ABSTRACT

A new simple approach in the design of digital algorithms for simultaneous reactive power and frequency estimation of local system under non-sinusoidal conditions and wide-range frequency deviations is presented. The algorithm is derived using weighted-least square (WLS) method. Cascade finite-impulse-response (FIR) comb digital filters are used to minimize the noise effect and to eliminate the presence of harmonic effects. The most important significance of this paper is mathematical model which transforms the problem of estimation into an over determined set of linear equations. The algorithm shows a very high level of robustness as well as high measurement accuracy over a wide range of frequency changes. To demonstrate the performance of the developed algorithm, computer simulated data records are processed.

Key words: Reactive power measurement, frequency measurement, parameter estimation, recursive algorithm, weighted-least square (WLS), finite-impulse-response (FIR) filter, digital phase shifter.

1. INTRODUCTION

Design and implementation of reactive power measurement instruments is currently dictated by the strong demands to the electrical energy savings in the transmission and distributions systems. The reactive power influences directly to the power factor and as a result overloads the transmission lines between the electrical energy sources and energy users and plays a vital role in the stable operation of power systems.

Transmission of electrical energy to loaders is performed at fundamental (system) frequency. In system management, the term fundamental (system frequency) reactive power appears to be the most appropriate indicator for active/reactive energy generation, flow, and absorption. It has a strong impact on the voltage profile and on the rms value of the line current. The nonsinusoidal reactive powers proposed to date would not help in determining, for example, how to control the system voltage through reactive power injection. The fundamental reactive power is usually the dominant elementary reactive power component. It is recommended to be measured separately from the rest of reactive power terms [1].

It should be noted that no reactive power meters available on the market, apart from expensive
harmonic power analyzers, specifically measures the fundamental reactive power [2].

Different approaches were proposed in the literature to the reactive power measurement. However, most of the algorithms for power measurements are sensitive to frequency variations. In order to solve this request, sophisticated and complex solutions have to be used.

One of numerical algorithms that consider the system frequency as an unknown parameter of the model to be estimated and, in this way, solve the problem of sensitivity to frequency variations in a wide range, has been presented in [3]. With the introduction of the power frequency in the vector of the unknown model parameters, the model itself becomes nonlinear and the strategies of nonlinear estimation must be used. In the first algorithm stage, the unknown signal parameters are estimated. By this, the voltage and current signals are processed independently on each other. In the second algorithm stage, the unknown power components are estimated. The implementation based on this technique generates accurate results but requires intensive computational effort.

In this paper, a simple algorithm for simultaneous estimation of the reactive power and frequency is prescribed. This paper proposes a general structure of a time-domain implementation of an accurate and computationally efficient algorithm for the fundamental reactive power measurement. The investigation has been simplified based on work published in [4], addressing the problem of simultaneous active power and frequency estimation. Consequently, this estimation problem has been transformed into an over determined set of linear equations and least square style weighted-least-square (WLS) algorithm has been employed. In addition, since the frequency of the signal is being estimated during measurement, it is not necessary to perform frequency insensitive phase shifting. It is easier to adjust phase shifter parameters in accordance to the actual frequency.

\[ v_{l,k} = V_1 \sin(\omega_1 k \Delta t + \gamma_1) \]  \hspace{1cm} (1)

\[ i_{l,k} = I_1 \sin(\omega_1 k \Delta t + \gamma_1) \]  \hspace{1cm} (2)

where \( V_1 \) and \( I_1 \) are amplitudes of the fundamental component of the voltage and current, respectively.

It follows that the instantaneous fundamental power is

\[ p_{l,k} = v_{l,k} i_{l,k} = P_1 - S_1 \cos(2\omega_1 k \Delta t + \lambda_1) \]  \hspace{1cm} (3)

where

\[ S_1 = V_1 I_1/2, \quad P_1 = S_1 \cos \phi_1, \quad \phi_1 = \gamma_1 - \gamma_1, \quad \lambda_1 = \psi_1 + \gamma_1, \quad (\omega_1 = 2\pi f_1. \quad f_1 \text{ is the fundamental frequency}) \]

The fundamental reactive power \( Q_1 \) could be calculated by the multiplication of the voltage and current signals defined as follows:

\[ v'_{l,k} = V_1 \sin(\omega_1 k \Delta t + \psi_1) \]  \hspace{1cm} (4)

\[ i'_{l,k} = I_1 \sin(\omega_1 k \Delta t + \psi_1) \]  \hspace{1cm} (5)

where the phases of the voltage and the current are changed in a way such that an additional phase shift of \(-\pi/2\), is introduced, that is \( \psi_1' = \psi_1 - \pi/2 \).

