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ON STRONGLY EDGE IRREGULAR FUZZY GRAPHS

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Abstract. In this paper strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graphs are introduced. A comparative study between strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graph is done. Also some properties of strongly edge irregular fuzzy graphs are studied and they are examined for strongly edge totally irregular fuzzy graphs.

1. Introduction

Euler first introduced the concept of graph theory in 1736. In 1965, Lofti A. Zadeh [14] introduced the concept of a fuzzy subset of a set as a method for representing the phenomena of uncertainty in real life situation. Azriel Rosenfeld introduced fuzzy graphs in 1975 [11]. It has been growing fast and has numerous applications in various fields. A. Nagoor Gani and K.Radha [6] introduced regular fuzzy graphs, total degree and totally regular fuzzy graphs. A. Nagoorgani and S. R. Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2008 [5]. S. P. Nandhini and E. Nandhini introduced the concept of strongly irregular fuzzy graphs and strongly totally irregular fuzzy graphs [8]. K. Radha and N. Kumaravel introduced the concept of edge degree, total edge degree and edge regular fuzzy graphs and discussed about the degree of an edge in some fuzzy graphs [10]. Noura Alshehri and M. Akram introduced the the concept of intuitionistic fuzzy planar graphs [9]. M. Akram and W. A. Dudek introduced the notions of bipolar fuzzy graphs [2]. Sovan Samanta and Madhumangal Pal introduced the concept of irregular bipolar fuzzy graphs [13].

Key words and phrases. degree and total degree of a vertex in fuzzy graphs, irregular fuzzy graph, totally irregular fuzzy graph, strongly irregular fuzzy graph, edge degree in fuzzy graph, total edge degree in fuzzy graph, edge regular fuzzy graphs.

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This is the background to introduce strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graphs and discussed some of its properties.

2. Preliminaries

We present some known definitions and results for ready reference to go through the work presented in this paper.

Definition 2.1. A *graph* G is an ordered triple $(V(G), E(G), \psi(G))$ consisting of a nonempty set $V(G)$ of vertices, a set $E(G)$, disjoint from $V(G)$, of edges and an incidence function $\psi(G)$ that associates with each edge of G an unordered pair of vertices of G. A graph G is *finite* if its vertex set and edge set are finite.

Definition 2.2. The *degree* $d_G(v)$ of a vertex v in G or simply $d(v)$ is the number of edges of G incident with vertex v.

Definition 2.3. Star $K_{1,n}$ with n spokes (having $n+1$ vertices with n pendant edges).

Definition 2.4. Barbell graph $B_{n,m}$ is defined by n pendant edges attached with one end of K_2 and m pendant edges attached with other end of K_2.

Definition 2.5. A fuzzy set A on a set X is characterized by a mapping m:$X \to [01]$, called the membership function. A fuzzy set is denoted as $A = (X, m)$.

Definition 2.6. A Fuzzy graph denoted by $G : (\sigma, \mu)$ on the graph $G^* : (V, E)$ is a pair of functions (σ, μ) where $\sigma : V \to [0, 1]$ is a fuzzy subset of a non empty set V and $\mu : V \times V \to [0, 1]$ is a symmetric fuzzy relation on σ such that for all u and v in V the relation $\mu(u, v) = \mu(uv) \leq \sigma(u) \wedge \sigma(v)$ is satisfied [11].

Definition 2.7. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The degree of a vertex u is $d_G(u) = \sum_{u \neq v} \mu(uv)$, for $uv \in E$ and $\mu(uv) = 0$, for uv not in E; this is equivalent to $d_G(u) = \sum_{uv \in E} \mu(uv)$ [6].

Definition 2.8. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The total degree of a vertex u is defined as $td(u) = \sum \mu(u, v) + \sigma(u) = d(u) + \sigma(u)$, $uv \in E$ [6].

Definition 2.9. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then G is said to be a irregular fuzzy graph if there exists a vertex which is adjacent to a vertices with distinct degrees [5].

Definition 2.10. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then G is said to be a totally irregular fuzzy graph if there exists a vertex which is adjacent to a vertices with distinct total degrees [5].

Definition 2.11. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then G is said to be a strongly irregular fuzzy graph if every pair of vertices have distinct degrees [8].

Definition 2.12. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. Then G is said to be a highly irregular fuzzy graph if every vertex in G is adjacent to the vertices having distinct degrees [5].

