A New Control Algorithm for Active Suspension Systems Featuring Hysteresis

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In this paper the vibration suppression problem relative to the vertical motion of road vehicles is investigated. In particular, the goal of the research is to design control laws for active suspension systems of road vehicles in order to reduce unwanted vibrations induced by a rough road profile. The road vehicle has been represented with a quarter-car model whereas the suspension system has been schematized using the Bouc-Wen hysteresis model. In addition, a control actuator has been collocated between the sprung mass and the unsprung mass. Since the problem under study is nonlinear, the time history of the control action has been obtained numerically using the iterative adjoint-based control optimization method. Simulations show that the designed control action significantly mitigates the vibrations induced by the road profile.

Keywords: *Nonlinear Dynamics, Quarter-Car Model, Bouc-Wen Hysteresis Model, Vibrations Suppression, Iterative Adjoint-Based Control Optimization.*

1. INTRODUCTION

Suspension systems of road vehicles are needed to guarantee a good ride quality for the vehicle occupants and at the same time to obtain a good vehicle stability [1-2]. Typically, suspension systems are realised using passive components. The design of a passive suspension system is performed making a trade-off between these two conflicting goals, namely the demand of a good ride quality and the need of a good vehicle stability [3-4]. The main drawback of a passive suspension system is that the suspension force can be only a function of the relative displacement and of the relative velocity between the sprung mass and the unsprung mass. On the other hand, active suspension systems can improve the performance of road vehicle suspensions circumventing these design constraints [5-6].

In this paper a new method to control nonlinear underactuated mechanical systems has been developed. This method originated from optimal control theory [7-10] and it is based on the iterative adjoint-based control optimization algorithm [11-17]. The proposed method has been applied to design control laws for active suspensions system of road vehicles in order to mitigate road-induced vibrations. The hysteresis phenomenon of the suspension system has been represented using Bouc-Wen hysteresis model.

2. SYSTEM MODEL

2.1 System Description

The system under study is a quarter-car model characterised by an hysteretic suspension system and equipped with an active controller showed in figure 1.

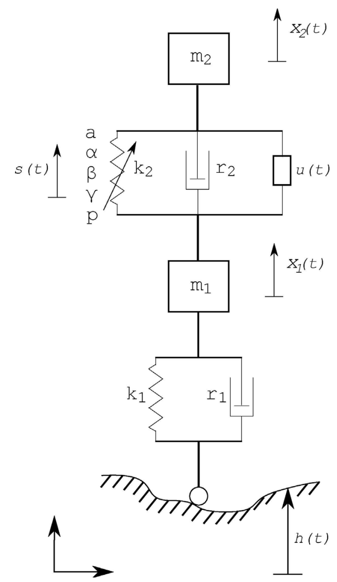

Figure 1. Quarter-Car Model with Hysteretic Suspension System and Active Controller

The displacement of the unsprung mass is denoted with x_1 whereas the displacement of the sprung mass is denoted with x_2. The unsprung mass is denoted with m1 whereas the sprung mass is denoted with m_2. The tyre stiffness is denoted with k1 whereas the tyre damping is denoted with r_1. The suspension system is realised by a nonlinear device interposed between the sprung mass and the unsprung mass. The suspension stiffness is denoted with k_2 whereas the suspension damping is denoted with r_2. The nonlinear suspension device provides a nonlinear elastic force field and a linear dissipative force field. The nonlinear suspension device is an hysteretic suspension system which is described using the Bouc-Wen model of hysteresis [18-20]. According to the Bouc-Wen model, the restoring force $\varphi(\Delta_2(t), s(t), t)$ can be expressed as:

