A Class of Selective CIC FIR Filter Functions

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Abstract — A new class of modified selective multiplierless Cascaded-Integrator-Comb (CIC) finite impulse response (FIR) filter functions with application in modern communication systems, which have superior performances compared to the classical CIC filter functions are presented here. A few design examples that illustrate the properties of the proposed filter class are also presented. Novel filter functions have a minimum attenuation of 166.52 dB, 258.56 dB and 350.46 dB, respectively. The achieved improvements versus classical CIC filters are 29.84 dB, 34.90 dB and 39.82 dB, respectively.

Keywords — CIC filters, FIR filters, linear phase, multiplierless structure, selective filters.

I. INTRODUCTION

THE term “Cascaded-Integrator-Comb (CIC)” filters was first reported in the early 1980s by E.B. Hogenauer [1]. Software defined radio has found an important place in modern communication systems where the majority of signal processing is performed in the digital domain. In software radio systems, sample rate changes can be very large. Such a requirement leads to large order digital filters, which can easily become a bottleneck in the overall system design. A CIC finite impulse response (FIR) filter that uses only additions/subtractions can be used to reduce the computational demands. Many papers have addressed the problem of some modifications of the classical CIC filters for sample rate conversion in software defined radio systems [2]-[4]. A number of demonstrated characteristics make the structures proposed in [2]-[3] a good candidate for software defined radio applications. Modified CIC filters are ideal for applications in which high sampling rates make the use of multipliers a computationally expensive option. This technique is especially useful for FPGA design [4] where multipliers are avoided because of the large silicon area required. The use of CIC filters is also important in applications in which large rate change factors require large amounts of coefficient storage or fast impulse response generation and the memory is either unavailable or too slow to perform the desired application. Other solutions proposed in the literature give some modified filter structures for sigma-delta analog-to-digital converters [5]-[6].

Because of the disadvantages of a CIC FIR filter (a not flat passband and a high passband drop), some solutions given in the literature try to improve the classical CIC magnitude response characteristic. Some authors use a compensation filter in the cascade with the original filter [7]-[12]. Then, some other use sharpening technique [13] or design a new class of filter functions as shown in [14]-[18].

The aim of this paper is to present a novel class of CIC FIR filter functions which is based on cascading CIC sections of different lengths (non-identical CIC sections). Design examples of the novel designed class of CIC filter functions are used to validate their characteristics by comparing them with those of the classical CIC filters under fair conditions: the same number of cascaded sections and the same group delay. The results show a substantial performance improvement of the proposed filter class compared to the classical CIC filters.

II. CLASSICAL CIC FILTER

The conventional CIC FIR filters are well known in literature. The normalized CIC FIR filter function of one section in z -domain is defined with

\[ H(N, z) = \frac{1}{N} \left( 1 + z^{-1} + z^{-2} + \cdots + z^{-(N-1)} \right) = \frac{1 - z^{-N}}{N \cdot (1 - z^{-1})} \]

There are two basic building blocks: a comb and an integrator. The filter function of a comb in z -domain is

\[ H_c(N, z) = (1 - z^{-N}) \]

therefore a difference equation in time domain is

\[ y_c(n) = x(n) - x(n-N) \]

Function

\[ H_i(z) = 1/(1 - z^{-1}) \]

represents the filter function of an integrator in z -domain and

\[ y_i(n) = x(n) + y(n-1) \]

is its time domain representation.

A poor magnitude characteristic of the CIC filter composed of one section, Eq. (1), is improved by cascading several identical CIC filters. The classical CIC FIR filter function of normalized amplitude response characteristic, represented in the z -domain, is defined as

\[ H(N, K, z) = \left( \frac{1 - z^{-N}}{N \cdot (1 - z^{-1})} \right)^K \]

where \( N \) is the decimation factor, and \( K \) is the number of sections (identical cascaded CIC filters of one section) [1].
The frequency response characteristic of CIC FIR filter function can be written in the form
\[
H(N, K, z) = e^{j\omega_0} e^{-jK(N-1)\omega_0/2} \left( \sin(N\omega_0/2) \right)^K \left( N \cdot \sin(\omega_0/2) \right). \tag{3}
\]

III. NOVEL CLASS OF CIC FIR FILTER FUNCTIONS

The novel class is designed as a cascade of four non-identical CIC FIR filter functions; one function \(H(N-1, z)\), two functions \(H(N, z)\), and one function \(H(N+1, z)\), of the form given by (1), as well as seven cascaded non-identical CIC FIR filter sections which are repeated an integer number of times.

