THE MATHEMATICAL MODEL OF AIR TORQUE POSITION DAMPERS WITH SINGLE BLADE

MATEMATIČKI MODEL ATP DAMPERA SA JEDNOM LOPATICOM

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ABSTRACT

The subject of the research presented in this paper is a mathematical model of ATP (Air Torque Position) damper used for indirect measurement of the flow velocity by measuring the position of the blade and the moment of air stream acting on the blade. The aim of the research is to verify the mathematical model for single blade ATP dampers as representatives of dampers with non-cascading blades. The case of ATP damper with straight pipeline section positioned in front and behind the damper was considered. Verification of the mathematical model was conducted experimentally. The experiment was performed on a laboratory facility which was set up in accordance with recommendations provided by standards for testing dampers used for control of the flow of air in HVAC (Heating, Ventilation and Air Conditioning) systems. The air flow velocity was measured with a hot-wire anemometer using the method of velocity measurement at one point of the conduit; blade attack angle was measured with electric potentiometer and the moment of the air stream acting on blade was measured by the mass sensor and leverage. Based on these measurements, the mathematical model was successfully validated and made ready to be used for accurate measurement of air velocity in single blade ATP dampers under different operating conditions.

Key words: ATP damper, HVAC, air velocity measurement.

INTRODUCTION

The mathematical model verified in this paper refers to the ATP damper. ATP damper (Air Torque Position) is a device that measures air velocity in indirect measurement of the blade angle of attack $\alpha$ and air stream moment $M$ acting on the blade. Federspiel developed mathematical model for ATP dampers (Federspiel, 2004a). During the development of the mathematical model, Federspiel considered the case of irrotational and incompressible flow of air around a single blade damper, Figure 1. He developed mathematical model for the case the axis of rotation is shifted from the axis of the blade in its longitudinal and transversal direction.

The mathematical model was developed on the basis of basic equations of fluid mechanics: continuity equation, Bernoulli equation and the momentum equation. Dampers on HVAC systems are installed in three positions: with a straight pipe section positioned in front of a damper placed at the end of the pipeline; with a straight pipe section positioned after a damper placed at the entrance of the pipeline and with straight pipe sections positioned in front of and behind the damper.

For all three positions of dampers in HVAC systems, Federspiel came to the same correlation between air velocity $v$ directly in front of the damper, the angle of attack of the blade $\alpha$ and moment of the air stream acting on the blade $M$: 
where: \( \rho \) is air density, \( A_s \) is cross-sectional area upstream of the damper and \( D_h \) is hydraulic diameter.

In the denominator of the mathematical model (eq. 1) is the density of the air stream \( \rho \). By placing the sensor and by measuring the pressure and temperature of the air stream directly in front of the blade of the ATP damper, density of the air stream \( \rho \) from the ideal gas law can be determined. In this way air velocity can be measured with the ATP damper at different operating conditions.

The correlation function in equation (eq. 1) is as follows:

\[
G(\alpha) = \left( \frac{D_h}{\sqrt{\frac{C_{Q,a} \cdot \tan \alpha}{C_{Q,a} + C_{Q,l}}} \cdot \frac{x}{D_h}} \right) \cdot \gamma, 
\]

where: \( \alpha \) represents blade angle of attack, \( x \) is the longitudinal distance from the axle to the center of pressure, \( y \) is the lateral distance from the axle to the center of pressure, \( C_{Q,a} \) is the longitudinal flow coefficient and \( C_{Q,l} \) is the lateral flow coefficient.

Federspiel conducted the verification of the mathematical model (eq. 1) for the damper with following characteristics: a square cross-section 0.61 x 0.61 m large, with straight pipe section after the damper located at the entrance of the pipeline and with four oppositely driven straight blades (cascading blades). He verified the mathematical model which can be used for accurate measuring of the air velocity under different operating conditions. The difference between the measured and modeled velocity is +/- 10 % of the measured velocity or +/- 5 % of the full scale (Federspiel, 2004b).