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$$\varphi(\Delta_2(t), s(t), t) = -\alpha k_2 \Delta_2(t) + \\ -(1-\alpha) k_2 s(t) \quad (0.1)$$

where $\Delta_2(t)$ is the relative displacement between the sprung mass and the unsprung mass, $s(t)$ is a function of time which denotes the hysteretic displacement and α denotes the ratio of the post-yield to pre-yield stiffness. The time evolution of the hysteretic displacement is described by the following differential equation:

$$\dot{s}(t) = \psi(\Delta_2(t), s(t), t) \quad (0.2)$$

The nonlinear function $\psi(\Delta_2(t), s(t), t)$ is defined as:

$$\psi(\Delta_2(t), s(t), t) = \\ = (a - (\beta\,\mathrm{sign}(s(t)\dot{\Delta}_2(t)) + \gamma) \cdot \\ \cdot (s(t)\,\mathrm{sign}(s(t)))^p)\dot{\Delta}_2(t) \quad (0.3)$$

where a, β, γ, and p are dimensionless parameters which regulate the shape of the hysteresis loops. Linearising the system around its stable equilibrium configuration, the resulting system natural frequencies are denoted respectively with $f_{n,1}$ and $f_{n,2}$ whereas the resulting system damping ratios are denoted respectively with ξ_1 and ξ_2. Considering a worst-case scenario, the quarter-car system is excited by a road profile h(t) which is assumed as a superposition of two harmonic displacements whose harmonic content is close to the linearised system natural frequencies. Indeed:

$$h(t) = H_1 \sin(2\pi f_1 t) + H_2 \sin(2\pi f_2 t) \quad (0.4)$$

where H_1 and H_2 denote the amplitudes of the road roughness whereas f_1 and f_2 denote the frequencies of the road roughness. A detailed list of all system data is reported in table 1.

2.2 Equations of Motion

The configuration of the system can be defined using a set of n2=2 degrees of freedom. Indeed, system generalized coordinates can be grouped in a vector q(t) as:

$$q(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} \quad (0.5)$$

where $x_1(t)$ denotes the displacement of the unsprung mass and $x_2(t)$ denotes the displacement of the sprung mass. System equations of motion can be derived using Lagrangian Dynamics [21-22] to yield:

$$M(q(t), t)\ddot{q}(t) = \\ = Q(q(t), \dot{q}(t), s(t), h(t), \dot{h}(t), t) \quad (0.6)$$

where $M(q(t), t)$ is system mass matrix and $Q(q(t), \dot{q}(t), s(t), h(t), \dot{h}(t), t)$ is the vector of generalised forces acting on the system. For the problem on hand, the system mass matrix and the vector of generalised forces are defined as follows:

$$M(q(t), t) = \begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \quad (0.7)$$

$$Q(q(t), \dot{q}(t), s(t), h(t), \dot{h}(t), t) = \\ = \begin{bmatrix} -k_1(x_1(t) - h(t)) - \varphi(\Delta_2(t), s(t), t) + \\ -r_1(\dot{x}_1(t) - \dot{h}(t)) - r_2(\dot{x}_1(t) - \dot{x}_2(t)) \\ \\ \varphi(\Delta_2(t), s(t), t) + r_2(\dot{x}_1(t) - \dot{x}_2(t)) \end{bmatrix} \quad (0.8)$$

If a control action $Q_c(u(t), t)$ is introduced on the system, the matrix form of system equations of motion becomes:

$$M(q(t), t)\ddot{q}(t) = \\ = Q(q(t), \dot{q}(t), s(t), h(t), \dot{h}(t), t) + \\ + Q_c(u(t), t) \quad (0.9)$$

For the system under study, the control input $u(t)$ is a force acting between the sprung and the unsprung mass. Therefore, the control action can be written as:

$$Q_c(u(t), t) = B_2(t) u(t) \quad (0.10)$$

where $B_2(t)$ is a Boolean matrix defined as follows:

$$B_2(t) = \begin{bmatrix} -1 \\ 1 \end{bmatrix} \quad (0.11)$$

Since the rank of the Boolean matrix $B_2(t)$ is $b_2 = 1$ and it is lesser than the number of system degrees of freedom $n_2 = 2$, the quarter-car model is an underactuated mechanical system.

