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ОРИГИНАЛЬНЫЕ НАУЧНЫЕ СТАТЬИ

ORIGINAL SCIENTIFIC PAPERS

A STUDY ON INTEGRAL TRANSFORMS OF THE GENERALIZED LOMMEL-WRIGHT FUNCTION

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Abstract:

Introduction/purpose: The aim of this article is to establish integral transforms of the generalized Lommel-Wright function.

Methods: These transforms are expressed in terms of the Wright Hypergeometric function.

Results: Integrals involving the trigonometric, generalized Bessel function and the Struve functions are obtained.

Conclusions: Various interesting transforms as the consequence of this method are obtained.

Key words: Generalized Lommel-Wright functions $J(z)$, Hankel transform, K-transform, Wright function, Whittaker function.

Introduction

The transform defined by the following integral equation

$$R_\nu\{f(x);p\} = g(p,\nu) = \int_0^{+\infty} (px)^{1/2} K_\nu(px) f(x) dx \quad (1)$$

is called the k transform with p as a complex parameter and $K_\nu(px)$ is called the Modified Bessel function of the third kind or the Macdonald function, see (Mathai et al, 2010, p.53). The Hankel transform of a function $f(x)$, denoted by $g(p,\nu)$ is defined as

$$g(p,\nu) = \int_0^{+\infty} (px)^{1/2} J_\nu(px) f(x) dx, \quad p > 0 \quad (2)$$

where $J_\nu(px)$ is called the Bessel-Maitland function or the Maitland-Bessel function (Mathai et al, 2010, p.22 and p.56).

The Wright hypergeometric function defined by the series (Srivastava & Manocha, 1984):

$${}_p\psi_q\left[\begin{array}{c}(\alpha_1,A_1),..,(\alpha_p,A_p);\\ \\(\beta_1,B_1)...,(\beta_q,B_q)\end{array} z\right] = \sum_{k=0}^{+\infty} \frac{\prod_{j=1}^{p}\Gamma(\alpha_j+A_jk)z^k}{\prod_{j=1}^{q}\Gamma(\beta_j+B_jk)k!}, \quad (3)$$

where the coefficients $A_1,....A_p$ and $B_1,....B_q$ are positive real numbers such that

$$1+\sum_{j=1}^{q} B_j - \sum_{j=1}^{p} A_j \geq 0, \quad (4)$$

can be slightly generalized (3) as given below.

$${}_p\psi_q\left[\begin{array}{c}(\alpha_1,1),..,(\alpha_p,1);\\ \\(\beta_1,1)...,(\beta_q,1)\end{array} z\right] = \frac{\prod_{j=1}^{p}\Gamma(\alpha_j)}{\prod_{j=1}^{q}\Gamma(\beta_j)} {}_pF_q\left[\begin{array}{c}\alpha_1,..,\alpha_p;\\ \\ \beta_1,...,\beta_q\end{array} z\right], \quad (5)$$

where ${}_pF_q$ is the generalized hypergeometric function defined by (Srivastava & Manocha, 1984; Rainville, 1960)

$$
{}_pF_q\left[\begin{matrix}\alpha_1,..,\alpha_p; \\ \\ \beta_1,...,\beta_q\end{matrix}\quad z\right] = \sum_{k=0}^{+\infty}\frac{(\alpha_1)_n,....,(\alpha_p)_n z^n}{(\beta_1)_n,....,(\beta_q)_n n!} =
$$

$$
{}_pF_q(\alpha_1,...,\alpha_p;\beta_1,....,\beta_q;z), \tag{6}
$$

where $(\lambda)_n$ is the well known Pochhammer symbol (Srivastava & Manocha, 1984).

The series representation of the generalized Lommel Wright function as (Kachhia & Prajapati, 2016);

$$
J_{\nu,\lambda}^{\mu,m}(z) = \sum_{k=0}^{+\infty}\frac{(-1)^k\Gamma(k+1)(\frac{z}{2})^{2k+\nu+2\lambda}}{\Gamma(\lambda+k+1)^m\Gamma(\nu+k\mu+\lambda+1)k!},
$$

$$
(z\in\mathbb{C}/(-\infty,0],\ m\in\mathbb{N},\ \nu,\lambda\in\mathbb{C},\mu>0). \tag{7}
$$

Also, we have the following relations of the generalized Lommel Wright functions with trigonometric functions and the generalized Bessel function $\mathbb{J}_{\nu,\lambda}^{\mu}(z)$ and the Struve function as follows:

$$
J_{1/2,0}^{1,1}(z) = \sqrt{\frac{2}{\pi z}}\sin(z) \tag{8}
$$

$$
J_{-1/2,0}^{1,1}(z) = \sqrt{\frac{2}{\pi z}}\cos(z) \tag{9}
$$

$$
J_{\nu,\lambda}^{\mu,1}(z) = \mathbb{J}_{\nu,\lambda}^{\mu}(z) \tag{10}
$$

$$
J_{\nu,1/2}^{1,1}(z) = H_\nu(z). \tag{11}
$$

The following known results of Mathai and Saxena (Mathai & Saxena, 1973):

$$
\int_0^{+\infty} x^{\delta-1}J_\eta(ax)dx = \frac{2^{\delta-1}a^{-\delta}\Gamma(\frac{\delta+\eta}{2})}{\Gamma(1+\frac{\eta-\delta}{2})},\quad \Re(\eta)<\Re(\delta)<3/2,\quad a>0 \tag{12}
$$

$$\int_0^{+\infty} x^{\delta-1} K_\eta(ax)dx = 2^{\delta-2}a^{-\delta}\Gamma(\delta \pm \eta)/2, \tag{13}$$

