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RENT PRICING DECISION SUPPORT MATHEMATICAL MODEL FOR FINANCE LEASES UNDER EFFECTIVE RISKS

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Original Scientific Paper

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Nowadays, leasing has become an increasingly important and popular method for equipment acquisition. But, because of the rent pricing difficulties and some risks that affect the lessor and lessee's decision making, there are many people that still tend to buy equipment instead of lease it. In this paper we explore how risk can affect the leasing issue support mathematical model. For this purpose, we consider three types of risk; Credit risk, Transaction risk and Risk based pricing. In particular, our focus was on how to make decision about rent pricing in a leasing problem with different customers, various quality levels and different pricing methods. Finally, the mathematical model has been solved by Genetic Algorithm that is a search heuristic to optimize the problem. This algorithm was coded in MATLAB® R2012a to provide the best set of results.

Keywords: leasing, pricing, transaction risk, risk based pricing, genetic algorithm.

INTRODUCTION

Nowadays, leasing is a good option for firms and companies to buy expensive services or assets which could be defined as: a process in which a fixed asset is borrowed by an individual or entity in return for certain and periodic payments in a certain period. Leasing is quite similar to debt, but it brings more benefits for both the lessee and the lessor compared to debt. For instance, the lessor has to pay less tax and the lessee can buy the asset at the end of the period in some kinds of leases. Another benefit for the lessor is during bankruptcy procedure because, he could return the leased asset to lessee and break to pay more money to him (Lin et al., 2013). In the leasing contract, the period of lease should be specified. Generally, the period of leasing should not be less than a year and more than 80% of the asset's life time. Four different types could be considered for leasing; financial lease, operating lease, sale and lease back and leveraged leasing. Each type has its own characteristics. For example, in operating lease the lessor is responsible for asset's maintenance while in financial lease it is not the lessor's duty to maintain the asset. Or in leveraged leasing the asset which leased, is often expensive so the lessee is forced to buy it by loan or debt and there are three parties in this kind of leasing. This paper has considered operating lease, because of some assumptions that has taken into account for modeling. However, in the leasing problem, we have to decide whether to purchase or lease, and this will be possible by considering the rent price. In this issue, many companies often prefer to lease rather than buy equipment and real estate especially when they need facilities in the short term or when less than an entire building is required (Ghayoot, 2003). In spite of leasing benefits in many circumstances, there are some risks that affect its quality. Some of the applicable risks associated with financing lease are Credit risk, Interest rate risk, Market risk, Operational risk, Liquidity risk, Transaction risk and Risk based pricing which are briefly described below:

- Credit risk: When a borrower fails to make his payments because of the credit lost.
- Interest rate risk: When an unexpected change in interest rates negatively affect the value of an investment.

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- Market risk: When an investor experiences losses due to factors that affect the overall performance of the financial markets.
- Operational risk: When a breakdown occurs in internal processes, people or system.
- Liquidity risk: When the difficulties of selling an asset arises.
- Transaction risk: When the exchange rate changes after a transaction agreement.
- Risk based pricing: When the differences between the pay back ability of people make lessee to allocate different interest rates on the same loan to different people.

This paper tries to describe a new leasing problem that is affected by three applicable risks; Credit risk, transaction risk and risk based pricing. Many times exchange rate fluctuations in developing countries make some problems for those who acquire an asset. But the transaction risk avoids them to lease that asset. Also, credit risk indicates that some lessees may face difficulties and do not be able to pay their lease price to the lessor. So, by considering risk based pricing, the lessor allocates different interest rate to different people i.e. those with higher credit score get a lower interest rate and vice versa. By applying these risks, we generate a mathematical model that helps the lessor decide on the proper price of each quality level that is defined for the asset, and clarify that which customer is qualified for these price and level of quality. Hence, we assume that in the early years, depreciation is calculated by Double Declining Balanced method (DDB) and from the year t, the calculating method is switched to Straight Line (SL). This assumption can save more money for the lesser. The remainder of the paper is organized as follows: Section 2 includes a literature review about leasing, and pricing-oriented papers within the context of related risks and their effects on decision making. Problem description and model formulation are given is Section 3. Section 4 includes the solution procedure. Experimental results are presented is Section 5 and finally Section 6 includes the conclusion and further study.

