GPSO ALGORITHM IN HCR GEARING GEOMETRY OPTIMIZATION

PRIMENA GPSO ALGORITMA ZA OPTIMIZACIJU PARAMETARA GEOMETRIJE HCR ZUPČASTIH PAROVA

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ABSTRACT

High contact ratio (HCR) gearing is a special kind of the basic involute profile gearing with higher contact ratio, since there are always at least two pairs of teeth in contact. Compared to standard gearings, HCR gearings have greater load capacity, higher resistance and a lower relative noise level. Their main drawbacks are more complicated design than standard involute profiles, greater predisposition for interference, pointed tip thickness and undercut of teeth during production. There is also a larger possibility of some interference or pointed tooth tip occurring. All these issues represent serious design constraints which need to be resolved in the design phase. In this paper, a new method for HCR gearing design is proposed, which uses Generalized Particle Swarm Optimization Algorithm (GPSO) for gearings parameters optimization. This recently introduced global optimization method enables finding of global optimal solution(s), ensuring that all relevant design and function constraints are satisfied.

Key words: gears, HCR gearing, geometry, optimization, GPSO algorithm.

REZIME

Standardni zupčasti parovi imaju tokom rada jedan ili dva zupca u zahvatu, odnosno prosečan broj zubaca u zahvatu je između jedan i dva. HCR zupčasti parovi predstavljaju posebnu, nestandardnu vrstu evolventnih zupčastih parova, kod kojih su u kontaktu uvek bar dva para zubaca, a jedinična visina zupca nije, kao kod standardnih zupčastih parova, jednaka jedinici, te je stoga i visina zubaca povećana. U poređenju sa standardnim, HCR zupčasti parovi dozvoljavaju veće opterećenje, otporniji su (pošto je opterećenje raspoređeno na više parova zubaca), pri radu stvaraju znatno manju buku i niži ukupni nivo vibracija, koji se dodatno mogu smanjiti ukoliko je broj zubaca u zahvatu celobrojna veličina. Navedene odlike HCR zupčastih parova omogućile su širok spektar potencijalne primene u inženjerstvu, pogotovo u situacijama kada je neophodno smanjiti masu ili dimenzije mehanizma za optimizacije rojem čestica. Ovaj nedavno predstavljeni globalni optimizacioni postupak omogućava pronađenje globalnog optimalnog rešenja i pokazao se veoma brzim i pouzdanim, obezbeđujući zadovoljenje svih relevantnih ograničenja po pitanju konstruisanja i funkcionalnosti.

Kljучне реčи: зуцаници, HCR зуцастые пары, геометрия, оптимизация, GPSO алгоритм.

INTRODUCTION

Gears are the oldest mechanical parts in mechanisms for power transmissions used in mechanical engineering. They were used every time when one wanted to transfer mechanical energy to a working machine. However, after a long period of using old-time power transmissions, modern gear technology was developed in the 20th century (Kuzmanović et al., 2010).

Standard gear transmissions are used with normal contact ratio, i.e. contact ratio between 1 and 2. There is a possibility of load sharing among the teeth, but there is a time during the mesh when one pair of teeth takes the entire load.

High contact ratio (HCR) gear pair has at least two pairs of teeth in contact. These gear pairs provide higher dynamic loads and reduced noise and vibration level. These characteristics enabled a broad field of potential engineering application of HCR gears, especially when decrease of transmission elements weight or dimensions is necessary, for example in aircrafts, passenger cars, ships and other transportation means.

It is well known that increasing the average number of teeth in contact leads to excluding or reduction of the vibration amplitude. First, it was established experimentally that dynamic loads decrease with increasing contact ratio in spur gearing (Kasuba, 1981). Moreover, in order to get a further reduction of the vibration, HCR gear profiles can be optimized. Sato et al. (Sato et al., 1983) found that HCR gears are less sensitive with respect to manufacturing errors. Moreover, they found that the best contact ratio, i.e. the average number of teeth in contact, should be about 2. This way, there are always exactly two pairs of teeth in contact, which means when one pair of teeth go out from the contact, another pair of teeth is coming in contact and applied force is considerably smaller since it is always divided on two pairs of teeth. Kahraman and Blankenship (Kahraman and Blankenship, 1999) published an experimental work on HCR gear vibration; they found that the best behavior is obtained with an integer contact ratio.