Federspiel came up with a mathematical model of the ATP damper which has a potentially universal character. In HVAC systems, dampers with non-cascading blades (number of blades less than three) are used very often to control the flow of air. They are placed in a pipeline in such a way that straight, flat pipeline sections are fitted in front of and behind the damper. The aim of this paper is to verify the proposed mathematical model precisely for this type of ATP dampers. As a representative of dampers with non-cascading blades, the ATP damper with a square cross-section and a single blade was used.

**Nomenclature:**

- \( A \) (\( m^2 \)) – cross-sectional area
- \( D_h \) (m) – hydraulic diameter
- \( F \) (N) – force
- \( g \) (m/s²) – gravitational constant
- \( C_{Q,a} \) – flow coefficient
- \( G(\cdot) \) – correlation function
- \( G(N) \) – weight
- \( H \) (m) – duct height
- \( l \) (m) – length of lever arm
- \( l_w \) (m) – arm of gravitational force
- \( L \) (m) – blade length
- \( m \) (kg) – mass
- \( M \) (N/m) – air stream moment
- \( \rho_w \) (Pa) – gauge
- \( \rho_a \) (Pa) – atmospheric pressure
- \( R \) (J/kgK) – gas constant
- \( R(\Omega) \) – electrical resistance
- \( t \) (°C) – temperature
- \( v \) (m/s) – velocity
- \( x \) (m) – longitudinal distance from the axle to the center of pressure
- \( y \) (m) – lateral distance from the axle to the center of pressure
- \( W(\text{m}) \) – blade thickness

**Greek symbols**

- \( \alpha \) (°) – blade angle of attack
- \( \gamma \) (·) – moment correction factor
- \( \delta (\text{m}) \) – distances between center of axle and damper blade normal to damper blade
- \( \Delta (\text{m}) \) – distances between center of axle and damper blade along to damper blade
- \( \rho (\text{kg/m}^3) \) – air density
- \( \phi(\cdot) \) – the position of the blade relative to the vertical

**Subscripts**

- \( a \) – longitudinal
- \( c \) – contraction position
- \( d \) – dead
- \( l \) – lateral
- \( n \) – downstream position
- \( mer \) – measured
- \( mod \) – model
- \( u \) – upstream position
- \( 1 \) – section for the purpose of flow velocity measuring, blade 1 – measuring section directly in front of the ATP damper blade, supporter of the blade
- \( I, II \) – two different positions of axis of rotation

**MATERIAL AND METHOD**

**Laboratory facility for ATP damper testing**

Verification of the mathematical model (eq. 1) was conducted experimentally with two independent series of measurements. Measuring range of flow velocity ranged from 0 to 10 m/s, while the blade angle of attack ranged from 0 to 90°. The mathematical model was calibrated (experimentally determined correlation function \( G(\alpha) \)) by one set of measurements, while the verification of the mathematical model was carried out by using a second set of measurements.

In the Laboratory of Fluid Mechanics, a laboratory facility for testing ATP dampers was installed in keeping with recommendations given by the standard (ANSI/AMCA 500 – D, 2007) for testing dampers used for control of the air flow rate in HVAC systems, Figure 2.

The air flow through the laboratory facility comes from an electric motor fan. The air flow is regulated by the control unit. From the fan, air enters a diffuser which converts the kinetic energy into the energy of air stream pressure and partially breaks the vortices that occur behind the fan. The air from the diffuser enters the laminarization chamber with a sieve which stabilizes the air stream. The chamber and the sieve additionally break vortices and provide uniform air stream in the straight section for the purpose of flow velocity measuring. Air velocity directly in front of the blade of the ATP damper cannot be accurately measured due to the presence of the blade in the air stream. Air velocity was determined from the equality of the mass air flow rate at the velocity measuring section (1) and the measuring section directly in front of the ATP damper blade (2).