LITERATURE REVIEW

The literature on finance lease modeling under effective risks is limited. Most of the existing studies were published in the last few years. These studies could be classified into three categories. The first category explains this problem theoretically. The advantages and disadvantages of leasing and owning real estate (Ghayoot, 2003) are considered in such studies. Although, some surveys has been conducted to illustrate the degree of leasing popularity among companies. For example, a survey on the leasing practices of Fortune 500_ An annual list published by Fortune magazine that ranks the top 500 U.S. corporations_ firms, was done in 1991. The primary focus of this survey is on the analytical issues involving financial leases. In this study, the author has found that most firms (88%) treat leasing as a financing decision. Also, about half of the firms view leasing as a substitute for debt (Redman and Tanner, 1991). In addition, a survey in the UK Capital Economic Organization in 2002 found that commercial and industrial lessees use their rented space more efficiently than owner-occupiers, saving 12% per employee. Jane-Raung Lin et al (2012) show that firms are not indifferent between debt and leases and the firm’s choices depend on their financial limitations. Firms with more tangible assets prefer leasing to borrowing. So, it is shown that this kind of firms use more operational leasing by surveying a sample of 4158 firms from Compustat North America for the period of 1991 to 2005 (Lin et al., 2013). In another survey, Redman et al. (Mukherjee, 1991) has worked on sources of funds to acquire real estate. In this matter they studied uses and evaluation of leases. In this issue, 218 members of international association of corporate real estate from twelve hundred members, answered to 23 questions about their firms. One of the concerns of this survey was the use of leasing by corporations. The most common benefits, considered by the managers were the conservation of cash, and tax benefits were the next one. Most of the firms used NPV method for leasing calculation. The alternative approach was ROR (Rate of Return) for them.

By surveying the literature, we observe that the main areas the finance lease can deal with, is the problem of making decision between lease or buy equipment. In this regard, Mkhatshwa (2011) has used the net present value (NPV) to decide whether to lease or purchase equipment. He has considered the effects of inflation rate on the NPV's of model. Based on the analysis considered in this study, certain conclusions can be made with considering the effects that inflation, interest rates and project life has on the decision to lease or purchase equipment. It is shown that when the percentage of own equity increases, the NPV or expected profit from a project will decrease at a constant rate. The analysis also gave further insight on the effect of the loan payback period, the project life and the changes in interest rate on net present value. Also, the diagrams of the above changes are provided to summarize the results of the analysis. Although, another study has been done that used the present values method after tax to compare the lease versus buy profitability, but the scrap value of the investment was not taken into account. However, for calculating the depreciation, three methods were examined: declining balance method, straight line method and realization method. Then these methods were used to calculate the net present value of depreciation and finally to calculate NPV (buy) and NPV (lease). By comparing them the right choice is easily achieved (Aho and Virtanen, 1981).

Another study that has used the NPV method was done by Levy and Sarnat (1979). They discussed the solution to the practical problem of neutralizing the risk differential induced by lease contracts. And also, they brought up the lease vs. buy problem. This problem stems of using different equations. The used method is NPV because the lease or buy decision is a type of capital budgeting problem requiring the application of present value techniques and taxes are important for making decision with calculating cash flow before and after tax. Finally, they showed that the correct solution requires a neutralization of differential financial risk implicit in the lease against purchase evaluation.

The second category gets one or two case studies to make a comparison between the cash flows of buy and lease to acquire equipment. For example, Akarakiri (1988) evaluated the method of equipment acquisition by means of leasing in Nigeria. He considered a case study of photocopy machine to compare the cash flow of lease, buy or rent, and finally he concluded that the break-even point plays an important role in making the right choice between these three options. So that, if the lifetime of the equipment is less than break-even point, a short-term lease or a month to month rental is suggested. If economic useful life of the equipment is greater than the required time to reach the break-even point, a long-term finance or a long-term sales contract should be considered. And finally, if the cumulative cash cost of the three options is up and down or involves uncertainties, rent or a short-term lease is suggested.

