ *(the first building)*



Slika 15. Vremenska promjena pomjeranja $u_7^{"2"}$ *i* $v_7^{"2"}$ *(prva zgrada)*
Figure 15. The time history response of displacements $u_7^{"2"}$ *and* $v_7^{"2"}$ *(the first building)*

Maksimalna pomjeranja karakterističnih tačaka "1" i "2" tavanice sedmog sprata prve zgrade u pravcu x i y ose ($u_{7\max}^{"1"}$ i $v_{7\max}^{"1"}$ odnosno $u_{7\max}^{"2"}$ i $v_{7\max}^{"2"}$) pri nezavisnom vibriranju zgrada iznose 24.2cm i 8.3cm, odnosno 18.8cm i 8.3cm (slike 14 i 15).

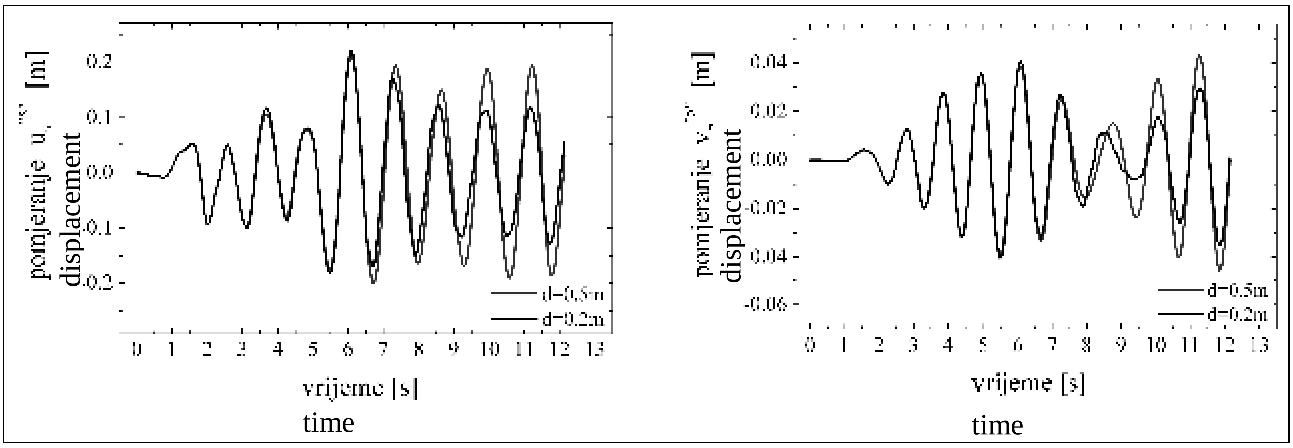
Maksimalna pomjeranja karakterističnih tačaka "5" i "6" tavanice sedmog sprata druge zgrade u pravcu x i y ose ($u_{7\max}^{"5"}$ i $v_{7\max}^{"5"}$ odnosno $u_{7\max}^{"6"}$ i $v_{7\max}^{"6"}$) pri nezavisnom vibriranju zgrada iznose 21.2cm i 4.6cm, odnosno 13.4cm i 4.6cm (slike 16 i 17).

Imajući u vidu da je pri nezavisnom vibriranju zgrada maksimalna vrijednost pomjeranja u pravcu x ose tavanice sedmog sprata za prvu zgradu 20.1cm, a za drugu 16.2cm i da se ta pomjeranja ostvaruju oko sedme sekunde (t=6.7-7.3s), tj. u sličnim trenucima vremena, zaključuje se da za ovaj slučaj širina dilatacione razdjelnice treba da iznosi 37cm, da ne bi došlo do sudara razmatranih zgrada. Zbog toga su izvršene odgovarajuće dodatne analize, koje se ovdje ne prikazuju, sa dilatacijama od d=35cm i d=37cm.