3. RESULTS AND DISCUSSION

3.1 Control Development

A regulation controller has been designed by using the iterative adjoint-based control optimization method [23-33]. The goal of the regulation controller is to reduce the vibration amplitudes of the sprung and unsprung masses induced by the road excitation. The synthesis of the regulation controller provides a feedforward control action and the corresponding evolution of the system state.

3.2 Regulation Controller Design

The regulation controller is an open-loop controller which has been designed using the iterative adjoint-based control optimization method. The control scheme which describes how the regulation controller acts on the system is represented in figure 2.

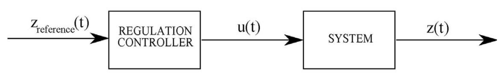

Figure 2. Control Scheme for Regulation Controller

The weight matrices which characterise the cost function have been defined as follows:

$$Q_T(T) = diag(10^4, 10^4, 10^4, 10^4, 10^4) \qquad (0.12)$$

$$Q_z(t) = diag(10^4, 10^4, 10^4, 10^4, 10^4) \qquad (0.13)$$

$$Q_u(t) = 10^{-4} \qquad (0.14)$$

where $Q_T(T)$ denotes the final state cost matrix, $Q_z(t)$ denotes the state cost matrix and $Q_u(t)$ denotes the input cost matrix. Figure 3 shows the iterative convergence towards the minimum of the cost function.

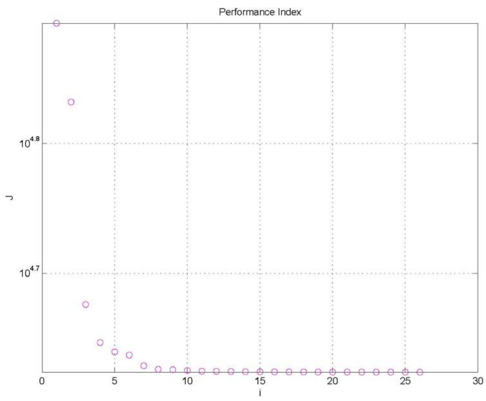

Figure 3. Cost Function - J

Figure 4 shows the time history of the resulting regulation controller.

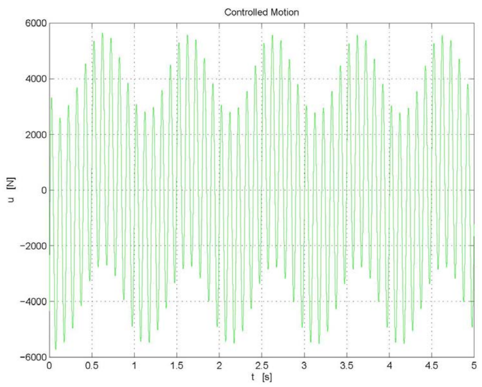

Figure 4. Regulation Controller - *u*(*t*)

Figure 5 represents the displacement of the unsprung mass when the system is uncontrolled.

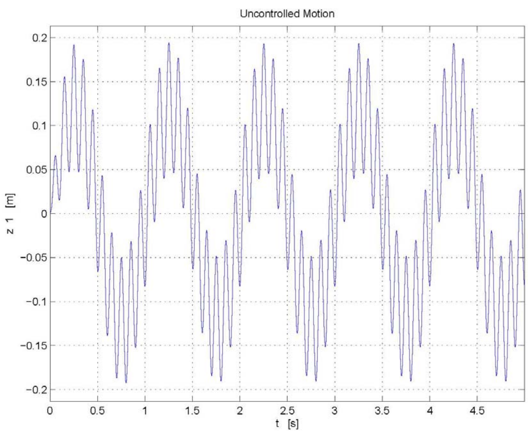

Figure 5. Uncontrolled Motion – x_1(*t*)

Figure 6 represents the displacement of the unsprung mass when the regulation controller acts on the system.