The third category tries to study the financing leasing by modeling (Aras et al., 2011; Mandell, 2002; Wheaton, 2000; Rabbani and Keyhanian, 2012). In the year 2010, a paper was published that considered a company which leases new products and sells remanufactured versions of them at the end of the lease periods. The authors formulate a profit maximizing model using the notion of consumer surplus. The resulting problem was solved by a Nelder –Mead simplex search method (Aras et al., 2011). Mandell (2002) has studied about calculating fairground rent. He introduced a view of fairground rents, which comes from Pareto criterion and provided two models for lessee and lessor and then combined them to get a better model and provide a decision rule for the lessor to choose between leasing and selling a piece of ground. In this issue, Wheaton has some studies but he has concentrated on the rent price in retail leasing with considering both lessor and lessee benefits. So, he showed that revenue percentages and fixed rent vary proportional and positively. A model is proposed where percentage rent gives the correct motivation to lessor, rather than to lessee (Wheaton, 2000).

There are some studies that considered risks in leasing problems. Rabbani and Keyhanian (2012) have analyzed credit risk and demand reduction risk in a leasing field. This study developed a mathematical model by considering manufacturer, lessee and lessor. The model was evaluated in terms of lessor. All the parameters are deterministic and the lessee would buy the asset at the end of the contract's period. Depreciation is considered in the model and the calculating method is straight line. The leasing game which solved by Nash Equilibrium is demonstrated with a numerical example. With considering this in mind, we have tried to describe a new leasing situation that is affected by three applicable risks; Credit risk, transaction risk and risk based pricing. Many times exchange rate fluctuations in developing countries make some problems for those who acquire an asset. But the transaction risk avoids them to lease that asset. Also, credit risk indicates that some lessees may face difficulties and do not be able to pay their lease price to the lessor. So, by considering risk based pricing, the lessor allocates different interest rate to different people i.e. those with higher credit score get a lower interest rate and vice versa. By applying these risks, we generate a mathematical model that helps the lessor decide on the proper price of each quality level that is defined for the asset, and clarify that which customer is qualified for these price and level of quality. Hence, we assume that in the early years, depreciation is calculated by Double Declining Balanced method (DDB) and from the year t, the calculating method is switched to Straight Line (SL). This assumption can save more money for the lesser. The remainder of the paper is organized as follows: Section 2 includes a literature review about leasing, and pricing-oriented papers within the context of related risks and their effects on decision making. Problem description and model formulation are given is Section 3. Section 4 includes the solution procedure. Experimental results are presented is Section 5 and finally Section 6 includes the conclusion and further study.

PROBLEM DEFINITION

There are three parties in a leasing problem; Producer, lessor and lessee. The problem is solved from the point of view of the lessor. The objective of the model is that the lessee maximizes the rate of return of the product which has the quality level q, and then decides on the proper price for each quality level. In other words, the lessor wants to know which price is appropriate for which quality level and which customer is qualified for these price and level of quality. Finally, this customer is the one who could minimize the payback period of the capital. Since, many risks threaten this issue, we consider three effective risks in the model; transaction risk, risk based pricing and credit risk. Nowadays, these risks are so prevalent and also, they restrict the model region. In this problem, the producer and the lessee are affected by the lessor's decisions because here, the main decisions are made by the lessor.

These assumptions are considered in this issue;

1. The values of all parameters are deterministic and constant over time.
2. Each lessee buys the product on the specified market value in the end of the leasing period.
3. The leasing contract period is long enough (at least 30 years).
4. The depreciation calculating method is Double Declining Balance (DDB) at first and switched to straight line after a specified year.
5. The rent price is constant.
6. Interest rate is deterministic and constant over time.