High contact ratio is obtained with increased addendum height, larger than in standard gearing. Proposed geometry of HCR gearings is very complicated due to the fact that there is a possibility of meshing occurrence during the production interference, much larger than interference happening in standard involute profiles. Also there is a higher risk of too small thickness of a tooth tip and significantly less favorable values of specific slips into the flanks (Kuzmanović et al., 2010). All these issues represent serious design constraints which need to be resolved in the design phase.
In this paper, a new procedure for HCR gear pairs design is proposed. It is based on global optimization and uses Generalized Particle Swarm Optimization Algorithm (GPSO) for gearing parameters optimization. This recently introduced global optimization method enables finding of global optimal values of gear geometry parameters, which enable desired contact ratio, ensuring at the same time that all relevant design and function constraints are satisfied.

**MATERIAL AND METHOD**

**Contact ratio of the standard involute gearing**

In Fig. 1, the driving pinion tooth is just coming into contact at point A with a tooth on a driven gear. The zone of action of meshing gear teeth shows that tooth contact begins and ends at the intersection of the two addendum circles with the line of action (points A and E).

The driving gear tip circle cuts the line of action across the point E, while the tip circle of the driven gear cuts this line across the point A. Before A, contact between the two teeth cannot take place as the driven gear does not exist. After the point E, the same thing occurs: The driven gear does not exist anymore and contact between teeth cannot take place. The straight line segment AE is called the path of contact. The segment AC is called the approach path while segment CE is called the recess path.

The distance between two corresponding flanks is called the base pitch. If the value of the base pitch is higher than the value of the path of contact, contact between the two following teeth does not occur, when the two mating teeth release, this continuity in meshing is not guaranteed. The correct working of the gear will be assured if the value of the path of contact is higher than that of the base pitch. The ratio between the length of contact $g_a$ and pitch on the base cylinder $p_{bt}$ given by the following formula (Bonfiglioli Riduttori, 1995):

$$\varepsilon_a = \frac{\text{length of contact}}{\text{base pitch}} = \frac{g_a}{p_{bt}}$$ (1)

is called the transverse contact ratio. So, contact ratio can be defined as the average number of teeth in contact at one time. It also results in the following equation (according to Fig. 1):

$$\varepsilon_a = \frac{g_a}{p_{bt}} = \frac{\sqrt{r_{a1}^2 - r_{bl}^2} + \sqrt{r_{a2}^2 - r_{b1}^2} - a_w \sin \alpha_{wl}}{P_{bt}}$$ (2)

where $r_{a1}$, $r_{a2}$, $r_{bl}$, and $r_{b1}$ are radiiuses of tip and base circles of driving and driven gears, respectively, $a_w$ is center distance and $\alpha_{wl}$ is working pressure angle (Bonfiglioli Riduttori, 1995). The tooth contact begins in A and terminates in E during the course of tooth action. In a pair of meshing spur gears, the line of contact along the width of the gears is parallel to the gear axes and shifts its position along the tooth profile curve from top to bottom region of tooth height or vice versa as the engagement proceeds during the course of action. Contact begins when the line of action intersect the tip circle of the driven gear. At the beginning of contact, the tooth of the driving pinion comes in contact with the top of the driven gear. The previous tooth pair is already in mesh so that the load is shared by these two pairs. This condition continues for a short time till the previous pair goes out of mesh. From this point onwards, the only pair takes the full load and continues to do so till a new pair comes in mating position. Thereafter, the load is again shared by the former pair and the new pair for a short while the former pair goes out of mesh. Contact ends when the line of action intersects the tip circle of the driver. At this point the tip of driver just leaves the flank of the driven gear.