$$\int_0^{+\infty} x^{\delta-1} \exp(-at)K_\eta(ax)dx = \frac{\Gamma(\delta \pm \eta)/2}{(2a)^{\delta-1}\Gamma(\delta+1/2)}, \tag{14}$$

$$\int_0^{+\infty} x^{\delta-1} \exp(1/2\ x)W_{(\eta,\alpha)}(x)dx = \frac{\Gamma(1/2 \pm \alpha + \delta)\Gamma(-\eta-\delta)}{\Gamma(1/2 \pm \alpha - \eta)}, \tag{15}$$

$$\int_0^{+\infty} x^{\delta-1} \exp(-1/2\ x)M_{(\eta,m)}(x)dx = \frac{\Gamma(2m+1)\Gamma(m+\delta+1/2)\Gamma(\eta-\delta)}{\Gamma(m-\delta+1/2)\Gamma(m+\eta+1/2)}, \tag{16}$$

$$\int_0^{+\infty} x^{\delta-1} W_{(\eta,\alpha)}(x)W_{(-\eta,\alpha)}(x)dx = \frac{\Gamma((\delta+1)/2 \pm \alpha)\Gamma(\delta+1)}{2\Gamma(1+\delta/2 \pm \eta)}. \tag{17}$$

Various generalizations and cases of the Lommel-Wright function have been investigated. For details, see (Paneva-Konovska, 2007; Menaria et al, 2016; Mondal & Nisar, 2017; Srivastava & Daoust, 1969; Kiryakova, 2000).

Integral formulas involving the Lommel-Wright functions have been developed by many authors. See e.g., (Choi & Agarwal, 2013; Choi et al, 2014; Jain et al, 2016; Chaurasia & Pandey, 2010). In this sequel, here, we aim at establishing a certain new generalized integral formula involving the generalized Lommel-Wright function $J^{\mu,m}_{\nu,\lambda}(z)$ interesting integral formulas which are derived as special cases.

Main results

This section deals with the evaluation of integrals formulas involving the Lommel-Wright function defined in (7) and the integrals involving the product of the Bessel function of first kind, the Kelvin‘s function and Whittaker function (Whittaker & Watson, 2013) with the generalized Lommel-Wright function.

THEOREM 1. *Let* $z \in \mathbb{C}/(-\infty, 0],\ m \in \mathbb{N},\ \nu, \lambda \in \mathbb{C}, \mu > 0$. *Then the Hankel transform of the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1} J_\eta(az) J_{\nu,\lambda}^{\mu,m}(b\, z^w) dz = \frac{1}{2}\left(\frac{b}{2}\right)^{\nu+2\lambda}\left(\frac{2}{a}\right)^{\rho+w(\nu+2\lambda)} \times$$
$$ {}_2\psi_{m+2}\left[\begin{array}{c}(1,1), (\frac{\eta+\rho+w\nu+2w\lambda}{2}, w); \\ (\lambda+1,1), ..., (\lambda+1,1), (\nu+\lambda+1,\mu), ((\frac{2+\eta-(\rho+w\nu+2w\lambda)}{2}), -w);\end{array} \left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \tag{18}$$

Proof. On using (7) in the integrand of (1) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1} J_\eta(az) J_{\nu,\lambda}^{\mu,m}(bz^w) dz =$$
$$\sum_{n=0}^{+\infty} \frac{(-1)^n \Gamma(n+1)(b/2)^{2n+\nu+2\lambda}}{\Gamma(\lambda+n+1)^m \Gamma(\nu+\lambda+n\mu+1) n!} \int_0^{+\infty} z^{\rho+w(2n+\nu+2\lambda)-1} J_\eta(az) dz.$$

Now using (12) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1} J_\eta(az) J_{\nu,\lambda}^{\mu,m}(bz^w) dz = \Big(1/2\Big)\Big(b/2\Big)^{\nu+2\lambda}\Big(2/a\Big)^{\rho+w(\nu+2\lambda)} \times$$
$$\sum_{n=0}^{+\infty} \frac{\Gamma(n+1)\Gamma(\eta+\rho+w\nu+2w\lambda+2wn)/2\Big(-b^2/4\Big)^n\Big(4/a^2\Big)^{wn}}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)\Gamma(2+\eta-\rho-w\nu-2w\lambda-2wn)/2\ n!}$$
$$= \frac{1}{2}\left(\frac{b}{2}\right)^{\nu+2\lambda}\left(\frac{2}{a}\right)^{\rho+w(\nu+2\lambda)} \times$$
$$ {}_2\psi_{m+2}\left[\begin{array}{c}(1,1), \left(\frac{\eta+\rho+w\nu+2w\lambda}{2}, w\right); \\ (\lambda+1,1), ..., (\lambda+1,1), (\nu+\lambda+1,\mu), \left(\left(\frac{2+\eta-(\rho+w\nu+2w\lambda)}{2}\right), -w\right);\end{array} \left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \tag{19}$$

□

THEOREM 2. *Let* $z \in \mathbb{C}/(-\infty, 0],\ m \in \mathbb{N},\ \nu, \lambda \in \mathbb{C}, \mu > 0$. *Then the K-Transform of the generalized Lommel-Wright function defined in (7) is given* *by*

$$\int_0^{+\infty} z^{\rho-1} K_\eta(az) J_{\nu,\lambda}^{\mu,m}(b\,z^w)dz = \frac{1}{4}\left(\frac{b}{2}\right)^{\nu+2\lambda}\left(\frac{2}{a}\right)^{\rho+w(\nu+2\lambda)}\times$$
$${}_2\psi_{m+1}\left[\begin{matrix}(1,1),(\frac{\rho+w\nu+2w\lambda\pm\eta}{2},w);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu);\end{matrix}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \quad (20)$$