Index:

M : Number of lessees that indexing with m
Q: Number of quality level of the assets indexing with q
U : Credit level or validity of the customers that indexing with u
P: Number of methods used by the lessor to pricing the rents. It is indexing with p
K : Number of the year

Parameters:

L_p : Equivalent uniform annual value of rent price that pricing by method p.
$O\&M_q$: Equivalent uniform annual value of maintenance costs of the product with quality level q.
MV_{pq} : Market value of the product with quality level q that pricing by method p.
SV_q : Salvage value at the end of the useful life of the product with quality level q.
θ_{pq} : Payback period of the leased product with quality level q that pricing by method p.
I_{pq} : Initial cost or price of the product with quality level q.
D_q : Amount of annual depreciation of the product with quality level q in kth year.
T_e : Effective rate of tax
i_u : Interest rate for customer with credit level u
N : Duration of leasing contract
p_b : The probability that a customer fails to pay rents in a period.

Decision variables:

χ_{mpq} : It is a binary variable. If the purchased product with quality level that pricing by p method q, is dedicated to the lessee m, this variable is equal to one. Otherwise, it is equal to zero.
z_{mu} : It is a binary variable. If the lessee m is in the level u of validity, this variable is equal to one. Otherwise, it is equal to zero.
f : Transaction risk ratio is a random variable with F distribution function.

Proposed model

$$\max \sum_{p=1}^{P}\sum_{q=1}^{Q}\sum_{m=1}^{M}\frac{1}{\theta_m}\chi_{mpq} \tag{1}$$

S.t:

$$\sum_{p=1}^{P}\sum_{q=1}^{Q}\chi_{mpq} = 1 \quad \forall m \in \{1,2,...,M\} \tag{2}$$

$$D_k = B\frac{2}{N}(1-\frac{2}{N})^{t-1} \le \frac{B-SV_q}{N} = D$$

$$(1-\frac{2}{N})^{t-1} \le \frac{B-SV_q}{2B} \tag{3}$$

$$(t-1)\ln(1-\frac{2}{N}) \le \ln(\frac{B-SV_q}{2B})$$

$$t = \left\lceil \frac{\ln\left(\frac{B-SV_q}{2B}\right)}{\ln\left(1-\frac{2}{N}\right)} + 1 \right\rceil \tag{4}$$

$$IR_m = \sum_{u=1}^{U} i_u z_{mu} \quad \forall m \in \{1,2,...,M\} \tag{5}$$

$$\sum_{u=1}^{U} Z_{mu} = 1 \quad \forall m \in \{1,2,...,M\} \tag{6}$$

$$E_p(L) = -p_m \times 0 + (1-p_m)\times L_p \quad \forall m \in \{1,2,...,M\} \quad \forall p \in \{1,2,...,p\} \tag{7}$$

$$\theta_m = \frac{\sum_p\sum_q I_{pq}}{(\sum_p\sum_q (1+f)MV_{pq})(\frac{A}{F}, IR_m, N) + (\sum_p E_p(L))\times(1-T_e) - (\sum_q O\ \&\ M_q)(1-T_e) + (\sum_q D_q)T_e} \tag{8}$$

$$\forall m \in \{1,2,...,M\}$$

$$x_{mpq} \in \{0,1\} \quad \forall m \in \{1,2,...,M\},\ \forall p \in \{1,2,...,p\},\ \forall q \in \{1,2,...,Q\}$$
$$z_{mu} \in \{0,1\} \quad \forall m \in \{1,2,...,M\},\ \forall u \in \{1,2,3\} \tag{9}$$

In this problem we have two variables. The binary variable x_{mpq} is to assign products to lessees. Another variable is z_{mu} that shows the credit level of the lessees. This variable is used to apply the risk based pricing on the model. This risk is the process in the financial services industry to allocate different interest rates on the same loan to different people, depending on their credit score and other factors which illustrate their ability of paying back the loan. Those with worse scores have a higher interest rate; those with better scores have a lower one. The idea of the process is to avoid the tragedy of the commons, which happens if everyone has the same interest rate. So, customers are categorized into u levels of validity. Each customer has a different interest rate according to his credit level. Furthermore each customer is only in one level, so the sum of the z_{mu} variables, should be equal to one.