Definition 2.13. Let $G : (\sigma, \mu)$ be a fuzzy graph. The degree of an edge uv is defined as $d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv)$. The minimum degree of an edge is $\delta_E(G) = \wedge\{d_G(uv) : uv \in E\}$. The maximum degree of an edge is $\Delta_E(G) = \vee\{d_G(uv) : uv \in E\}$ [10].

Definition 2.14. Let $G : (\sigma, \mu)$ be a fuzzy graph. The total degree of an edge uv is defined as $td_G(uv) = d_G(u) + d_G(v) - \mu(uv)$. The minimum total degree of an edge is $\delta_{tE}(G) = \wedge\{td_G(uv) : uv \in E\}$. The maximum total degree of an edge is $\Delta_{tE}(G) = \vee\{td_G(uv) : uv \in E\}$ [10].

Definition 2.15. The degree of an edge uv in the underlying graph is defined as $d_G(uv) = d_G(u) + d_G(v) - 2$ [1].

Definition 2.16. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a neighbourly edge irregular fuzzy graph if every pair of adjacent edges having distinct degrees [12].

Definition 2.17. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a neighbourly edge totally irregular fuzzy graph if every pair of adjacent edges having distinct total degrees [12].

3. Strongly Edge Irregular Fuzzy Graphs and Strongly Edge Totally Irregular Fuzzy Graphs

In this section, we introduce strongly edge irregular fuzzy graphs and strongly edge totally irregular fuzzy graphs and study some properties of strongly edge irregular fuzzy graphs through various examples.

Definition 3.1. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a strongly edge irregular fuzzy graph if every pair of edges having distinct degrees (or) no two edges have same degree.

Definition 3.2. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^* : (V, E)$. Then G is said to be a strongly edge totally irregular fuzzy graph if every pair of edges having distinct total degrees (or) no two edges have same total degree.

Example 3.1. Graph which is both strongly edge irregular fuzzy graph and strongly edge totally irregular fuzzy graph. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$ which is a cycle of length five.

From Figure 1, $d_G(u) = 0.6$, $d_G(v) = 0.3$, $d_G(w) = 0.5$, $d_G(x) = 0.7$, $d_G(y) = 0.9$. Degrees of the edges are calculated as follows

$$d_G(uv) = d_G(u) + d_G(v) - 2\mu(uv) = 0.6 + 0.3 - 2(0.1) = 0.7,$$
$$d_G(vw) = d_G(v) + d_G(w) - 2\mu(vw) = 0.3 + 0.5 - 2(0.2) = 0.4,$$
$$d_G(wx) = d_G(w) + d_G(x) - 2\mu(wx) = 0.5 + 0.7 - 2(0.3) = 0.6,$$
$$d_G(xy) = d_G(x) + d_G(y) - 2\mu(xy) = 0.7 + 0.9 - 2(.4) = 0.8,$$
$$d_G(yu) = d_G(y) + d_G(u) - 2\mu(yu) = 0.9 + 0.6 - 2(.5) = 0.5.$$

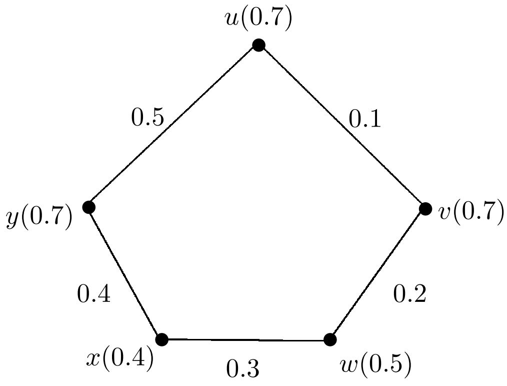

Figure 1.

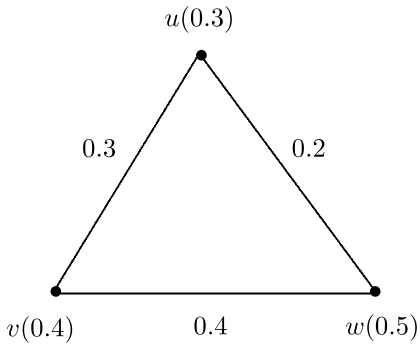

Figure 2.