The filter function of normalized amplitude response characteristic of a designed novel class of CIC FIR filter functions can be written in a non-recursive form as follows
\[
H(N, K, L, z) = \left( \frac{1}{N+1} \sum_{r=0}^{\frac{N}{2}} z^{-r} \right) \left( \frac{1}{N-\frac{N}{2}} \sum_{r=0}^{\frac{N}{2}} z^{-r} \right) \left( \frac{1}{N+\frac{N}{2}} \sum_{r=0}^{\frac{N}{2}} z^{-r} \right) \left( \frac{1}{N-\frac{N}{2}} \sum_{r=0}^{\frac{N}{2}} z^{-r} \right)^L \tag{4}
\]

where \(N\) and \(L\) are free integer parameters, and \(K = 7L + 4\). This form is suitable for hardware realization, because it is unconditionally stable.

The novel class of CIC FIR filter functions is designed based on the classical CIC filters, by spreading the delays in the CIC filter comb stages. The delays are set around one specific value of \(N\). The recursive form of a novel class of CIC FIR filter functions with a normalized amplitude response characteristic is
\[
H(N, K, L, z) = \frac{1 - z^{-N+1}}{(N-1)(1-z^{-1})N(1-z^{-1})(N+1)(1-z^{-1})} \cdot \frac{1 - z^{-N}}{(N-3)(1-z^{-1})(N-2)(1-z^{-1})(N-1)(1-z^{-1})} \cdot \frac{1 - z^{-N}}{(N-1)(1-z^{-1})(N+2)(1-z^{-1})(N+3)(1-z^{-1})}^{L}, \tag{5}
\]

and \(K = 7L + 4\).

A. Frequency Response Characteristic

The frequency response of the designed novel class of FIR filter functions is obtained by evaluating the filter function in the \(z\)-plane at the sample points defined by setting \(z = e^{j\omega}\), where \(\omega = 2\pi f\) has units of radians per second. Using Euler’s identity, it can be separated into a real-valued magnitude \(A(N, K, L, \omega)\) and a real-valued phase angle \(\phi(N, K, L, \omega)\) for each frequency \(\omega\),
\[
H(N, K, L, z = e^{j\omega}) = e^{j\phi(N, K, L, \omega)} \cdot A(N, K, L, \omega) \tag{6}
\]

where the parameter \(K = 7L + 4\).

The normalized amplitude response characteristic of the proposed filter functions is defined in the form
\[
A(N, K, L, \omega) = \sin((N-1)\omega/2) \sin(N\omega/2) \sin(N+1)\omega/2) / ((N-1)\sin(\omega/2)N\sin(\omega/2)(N+1)\sin(\omega/2))
\]

\[
\sin((N-3)\omega/2) \sin((N-2)\omega/2) \sin((N-1)\omega/2) / ((N-3)\sin(\omega/2)(N-2)\sin(\omega/2)(N-1)\sin(\omega/2))
\]

\[
\sin(N\omega/2) \sin(N+1)\omega/2) / ((N+1)\sin(\omega/2)(N+2)\sin(\omega/2)(N+3)\sin(\omega/2))
\]

and \(K = 7L + 4\). \(7\)

The linear phase response characteristic of the proposed novel class of the modified CIC FIR filter has the form
\[
\tau(N, K, L, \omega) = -(N-1) \cdot K \cdot \omega / 2 + 2 \cdot \nu \cdot \pi, \quad \nu = 0, 1, 2, ..., \quad \text{and} \quad K = 7L + 4. \tag{8}
\]

It is independent of the frequency. For \(N\) being an odd number, the group delay is an integer multiple of \(K / 2\). If \(N\) is an even number, the group delay is equal to an integer plus half a multiple of \(K / 2\).

B. Selection of the Design Parameters

The choice of free integer parameters \(N\) and \(L\) is done in the same way as for CIC filters, there are the same restrictions on the group delay response. The parameter \(K\) can take different integer values, \(K = 7L + 4\).