The objective function in this problem is maximizing the rate of return of the leased product with q quality level and p method of pricing for m customer (1). Therefore, x_{mpq} variable is needed, to be sure that if there is not a customer for a product with q quality level and p method of pricing, we will not calculate the rate of return for that.

There are number of constraints for the main problem that restrict the solution region. Equation (2) ensures that each customer can leased only one product with one specified quality level and only one specified pricing method. So this method is used for all customers.

Depreciation is an important issue in calculating rent price and also the present value, because the amount of depreciation is including tax saving for the lessor. To calculate depreciation, a switching model is used as showed in section (3) of the model. Due to this saving, an increase in depreciation is desirable. For early years of cash flow, DDB method is better because the amount of depreciation is more than end of the cash flow. But from point t, calculated in equation (4) of the model, method is switched to straight line (SL) for calculating depreciation. From this point calculated depreciation by SL method is more than by DDB. From this point onwards, the straight-line method for depreciation is calculated.
Effective risks

Risk based pricing is considered in the problem. So, as showed in equation (5), for calculating equivalent uniform annual value of products, we should use desired interest rate named as IR_m. Also, with equation (6) ensure that there is only one interest rate for each customer.

The effect of Transaction Risk is on the market value of the product. The lessee, due to this risk should pay the specified ratio of equivalent uniform annual value of product's market value. So, $f \times MV_{pq}(\frac{A}{F}, i_u, N)$ is added to the model to determine this risk in calculating.

We assume that the contract period (N) is long enough so the financial factor $\left(A/P, \mathrm{IR}_m, N\right)$ tends to IR_m:

$$(\frac{P}{A}, IR_m, N) = \frac{1-(1+IR_m)^{-N}}{IR_m}$$

$$\lim_{N\to\infty}(\frac{A}{P}, IR_m, N) = \lim_{N\to\infty}\frac{IR_m}{1-(1+IR_m)^{-N}} = IR_m$$

Also, it could be said that $\theta = \frac{A}{P}$. So, the inverse of θ, $\frac{1}{\theta}$ is approximately equivalent to the internal rate of return (IRR) of the financial flow (Rabbani and Keyhanian, 2012). This concept explains that the objective function maximizes the IRR of all leasing contracts.

To reduce the losses resulting from Credit Risk, we calculate rent price for each lessee as in equation (7); If the customer does not pay the rent, L_p is 0 and the probability is p_m. But if the customer pays the rent L_p is not zero with $1-p_m$ probability.

As showed in equation (8), to calculate payback period of the asset, we want to know when the initial value of the product will be covered, without considering time's effect on the cash flow. So, we have to divide the initial cost of asset to the equivalent uniform annual costs and revenues with considering tax rate. z_{mu} and x_{mpq} are binary variables in this problem (9).

SOLUTION METHODOLOGY

The majority of the studies that provide a model for the leasing problem did not solve their model and focus their attention on strategic and theory of the concept. The only study that tries to solve the problem has used Nelder-Mead method. In an attempt to offer an easily implementable method that can provide good results for solving our model, a meta heuristic algorithm based on genetic algorithm has been suggested. Because our model is a multi-dimensional problem and the genetic algorithm execution technique is not dependent on the error surface so, it can easily optimize such problems. Also, the number of parameters in the model is large and GA can find a good solution for this kind of models and improve it. The initial values of the model are taken from Iranian leasing official website (Table 1) in which a rent calculating is provided. We choose PEUGOET206 in three levels; type 2, 5 and 6, for analyzing.