We noted that every pair of edges having distinct degrees. Hence G is strongly edge irregular fuzzy graph. Total degrees of the edges are calculated as follows

$$td_G(uv) = d_G(uv) + \mu(uv) = 0.7 + 0.1 = 0.8,$$
$$td_G(vw) = d_G(vw) + \mu(vw) = 0.4 + 0.2 = 0.6,$$
$$td_G(wx) = d_G(wx) + \mu(wx) = 0.6 + 0.3 = 0.9,$$
$$td_G(xy) = d_G(xy) + \mu(xy) = 0.8 + 0.4 = 1.2,$$
$$td_G(yu) = d_G(yu) + \mu(yu) = 0.5 + 0.5 = 1.$$

It is noted that every pair of edges in G having distinct total degrees. So, G is strongly edge totally irregular fuzzy graph. Hence G is both strongly edge irregular fuzzy graph and strongly edge totally irregular fuzzy graph.

Example 3.2. Strongly edge irregular fuzzy graphs need not be strongly edge totally irregular fuzzy graphs.

From Figure 2 $d_G(u) = 0.5$, $d_G(v) = 0.7$, $d_G(w) = 0.6$. Also, $d_G(uv) = 0.6$, $d_G(vw) = 0.5$, $d_G(wu) = 0.7$, $td_G(uv) = 0.9$, $td_G(vw) = 0.9$, $td_G(wu) = 0.9$. We

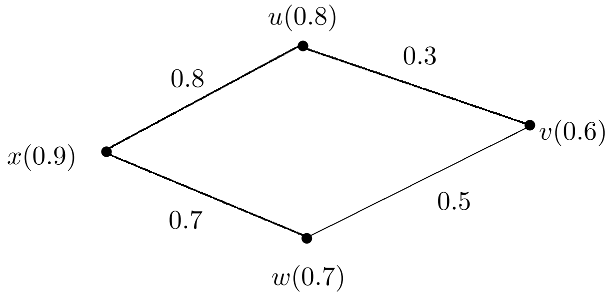

FIGURE 3.

noted that G is strongly edge irregular fuzzy graph, since every pair of edges having distinct degrees. Also, G is not strongly edge totally irregular fuzzy graph, since all the edges having same total degree. Hence strongly edge irregular fuzzy graph need not be strongly edge totally irregular fuzzy graph.

Example 3.3. Strongly edge totally irregular fuzzy graph need not be strongly edge irregular fuzzy graph. Consider $G^* : (V, E)$ where $V = \{u, v, w, x\}$ and $E = \{uv, vw, wx, xu\}$.

From Figure 3, $d_G(u) = 1.1$, $d_G(v) = 0.8$, $d_G(w) = 1.2$, $d_G(x) = 1.5$. $d_G(uv) = 1.3$, $d_G(vw) = 1$, $d_G(wx) = 1.3$, $d_G(xu) = 1$. It is noted that $d_G(uv) = d_G(wx)$. Hence G is not strongly edge irregular fuzzy graph. Also, $td_G(uv) = 1.6$, $td_G(vw) = 1.5$, $td_G(wx) = 2$, $td_G(uv) = 1.8$. It is observed that $td_G(uv) \neq td_G(vw) \neq td_G(wx) \neq td_G(xu)$. So, G is strongly edge totally irregular fuzzy graph. Hence strongly edge totally irregular fuzzy graph need not be strongly edge irregular fuzzy graph.

Theorem 3.1. *Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is constant function. If G is strongly edge irregular fuzzy graph, then G is strongly edge totally irregular fuzzy graph.*

Proof. Assume that μ is a constant function, let $\mu(uv) = c$ for all $uv \in E$, where c is constant. Let uv and xy be any pair of edges in E. Suppose that G is strongly edge irregular fuzzy graph. Then $d_G(uv) \neq d_G(xy)$, where uv and xy are any pair of edges in $E \Rightarrow d_G(uv) \neq d_G(xy) \Rightarrow d_G(uv)+c \neq d_G(xy)+c \Rightarrow d_G(uv)+\mu(uv) \neq d_G(xy)+\mu(xy)$ $\Rightarrow td_G(uv) \neq td_G(xy)$, where uv and xy are any pair of edges in E. Hence G is strongly edge totally irregular fuzzy graph. □

Theorem 3.2. *Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is constant function. If G is strongly edge totally irregular fuzzy graph, then G is strongly edge irregular fuzzy graph.*