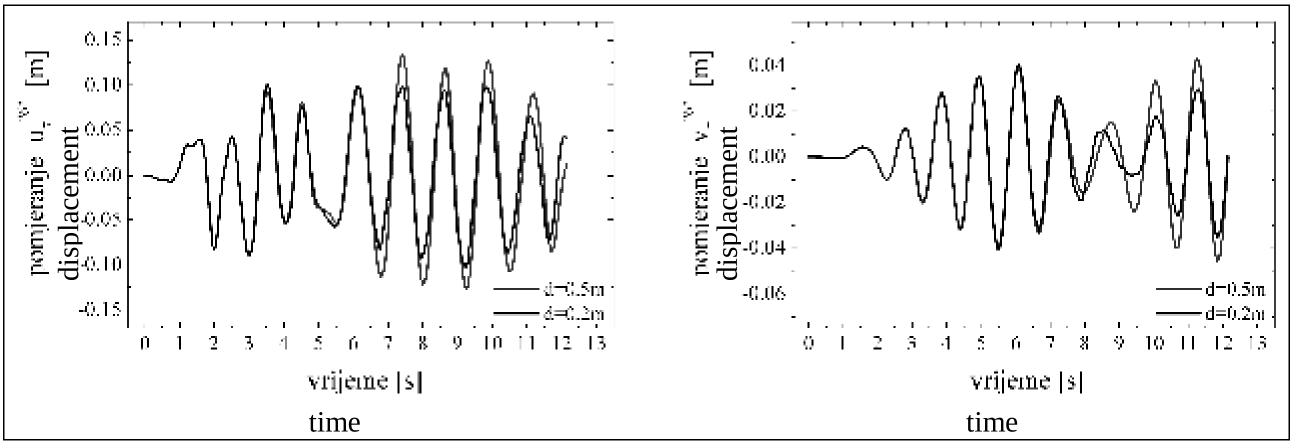
The maximum displacements of the characteristic points "1" and "2" of the slab of the seventh floor of the first building in directions of axes x and y (i.e. $u_{7\max}^{"1"}$ and $v_{7\max}^{"1"}$, or $u_{7\max}^{"2"}$ and $v_{7\max}^{"2"}$) during the independent vibrations of buildings are equal to 24.2cm and 8.3cm, and also 18.8cm and 8.3 cm (Figs. 14 and 15).

The maximum displacements of the characteristic points "5" and "6" of the slab of the seventh floor of the second building in directions of axes x and y (i.e. $u_{7\max}^{"5"}$ and $v_{7\max}^{"5"}$, or $u_{7\max}^{"6"}$ and $v_{7\max}^{"6"}$), during the independent vibrations of buildings are equal to 21.2cm and 4.6cm, and also 13.4cm and 4.6cm (Figs. 16 and 17).

Having in mind that in the independent vibrations of buildings the maximum values of displacements in direction of x axis for the first building is 20.1cm and 16.2cm for the second building and also that the maximum displacements are occuring about the seventh second (t = 6.7 to 7.3s), i.e. in the similar time, one may conclude that for this case the separation joint should be



Slika 16. Vremenska promjena pomjeranja $u_7^{"5"}$ *i* $v_7^{"5"}$ *(druga zgrada)*
Figure 16. The time history response of displacements $u_7^{"5"}$ *and* $v_7^{"5"}$ *(the second building)*



Slika 17. Vremenska promjena pomjeranja $u_7^{"6"}$ *i* $v_7^{"6"}$ *(druga zgrada)*
Figure 17. The time history response of displacements $u_7^{"6"}$ *and* $v_7^{"6"}$ *(the second building)*

U slučaju kada je dilataciona razdjelnica između razmatranih zgrada iznosila 35cm sudar zgrada, odnosno tavanica sedmog sprata, dogodio se dva puta i to u trenucima vremena t=6.62s i t=6.72s, dok za d=37cm nije bilo sudara zgrada.