Calculating transaction risk ratio:

The f value, a random variable with F distribution function, is calculated by using the available data. To calculate index of real rate of exchange (Figure 1), we used annual data and following equation;

$$RER = \frac{CPIW}{CPII} \times NER$$

RER: Real Exchange Rate
CPII: Consumer Price Index of Iran
CPIW: Consumer Price Index of World
NER: Nominal Exchange Rate

Table 1 Exchange rate data (Oladi et al., 2007).

Year	Nominal exchange rate	CPI	Real exchange rate
53	73.6	0.12841	9.451
54	74.4	0.130027	9.674
55	79.6	0.185892	14.797
56	78	0.159962	12.477
57	91	0.240901	21.922
58	159.5	0.130376	20.795
59	234	0.134188	31.4
60	395	0.136494	53.915
61	475	0.160669	76.318
62	403.6	0.140491	56.702
63	610.6	0.1164	71.073
64	639.6	0.100775	64.456
65	815.2	0.126435	103.069
66	1134.6	0.10279	116.625
67	954.2	0.072663	69.335
68	1431.3	0.064824	92.783
69	1525.8	0.083903	128.019
70	1535.2	0.081902	125.735
71	1624.5	0.096018	155.981
72	1698.8	0.141129	239.749
73	2602.2	0.12427	323.374
74	4049.3	0.111507	451.527
75	4215.2	0.115848	488.323
76	4781.5	0.104076	497.637
77	6468.3	0.098305	635.864
78	8657.6	0.11227	971.993
79	8188.1	0.104968	859.488
80	8008	0.10836	867.746
81	8019	0.114762	920.274
82	8323	0.116384	968.663
83	8747	0.11366	994.186

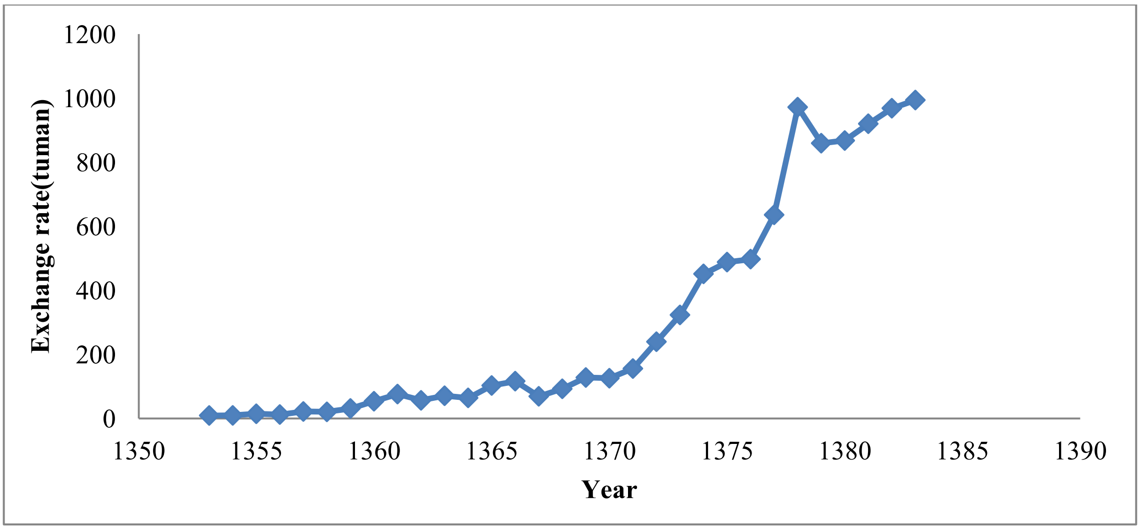

Figure 1: Real exchange rate plot

The value of IP (investment by privet sectors), IG (investment by government), GDP (gross domestic product) and R (interest rate) are shown in Table 2.