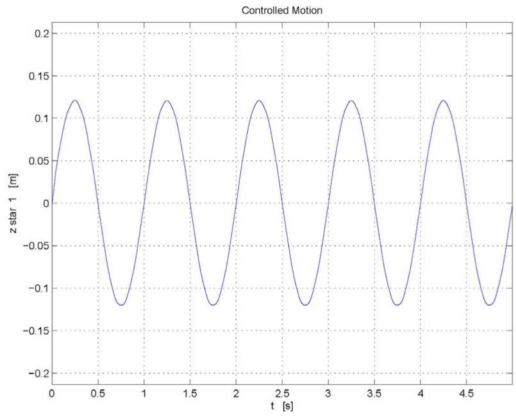

Figure 6. Controlled Motion - x_1(*t*)

Figure 7 represents the displacement of the sprung mass when the system is uncontrolled.

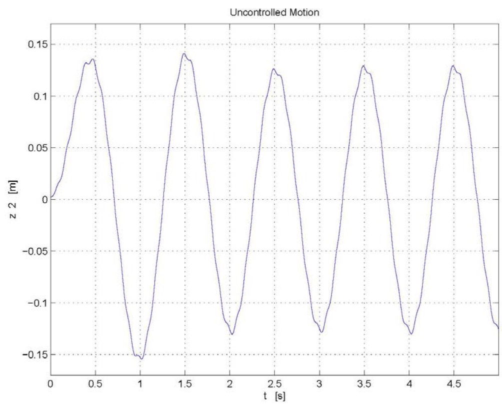

Figure 7. Uncontrolled Motion – x_2(*t*)

Figure 8 represents the displacement of the sprung mass when the regulation controller acts on the system.

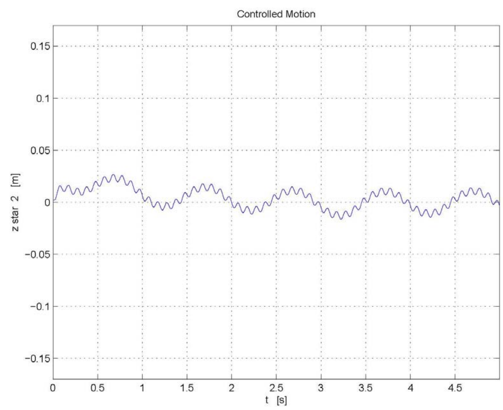

Figure 8. Controlled Motion - x_2(*t*)

These figures show that the effect of the action of the regulation controller is a considerable amplitude reduction of both the unsprung mass displacement and of the sprung mass displacement. The amplitude reductions of both displacements ε_{x_1} and ε_{x_2} can be quantified comparing the maximum amplitudes of system motion with and without the controller as follows:

$$\varepsilon_{x_1} = \frac{X_{1,u} - X_{1,c}}{X_{1,u}} = \mathbf{37.62\ \%} \qquad (0.15)$$

$$\varepsilon_{x2} = \frac{X_{2,u} - X_{2,c}}{X_{2,u}} = \mathbf{88.46\ \%} \qquad (0.16)$$

where $X_{1,u}$ and $X_{2,u}$ denote the maximum amplitudes of system steady-state displacements when there is no control action whereas $X_{1,c}$ and $X_{2,c}$ denote the maximum amplitudes of system steady-state motion when the regulation controller acts on the system.