Proof. On using (7) in the integrand of (2) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1} K_\eta(az) J_{\nu,\lambda}^{\mu,m}(bz^w)dz =$$
$$\sum_{n=0}^{+\infty}\frac{(-1)^n\Gamma(n+1)(b/2)^{2n+\nu+2\lambda}}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}\int_0^{+\infty} z^{\rho+w(2n+\nu+2\lambda)-1}K_\eta(az)dz.$$

Now using (13) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1} K_\eta(az) J_{\nu,\lambda}^{\mu,m}(bz^w)dz = \left(1/4\right)\left(b/2\right)^{\nu+2\lambda}\left(2/a\right)^{\rho+w(\nu+2\lambda)}\times$$
$$\sum_{n=0}^{+\infty}\frac{\Gamma(n+1)\Gamma(\rho+w\nu+2w\lambda\pm\eta+2wn)/2\left(-b^2/4\right)^n\left(4/a^2\right)^{nw}}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}$$
$$=\frac{1}{4}\left(\frac{b}{2}\right)^{\nu+2\lambda}\left(\frac{2}{a}\right)^{\rho+w(\nu+2\lambda)}$$
$${}_2\psi_{m+1}\left[\begin{matrix}(1,1),(\frac{\rho+w\nu+2w\lambda\pm\eta}{2},w);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu);\end{matrix}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \quad (21)$$

□

THEOREM 3. *Let* $z\in\mathbb{C}/(-\infty,0],\ m\in\mathbb{N},\ \nu,\lambda\in\mathbb{C},\mu>0.$ *Then the K-Transform of the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1}\exp(-az) K_\eta(az) J_{\nu,\lambda}^{\mu,m}(b\,z^w)dz = \frac{2a\sqrt{\pi}\left(b/2\right)^{\nu+2\lambda}}{(2a)^{\rho+w(\nu+2\lambda)}}\times$$
$${}_2\psi_{m+2}\left[\begin{matrix}(1,1),(\rho+w\nu+2w\lambda\pm\eta,2w);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu),(\rho+w\nu+2w\lambda+1/2,2w);\end{matrix}\right.$$

$$\left(\frac{-b^2}{4(4a^2)^w}\right)\Bigg]. \tag{22}$$

Proof. On using (7) in the integrand of (3) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1}\exp(-az)K_\eta(az)J_{\nu,\lambda}^{\mu,m}(bz^w)dz = \sum_{n=0}^{+\infty}\frac{(-1)^n\Gamma(n+1)(b/2)^{2n+\nu+2\lambda}}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}\times \int_0^{+\infty} z^{\rho+w(2n+\nu+2\lambda)-1}\exp(-az)K_\eta(az)dz. \tag{23}$$

Now using (14) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1}\exp(-az)K_\eta(az)J_{\nu,\lambda}^{\mu,m}(bz^w)dz = \frac{2a\sqrt{\pi}\left(b/2\right)^{\nu+2\lambda}}{(2a)^{\rho+w(\nu+2\lambda)}}\times \sum_{n=0}^{+\infty}\frac{\Gamma(n+1)\Gamma(\rho+w\nu+2w\lambda\pm\eta+2wn)(\frac{-b^2}{4(4a)^w})^n}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)(\rho+w\nu+2w\lambda+1/2,2w)n!}$$

$$= \frac{2a\sqrt{\pi}(b/2)^{\nu+2\lambda}}{(2a)^{\rho+w(\nu+2\lambda)}}\times {}_2\psi_{m+2}\left[\begin{matrix}(1,1),(\rho+w\nu+2w\lambda\pm\eta,2w);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu),(\rho+w\nu+2w\lambda+1/2,2w);\end{matrix}\left(\frac{-b^2}{4(4a^2)^w}\right)\right]. \tag{24}$$

□

THEOREM 4. *Let* $z\in\mathbb{C}/(-\infty,0]$, $m\in\mathbb{N}$, $\nu,\lambda\in\mathbb{C},\mu>0$. *Then the product of the Whittaker function and the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1}\exp(az/2)W_{\eta,\alpha}(az)J_{\nu,\lambda}^{\mu,m}(w\ z^\theta)dz = \frac{\left(w/2\right)^{\nu+2\lambda}}{(a)^{\rho+\theta(\nu+2\lambda)}\Gamma(1/2\pm\alpha-\eta)}\ {}_3\psi_{m+1}\left[\begin{matrix}(1,1),(1/2\pm\alpha+\rho+\nu\theta+2\lambda\theta,2\theta),(-\eta-\rho-\nu\theta-2\theta\lambda,-2\theta);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu);\end{matrix}\right.$$

$$\left(\frac{-w^2}{4(a^2)^\theta}\right)\Bigg]. \tag{25}$$

Proof. Putting $az = x, adz = dx$ as $z \to 0, x \to 0$ and $z \to +\infty, x \to +\infty$ and using (7) in the integrand of (4) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1}\exp(az/2)W_{\eta,\alpha}(az)J_{\nu,\lambda}^{\mu,m}(wz^\theta)dz =$$
$$\frac{\left(w/2\right)^{\nu+2\lambda}}{(a)^{\rho+\theta(\nu+2\lambda)}}\sum_{n=0}^{+\infty}\frac{\Gamma(n+1)\left(\frac{-w^2}{4(a^2)^\theta}\right)^n}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}\times$$
$$\int_0^{+\infty} x^{\rho+\theta(2n+\nu+2\lambda)-1}\exp(x/2)W_{\eta,\alpha}(x)dx.$$