Proof. Assume that μ is a constant function, let $\mu(uv) = c$ for all $uv \in E$, where c is constant. Let uv and xy be any pair of edges in E. Suppose that G is strongly edge totally irregular fuzzy graph. Then $td_G(uv) \neq td_G(xy)$, where uv and xy are any pair of edges in $E \Rightarrow td_G(uv) \neq td_G(xy) \Rightarrow d_G(uv) + \mu(uv) \neq d_G(xy) + \mu(xy) \Rightarrow$

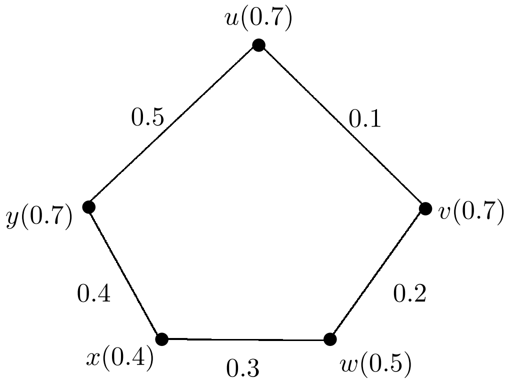

FIGURE 4.

$d_G(uv) + c \neq d_G(xy) + c \Rightarrow d_G(uv) \neq d_G(xy)$, where uv and xy are any pair of edges in E. Hence G is strongly edge irregular fuzzy graph. □

Remark 3.1. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. If G is both strongly edge irregular fuzzy graph and strongly edge totally irregular fuzzy graph. Then μ need not be a constant function.

Example 3.4. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$ which is a cycle of length five.

From Figure 4, $d_G(u) = 0.6, d_G(v) = 0.3$, $d_G(w) = 0.5, d_G(x) = 0.7$, $d_G(y) = 0.9$. Also, $d_G(uv) = 0.7$, $d_G(vw) = 0.4$, $d_G(wx) = 0.6$, $d_G(xy) = 0.8$, $d_G(yu) = 0.5$. It is noted that every pair of edges in G having distinct degrees. Hence G is strongly edge irregular fuzzy graph. Also, $td_G(uv) = 0.8$, $td_G(vw) = 0.6$, $td_G(wx) = 0.9$, $td_G(xy) = 1.2$, $td_G(yu) = 1$. Note that every pair of edges in G having distinct total degrees. Hence G is both strongly edge irregular fuzzy graph and strongly edge totally irregular fuzzy graph. But μ is not constant function.

Theorem 3.3. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. If G is strongly edge irregular fuzzy graph, then G is neighbourly edge irregular fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. Let us assume that G is strongly edge irregular fuzzy graph then every pair of edges in G have distinct degrees so every pair of adjacent edges have distinct degrees. Hence G is neighbourly edge irregular fuzzy graph. □

Theorem 3.4. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. If G is strongly edge totally irregular fuzzy graph, then G is neighbourly edge totally irregular fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^*(V, E)$. Let us assume that G is strongly edge totally irregular fuzzy graph then every pair of edges in G have distinct total

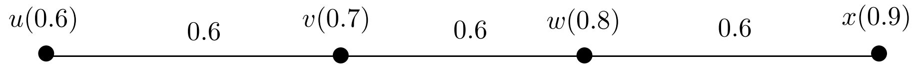

FIGURE 5.

degrees so every pair of adjacent edges have distinct total degrees. Hence G is neighbourly edge total irregular fuzzy graph. □

Remark 3.2. Converse of the above Theorems 3.3 and 3.4 need not be true.

Example 3.5. Consider $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a path on four vertices.

From Figure 5, $d_G(u) = 0.6$, $d_G(v) = 1.2$, $d_G(w) = 1.2$, $d_G(x) = 0.6$, $d_G(uv) = 0.6$, $d_G(vw) = 1.2$, $d_G(wx) = 0.6$. Here, $d_G(uv) \neq d_G(vw)$ and $d_G(vw) \neq d_G(wx)$. Hence G is neighbourly edge irregular fuzzy graph. But G is not strongly edge irregular fuzzy graph, since $d_G(uv) = d_G(wx)$. Also, $td_G(uv) = 1.2$, $td_G(vw) = 1.8$, $td_G(wx) = 1.2$. Note that $td_G(uv) \neq td_G(vw)$ and $td_G(vw) \neq td_G(wx)$. Hence G is neighbourly edge totally irregular fuzzy graph. But G is not strongly edge totally irregular fuzzy graph, since $td_G(uv) = td_G(wx)$.