At larger contact ratio than 1, there is possibility of load sharing among the teeth. For contact ratio between 1 and 2, which are common for standard spur gears, there will still be periods during the mesh when one pair of teeth takes the entire load. However, these will occur toward the center of the mesh region where the load is applied at a lower position on the tooth, rather than at its tip. This means that when one pair of teeth is just entering contact at A, another pair, already in contact, will not yet have reached point E. The minimum acceptable contact ratio for smooth operation is 1.2. Gears should not generally be designed having contact ratios less than about 1.2, because inaccuracies in mounting might reduce the contact ratio even more, increasing the possibility of impact between the teeth as well as an increase in the noise level. To ensure smooth and continuous operation, the contact ratio must be as high as possible, which the limiting factors permit. A minimum contact ratio of 1.4 is preferred, and larger is better. Contact ratios for conventional gearing are generally in the range 1.4 to 1.6 (Hasssan, 2009); so the number of tooth engagements is either one or two. For example, contact ratio of 1.6 means two pairs of teeth are in contact 60% of the time and one pair carries the load 40% of the time.

**HCR involute gear profile**

Special kind of basic involute profile of non-standard gearing is called high contact ratio (HCR) gearing, when the contact ratio is higher and there are always at least two pairs of teeth in contact ($\varepsilon_a \geq 2$) and where unit addendum height is not equal one like for standard gearing, so the tooth height is increased and it is bigger than one ($h_a > 1$). When HCR gearing is used, there is a greater risk of interference due to a greater height of tooth. Advantage of the HCR gearing is a higher resistance (load distribution is shared on the more pairs of teeth at the same time) and lower relative noise level of gearing, which can be significantly reduced by using integer HCR factor $\varepsilon_a$.

The value of contact ratio is the main indicator of HCR gearing, which differs from the commonly used standard profiles. The geometry of involute HCR gearing is presented on Fig. 2.

![Fig. 1. Contact ratio (Vereš et al., 2011)](image)

![Fig. 2. Geometry of involute HCR gearing (Vereš et al., 2011)](image)
comparing these both kind of gearing, it is clearly obvious that in the case of involute gearing the maximum force is applied when one pair of teeth is in contact (between points BD, Fig. 1), which can be considered 100 % of the value of the force $F$. The biggest applied force in the HCR gearing (between points BB’ and DD’) can be considered about 50 %, when two pairs of teeth are in contact. Consequently, the size of the applied force is decreased when three pairs of teeth are in contact. So, the value of involute gearing $2/3F$ is decreased to $1/3F$, and value $1/3F$ is decreased to $1/6F$, which means the load distribution is more favorable in the HCR gearing.

The possibilities of optimization of hcr geometrical parameters

According to results of different measurements of gear pair, reduction of noise and vibration proved to be best using HCR gearing with the value of contact ratio $e_0 = 2$ (Vereš et al., 2012). The increase in contact ratio can be implemented in two ways: by decreasing pressure angle and by increasing tooth height. Obviously, the use of a standard pressure angle and standard tools is preferable (Podzharov et al., 2003). Therefore, the most favorable solution is obtained by increasing addendum height, obviously, the use of a standard pressure angle and standard tools is preferable (Podzharov et al., 2003). Therefore, the most favorable solution is obtained by increasing addendum height, but however there are a lot of geometrical and manufacturing constraints that have to be satisfied, which limits increasing the contact ratio. The equation (1) shows that $e_0 = f(g_a, p_{ho})$. Tooth pitch on the base circle of standard involute gearing is equal to base pitch on HCR gearing, and it is considered as constant. This means that achieving the greatest value of the contact ratio $e_0$ has to be obtained by the greatest possible increase of length of line of action $g_a$. The length of line of action $g_a$ is calculated in following way:

$$g_a = \sqrt{r_{a1}^2 - r_{a2}^2} + \sqrt{r_{a1}^2 - r_{a2}^2} - a_n \sin \alpha_{var}$$  \hspace{1cm} (3)