By multiplying the voltage and current signals defined in (4) and (5), respectively, it follows that:

\[ p'_{l,k} = v'_{l,k} i'_{l,k} = Q_1 - S_1 \cos(2\omega_1 k \Delta t + \lambda_1') \]  \hspace{1cm} (6)

\[ S_1 = V_1 I_1/2, \quad Q_1 = S_1 \cos \phi'_1, \quad \phi'_1 = \psi_1' - \pi/2, \quad \lambda_1' = \psi_1' + \gamma_1' \]

The three consecutive samples of the instantaneous fundamental power signal (6) are connected with the following relationship [4]:

\[ y_k = \mathbf{z}_k^T \mathbf{x}_k \]  \hspace{1cm} (7)

where

\[ y_k = p'_{l,k} + p'_{l,k-2}, \quad \mathbf{z}_k = \begin{bmatrix} 2 \\ -2 p'_{l,k-1} \end{bmatrix} \]

\[ \mathbf{x}_k = \begin{bmatrix} x_{l,k} \\ x_{2,k} \end{bmatrix} = \begin{bmatrix} Q_1[1 - \cos(2\omega_1 k \Delta t)] \\ \cos(2\omega_1 k \Delta t) \end{bmatrix} \]

By substituting \( k = 1, 2, \ldots , K \) (K>2) in (7) yield the following system of linear equations:

\[ \mathbf{y} = \mathbf{Zx} \]  \hspace{1cm} (8)

where
The forgetting factor (0<\phi<1) is involved to give greater impact to the newer samples. The model is linear and a linear algorithm for parameter estimation is used. The system (9) is over determined, i.e. the number of equations is much larger then the number of the variables. Therefore, the recursive WLS algorithms can be used.

Having obtained vector \( x_K \), the frequency and fundamental reactive power are then derived by following equations:

\[
f_{1,k} = \frac{\arccos(x_{1,k})}{4 \pi \Delta t}
\]

\[
Q_{1,k} = \frac{x_{1,k}}{1-x_{2,k}}
\]

It can be noticed that the algorithm is very simple and suitable for implementation.

The accuracy of this algorithm depends on the forgetting factor \( \phi^2 \). A higher value of \( \phi^2 \) means better accuracy but low convergence rate. The compromise is made by performing the adaptation during the estimation process. In order to improve dynamic properties of the algorithm, instead of setting the forgetting factor to a specific constant value, the factor is heuristically tuned to the system dynamics, providing the algorithm with faster convergence and better accuracy. A forgetting factor tuning procedure is implemented by introducing a suitable tuning function in [4], [5]. In that case, \( \phi^2 \) can be calculated as follows:

\[
\phi^2 = \phi_{\text{min}} + \frac{\phi_{\text{max}} - \phi_{\text{min}}}{1 + |R_K / R_0|^p}
\]

where \( R_K \) is the covariance of the estimation error in the step \( K \) and \( \phi_{\text{min}}, \phi_{\text{max}}, R_0 \) and \( p \) are the chosen values. These parameters have to be heuristically selected, depending on the signal parameters dynamic.

We can define a covariance of the estimation error as follows:

\[
R_K = e_k^T e_k = (y_K - Z_k x_k)^T (y_K - Z_k x_k)
\]

where \( e_k = y_k - Z_k x_k \)

Using the forms of matrices \( y \) and \( Z \) that are given in (9), we obtain:

\[
R_K = \phi^2_{K-1} R_{K-1} + (y_K - Z_k^T x_k)^2
\]

The Fig. 1 shows the global block diagram of the algorithm. The input current \( i_k \) and voltage \( v_k \) are filtered by band-pass filters and phase shifter and multiplied in order to calculate instantaneous power \( p'_{1k} \), which is brought to the input of the block of estimation algorithm. As a result, the fundamental reactive power and frequency are obtained.

**Fig. 1. Block diagram of measurement (estimation) of fundamental reactive power and frequency**

### 3. ADAPTIVE FILTERING OF POWER SYSTEM SIGNALS

A number of measurement algorithms apply orthogonal signal components obtained by two orthogonal FIR filters. The frequency response of the filters must have nulls at the higher order harmonic frequencies that are expected to be present in the signal and must have a unity gain at the main harmonic frequency. In a case of time-varying frequency, the filter parameters have to be adapted during frequency estimation.

The cascade structure is a particularly useful one for implementing FIR filters. In general, the idea is to search the second-order subsections that eliminate the DC component and all harmonic except the measured one for which has to have unity gain [6], [7]. The complete filter can be realized as cascade of all these subsections.