Theorem 3.5. *Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is constant function. If G is strongly edge irregular fuzzy graph, then G is an irregular fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. Assume that μ is a constant function, let $\mu(uv) = c$ for all $uv \in E$, where c is constant. Let us suppose that G is strongly edge irregular fuzzy graph. Then every pair of edges having distinct degrees. Let uv and vw are adjacent edges in G having distinct degrees. Then $d_G(uv) \neq d_G(vw) \Rightarrow d_G(u) + d_G(v) - 2\mu(uv) \neq d_G(v) + d_G(w) - 2\mu(uv) \Rightarrow d_G(u) + d_G(v) - 2c \neq d_G(v) + d_G(w) - 2c \Rightarrow d_G(u) + d_G(v) \neq d_G(v) + d_G(w) \Rightarrow d_G(u) \neq d_G(w)$ so there exists a vertex v which is adjacent to a vertices u and w having distinct degrees. Hence G is an irregular fuzzy graph. □

Theorem 3.6. *Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is constant function. If G is strongly edge totally irregular fuzzy graph, then G is an irregular fuzzy graph.*

Proof. Proof is similar to the above Theorem. □

Remark 3.3. Converse of the above Theorems 3.5 and 3.6 need not be true.

Example 3.6. Consider $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a path on four vertices.

From Figure 6, $d_G(u) = 0.2$, $d_G(v) = 0.4$, $d_G(w) = 0.4$, $d_G(x) = 0.2$. Here, G is an irregular fuzzy graph. Also, $d_G(uv) = 0.2$, $d_G(vw) = 0.4$, $d_G(wx) = 0.2$, $td_G(uv) = 0.4$, $td_G(vw) = 0.8$, $td_G(wx) = 0.4$. It is noted that $d_G(uv) = d_G(wx)$. Hence G is not strongly edge irregular fuzzy graph. Also, $td_G(uv) = td_G(wx)$. Hence G is not strongly edge totally irregular fuzzy graph.

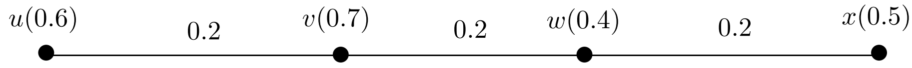

FIGURE 6.

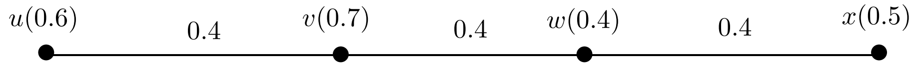

FIGURE 7.

Theorem 3.7. *Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is strongly edge irregular fuzzy graph, then G is highly irregular fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$. Assume that μ is a constant function, let $\mu(uv) = c$ for all $uv \in E$, where c is constant. Let v be any vertex adjacent with u, w and x. Then uv, vw and vx are adjacent edges in G. Let us suppose that G is strongly edge irregular fuzzy graph then every pair of edges in G have distinct degrees so every pair of adjacent edges in G have distinct degrees hence $d_G(uv) \neq d_G(vw) \neq d_G(vx) \Rightarrow d_G(v) + d_G(u) - 2\mu(uv) \neq d_G(v) + d_G(w) - 2\mu(vw) \neq d_G(v)+d_G(x)-2\mu(vx) \Rightarrow d_G(v)+d_G(u)-2c \neq d_G(v)+d_G(w)-2c \neq d_G(v)+d_G(x)-2c \Rightarrow d_G(v) + d_G(u) \neq d_G(v) + d_G(w) \neq d_G(v) + d_G(x) \Rightarrow d_G(u) \neq d_G(w) \neq d_G(x)$ so the vertex v is adjacent to the vertices with distinct degrees. Hence G is highly irregular fuzzy graph. □

Theorem 3.8. *Let $G : (\sigma, \mu)$ be a connected fuzzy graph on $G^*(V, E)$ and μ is a constant function. If G is strongly edge totally irregular fuzzy graph, then G is highly irregular fuzzy graph.*

Proof. Proof is similar to the above Theorem. □

Remark 3.4. Converse of the above Theorems 3.7 and 3.8 need not be true.