Table 2: Investment data (Oladi et al., 2007)

IP	IG	GDP (p)	R
42209.27	25242.26	196581	8
70552.58	26800.4	206113.8	8
84476.73	45047.35	242326	8
74384.27	37854.59	236645.3	9
39444.75	52803.57	219191	9
36869.62	27369.13	209919.4	7
42933.55	23905.57	178149	7
38311.4	23242.35	170281.2	7
35316.07	30795.15	191666.8	7
65389.95	29907.12	212876.5	7
63951.33	24271.27	208515.9	7.2
49447.15	21433.67	212686.3	6
35484.83	24431.72	193235.4	6
38763.09	19664.99	191312.4	6
33797.14	13139.18	180822.5	6
38004.3	12414.03	191502.6	6
39064.61	18336.03	218537.7	6.5
63947.13	20068.9	245036.4	6.5
56622.66	24467.51	254822.5	7.5
36233.32	36496.05	258601.4	8
30095.11	32294.75	259876.3	8
29728.18	29832.05	267534.2	8
41586.88	32878.48	283806.6	8
53398.5	30366	291768.7	8
56979.98	29505.14	300139.6	8
57269.42	34235.91	304941.2	8
61670.41	33596.98	320068.9	8
72942	35820	330565	7
81022	40804	355554	7
90764	44207	379838	7
995246	452603	398234	7

To calculate index of real rate of exchange we use data in Table 3 and 4, and following equation.

$$\beta = b_0 + \frac{1}{IP^* - IP}(b_1 IG + b_2 R + b_3 P + b_4 h^2)$$

By using Garch method to calculate β, we consider following value for b_1 to b_4.
b_1=0.02, b_2=0.02, b_3=0.01, b_4=0.05
β=0.165

So, we use this value as *f*, uncertainty of the real exchange rate in our model.

A Genetic algorithm is an optimization method based on the biological analogy of "survival of the fittest" that used to find true or approximate solutions to optimization and search problems. These algorithms are categorized as global search heuristics. In contrast to simulated annealing where only one model is disconcerted and walked through the model space in genetic algorithms a group of models is always considered. Through analogies of genetic reproduction, crossover, mutation the quality of the average population and the individuals is improved over several generations. Genetic algorithm scan be considered as a special case of the more general evolutionary algorithms. Figure 2 shows the general genetic algorithm flowchart.