The attenuation in the stopband region is closely related to the parameter \(L\). By increasing \(L\) for a constant value of \(N\), a higher stopband attenuation is achieved. The constant group delay \(\tau\) is equal for the classical CIC filters (Eq. (3)) and the novel modified CIC filter functions (Eq. (9)). The values of constant group delay for different values of parameters \(N\), \(L\), and \(K = 7L + 4\) are given in Table 1.

<table>
<thead>
<tr>
<th>(N)</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau[x])</td>
<td>(L = 1, K = 11)</td>
<td>27.5</td>
<td>33</td>
<td>38.5</td>
<td>44</td>
<td>49.5</td>
</tr>
<tr>
<td>(L = 2, K = 18)</td>
<td>45</td>
<td>54</td>
<td>63</td>
<td>72</td>
<td>81</td>
<td>90</td>
</tr>
<tr>
<td>(L = 3, K = 25)</td>
<td>62.5</td>
<td>75</td>
<td>87.5</td>
<td>100</td>
<td>112.5</td>
<td>125</td>
</tr>
</tbody>
</table>

IV. FREQUENCY RESPONSE CHARACTERISTICS

A few illustrative examples are designed and simulations are conducted in order to compare the frequency response characteristics of the classical CIC filter functions and the novel designed classes of selective CIC FIR filter functions. These functions are compared in fair conditions: the same level of constant group delay \(\tau\), as well as the number of delay elements. The novel filter
functions \( H(N, K, L, z) \) from Eq. (5) and the classical CIC filters \( H(N, K, z) \) from Eq. (2) have the same number of cascaded sections \( K \), with a difference that the CIC filters have an identical structure in all cascades, and the proposed functions have a cascade connections of non-identical CIC sections. The choice of free parameters \( N \) and \( L \) is done in the same way as for CIC filters, there are the same restrictions on the group delay response. The parameter \( K \) can take values \( K = 7L + 4 \). Generally, the main task of the comparison was to vary the free parameters and to compare obtained filter characteristics.

Fig. 1. Comparison of normalized magnitude response characteristics in dB of classical CIC filter (dashed lines), and proposed novel class of CIC FIR filter functions (solid lines), for \( N = 6 \).

A detailed analysis of the normalized magnitude response characteristics in dB, defined for the classical CIC filters as
\[
\alpha_{\text{CIC}}(f) = -20 \cdot \log_{10} |H(N, K, e^{j2\pi f})|
\]
and for the novel class of CIC FIR filter functions as
\[
\alpha(f) = -20 \cdot \log_{10} |H(N, K, L, e^{j2\pi f})|
\]
versus normalized frequency \( f = \omega/(2\pi) \), is depicted in Fig. 1. The maximum attenuation in the passband is \( \alpha_{\text{max}} = 0.28 \text{dB} \).

Fig. 2. Zooms of normalized magnitude response characteristics in dB of classical CIC filter (dashed lines), and proposed novel class of CIC FIR filter functions (solid lines), for \( N = 6 \).
Fig. 3. Zooms of normalized magnitude response characteristics in dB of classical CIC filter (dashed lines), and proposed novel class of CIC FIR filter functions (solid lines), for $K = N$.

The filter functions have the same level of constant group delay, as well as the number of delay elements, but the novel first designed class gives higher insertion losses in stopband, and it has higher selectivity.

Zooms of the normalized magnitude response characteristics of classical CIC filter and proposed class of CIC FIR filter functions are given in Figs. 2 and 3, for different values of parameter $N$. The novel class of CIC filter functions has two peaks in the transition area of the classical filter (at a frequency between the passband $f_{cp}$ and stopband $f_{cs}$ cut-off frequencies). Note that the attenuations of the novel class in the stopband area are higher than the attenuations of the classical CIC filter in the stopband area. For $N = 6$, in case of $K = 11$ (Fig. 2a), a classical CIC filter has an attenuation of 136.68 dB and a novel class 166.52 dB. An improvement of 29.84 dB or approximately about 21.8 % is achieved. In case of $K = 18$ (Fig. 2b), an improvement of 34.90 dB or approximately about 15.6 % is achieved. In case of $K = 25$ (Fig. 2c), a significant improvement of 39.82 dB or approximately about 12.8 % is achieved. For $N = 11$, in case of $K = 11$ (Fig. 3a), the achieved improvement is 43.19 dB or approximately about 33.73 %. In case of $K = 18$ (Fig. 3b), the improvement is 73.20 dB or about 33.08 %. For $K = 25$ (Fig. 3c), the improvement is 102.49 dB or approximately about 32.80 %. Note that for a higher filter order significant improvements of about 33 % are achieved. From Figs. 2 and 3, it is also evident that the normalized stopband cut-off frequencies are practically identical for different values of integer parameter $L$, but minimum attenuation increases rapidly by increasing its value.