Now using (15) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1}\exp(az/2)W_{\eta,\alpha}(az)J_{\nu,\lambda}^{\mu,m}(wz^\theta)dz = \frac{(w/2)^{\nu+2\lambda}}{(a)^{\rho+\theta(\nu+2\lambda)}\Gamma(1/2\pm\alpha-\eta)}\times$$
$$\sum_{n=0}^{+\infty}\left[\frac{\Gamma(n+1)\Gamma(1/2\pm\alpha+\rho+\theta\nu+2\theta\lambda+2\theta\, n)\Gamma(-\eta-\rho-\theta\nu-2\theta\lambda-2\theta\, n)}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}\right.$$
$$\left.\times\left(\frac{-w^2}{4(a^2)^\theta}\right)^n\right] = \frac{\left(w/2\right)^{\nu+2\lambda}}{(a)^{\rho+\theta(\nu+2\lambda)}\Gamma(1/2\pm\alpha-\eta)}\times$$
$${}_3\psi_{m+1}\left[\begin{array}{c}(1,1),(1/2\pm\alpha+\rho+\theta\nu+2\theta\lambda,2\theta),(-\eta-\rho-\nu\theta-2\theta\lambda,-2\theta);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu);\end{array}\right.$$
$$\left.\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{26}$$

□

THEOREM 5. *Let* $z \in \mathbb{C}/(-\infty,0],\ m \in \mathbb{N},\ \nu,\lambda \in \mathbb{C}, \mu > 0$. *Then the product of the Whittaker function and the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1}\exp(-az/2)M_{\eta,\alpha}(az)J_{\nu,\lambda}^{\mu,m}(w\, z^\theta)dz =$$

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$$\frac{\left(w/2\right)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}\Gamma(2\alpha+1)}{(a)^{\rho}\Gamma(\alpha+\eta+1/2)}\times$$
$$_3\psi_{m+2}\left[\begin{matrix}(1,1),(\alpha+\rho+1/2+\nu\theta+2\lambda\theta,2\theta),(\eta-\rho-\nu\theta-2\theta\lambda,-2\theta);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu),(\alpha-\rho-\theta\nu-2\theta\lambda+1/2,-2\theta);\end{matrix}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \quad (27)$$

Proof. Putting $az = x, adz = dx$ as $z \to 0, x \to 0$ and $z \to +\infty, x \to +\infty$ and using (7) in the integrand of (5) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1}\exp(-az/2)M_{\eta,\alpha}(az)J_{\nu,\lambda}^{\mu,m}(wz^{\theta})dz = \frac{\left(w/2\right)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}}\times$$
$$\sum_{n=0}^{+\infty}\frac{\Gamma(n+1)\left(\frac{-w^2}{4(a^2)^{\theta}}\right)^n}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)n!}\times$$
$$\int_0^{+\infty} x^{\rho+\theta(2n+\nu+2\lambda)-1}\exp(-x/2)M_{\eta,\alpha}(x)dx.$$

Now using (16) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1}\exp(-az/2)M_{\eta,\alpha}(az)J_{\nu,\lambda}^{\mu,m}(wz^{\theta})dz =$$
$$\frac{(w/2)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}\Gamma(2\alpha+1)}{(a)^{\rho}\Gamma(\alpha+\eta+1/2)}\times$$
$$\sum_{n=0}^{+\infty}\left[\frac{\Gamma(n+1)\Gamma(\alpha+\rho+\theta\nu+2\theta\lambda+1/2+2\theta\, n)\Gamma(\eta-\rho-\theta\nu-2\theta\lambda-2\theta\, n)}{\Gamma(\lambda+n+1)^m\Gamma(\nu+\lambda+n\mu+1)\Gamma(\alpha-\rho-\theta\nu-2\theta\lambda-2n\theta+1/2)n!}\times\right.$$
$$\left.\left(\frac{-w^2}{4(a^2)^{\theta}}\right)^n\right] = \frac{\left(w/2\right)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}\Gamma(2\alpha+1)}{(a)^{\rho}\Gamma(\alpha+\eta+1/2)}\times$$
$$_3\psi_{m+2}\left[\begin{matrix}(1,1),(\alpha+\rho+1/2+\nu\theta+2\lambda\theta,2\theta),(\eta-\rho-\nu\theta-2\theta\lambda,-2\theta);\\(\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu),(\alpha-\rho-\theta\nu-2\theta\lambda+1/2,-2\theta);\end{matrix}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \quad (28)$$

□

THEOREM 6. *Let* $z \in \mathbb{C}/(-\infty, 0],\ m \in \mathbb{N},\ \nu, \lambda \in \mathbb{C}, \mu > 0.$ *Then the product of the Whittaker function and the generalized Lommel-Wright function defined in (7) is given by*

$$\int_0^{+\infty} z^{\rho-1} W_{\eta,\alpha}(az) W_{-\eta,\alpha}(az) J_{\nu,\lambda}^{\mu,m}(w\ z^{\theta}) dz = \frac{\left(w/2\right)^{\nu+2\lambda} (1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}}$$

$${}_3\psi_{m+2}\left[\begin{array}{c}(1,1), (\frac{\rho+\theta(\nu+2\lambda)+1}{2} \pm \alpha, \theta), (\rho+\theta(\nu+2\lambda)+1, 2\theta);\\ (\lambda+1,1), ..., (\lambda+1,1), (\nu+\lambda+1,\mu), 2(1+\frac{\rho+\theta(\nu+2\lambda)}{2} \pm \eta, \theta);\end{array}\right.$$