Air velocity \( v_1 \) was measured with a hot-wire anemometer, one point method as recommended by the standard (ISO 7145, 1982). A trolley with an adjustable tripod was used for the purpose of positioning of the hot wire anemometer. In addition to the flow velocity in the straight pipe section, air stream...
temperature $t_1$ was measured with a glass thermometer and gauge $p_{m1}$ was measured with a manometer. Directly in front of the ATP damper blade, air stream temperature $t_2$ was measured with a thermometer and gauge $p_{m2}$ was measured with a manometer. Atmospheric pressure was measured with a digital barometer.

Air density in the pipeline at the locations (1) and (2) was determined on the basis of the ideal gas law:

$$\rho_1 = \frac{p_{a1} + p}{RT_1},$$

$$\rho_2 = \frac{p_{a2} + p}{RT_2},$$

where $R$ is a gas constant. At the section (1), cross-section is circular presented with a diameter $D$ and at the section (2) cross-section is a square presented with side $b$.

From the equality of the mass flow rate in sections (1) and (2) the average air velocity in section (2) can be determined, directly in front of the blade of the ATP damper:

$$v_2 = v = \frac{\rho_1 v_1 A_1}{\rho_2 A_2},$$

where $A_1$ and $A_2$ are cross-sectional areas in sections (1) and (2).

Prerequisite for presented indirect determination of the velocity in front of a blade was to have good sealing of flange joints secured. The sealing of flange joints was secured by rubber seals. According to the recommendations from the ANSI / AMCA 500 – D 2007 standard, length of the straight pipeline section in front of the of damper is 3m, and 2m after the damper. Ducts are on the bracket with adjustable height, while the fan and the laminarization chamber are placed on the trolley with adjustable height regulated with a screw. In this way, leveling of the fan, laminarization chamber and the duct of the laboratory facility was fully enabled.

**Laboratory ATP damper with a single blade**

The moment of the air stream acting on the blade of the damper $M$ was measured by the moment meter. The moment meter consists of a lever arm of length $l$, weighing cell for mass $m$ measuring and of electronics that display the measured moment $M_{mer}$. Figure 3. The blade is connected rigidly to the shaft; the shaft is bolted solidly to the lever, while the lever is rigidly attached to the weighing cell with a spherical joint to measure the mass. In this way, the moment of air stream acting on the blade $M$ is transferred to the moment meter. The idea is to transfer the moment of the air stream as much as possible to the moment meter in order to get a more accurate measurement of the moment. However, during the transfer of the moment to the meter, one part of the moment is lost to the overcoming "parasitic" moments (moment loss due to the deflection of the blade, moment of friction in the bearing, resistance, moment of transfer mechanism, etc.). Due to the above mentioned, the measured moment $M_{mer}$:

$$M_{mer} = m \cdot g \cdot l,$$

has a lower value than the moment of air stream acting on the blade $M$. The axis of rotation is displaced from the axis of the blade in order to obtain higher moment of air stream acting on the blade. According to Kirchhoff (Kirchhoff, 1869), position of the center of pressure $D$ changes with a blade angle of attack $\alpha$. The pressure will be higher on that end of the blade which first meets the air stream. For this reason, the rotary axis $O$ is shifted from the axis of the blade towards the opposite end of the blade that first meets the air stream, Figure 4. In this way, under the same conditions, lever arm of the air stream $\Delta I > \Delta II$ is increased, and therefore the moment of air stream acting on the front surface of the damper blade is also increased, $M_{I1} > M_{I2}$.

![Fig. 2. Schematic representation of the laboratory testing facility for ATP dampers](image)

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![Fig. 3. Scheme of the moment meter of the ATP damper](image)

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![Fig. 4. The position of the center of pressure for the ATP damper blade](image)

**Fig. 4. The position of the center of pressure for the ATP damper blade**
The calibration of the moment meter is done with weights of the known mass Figure 3. Since the weighing cell for mass measurement has linear characteristic, the resulting characteristic of the moment meter is also linear, Figure 5.