Example 3.7. Consider $G : (\sigma, \mu)$ be a fuzzy graph such that $G^* : (V, E)$ is a path on four vertices.

From Figure 7, $d_G(u) = 0.4$, $d_G(v) = 0.8$, $d_G(w) = 0.8$, $d_G(x) = 0.4$. Hence G is highly irregular fuzzy graph. Note that $d_G(uv) = 0.4$, $d_G(vw) = 0.8$, $d_G(wx) = 0.4$. So, G is not strongly edge irregular fuzzy graph. $td_G(uv) = 0.8$, $td_G(vw) = 1.2$, $td_G(wx) = 0.8$. So, G is not strongly edge totally irregular fuzzy graph.

Theorem 3.9. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a path on $2m$ $(m > 1)$ vertices. If the membership value of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$ are respectively $c_1, c_2, c_3, \ldots, c_{2m-1}$ such that $c_1 < c_2 < c_3 < \ldots, c_{2m-1}$, then G is both strongly edge irregular fuzzy graph and strongly edge totally irregular fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a path on $2m$, $m > 1$, vertices. Let $e_1, e_2, e_3, \ldots, e_{2m-1}$ be the edges of a path G^* in that order. Let membership values of the edges $e_1, e_2, e_3, \ldots, e_{2m-1}$ are respectively $c_1, c_2, c_3, \ldots, c_{2m-1}$ such that $c_1 < c_2 < c_3 < \cdots < c_{2m-1}$, then

$$\begin{aligned}
d_G(v_i) &= c_{i-1} + c_i, && \text{for } i = 2, 3, 4, 5, \ldots, 2m-1,\\
d_G(v_1) &= c_1,\\
d_G(v_{2m}) &= c_{2m-1},\\
d_G(e_i) &= c_{i-1} + c_{i+1}, && \text{for } i = 2, 3, 4, 5, \ldots, 2m-2,\\
d_G(e_1) &= c_2,\\
d_G(e_{2m-1}) &= c_{2m-2},
\end{aligned}$$

hence G is strongly edge irregular fuzzy graph. Since

$$\begin{aligned}
td_G(e_i) &= c_{i-1} + c_{i+1} + c_i, && \text{for } i = 2, 3, 4, 5, \ldots, 2m-2,\\
td_G(e_1) &= c_2 + c_1,\\
td_G(e_{2m-1}) &= c_{2m-2} + c_{2m-1},
\end{aligned}$$

G is strongly edge totally irregular fuzzy graph. □

Theorem 3.10. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a cycle on n ($n \geq 4$) vertices. If the membership value of the edges $e_1, e_2, e_3, \ldots, e_n$ are respectively $c_1, c_2, c_3, \ldots, c_n$ such that $c_1 < c_2 < c_3 < \ldots, c_n$, then G is both strongly edge irregular fuzzy graph and strongly edge totally irregular fuzzy graph.*

Proof. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a cycle on n ($n \geq 4$) vertices. Let $e_1, e_2, e_3, \ldots, e_n$ be the edges of a cycle G^* in that order. Let membership values of the edges $e_1, e_2, e_3, \ldots, e_n$ are respectively $c_1, c_2, c_3, \ldots, c_n$ such that $c_1 < c_2 < c_3 < \cdots < c_n$, then

$$\begin{aligned}
d_G(v_i) &= c_{i-1} + c_i, && \text{for } i = 2, 3, 4, 5, \ldots, n,\\
d_G(v_1) &= c_1 + c_n\\
d_G(e_i) &= c_{i-1} + c_{i+1}, && \text{for } i = 2, 3, 4, 5, \ldots, n-1,\\
d_G(e_1) &= c_2 + c_n,\\
d_G(e_n) &= c_1 + c_{n-1},
\end{aligned}$$

hence G is strongly edge irregular fuzzy graph. Since

$$\begin{aligned}
td_G(e_i) &= c_{i-1} + c_{i+1} + c_i, && \text{for } i = 2, 3, 4, 5, \ldots, n-1,\\
td_G(e_1) &= c_2 + c_n + c_1,\\
td_G(e_n) &= c_1 + c_{n-1} + c_n,
\end{aligned}$$