$$\left.\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{29}$$

Proof. Putting $az = x, adz = dx$ as $z \to 0, x \to 0$ and $z \to +\infty, x \to +\infty$ and using (7) in the integrand of (6) which is verified by uniform convergence of the involved series under the given conditions, we get

$$\int_0^{+\infty} z^{\rho-1} W_{-\eta,\alpha}(az) W_{\eta,\alpha}(az) J_{\nu,\lambda}^{\mu,m}(wz^{\theta}) dz = \frac{\left(w/2\right)^{\nu+2\lambda} (1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}} \times$$

$$\sum_{n=0}^{+\infty} \frac{\Gamma(n+1)\left(\frac{-w^2}{4(a^2)^{\theta}}\right)^n}{\Gamma(\lambda+n+1)^m \Gamma(\nu+\lambda+n\mu+1) n!} \times$$

$$\int_0^{+\infty} x^{\rho+\theta(2n+\nu+2\lambda)-1} W_{-\eta,\alpha}(x) W_{\eta,\alpha}(x) dx.$$

Now using (17) in the above equation we get

$$\int_0^{+\infty} z^{\rho-1} W_{-\eta,\alpha}(az) W_{\eta,\alpha}(az) J_{\nu,\lambda}^{\mu,m}(wz^{\theta}) dz = \frac{(w/2)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}} \times$$

$$\sum_{n=0}^{+\infty} \frac{\Gamma(n+1)\Gamma(\frac{\rho+\theta(\nu+2\lambda+2n+1)}{2} \pm \alpha)\Gamma(\rho+\theta\nu+2\theta\lambda+2\theta\ n)(\frac{-w^2}{4(a^2)^{\theta}})^n}{\Gamma(\lambda+n+1)^m \Gamma(\nu+\lambda+n\mu+1) 2\Gamma(1+\frac{\rho+\theta(\nu+2\lambda+2n)}{2} \pm \eta) n!}$$

$$= \frac{\left(w/2\right)^{\nu+2\lambda} (1/a)^{\theta(\nu+2\lambda)}}{(a)^{\rho}} \times$$

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$$
{}_3\psi_{m+2}\left[\begin{array}{c}(1,1),(\frac{\rho+\theta(\nu+2\lambda)+1}{2}\pm\alpha,\theta),(\rho+\theta(\nu+2\lambda)+1,2\theta);\\ (\lambda+1,1),...,(\lambda+1,1),(\nu+\lambda+1,\mu),2(1+\frac{\rho+\theta(\nu+2\lambda)}{2}\pm\eta,\theta);\end{array}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \quad (30)
$$

□

Special cases

In this section, we get some integral formulas involving a trigonometric function and the generalized Lommel-Wright function as follows:

Corollary 1. *If we take $m=1,\mu=1,\lambda=0$ and $\nu=1/2$ in (1) and then by using (8), we derive the following integral formula:*

$$
\int_0^{+\infty} z^{\rho-w/2-1}J_\eta(az)\sin(b\,z^w)dz=\Big(b/4\Big)\sqrt{\pi}\left(\frac{2}{a}\right)^{\rho+w/2}\times
$$

$$
{}_1\psi_2\left[\begin{array}{c}(\frac{\eta+\rho+w/2}{2},w);\\ (3/2,1),((\frac{2+\eta-(\rho+w/2)}{2}),-w);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \quad (31)
$$

Corollary 2. *If we take $m=1,\mu=1,\lambda=0$ and $\nu=1/2$ in (2) and then by using (8), we obtain:*

$$
\int_0^{+\infty} z^{\rho-w/2-1}K_\eta(az)\sin(b\,z^w)dz=\Big(b/8\Big)\sqrt{\pi}\left(\frac{2}{a}\right)^{\rho+w/2}\times
$$

$$
{}_1\psi_1\left[\begin{array}{c}(\frac{\rho+w/2\pm\eta}{2},w);\\ (3/2,1);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \quad (32)
$$

Corollary 3. *If we take $m=1,\mu=1,\lambda=0$ and $\nu=1/2$ in (3) and then by using (8), we obtain:*

$$
\int_0^{+\infty} z^{\rho-w/2-1}\exp(-az)K_\eta(az)\sin(b\,z^w)dz=\left(\frac{\pi\;ab}{(2a)^{\rho+w/2}}\right)\times
$$

$$
{}_1\psi_2\left[\begin{array}{c}(\rho+w/2\pm\eta,2w);\\ (3/2,1),(\rho+w/2+1/2,2w);\end{array}\left(\frac{-b^2}{4(4a^2)^w}\right)\right]. \quad (33)
$$

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COROLLARY 4. *If we take $m = 1, \mu = 1, \lambda = 0$ and $\nu = 1/2$ in (4) and then by using (8), we obtain:*

$$\int_0^{+\infty} z^{\rho-\theta/2-1}\exp(az/2)W_{\eta,\alpha}(az)\sin(w\ z^{\theta})dz =$$
$$\left(\frac{w/2\sqrt{\pi}}{(a)^{\rho+w/2}(\Gamma(1/2\pm\alpha-\eta))}\right)\times$$
$${}_2\psi_1\left[\begin{array}{c}(1/2\pm\alpha+\rho+\theta/2,2\theta),(\eta-\rho-\theta/2,-2\theta);\\(3/2,1);\end{array}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \quad (34)$$

COROLLARY 5. *If we take $m = 1, \mu = 1, \lambda = 0$ and $\nu = 1/2$ in (5) and then by using (8), we obtain:*

$$\int_0^{+\infty} z^{\rho-\theta/2-1}\exp(-az/2)M_{\eta,\alpha}(az)\sin(w\ z^{\theta})dz =$$
$$\left(\frac{w/2\sqrt{\pi}(1/a)^{\theta/2}\Gamma(2\alpha+1)}{(a)^{\rho}(\Gamma(\alpha+\eta+1/2))}\right)\times$$
$${}_2\psi_2\left[\begin{array}{c}(\alpha+\rho+\theta/2,2\theta),(\eta-\rho-\theta/2,-2\theta);\\(3/2,1),(\alpha-\theta+1/2,-2\theta);\end{array}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \quad (35)$$