where the tip diameters of driving and driven gear are as follows:

$$r_{a1} = r_t + \left(h_{a1} + x_1 \cdot m_a\right)$$  \hspace{1cm} (4)

$$r_{a2} = r_t + \left(h_{a2} + x_2 \cdot m_a\right)$$  \hspace{1cm} (5)

with $m_a$ being gear module. From equations (4) and (5), it is clear that the length of the line of action $g_a$ is directly dependent on the addendum height $h_{a1}$, $h_{a2}$ and factors of correction $x_1$, $x_2$. Optimization of geometric parameters of HCR gearing can be based on optimization criterion which provides the desired value of contact ratio ($e_0 = 2$) for a given centre distance $a_n$. The main optimization parameter at this level can be addendum heights of teeth $h_{a1}$, $h_{a2}$, and factors of correction $x_1$, $x_2$. For a given distance between centers of the wheels, a new parameter $x_1$ can be defined as relationship between $x_1$ and $x_2$, which additionally reduces the number of parameters.

Addendum heights $h_{a1}$ and $h_{a2}$ can be found from following equations:

$$h_{a1} = h_{a1}^* \cdot m_a$$  \hspace{1cm} (6)

$$h_{a2} = h_{a2}^* \cdot m_a$$  \hspace{1cm} (7)

where $h_{a1}^*$, $h_{a2}^*$ are addendum heights for module equal to one. Further it follows:

$$x_2 = x_1 - x_1$$  \hspace{1cm} (8)

So, that means contact ratio is the aim function of both addendum heights and correction factor of pinion $e_0 = f(h_{a1}^*, h_{a2}^*, x_1) = \max$, i.e. optimization parameters $h_{a1}^*, h_{a2}^*, x_1$ make a nonlinear optimization of triple constraint, with limitations requirements defined for (Kuzmanović, 2010): - removal of interference during production and meshing interference, - minimum arc thickness of the tooth tip $x_{a1,2}$.

All these constraints will be shortly described and mathematically defined, due to lack of space. For more detailed explanation, see (Kuzmanović, 2010; Vereš et al., 2009).

Interference during the production

This interference occurs in the production process of gear forming when the tooth of the rack tool is in collision with a produced transition curve of the gear wheel, resulting in a so-called undercut tooth.

This phenomenon largely depends on the method of manufacturing process. Unfavorable conditions arise with manufacturing by tool rack, so if it is not known in advance the means of production, the production interference of gearing for production by tool rack should always be checked.

Interference during the production will not occur if following conditions are satisfied (Vereš et al., 2009):

$$r_{a1} \cdot \tan \alpha_n - \frac{m_a}{\sin \alpha_n} (h_{a1}^* - x_1) \geq 0$$  \hspace{1cm} (9)

$$r_{a2} \cdot \tan \alpha_n - \frac{m_a}{\sin \alpha_n} (h_{a2}^* - x_1 + x_2) \geq 0$$  \hspace{1cm} (10)

where $\alpha_n$ is transverse pressure angle [13].

Meshing interference

Meshing interference is referred in the case of a collision between curves of teeth profiles as interference between these curves. It means that the meshing interference may occur as a collision of head of gear and the transition curve of pinion and /
or head of pinion and the transition curve of gear wheel (Vereš et al., 2009).
In order to avoid meshing interference, following conditions must be fulfilled:

\[ \sqrt{r_1^2 + (a_n \cdot \sin \alpha_{at} - g_{f1})^2} \geq r_1 \]  
\[ \sqrt{r_2^2 + (a_n \cdot \sin \alpha_{at} - g_{f2})^2} \geq r_2 \]

where \( g_{f1} \) and \( g_{f2} \) are interferences during the production (Vereš et al., 2009).