5 ZAVRŠNE NAPOMENE

Na osnovu prikazanog numeričkog modela mogućeg sudara susjednih nesimetričnih višespratnih zgrada istih spratnih visina usled uticaja zemljotresa prikazanog preko zadatog akcelerograma, razvijen je kompjuterski program Sudar_3D. Program, koji osim što daje vremenske odgovore višespratnih nesimetričnih zgrada usled zadatog akcelerograma, uključujući i analizu mogućeg sudara zgrada, može da ima još i praktičnu primjenu pri određivanju potrebnih širina dilatacionih razdjelnica između susjednih zgrada.

equal et least 37cm, in order to prevent pounding. herefore, the corresponding analyses were also onducted for dillatation joints of d=35cm and d=37cm, ut they are not presented here.

In the case when the separation joint was 35cm the pounding of buildings, i.e. slabs of the seventh floor, has occured two times, namely at times t=6.62s and t=6.72s, while for the case of d=37cm there were no pounding.

5 THE FINAL REMARKS

In accordance with the presented analysis of the possible pounding between buildings with the same story heights during an earthquake, described by the given accelerogram, the corresponding computer program Impact_3D was developed. The code, besides producing the time history response of multi-story non-symmetric buildings due to a given accelerogram, may have also the practical aspect in determination of the necessary separation gaps between neighboring buildings.

U cilju ilustracije numeričkog postupka razmatrane su dvije susjedne višespratne nesimetrične zgrade istih spratnih visina, sa sedam odnosno deset spratova (slike 7 i 8). Prikazani su vremenski odgovori razmatranih zgrada usled djelovanja iste seizmičke pobude (za zadati akcelerogram koji odgovara frekventnom zapisu zemljotresa El Centro, iz decembra 1940. godine, komponenta NS) za dominantni pravac zemljotresa β=0°. Pri tome su posebno posmatrane dvije situacije, prva kada je dilataciona razdjelnica dovoljnih dimenzija, pa zgrade osciluju nezavisno i druga kada je dilataciona razdjelnica nedovoljnih dimenzija, pa dolazi do sudara zgrada, odnosno njihovih tavanica na istom visinskom nivou (slike 10-17). Zatim je određena potrebna minimalna širina dilatacione razdjelnice pri kojoj ne bi došlo do sudara razmatranih zgrada usled zadatog zemljotresa.

Analizom navedenih primjera može da se zaključi da vremenski odgovori susjednih nesimetričnih zgrada, kao i potrebna širina dilatacione razdjelnice, zavise kako od međusobnog položaja zgrada, njihovih krutosti, rasporeda masa, tako i prirode zemljotresa. To znači, od datog akcelerograma (frekventnog sastava i maksimalnog ubrzanja) i njegovog dominantnog pravca djelovanja, kao i od usvojene vrijednosti koeficijenta sudara, tj. lokalne disipacije energije u zoni sudara.

In order to illustrate the numerical procedure two neighboring multi-story non-symmetric building of the same story heights, with seven and ten floors are considered (Figs. 7 and 8). The time history responses of considered buildings, due to the same earthquake excitation (given accelerogram of El Centro earthquake, from Dec. 1940, component N-S) for the dominant direction in the x axis, β=0°. Two cases were considered, the first one when the separation joint is sufficient, so the building are vibrating independently, and the second one when the separation joint is unsufficient, so the pounding between buildings (i.e. between slabs at the same level) during earthquake is occuring (Figs. 10-17). Also, the minimum possible separation distance is determined in order to prevent pounding in the case of considered buildings due to the given earthquake.

After the analysis of conducted numerical examples one may conclude that the time response of adjacent non-symmetric buildings and the necessary separation distnce between buildings depend on the configuration of buildings, their dynamic properties (stiffness and mass distributions) and also on the nature of considered earthquake. It means on the given accelerogram (its frequency contnet and the maximum acceleration) and the dominant direction of the soil movement, and also on the value of the coefficient of impact, i.e. on the local dissipation of energy in the zone of impact.