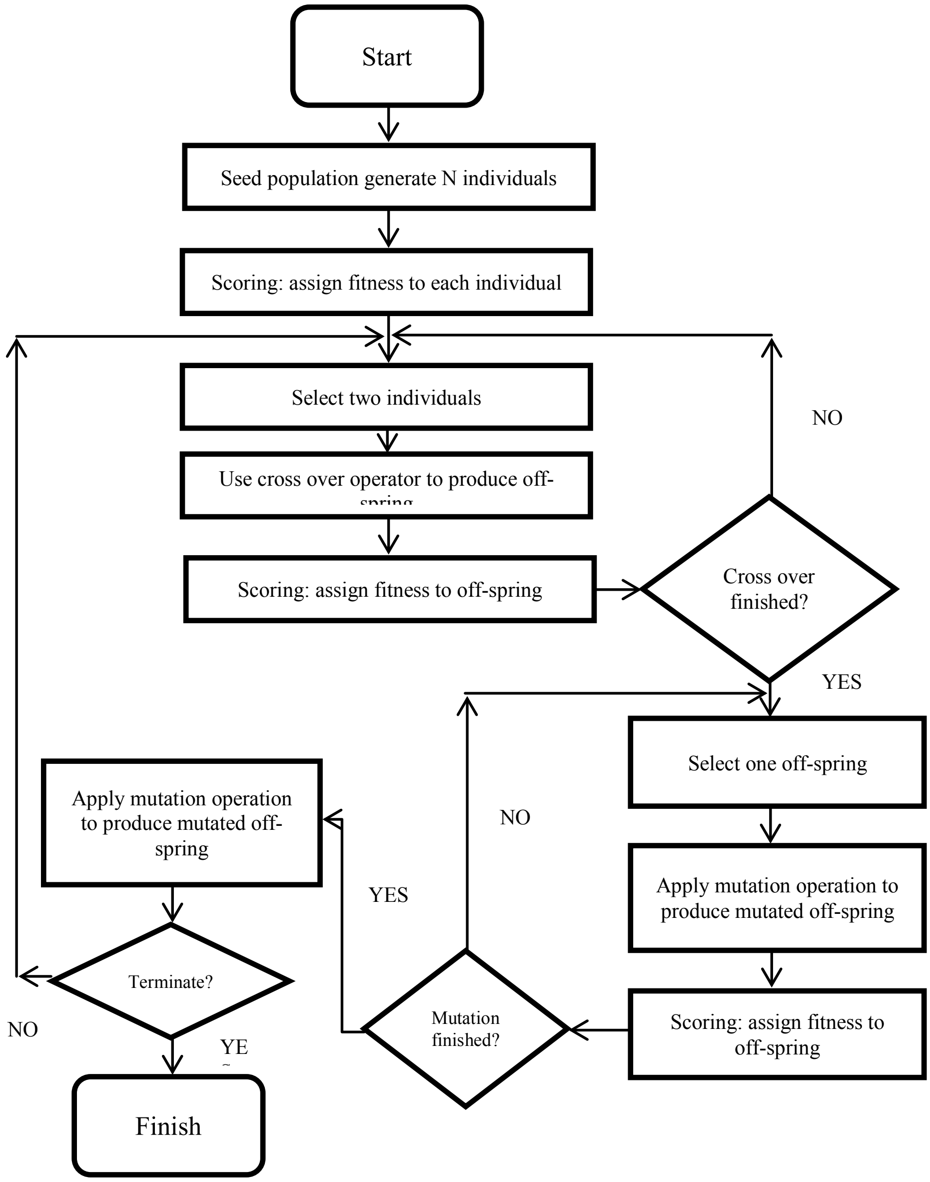

Figure 2 Genetic algorithm flowchart

We run the model and, answer is showed in Tables 3 and 4. In this answer for example the first customer has maximum credit level according to Z_{mu} results. So, as expected, the best quality level with high pricing method is selected. The second customer has maximum credit level too. Selected quality for this customer is medium and the pricing method is the second one. For the third customer, validity is in second level. So, there is about 1 percent probability that the customer will be failed to pay rents. The medium quality level is selected and the pricing method is the third one. In this run payback period with GA method is 29.243 year and the payback period model is 27.396. Up to 10 percent error is acceptable in GA method.

Table3: Optimum values for x_{mpq}

x_{mpq}			q		
			1	**2**	**3**
m=1	p	**1**	0	0	0
		2	0	0	0
		3	1	0	0
m=2	p	**1**	0	0	0
		2	0	1	0
		3	0	0	0
m=3	p	**1**	0	0	0
		2	0	0	0
		3	0	1	0

Table4: Optimum values for z

Z_{mu}	M		
u	**1**	**2**	**3**
1	1	0	0
2	0	0	0
3	0	0	0
Z_{mu}	M		
u	**1**	**2**	**3**
1	0	1	0
2	0	0	0
3	0	0	0
Z_{mu}	M		
u	**1**	**2**	**3**
1	0	0	0
2	0	0	1
3	0	0	0

CONCLUTION AND FUTURE STUDY

Nowadays, leasing has become a common way for equipment acquisition. But because of the lack of awareness of related risks, wrong decisions are made by managers and this decisions cause many consequences on organization's performance. This study tries to consider many aspects of leasing problem and its related risks; Credit risk, Transaction risk and Risk based pricing. For this purpose, we have provided a model that helps how to allocate customers to quality levels and pricing methods. In other words, this problem illustrates that without considering related risks, the decisions does not assures the best results. In this study, by using a genetic algorithm we solve the problem. For further study, other methods could be suggested for solving the model such as Sequential Unconstrained Minimization Technique (SUMT) or other meta-heuristic methods to optimize the model with a better performance. Also, there are some other risks like liquidity risk, operational risk, portfolio risk and etc. that could be considered in model and study its effects on lessee and lessor's decisions.

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