The moment correction factor $\gamma$ that takes into account the parasitic moments of the ATP damper is defined as the ratio of the moment of air stream on the blade $M$ and the measured moment $M_{\text{meas}}$:

$$\gamma = \frac{M}{M_{\text{meas}}}. \quad (7)$$

The moment correction factor is determined by using the weight of the known mass $m$ which is hanging by the shaft of the moment meter, Figure 3. In this way the real value of the moment acting on the meter $M$ is collected, while the moment collected from the digital weighing display $M_{\text{meas}}$ is reduced by the value of parasitic moments.

![Fig. 5. The calibration curve of the moment meter for the lever arm of l=100 mm of length](image)

The blade and its supporter produce a dead moment of blade $M_d$ with their weight, which is tarred as useless on weighing cell electronics, Figure 6:

$$M_d = g\delta \left[ m_1 \left(1 - \frac{\Delta}{\delta} \cot \alpha \right) + \frac{m_2}{2} \right] \sin \alpha. \quad (8)$$

where $m_1$ and $m_2$ signify mass of the blade and the supporter, respectively.

![Fig. 6. The dead moment of the ATP damper blade and its supporter](image)

The rotary potentiometer and a digital multimeter were used to measure the blade angle of attack. The rotary potentiometer is calibrated in accordance with the protractor that monitors the position of the blade with the axis of rotation displaced from the axis of the blade, Figure 7. The position of the blade can be defined with the angle $\alpha$ (defined by the position of the blade relative to the horizontal) or angle $\phi$ (defined by the position of the blades relative to the vertical), with the following relationship between the angles:

$$\alpha + \phi = 90^\circ. \quad (9)$$

![Fig. 7. The blade angle of attack of the ATP damper](image)

Figure 8 presents the appearance of the formed protractor for measuring the position of the blade using the blade angle of attack $\alpha$.

![Fig. 8. The ATP damper protractor](image)

During calibration, a $30^\circ$ deflection of the rotary potentiometer was made and hence the area with a nonlinear characteristic of rotary potentiometer was left behind. In this way, approximately linear relationship between the blade angle of attack $\alpha$ and electric resistance of rotary potentiometer $R$ was achieved, Figure 9.

The ATP damper with a square cross-section (side length $b=250$ mm) was tested. The damper blade is made of galvanized steel, the length of the blade is $L=248$ mm, while its thickness is $W=0.75$ mm. Along the blades, at a distance of 31 mm, on both
sides of the longitudinal axis, there are two reinforcements, 10 mm wide and 2 mm thick. The distances between the center of the axle and the damper blade, perpendicular and along the damper blade (Figure 1) are $\delta=20$ mm and $\Delta=105$ m.

**RESULTS AND DISCUSSION**

In order to achieve measuring of air flow velocity as accurate as possible, and verify the proposed mathematical model, the moment of air currents acting on the blade must be transferred to the moment meter as much as possible. At the same time, moment spent in the transmission from the blade to the moment meter must be as low as possible. To meet this requirement, it is necessary to analyze two parameters of the ATP damper operation: dead moment of the blade $M_d$ and the moment correction factor $\gamma$.

In Figure 10, the dead moment $M_d$, in terms of the blade attack angle $\alpha$ of the ATP damper is presented. This is a moment produced by the weight of the blade and its supporter. In spite of the fact that dead moment is tarred as useless on weighing electronics, the construction of the ATP damper blade should be such to diminish the dead moment as much as possible. Low dead moment is needed for two reasons. The first reason is that a large dead moment (in case of dead moment greater than the maximum moment that can be measured by the moment meter under the influence of air stream) can permanently deform the strain gauge for cell mass measuring. Another reason is that higher dead moment creates greater load of bearing and of the whole mechanism of the moment transfer, which increases the proportion of parasitic moments.