COROLLARY 6. *If we take $m = 1, \mu = 1, \lambda = 0$ and $\nu = 1/2$ in (6) and then by using (8), we obtain:*

$$\int_0^{+\infty} z^{\rho-\theta/2-1}W_{\eta,\alpha}(az)W_{-\eta,\alpha}(az)\sin(w\ z^{\theta})dz = \left(\frac{w/2\sqrt{\pi}(1/a)^{\theta/2}}{(a)^{\rho}}\right)$$
$${}_2\psi_2\left[\begin{array}{c}(\frac{\rho+\theta/2+1}{2}\pm\alpha,\theta),(\rho+\theta/2+1,2\theta);\times\\(3/2,1),2(1+\frac{\rho+\theta/2}{2}\pm\eta,\theta);\end{array}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \quad (36)$$

COROLLARY 7. *If we take $m = 1, \mu = 1, \lambda = 0$ and $\nu = -1/2$ in (1) and then by using (9), we derive the following integral formula:*

$$\int_0^{+\infty} z^{\rho-w/2-1}J_{\eta}(az)\cos(z)dz = \sqrt{\left(\frac{\pi}{4}\right)}\left(2/a\right)^{\rho-w/2}\times$$
$${}_1\psi_2\left[\begin{array}{c}(\frac{\eta+\rho-w/2}{2},w);\\(1/2,1),((\frac{2+\eta-(\rho-w/2)}{2}),-w);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^{w}\right]. \quad (37)$$

Corollary 8. *If we take* $m=1,\mu=1,\lambda=0$ *and* $\nu=-1/2$ *in* (2) *and then by using* (9)*, we obtain:*

$$\int_0^{+\infty} z^{\rho-w/2-1} K_\eta(az)\cos(z)dz = 1/4\sqrt{\pi}\left(\frac{2}{a}\right)^{\rho-w/2} \times \\ {}_1\psi_1\left[\begin{matrix}(\frac{\rho-w/2\pm\eta}{2},w);\\(1/2,1);\end{matrix}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \tag{38}$$

Corollary 9. *If we take* $m=1,\mu=1,\lambda=0$ *and* $\nu=-1/2$ *in* (3) *and then by using* (9)*, we obtain:*

$$\int_0^{+\infty} z^{\rho-w/2-1}\exp(-az)K_\eta(az)\cos(b\,z^w)dz = \left(\frac{2a\pi}{(2a)^{\rho-w/2}}\right)\times \\ {}_1\psi_2\left[\begin{matrix}(\rho-w/2\pm\eta,2w);\\(1/2,1),(\rho-w/2+1/2,2w);\end{matrix}\left(\frac{-b^2}{4(4a^2)^w}\right)\right]. \tag{39}$$

Corollary 10. *If we take* $m=1,\mu=1,\lambda=0$ *and* $\nu=-1/2$ *in* (4) *and then by using* (9)*, we obtain:*

$$\int_0^{+\infty} z^{\rho-\theta/2-1}\exp(az/2)W_{\eta,\alpha}(az)\cos(w\,z^\theta)dz = \\ \left(\frac{\sqrt{\pi}}{(a)^{\rho-\theta/2}(\Gamma(1/2\pm\alpha-\eta))}\right)\times \\ {}_2\psi_1\left[\begin{matrix}(1/2\pm\alpha+\rho-\theta/2,2\theta),(\eta-\rho+\theta/2,-2\theta);\\(1/2,1);\end{matrix}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{40}$$

Corollary 11. *If we take* $m=1,\mu=1,\lambda=0$ *and* $\nu=-1/2$ *in* (5) *and then by using* (9)*, we obtain:*

$$\int_0^{+\infty} z^{\rho-\theta/2-1}\exp(-az/2)M_{\eta,\alpha}(az)\cos(w\,z^\theta)dz = \\ \left(\frac{\sqrt{\pi}(1/a)^{-\theta/2}\Gamma(2\alpha+1)}{(a)^\rho(\Gamma(\alpha+\eta+1/2))}\right)\times \\ {}_2\psi_2\left[\begin{matrix}(\alpha+\rho-\theta/2+1/2,2\theta),(\eta-\rho+\theta/2,-2\theta);\\(1/2,1),(\alpha-\rho+\theta+1/2,-2\theta);\end{matrix}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{41}$$

COROLLARY 12. *If we take $m=1, \mu=1, \lambda=0$ and $\nu=-1/2$ in (6) and then by using (9), we obtain:*

$$\int_0^{+\infty} z^{\rho-\theta/2-1} W_{\eta,\alpha}(az) W_{-\eta,\alpha}(az)\cos(w\, z^{\theta})dz = \left(\frac{\sqrt{\pi}(1/a)^{-\theta/2}}{(a)^{\rho}}\right)\times {}_2\psi_2\left[\begin{array}{c}(\frac{\rho-\theta/2+1}{2}\pm\alpha,\theta),(\rho-\theta/2+1,2\theta);\\(1/2,1),2(1+\frac{\rho-\theta/2}{2}\pm\eta,\theta);\end{array}\left(\frac{-w^2}{4(a^2)^{\theta}}\right)\right]. \tag{42}$$