The locations of zeros in the $z$-plane along with their multiplicities for classical and proposed filter functions are shown in Figs. 4 and 5, for $K = 18$ cascades and case of $N = 6$ and $N = 9$, respectively. The CIC filter function has $N - 1$ different zeros, $z_r = e^{j2\pi r/N}$, $r = 1, 2, ..., N - 1$. All zeros are multiple with multiplicity $K$ and lie on the unit circle. The total number of zeros is $N\cdot K$. The zeros of the proposed filter functions are more evenly distributed with their multiplicities and therefore reduced, as can be seen in Figs. 4b and 5b.

(a) Classical CIC filter

(b) Proposed filter, $L = 2$

Fig. 4. Locations and multiplicities of filter function zeros in $z$-plane for $N = 6$, and $K = 18$ cascades

(a) Classical CIC filter

(b) Proposed filter, $L = 2$

Fig. 5. Locations and multiplicities of filter function zeros in $z$-plane for $N = 9$, and $K = 18$ cascades.
Improvement of the attenuation in the stopband of the proposed novel class of CIC filter functions over classical CIC filters, versus filter order $N \in \{5 - 20\}$, obtained for different values of parameter $L$ is shown in Fig. 6. Data given in Fig. 6 show that the novel filter class has much better performances than the classical CIC filter, especially for lower filter orders $N \in \{5 - 13\}$.

Fig. 6. Improvement of the attenuation in the stopband of the proposed novel class over classical CIC filters.

Fig. 7 presents two-dimensional (2D) contour plots of normalized magnitude response characteristics (overall and lower frequency part zoomed) of the classical CIC filters and the proposed novel class of CIC FIR filter functions. As the value of the parameter $N$ increases, as well as the normalized frequency, the benefits of the proposed filter class become less apparent, and the characteristics closely resemble those of the classical CIC filters. Therefore, it can be concluded that the proposed filter class is more efficient in the lower part of frequency range and for smaller values of parameter $N$.

Fig. 7. 2D contour plots of magnitude frequency response characteristics for classical and proposed first class of CIC FIR filters for $N \in \{4 - 24\}$ and $K=18$.

In Fig. 8, a three-dimensional (3D) plot of normalized magnitude response characteristic of a novel class of CIC FIR filter functions is shown. The Figure shows a normalized magnitude response in frequency domain as a function of parameter $N \in \{4 - 17\}$, for case of $K=18$. It is worth noting that with the increase in the value of the parameter $N$ the passband becomes narrower, as is expected. The number of transfer function zeros is increased and this is clearly visible in the branching of high loss regions in magnitude response characteristics, especially for the smaller values of the parameter $N$ and towards higher frequencies.

V. CONCLUSION

This paper proposes a novel class of CIC FIR filter functions which is designed based on the classical CIC filters, by spreading the delays in the CIC filter comb stages. The delays are set around one specific value of filter order $N$.

An important measure of the superiority of the proposed filter class performances is to compare the proposed filter characteristics to the characteristics of the classical CIC filters. A way to do a fair comparison is to design the filter functions for the same value of the group delay. Performance comparisons are done with a few design examples. The differences of the attenuations of the proposed modified filter class and the classical CIC filters are given to show the achieved improvement of the attenuation in dB in the stopband.

The possibilities of the application of filters in modern communications systems are very wide. They can be used in software defined radio systems [2] - [3], FPGA design [4], sigma-delta analog-to-digital converters [5] - [6], [19] - [22], etc.

The novel CIC filter functions have integer coefficients of the impulse responses and linear phase response characteristics. For a proper understanding of complex phenomena in pharmacy, medicine and cosmos, it is necessary to use filters with a linear phase response and a
maximum possible attenuation in the stopband region. In some applications, the passband of the filter needs to be extremely small. New demands of high-resolution devices will be largely defined by the quality of filter function. In the 70's, professional equipment was able to perform measurements up to 160 dB.

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