G is strongly edge totally irregular fuzzy graph. □

Theorem 3.11. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a star $K_{1,n}$. If the membership values of no two edges are the same, then G is strongly edge irregular fuzzy graph and G is totally edge regular fuzzy graph.*

Proof. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices adjacent to the vertex x. Let $e_1, e_2, e_3, \ldots, e_n$ be the edges of a star G^* in that order having membership values $c_1, c_2, c_3, \ldots, c_n$ such that $c_1 \neq c_2 \neq c_3, \ldots, \neq c_n$. Then $d_G(e_i) = (c_1 + c_2 + \cdots + c_n) + c_i - 2c_i, (1 \leq i \leq n)$. $d_G(e_i) = (c_1 + c_2 + c_3 \cdots + c_n) - c_i, (1 \leq i \leq n)$. All edges $e_i, (1 \leq i \leq n)$ having distinct degrees. Hence G is strongly edge irregular fuzzy graph. Also, $td_G(e_i) = (c_1+c_2+\cdots+c_n)+c_i-c_i$, $1 \leq i \leq n$ and $td_G(e_i) = (c_1+c_2+c_3 \cdots+c_n)$, $1 \leq i \leq n$. All edges e_i, $1 \leq i \leq n$, having same total degree. Hence G is totally edge regular fuzzy graph. □

Theorem 3.12. *Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$, a Barbell graph $B_{n,m}$. If the membership values of no two edges are the same, then G is strongly edge irregular fuzzy graph and G is not strongly edge totally irregular fuzzy graph.*

Proof. Let $v_1, v_2, v_3, \ldots, v_n$ be the vertices adjacent to the vertex x. Let $e_1, e_2, e_3, \ldots, e_n$ be the edges incident with the vertex x in that order having membership values $c_1, c_2, c_3, \ldots, c_n$ such that $c_1 < c_2 < c_3, \ldots, < c_n$. Let $u_1, u_2, u_3, \ldots, u_n$ be the vertices adjacent to the vertex y. Let $f_1, f_2, f_3, \ldots, f_m$ be the edges incident with the vertex y in that order having membership values $h_1, h_2, h_3, \ldots, h_m$ such that $c_1 < c_2 < c_3 \cdots < c_n < h_1 < h_2 < h_3, \cdots < h_m < c$, where c is the membership value of the edge xy. Then $d_G(xy) = (c_1+c_2+c_3 \cdots+c_n)+c+(h_1+h_2+h_3 \cdots+h_m)+c-2c$ and

$$
\begin{aligned}
d_G(xy) &= (c_1 + c_2 + c_3 \cdots + c_n) + (h_1 + h_2 + h_3 \cdots + h_m), \\
td_G(xy) &= (c_1 + c_2 + c_3 \cdots + c_n) + (h_1 + h_2 + h_3 \cdots + h_m) + c, \\
d_G(e_i) &= (c_1 + c_2 + c_3 \cdots + c_n + c) + c_i - 2c_i, && 1 \leq i \leq n, \\
d_G(e_i) &= (c_1 + c_2 + c_3 \cdots + c_n + c) - c_i, && 1 \leq i \leq n, \\
d_G(f_i) &= (h_1 + h_2 + h_3 \cdots + h_m + c) + h_i - 2h_i, && 1 \leq i \leq m, \\
d_G(f_i) &= (h_1 + h_2 + h_3 \cdots + h_m + c) - h_i, && 1 \leq i \leq m.
\end{aligned}
$$

Note that every pair of edge having distinct degrees. Hence G is strongly edge irregular fuzzy graph. We have

$$
\begin{aligned}
td_G(e_i) &= (c_1 + c_2 + c_3 \cdots + c_n) + c + c_i - c_i, && 1 \leq i \leq n, \\
td_G(e_i) &= (c_1 + c_2 + c_3 \cdots + c_n) + c, && 1 \leq i \leq n, \\
td_G(f_i) &= (h_1 + h_2 + h_3 \cdots + h_m) + c + h_i - h_i, && 1 \leq i \leq m, \\
td_G(f_i) &= (h_1 + h_2 + h_3 \cdots + h_m) + c, && 1 \leq i \leq m.
\end{aligned}
$$

Note that all e_i, $1 \leq i \leq n$, having same total degrees and all f_i, $1 \leq i \leq n$, having same total degrees. Hence G is not strongly edge totally irregular fuzzy graph. □

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