**Minimum thickness of the tooth head circle**
Changing addendum height, \( h_{at} \), will certainly influence the total thicknesses of the tooth on tip circle of \( S_{at} \) (drive gear) and \( d_{at} \) (driven gear). Greater tooth height, as well as a positive correction factor, may affect the thickness of the tooth on tip circle under the permissible value. Minimum tooth tip thickness is defined as [10, 13]:

\[ s_{ai} = 2r_{at} \left( \frac{s_{at} + \sin \alpha_{at} - \sin \alpha_{ai}}{2r_{at}} \right) \geq 0.4m_n \]  
\[ s_{a2} = 2r_{at} \left( \frac{s_{at} + \sin \alpha_{at} - \sin \alpha_{a2}}{2r_{at}} \right) \geq 0.4m_n \]  

**Generalized particle swarm optimization algorithm**
The Particle Swarm Optimization (PSO) algorithm is relatively novel, yet well studied and proven optimizer based on the social behavior of animals moving in large groups (particularly birds) (Kennedy and Eberhart, 1995). Compared to other evolutionary techniques, PSO has only a few adjustable parameters, and it is computationally inexpensive and very easy to implement (Ratnaveera et al., 2004).

PSO uses a set of particles called swarm to investigate the search space. Each particle is described by its position \( x \) and velocity \( v \). The position of each particle is a potential solution, and the best position that each particle achieved during the entire optimization process is memorized \( p \). In our example, position of the particle \( x \) is the vector of gear geometry parameters that need to be calculated, i.e.:

\[ x = \left[ h_{ai} \ h_{a2} \ x_1 \right] \]  

The swarm as a whole memorizes the best position ever achieved by any of its particles \( g \). The position and the velocity of each particle in the \( k \)-th iteration are updated as

\[ v[k+1] = w \cdot v[k] + c_p \cdot r_p \cdot (p[k] - x[k]) + c_g \cdot r_g \cdot (g[k] - x[k]) \]

\[ x[k+1] = x[k] + v[k+1] \]

Acceleration factors \( c_p \) and \( c_g \) control the relative impact of the personal (local) and common (global) knowledge on the movement of each particle. Inertia factor \( w \), introduced for the first time in (Shi and Eberhart, 1999), keeps the swarm together and prevents it from diversifying excessively and therefore diminishing PSO into a pure random search. Random numbers \( r_p \) and \( r_g \) are mutually independent and uniformly distributed on the range \([0, 1]\).

Particle swarm, however, has also some disadvantages. Most important of them is its inability to independently control various aspects of the search, such as stability, oscillation frequency and the impact of personal and global knowledge (Rapać and Kanović, 2009). The new algorithm, named Generalized PSO (GPSO), which is described and analyzed in details in (Kanović et al., 2011), overcomes the above mentioned flaw. It considers each particle within the swarm as a second-order linear stochastic system with two inputs and one output. The output of such a system is the current position of the particle \( x \), while its inputs are personal and global best positions \( p \) and \( g \), respectively. Such systems are extensively studied in engineering literature (Åström and Wittenmark, 1997). The stability and response properties of such a system can be directly related to its performance as an optimizer, i.e., its explorative and exploitative properties. This way, the above mentioned flaw of PSO can be overcome. GPSO algorithm proved to be very efficient for various engineering applications (Kanović et al., 2013; Kanović et al., 2014). GPSO uses canonical equation, often used in control theory (Åström and Wittenmark, 1997):

\[ x[k+1] = -2\zeta \rho \cdot [k+1] + \rho^{2} \cdot x[k] - [1 - (1 - c) \cdot g[k]] \]

where \( \rho \) is the eigenvalues module, and \( \zeta \) is the cosine of their arguments. Parameter \( c \) is introduced to replace both \( b_p \) and \( b_g \). The parameters in this equation allow a more direct and independent control of the various aspects of the search procedure.

Based on many analyses of PSO and its parameters, two parameters adjusting schemes, GPSO1 and GPSO2 were proposed in (Kanović et al., 2011). In both of them, it is recommended to linearly decrease \( \rho \) from about 0.95 to about 0.6, and \( c \) from about 0.8 to approximately 0.2. In the first scheme (GPSO1), \( \zeta \) was adopted as a stochastic parameter with uniform distribution ranging from -0.9 to 0.2, while in the second scheme (GPSO2), \( \zeta \) was uniformly distributed in the range \([-0.9, 0.6]\).