4. CONCLUSIONS

Authors' research is focused on the development of new and effective methods to design control strategies for nonlinear mechanical systems [34-44]. In this paper authors propose a general and effective method to control nonlinear underactuated mechanical systems. The proposed method is based on the iterative adjoint-based control optimization algorithm. This method has been utilized to design control laws for active suspension systems of road vehicles which present the hysteresis phenomenon. The system analyzed has been idealized using a quarter-car model whereas the suspension systems has been schematized using the Bouc-Wen hysteresis model. The quarter-car system has been excited by a rough road profile which has been assumed as a superposition of two harmonic functions whose harmonic content is close to the linearized system natural frequencies. An active control system has been collocated between the sprung mass and the unsprung mass in order to reduce the amplitude of the vibrations caused by the roughness of the road profile. The problem on hand has been solved designing a regulation controller using the iterative adjoint-based control optimization method and the resulting regulation controller is an open-loop controller. The synthesis of the regulation controller provides a feedforward control action which drastically reduces the amplitude of the displacements relative to the sprung mass and to the unsprung mass. Authors believe that the proposed method represents a viable solution to control nonlinear underactuated mechanical systems.

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НОВИ АЛГОРИТАМ УПРАВЉАЊА АКТИВНИМ СИСТЕМИМА ВЕШАЊА СА КАРАКТЕРИСТИКАМА ХИСТЕРЕЗЕ

Domenico Guida, Carmine M. Pappalardo

У овом раду се истражује проблем пригушивања вибрација у односу на вертикално кретање друмских возила. Истакнуто је да је циљ истраживања пројектовање закона управљања за активне системе вешања да би се редуковале непожељне вибрације које изазива нераван профил друма. Друмско возило је представљено моделом четвртине возила, док је шема система вешања изражена коришћењем Бок-Вановог модела хистерезе. Поред тога, актуатор управљачког система је постављен између оптерећене и неоптерећене масе. Пошто је проблем истраживања нелинеаран, управљање по времену је

добијено нумерички применом итеративне методе оптимизације спрегнутог управљања. Симулације показују да пројектовано управљање знатно ублажава вибрације изазване профилом друма

Table 1. System Data.

DESCRIPTION	SYMBOLS	DATA [UNITS]
Unsprung Mass	m_1	$40\,[kg]$
Sprung Mass	m_2	$300\,[kg]$
Tyre Stiffness	k_1	$140000\,[kg \cdot s^{-2}]$
Suspension Stiffness	k_2	$12000\,[kg \cdot s^{-2}]$
Tyre Damping	r_1	$80\,[kg \cdot s^{-1}]$
Suspension Damping	r_2	$800\,[kg \cdot s^{-1}]$
Suspension Post-yield to Pre-yield Stiffness Ratio	α	$0.5\,[\backslash]$
Bouc-Wen Model Hysteresis Parameter	a	$1\,[\backslash]$
Bouc-Wen Model Hysteresis Parameter	β	$500\,[\backslash]$
Bouc-Wen Model Hysteresis Parameter	γ	$100\,[\backslash]$
Bouc-Wen Model Hysteresis Parameter	p	$2.5\,[\backslash]$
First Mode Natural Frequency	$f_{n,1}$	$0.9716\,[s^{-1}]$
Second Mode Natural Frequency	$f_{n,2}$	$9.7552\,[s^{-1}]$
First Mode Damping Ratio	ξ_1	$0.1870\,[\backslash]$
Second Mode Damping Ratio	ξ_2	$0.1825\,[\backslash]$
Ground Displacement Amplitude 1	H_1	$0.12\,[m]$
Ground Displacement Amplitude 2	H_2	$0.03\,[m]$
Ground Displacement Frequency 1	f_1	$1\,[s^{-1}]$
Ground Displacement Frequency 2	f_2	$10\,[s^{-1}]$
Initial Displacement of Unsprung Mass	$x_{1,0}$	$0.001\,[m]$
Initial Displacement of Sprung Mass	$x_{2,0}$	$0.002\,[m]$
Initial Velocity of Unsprung Mass	$v_{1,0}$	$0.03\,[m \cdot s^{-1}]$
Initial Velocity of Sprung Mass	$v_{2,0}$	$0.04\,[m \cdot s^{-1}]$
Initial Hysteretic Displacement	s_0	$0.001\,[m]$
Time Span	T	$5\,[s]$
Time Step	Δt	$0.001\,[s]$