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REZIME

SUDAR SUSJEDNIH NESIMETRIČNIH VIŠESPRATNIH ZGRADA USLED UTICAJA ZEMLJOTRESA

Ljiljana ŽUGIĆ
Stanko BRČIĆ

Usled dejstva zemljotresa susjedne zgrade izgrađene u skladu sa postojećim tehničkim propisima načelno osciluju nezavisno. Međutim, ako se njihove relevantne dinamičke karakteristike bitno razlikuju i ako je dilataciona razdjelnica između njih nedovoljnih dimenzija može da dođe do njihovog sudara, tj. sudara tavanica susjednih zgrada na istom visinskom nivou ako su susjedne zgrade istih spratnih visina.

U radu je analiziran mogući sudar susjednih višespratnih zgrada istih spratnih visina usled djelovanja iste seizmičke pobude sa zadatim dominantnim pravcem djelovanja. Zgrade su nesimetrične s obzirom na njihovu krutost i/ili masu, pa je njihov matematički model trodimenzionalan, tj. svaka tavanica vrši ravno kretanje u svojoj horizontalnoj ravni, sa po tri stepena slobode kretanja.

Mogući sudar zgrada je analiziran kombinacijom direktne numeričke integracije odgovarajućih diferencijalnih jednačina kretanja i klasične analize sudara dva kruta tijela pri ravnom kretanju. U slučaju sudara pojedinih tavanica susjednih zgrada nagla promjena u kinematičkom stanju neposredno prije i neposredno poslije sudara određena je rješavanjem odgovarajućih jednačina sudara tavanica. Pri formiranju jednačina sudara tavanica korišćene su uobičajne pretpostavke teorije sudara u klasičnoj mehanici, pri čemu je lokalna disipacija energije u zoni sudara uzeta u obzir uvođenjem koeficijenta sudara.

U cilju numeričke realizacije ovog problema razvijen je odgovarajući kompjuterski program, koji osim što daje vremenske odgovore susjednih višespratnih nesimetričnih zgrada usled zadatog akcelerograma za dominantni pravac zemljotresa, uključujući i analizu mogućeg sudara zgrada, može imati još i praktičnu primjenu pri određivanju potrebnih širina dilatacionih razdjelnica između susjednih zgrada sa ciljem sprečavanja mogućeg sudara za dati zemljotres.

Ključne riječi: sudar zgrada, nesimetrična zgrada, koeficijent sudara, vremenska analiza, uticaj zemljotresa

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SUMMARY

POUNDING OF ADJACENT NON-SYMMETRIC MULTISTORY BUILDINGS DUE TO AN EARTHQUAKE

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Adjacent buildings, if constructed in accordance with the building codes, in event of an earthquake will oscillate independently. However, if their relevant dynamic characteristics are substantially different and if the separation gap between them is insufficient, the pounding between adjacent floor slabs at the same level may happen.

The paper is analyzing a possible pounding of adjacent multistory buildings with the same story heights due to the same earthquake excitation with a given dominant direction. Buildings are non-symmetric with respect to their stiffness and/or mass distributions, so the mathematical model is three-dimensional, i.e. each floor slab is undergoing the planar motion in its plane with three dofs each.

Possible pounding between buildings is analyzed by combination of a direct numerical integration of the corresponding differential equations of motion and the classical analysis of an impact of two rigid bodies in planar motion. In the case of pounding between adjacent slabs the sudden change in kinematic state immediately before and after the collision is determined by solution of the corresponding collision equations. Formulation of the impact equations is based upon the usual assumptions of the impact theory in classical mechanics, with introduction of the coefficient of impact to account the local dissipation of energy in the zone of collision.

The corresponding computer code is developed in order to perform numerical simulations. Besides determination of the time history of adjacent multistory non-symmetric buildings due to earthquake acceleration with a given dominant direction, including the analysis of the possible pounding, the code may also have the practical application to determine the necessary dilatation gap between buildings in order to prevent the pounding for a given earthquake.

Key words: pounding between buildings, non-symmetric buildings, coefficient of impact, time history analysis, earthquake effect