Reduction of the weight of the blade and its supporter also reduces the dead moment. However, deflection of the blade increases. In this way, the proportion of parasitic moments is increased, since a part of the air stream moment is spent on deflection of the blade instead of being transferred to the moment meter. Selecting galvanized sheet thickness 0.75 mm for the blade ensures a light-weight blade which therefore diminishes the dead moment acting on the measuring blade. At the same time, placing reinforcements significantly reduced deflection of the blade influenced by air stream. In this way, two opposing requirements were met: the blade's dead moment is as little as possible, as well as blade deflection.

In Figure 11, values of the moment correction factor $\gamma$ for different angles of the ATP damper calculated by equation (eq. 7) are presented. Values of the moment correction factor range from 1.02 to 1.03 in the whole range of angle of blade attack. Practically, the share of parasitic moments in the total moment of the air stream is only 2 to 3 %. In this way a much larger part of the moment of air stream acting on blades is transferred to the moment meter.

The first set of measured data was used to determine the correlation function (2), i.e. to calibrate the mathematical model (1). Practically, correlation function (2) has been determined experimentally. Figure 12 shows how a squared correlation function $G^2(\alpha)$ depends on the blade angle of attack of the ATP damper. It can be observed that the correlation function depends only on the blade angle of attack.

Mathematical model for the single blade ATP damper created thusly is semi-empirical. It can be observed by the definition of the correlation function (2). Correlation function can be calculated only if longitudinal flow coefficient $C_{Q,a}$ is known, as well as the lateral flow coefficient $C_{Q,l}$ which can only be obtained experimentally.

With the second set of data, verification of the previously calibrated mathematical model was conducted. The velocity of the second series of measurements and the flow velocity obtained from a calibrated mathematical model were compared. Figure 13 shows the accuracy of measurement (the difference of the measured velocity $v_{meas}$ and the model velocity $v_{mod}$ divided by measured velocity $v_{meas}$). With several points excluded, it can be observed that this difference is within the limits of +/- 10 %. It should be noted that this is accuracy over the entire measuring range of angle of attack and velocity. It is interesting to see in which positions of the blade angle of attack this difference extends beyond the boundaries of +/- 10 %.
Figure 14 presents the dependence of the difference between the measured air velocity and the model air velocity $\Delta v$ and the ATP damper blade angle of attack $\alpha$. It can be noticed that for $\alpha=0^\circ$ a significant difference between the measured and model velocity appears. Precisely at this position of the blade angle of attack the accuracy exceeds limits of +/- 10%.

This happens when the position of the blade is horizontal and when the ATP damper is fully opened. In this position, the moment of air stream acting on the blade is very small. The sensitivity of the measuring unit is such that, at this position of the blade, supporter has a significantly greater impact than the moment acting on the blade itself.

In this way, the mathematical model for single blade ATP damper with straight pipeline sections in front and behind, which can be successfully used for accurate measurement of air velocity at different operating conditions, was successfully validated.

This type of damper could be used in different applications. In the spirit of the journal theme it would be interesting its application for air velocity measurement, in front of the layer of material in the process of convective drying (Pavkov, et al., 2010, 2011).

CONCLUSION

In order to prove the universal applicability of the existing mathematical model in all variants of the ATP damper, verification was conducted for ATP dampers with blades that do not form a cascade and which have straight pipeline sections in front and behind. This kind of regulation damper can often be found on HVAC systems.

A single blade ATP damper, as a representative of dampers with non-cascading blades, was used. It was found that the accuracy (difference between the measured and the model velocity divided by the measured velocity) was within the limits of +/- 10%. It should be noted that this is accuracy over the entire measuring range of angle of attack and velocity.

Successful verification of the mathematical model for instances when single blade ATP dampers with straight pipeline sections in front and behind are used is the scientific contribution offered by this paper. The presented case of ATP damper with the verified mathematical model can be successfully used for accurate measurement of air velocity at different operating conditions.

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