The second order subsection that eliminates DC component and frequency \( \omega/2 = \pi / \Delta t \) and has a unity gain at the fundamental frequency \( \omega_1 \) is given by the following Z-domain transfer function [6]

\[
H_{10}(z) = \frac{1 - z^{-2}}{1 - z_1^{-2}},
\]

where \( |1 - z_1^{-2}| = 2 \sin (\omega_1 \Delta t) \), \( z^{-1} = e^{-j2\pi \omega / f_s} \), \( \omega = 2\pi f \), \( f_s \) is the sampling frequency, \( \Delta t = 1 / f_s \), and...
\[ z_{i}^{l} = e^{-j2\pi f_{i}^{l} T_{0}}, \quad \omega_{i} = 2\pi f_{i}, \quad f_{s}, f_{1} \text{ is the fundamental frequency.} \]

The subsection that rejects the harmonic \( \omega_{i} \) and has a unity gain at the fundamental frequency \( \omega_{1} \) is shown as follows

\[
H_{i}(z) = \frac{1-2\cos(\omega_{i} T)z^{-1} + z^{-2}}{1-2\cos(\omega_{1} T)z^{-1} + z^{-2}} \quad i = 2, 3, \ldots, M 
\] (17)

where

\[ |1-2\cos(\omega_{i} T)z^{-1} + z^{-2}| = 2|\cos(\omega_{1} T) - \cos(\omega_{i} T)| \]

is used to adjust the gain. \( M \) denotes the maximum integer part of the \( fs/(2f_{1}) \). It is equal to the number of the subsections in the cascade.

The transfer function of the complete filter is given as follows

\[
H_{i}(z) = H_{10}(z) \prod_{i=2}^{M} H_{i}(z) 
\] (18)

It can be seen that calculation of the filter coefficients can be easily performed if the fundamental frequency \( \omega_{i} \) is known. Therefore, this design algorithm is convenient for the use in our algorithm where \( \cos(2\omega_{i} T) \) is estimated directly. In this case, the \( \cos(i\omega_{1} T) \) can be easily calculated using trigonometric formulas.

Due to its cascade structure, the filter (18) is convenient for designing different variations by adding or omitting certain subsections.

The frequency responses of the filters for the first harmonic for different fundamental frequencies and a sampling frequency of \( f_{s}=800 \text{ Hz} \) are shown in Fig. 2.

![Fig. 2. Frequency responses of the first harmonic filter for different fundamental frequencies and a sampling frequency of \( f_{s}=800 \text{ Hz} \)](image)

The measurement of the reactive power under sinusoidal conditions, using the instantaneous fundamental power signal (6), requires the introduction of an additional phase shift of \( \pi/2 \) between the voltage and current signals. This phase shift should be equal to \( \pi/2 \) exactly and should be independent on the frequency of the input signals. A phase shift corrector with the following transfer function can perform a phase shift of \(-\pi/2\) on the fundamental frequency [6]:

\[
H_{PHSH}(z) = \frac{-\cos(\omega_{1} T) + z^{-1}}{\sin(\omega_{1} T)} \quad \frac{\pi}{2} 
\] (19)

The cascade connection of the phase shift corrector (19) and filter (18) gives a new filter that, together with filter (18), forms an orthogonal pair for the fundamental frequency [6]:

\[
H_{2}(z) = H_{PHSH}(z, -\frac{\pi}{2})H_{0}(z) \prod_{m=2}^{M} H_{m}(z) 
\] (20)

4. PERFORMANCE EVALUATION THROUGH SIMULATION

Testing of the proposed numerical algorithm is performed using computer simulations by standard test signals. A sampling frequency of \( f_{s}=800 \text{ Hz} \) has been chosen. Using static tests, it has been established that zero errors have been occurred. The following algorithm parameters were used: \( \varphi_{\min} = 0.5 \), \( \varphi_{\max} = 0.999 \), \( R_{0} = 10^{-6} \), \( p = 2 \).

In order to test dynamic characteristic of the algorithm, the following tests have been performed:

**Test #1: Disturbance – Step change of the frequency**

\[
u(t) = 100 \sin \omega_{1} t \]
\[
i(t) = \sin(\omega_{1} t + \pi/2) \quad f = 50 \text{ Hz} \]
\[
t \leq 0s \quad t > 0s
\]

**Test #2: Disturbance – Step change of the voltage amplitude**

\[
u(t) = \begin{cases} 100 \sin \omega_{1} t & t \leq 0s \\ 50 \sin \omega_{1} t & t > 0s \end{cases} \]

\[
i(t) = \sin(\omega_{1} t + \pi/2), \quad f = 50 \text{ Hz}
\]

**Test #3: Disturbance – Step change of the current phase shift**

\[
u(t) = 100 \sin \omega_{1} t \]
\[
i(t) = \begin{cases} \sin \omega_{1} t & t \leq 0s \\ \sin(\omega_{1} t + \pi/3) & t > 0s \end{cases} \quad f = 50 \text{ Hz}
\]
Test #4: Disturbance – Continuous change of the voltage amplitude

\[ u(t) = \begin{cases} 
100 \sin \omega t & t \leq 0s \\
(1-0.5\sin \pi t) \sin \omega t & t > 0s 
\end{cases} \]

\[ i(t) = \sin (\omega t + \pi/2), f = 50Hz. \]