COROLLARY 13. *If we take $m=1$ in (1) and then by using (10), we derive the following integral formula:*

$$\int_0^{+\infty} z^{\rho-1} J_{\eta}(az) J^{\mu,1}_{\nu,\lambda}(b\, z^{w})dz = \left(1/2\right)\left(b/2\right)^{\nu+2\lambda}\left(2/a\right)^{\rho+w(\nu+2\lambda)}\times {}_2\psi_3\left[\begin{array}{c}(1,1)(\frac{\eta+\rho+w\nu+2w\lambda}{2},w);\\(\lambda+1,1),(\nu+\lambda+1,\mu),((\frac{2+\eta-(\rho+w\nu+2w\lambda)}{2}),-w);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^{w}\right]. \tag{43}$$

COROLLARY 14. *If we take $m=1$ in (2) and then by using (10), we obtain:*

$$\int_0^{+\infty} z^{\rho-1} K_{\eta}(az) J^{\mu,1}_{\nu,\lambda}(b\, z^{w})dz = \left(1/4\right)\left(b/2\right)^{\nu+2\lambda}\left(2/a\right)^{\rho+w(\nu+2\lambda)}\times {}_2\psi_2\left[\begin{array}{c}(1,1),(\frac{\rho+w\nu+2w\lambda\pm\eta}{2},w);\\(\lambda+1,1),(\nu+\lambda+1,\mu);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^{w}\right]. \tag{44}$$

COROLLARY 15. *If we take $m=1$ in (3) and then by using (10), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}\exp(-az) K_{\eta}(az) J^{\mu,1}_{\nu,\lambda}(b\, z^{w})dz = \left(\frac{2a\sqrt{\pi}(b/2)^{\nu+2\lambda}}{(2a)^{\rho+w(\nu+2\lambda)}}\right)\times {}_2\psi_3\left[\begin{array}{c}(1,1),(\rho+w\nu+2w\lambda\pm\eta,2w);\\(\lambda+1,1),(\nu+\lambda+1,\mu),(\rho+w(\nu+2\lambda)+1/2,2w);\end{array}\left(\frac{-b^2}{4(4a^2)^{w}}\right)\right]. \tag{45}$$

COROLLARY 16. *If we take $m=1$ in (4) and then by using (10), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}\exp(az/2) W_{\eta,\alpha}(az) J^{\mu,1}_{\nu,\lambda}(w\, z^{\theta})dz =$$

$$\left(\frac{(w/2)^{\nu+2\lambda}}{(a)^{\rho+\theta(\nu+2\lambda)}(\Gamma(1/2\pm\alpha-\eta))}\right)\times$$

$$ {}_3\psi_2\left[\begin{array}{c}(1,1)(1/2\pm\alpha+\rho+\theta\nu+2\theta\lambda,2\theta),(\eta-\rho-\theta\nu-2\theta\lambda,-2\theta);\\ (\lambda+1,1),(\nu+\lambda+1,\mu);\end{array}\right.$$

$$\left.\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{46}$$

COROLLARY 17. *If we take $m=1$ in (5) and then by using (10), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}\exp(-az/2)M_{\eta,\alpha}(az)J^{\mu,1}_{\nu,\lambda}(w\,z^\theta)dz=$$

$$\left(\frac{(w/1)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}\Gamma(2\alpha+1)}{(a)^\rho(\Gamma(\alpha+\eta+1/2))}\right)\times$$

$$ {}_3\psi_3\left[\begin{array}{c}(1,1),(\alpha+\rho+\theta\nu+2\theta\lambda+1/2,2\theta),(\eta-\rho-\theta\nu-2\theta\lambda,-2\theta);\\ (\lambda+1,1),(\nu+\lambda+1,\mu),(\alpha-\rho-\theta\nu-2\theta\lambda+1/2,-2\theta);\end{array}\right.$$

$$\left.\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{47}$$

COROLLARY 18. *If we take $m=1$ in (6) and then by using (10), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}W_{\eta,\alpha}(az)W_{-\eta,\alpha}(az)J^{\mu,1}_{\nu,\lambda}(w\,z^\theta)dz=\left(\frac{(w/2)^{\nu+2\lambda}(1/a)^{\theta(\nu+2\lambda)}}{(a)^\rho}\right)\times$$

$$ {}_3\psi_3\left[\begin{array}{c}(1,1),(\frac{\rho+\theta(\nu+2\lambda)+1}{2}\pm\alpha,\theta),(\rho+\theta(\nu+2\lambda)+1,2\theta);\\ (\lambda+1,1),(\nu+\lambda+1,\mu),2(1+\frac{\rho+\theta(\nu+2\lambda)}{2}\pm\eta,\theta);\end{array}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \tag{48}$$

COROLLARY 19. *If we take $m=1,\mu=1$ and $\lambda=1/2$ in (1) and then by using (11), we derive the following integral formula:*

$$\int_0^{+\infty} z^{\rho-1}J_\eta(az)H_\nu(b\,z^w)dz=\Big(1/2\Big)\Big(b/2\Big)^{\nu+1}\Big(2/a\Big)^{\rho+w(\nu+1)}\times$$

$$ {}_2\psi_3\left[\begin{array}{c}(1,1)(\frac{\eta+\rho+w\nu+w}{2},w);\\ (3/2,1),(\nu+3/2,1),((\frac{2+\eta-(\rho+w\nu+w)}{2}),-w);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^w\right]. \tag{49}$$