In the present research, GPSO algorithm was implemented in MATLAB, a high-performance language for technical computing, which integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. The algorithm was implemented as a function, which allows to user to specify the values of all adjustable parameters and to define many available algorithm options. The input parameters are the name of the function which is optimized, number of variables and algorithm parameters options, while output parameters are the optimal function value, its coordinates (i.e. the value of optimal parameters \( h_{ai}, h_{a2}, x_1 \)) and some other optional parameters.

**Optimization criterion formulation**
The goal of the optimization is to calculate gear construction parameters \( h_{at}, h_{a2} \) and \( x_1 \) which provide optimal value of the contact ratio, \( e_c = 2 \) and, at the same time, satisfy all constraints described by \((9) – (14)\). Therefore, the optimization criterion in this particular application was defined using penalty function method:

\[ F = (e_c - 2)^2 + \sum_{i=1}^{n} p_i C_i^2 \]

where \( p_i \) is penalty factor for \( i \)-th constraint, and \( C_i \) is the value of deviation for \( i \)-th constraints, calculated in following way

\[ C_i = \begin{cases} 0 & \text{if constraint is satisfied} \\ \text{deviation} & \text{if constraint is not satisfied} \end{cases} \]

Penalty functions apply for all constraints defined by \((9) – (14)\), and penalty factors are all set to 10, which, as experiments and calculation results proved, ensured convergence of optimization procedure and satisfaction of all constraints. From (18), it is obvious that best possible solution is obtained for \( F=0 \), which means that all constraints are satisfied and \( e_c=2 \).
RESULTS AND DISCUSSION

Using GPSo algorithm, a solution of HCR value is obtained. Calculations were conducted using 100, 200 and 500 iterations, and the obtained results are presented in Table 1. Solutions have high accuracy, which goes till 10^-15.

Given data:
\[ z_1 = 21; z_2 = 51; m_n = 4 \text{ mm}; \alpha_n = 144 \text{ mm}; \]
\[ \varepsilon = 0.349065850398866 \text{ rad}, \beta = 0 \]

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<th>( \hat{h}_1 )</th>
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</table>

One can notice that even 100 iterations provide satisfactory results, with 10^-7 accuracy. If more accurate value of \( \varepsilon \) is requested, more iterations are needed, as shown above. It should also be noted that this procedure provides more optimal solutions, so the designer has the opportunity to choose the most appropriate one.

CONCLUSION

High contact ratio (HCR) gear pair has at least two pairs of teeth in contact. High contact ratio is obtained with increased addendum height, so larger than in standard gearing. Proposed geometry of HCR gearing is much complicated due to the fact there is larger possibility of occurring meshing and during the production interference, much larger than interference happening in standard involute profiles. Also, there is a higher risk of too small thickness of a tooth tip and significantly less favorable values of specific slips into the flanks.

According to results of different measurements of gear pair, reduction of noise proved to be the best using HCR gearing with the value of contact ratio \( \varepsilon \) = 2. Decrease in noise is caused by \( \varepsilon \) = 2 because there are always two pairs of teeth in contact, which means when one pair of teeth go out from the contact, another pair of teeth is coming in contact and applied force is considerably smaller since it is divided on two pairs of teeth.

Due to increased addendum height, there is larger possibility of occurring some interference or pointed tooth tip. There are geometrical contraints for gear parameters that must be satisfied in order to avoid these irregularities, which complicates design process of HCR gears.

In this paper, a new technique is presented for optimal gear parameter calculation. Optimal values of parameters \( h_{a1}, h_{a2} \) and \( x_1 \) are determined using Generalized Particle Swarm Optimization (GPSo), a relatively novel, robust an efficient optimization methods, such as genetic algorithm, since it provides same quality of solution with less computational effort.

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