Test #5: Disturbance – Continuous change of the frequency

\[ u(t) = 100 \sin \omega t \]

\[ i(t) = \sin (\omega t + \pi/2) \]

\[ f = \begin{cases} 
50Hz & t \leq 0s \\
50 + \sin (10 \pi t) & t > 0s 
\end{cases} \]

The white noise with a Signal-to-Noise-Ratio (SNR) of 60 dB was added to sinusoidal signals. As shown in Table I and Fig. 3, the algorithm has possibility to trade a speed for accuracy by specifying the forgetting factor \( \phi^2 \). The estimation can be either fast and less accurate (for smaller \( \phi^2 \)) or slow and more accurate (for higher \( \phi^2 \)). A compromise between accuracy and convergence for the presented example should be achieved in the adaptation of \( \phi^2 \) during estimation procedure. This way, we have obtained a technique that provides accurate estimates for SNR=60 dB with the frequency and reactive power errors in the range of 0.002 Hz and 0.03%, respectively, in about 25 ms and requires modest computations.

### Table I. Maximum errors and convergence times

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Maximum frequency estimation error</th>
<th>Maximum power estimation error</th>
<th>Convergence time</th>
</tr>
</thead>
<tbody>
<tr>
<td>ML((\phi^2=0.5))</td>
<td>0.0172 Hz</td>
<td>0.088%</td>
<td>26 ms</td>
</tr>
<tr>
<td>ML((\phi^2=0.8))</td>
<td>0.0104 Hz</td>
<td>0.073%</td>
<td>55 ms</td>
</tr>
<tr>
<td>ML((\phi^2=0.9))</td>
<td>0.0066 Hz</td>
<td>0.053%</td>
<td>100 ms</td>
</tr>
<tr>
<td>ML((\phi^2=\text{adapted}))</td>
<td>0.0011 Hz</td>
<td>0.030%</td>
<td>28 ms</td>
</tr>
</tbody>
</table>

**Fig. 3.** Estimations for Test #1 (a) without noise presence, and (b) with SNR=60 dB
Fig. 4. Parameter estimations for Test #2 with SNR=60 dB

Fig. 5. Parameter estimations for Test #3 with SNR=60 dB

Fig. 6. Parameter estimations for Test #4 with SNR=60 dB
The ability of the frequency and reactive power estimation over a wide range of frequency changes is investigated using sinusoidal test signals with the following time dependence: \( f(t) = 50 + \sin(10 \pi t) \). The estimation is computed for \( \phi^2 \in \{0.5, 0.8, 0.9\} \) and shown in Fig. 7. Good dynamic responses and high accuracy of the reactive power and frequency measurement can be noticed.

Fig. 8 shows the maximum errors observed in frequency and reactive power estimates when input signals of 30, 50 and 70 Hz having SNR in the range of 40–80 dB were used. In case of signals with low signal-to-noise ratio (SNR), the accuracy of the algorithm could further be improved by oversampling of the input signal combined with high-order antialiasing filters and decimation or/and with interpolation and anti-imaging filters [8]. Oversampling of the analog signal has become popular in the DSP industry to improve resolution of analog-to-digital conversion (ADC). Oversampling uses a sampling rate, which is much higher than the Nyquist rate. It reduces the ADC noise floor with possible noise shaping so that a low-resolution ADC can be used. Each doubling of the sampling rate increases the resolution by a half bit [8].

To demonstrate the effectiveness of the proposed technique in the presence of harmonics, an input signal having the fundamental frequency of 30, 50 and 70 Hz, a third harmonic component in the range of 0%–20%, and a fifth harmonic component equal to half of the third component have been used. Fig. 9 shows the maximum errors for the proposed technique. The errors of the estimated values are nearly zero, despite the presence of harmonic components.
5. CONCLUSION

The design of a novel recursive method for the measurement of the reactive power and local system frequency has been described in the paper. The proposed technique is suitable for measurement for a wide range of frequency changes. It is useful in designing microprocessor-based devices that need accurate and fast measuring of power system parameters. The chosen model is linear and a linear algorithm for parameter estimation has been used. Therefore, the derived algorithm is very simple and requires modest resources for implementation.

The simulations have shown that proposed technique provides accurate estimates. The obtained results confirm a good dynamic response of the algorithm as well as accuracy of the reactive power and frequency estimations. The algorithm calculates a new estimate of the quantities at every sampling interval irrespective of the phase of the input signal. The algorithm is capable of accurately tracking frequency under dynamic power system conditions and provides high-speed measurement, exhibiting a delay of only 25 ms. The algorithm has shown an immunity to the presence of harmonics.

6. REFERENCES