COROLLARY 20. *If we take $m=1,\mu=1$ and $\lambda=1/2$ in (2) and then by using (11), we obtain:*

$$\int_0^{+\infty} z^{\rho-1}K_\eta(az)H_\nu(b\,z^w)dz=\Big(1/4\Big)\Big(b/2\Big)^{\nu+1}\Big(2/a\Big)^{\rho+w(\nu+1)}\times$$

$$ {}_2\psi_2\left[\begin{array}{c}(1,1),(\frac{\rho+w\nu+w\pm\eta}{2},w);\\(3/2,1),(\nu+3/2,\mu);\end{array}\left(\frac{-b^2}{4}\right)\left(\frac{4}{a^2}\right)^{w}\right]. \quad (50) $$

COROLLARY 21. *If we take $m=1,\mu=1$ and $\lambda=1/2$ in (3) and then by using (11), we obtain:*

$$ \int_0^{+\infty} z^{\rho-1}\exp(-az)K_\eta(az)H_\nu(b\,z^w)dz=\left(\frac{2a\sqrt{\pi}(b/2)^{\nu+1}}{(2a)^{\rho+w(\nu+1)}}\right)\times $$
$$ {}_2\psi_3\left[\begin{array}{c}(1,1),(\rho+w\nu+w\pm\eta,2w);\\(3/2,1),(\nu+3/2,1),(\rho+w(\nu+1)+1/2,2w);\end{array}\left(\frac{-b^2}{4(4a^2)^w}\right)\right]. \quad (51) $$

COROLLARY 22. *If we take $m=1,\mu=1$ and $\lambda=1/2$ in (4) and then by using (11), we obtain:*

$$ \int_0^{+\infty} z^{\rho-1}\exp(az/2)W_{\eta,\alpha}(az)H_\nu(w\,z^\theta)dz=\left(\frac{(w/2)^{\nu+1}}{(a)^{\rho+\theta(\nu+1)}(\Gamma(1/2\pm\alpha-\eta))}\right) $$
$$ {}_3\psi_2\left[\begin{array}{c}(1,1)(1/2\pm\alpha+\rho+\theta\nu+\theta,2\theta),(\eta-\rho-\theta\nu-\theta,-2\theta);\\(3/2,1),(\nu+3/2,1);\end{array}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \quad (52) $$

COROLLARY 23. *If we take $m=1,\mu=1$ and $\lambda=1/2$ in (5) and then by using (11), we obtain:*

$$ \int_1^{+\infty} z^{\rho-1}\exp(-az/2)M_{\eta,\alpha}(az)H_\nu(w\,z^\theta)dz= $$
$$ \left(\frac{(w/2)^{\nu+1}(1/a)^{\theta(\nu+1)}\Gamma(2\alpha+1)}{(a)^\rho(\Gamma(\alpha+\eta+1/2))}\right)\times $$
$$ {}_3\psi_3\left[\begin{array}{c}(1,1),(\alpha+\rho+\theta\nu+\theta+1/2,2\theta),(\eta-\rho-\theta\nu-\theta,-2\theta);\\(3/2,1),(\nu+3/2,1),(\alpha-\rho-\theta\nu-\theta+1/2,-2\theta);\end{array}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \quad (53) $$

COROLLARY 24. *If we take $m=1,\mu=1$ and $\lambda=1/2$ in (6) and then by using (11), we obtain:*

$$ \int_0^{+\infty} z^{\rho-1}W_{\eta,\alpha}(az)W_{-\eta,\alpha}(az)H_\nu(w\,z^\theta)dz=\left(\frac{(w/2)^{\nu+1}(1/a)^{\theta(\nu+1)}}{(a)^\rho}\right)\times $$

$$ {}_3\psi_3\left[\begin{array}{c}(1,1),(\frac{\rho+\theta(\nu+1)+1}{2}\pm\alpha,\theta),(\rho+\theta(\nu+1)+1,2\theta);\\(3/2,1),(\nu+3/2,1),2(1+\frac{\rho+\theta(\nu+1)}{2}\pm\eta,\theta);\end{array}\left(\frac{-w^2}{4(a^2)^\theta}\right)\right]. \quad (54) $$

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ИССЛЕДОВАНИЕ ИНТЕГРАЛЬНЫХ ПРЕОБРАЗОВАНИЙ ОБОБЩЕННЫХ ФУНКЦИЙ ЛОММЕЛЯ-РАЙТА

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РУБРИКА ГРНТИ: 27.23.21 Интегральные преобразования. Операционное исчисление,
27.23.25 Специальные функции,
27.27.19 Функции многих комплексных переменных

ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: Целью данной статьи является установление интегральных преобразований обобщенной функции Поммеля-Райта.

Методы: Эти преобразования выражаются в терминах гипергеометрической функции Райта.

Результаты: В результате получены интегралы с тригонометрическими, обобщенными функциями Бесселя и Струве.

Выводы: Вследствие применения данного метода получаются различные интересные преобразования.

Ключевые слова: обобщенные функции Поммеля-Райта J(z), преобразование Ханкеля, К-преобразование, функция Райта, функция Уиттекера.

СТУДИЈА О ИНТЕГРАЛНИМ ТРАНСФОРМАЦИЈАМА ГЕНЕРАЛИЗОВАНЕ ФУНКЦИЈЕ ЛОМЕЛА И РАЈТА

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ОБЛАСТ: математика
ВРСТА ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: Циљ овог рада јесте успостављање интегралних трансформација генерализоване функције Ломела и Рајта.

Методе: Интегралне трансформације изражене су помоћу Рајтове хипергеометријске функције.

Резултати: Добијени су интеграли који укључују тригонометријске, генерализоване Беселове и Струвеове функције.

Закључак: Као последице ове методе добијају се разне занимљиве трансформације.

Кључне речи: генерализоване функције Ломела и Рајта J(z), Ханкелова трансформација, K-трансформација, Рајтова функција, Витакерова функција.

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