DIBR – FUZZY MARCOS MODEL FOR SELECTING A LOCATION FOR A HEAVY MECHANIZED BRIDGE

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Abstract:

Introduction/purpose: The paper presents the DIBR-FMARCOS model of multi-criteria decision-making for defining a location for placing a bridge over an obstacle using a heavy mechanized bridge (TMM-3). After the application of the proposed model, the sensitivity analysis of the output results was performed and it was concluded that the model is stable, i.e. that the model gives consistent results and that its application is possible in real situations.

Methods: The DIBR method was used to determine the weight coefficients of the criteria, while the ranking of alternatives was performed by the FuzzyMARCO method.

Results: The application of this model has led to the selection of the location for placing a bridge from the TMM-3 set, based on the defined criteria. After applying the proposed model, the sensitivity analysis of the output results was performed and the consistency of the output results of the method was proven.
Conclusion: Finally, it was concluded that the proposed model can be applied in practice, because it gives stable output results. It was also concluded that the DIBR method facilitates the process of obtaining the weight coefficients of the criteria, and the FMARCOS method copes well with unclear and inaccurate input data and has good stability. This model can be further improved by more detailed operationalization of the criteria, as well as by the use of other different methods for determining the weights of the criteria and ranking.

Key words: location, bridge, MCDM, DIBR, Fuzzy, MARCOS.

Introduction

Ensuring the attack pace during offensive operations is an imperative of every army in the world. During their deployment, military units often encounter obstacles, both artificial and natural, which can have a direct impact on the possibility of continuing the ongoing operation, and ultimately on the final outcome of the entire operation. In order to deal with this problem, military units employ bridge systems to overcome obstacles fast and efficiently thus creating conditions for further deployment of their own troops. The Serbian Army has TMM-3 sets of heavy mechanized bridges in its engineering units as one option for surmounting obstacles.

The TMM-3 heavy mechanized bridge is intended for the construction of bridges over natural and artificial obstacles up to 40 m wide and up to 3 m deep for enabling the crossing of tracked vehicles weighing up to 60 tons and wheeled vehicles with the axle pressure up to 11 tons.

The TMM-3 set consists of four bridge-builders (KRAZ-255B vehicle) with four track-type bridge blocks, and all operations during the assembly or disassembly of the bridge are performed by the crew of this vehicle. The length of each bridge block is 10.5 m, while the width of the road is 3.8 m (SSNO, 1973; Weaponsystem.net, 2021).

In order to set up a bridge from the TMM-3 set, it is necessary that a chosen location on an obstacle meet certain conditions. These conditions depend on the characteristics of the obstacle itself and on the specifications and constructional features of the bridge block and the bridge-builder.

In order to define the optimal location for erecting a bridge over an obstacle, a choice must be made among several different locations on the obstacle that meet the minimum requirements for the use of this system, for which the application of multicriteria decision-making methods is suitable.
Since the characteristics of obstacles are diverse, and do not always represent quantitative properties, i.e. the input data are very often of a qualitative type, it is desirable to use one of the ways of defining uncertainty to describe certain properties, such as the fuzzy set theory.

The application of this theory, in combination with some of the methods of multi-criteria decision-making which treat different problems, has been presented in many papers: for selecting the most efficient procedure for rectification of the optical sight of the long-range rifle in the MCDM model with the AHP and VIKOR methods (Radovanovic et al., 2020), for supplier selection in combination with the PIPERCIA and SAW methods (Dalić et al., 2020), for evaluating websites with the WASPAS method (Stanujkić & Karabašević, 2018), as an auxiliary tool in optimizing the procurement process in order to achieve additional savings by developing stronger cooperation with the optimal supplier,
applying the AHP and TOPSIS methods (Chatterjee & Stević, 2019), to select a supplier using the ELECTRE method (Milovanović et al, 2021), for choosing directions of action of the Group for additional hindering with the AHP and ANP methods (Pamučar et al, 2012), to select a construction machine in the neuro-fuzzy model (Božanić et al, 2021), for risk assessment of natural disasters (Pamučar et al, 2016), to resolve the problems of inadequate, indistinct, and discrepant information (Zulqarnain et al, 2021), for a third party reverse logistic provider (3PRLP) optimization problem (Riaz et al, 2021), etc.

This paper presents the application of the MCDM model for determining the best location for bridge construction over an obstacle using the TMM-3, in which the DIBR methods and the MARCOS method, modified by triangular fuzzy numbers, were applied. The overview of the applied methodology is given in Figure 1.

Literature review

Different methods are used to solve different problems of multicriteria decision making. This part of the paper gives an overview of the literature focusing on the location choice as well as on the DIBR and MARCOS methods.

The problem of location selection has been elaborated in many papers in which multicriteria decision-making methods are applied. Yücenur and Ipekçı select the location for an offshore power plant using the SWARA and WASPAS methods (Yücenur & Ipekçı, 2021). Mihajlović et al (2019) evaluate the locations for the logistics and distribution center in the southern and eastern region of Serbia using the AHP and WASPAS methods, while Kaya (2021) consider the problem of choosing a location for a small hotel in a case study of Cappadocia in Turkey, using the PIPRECIA and ARAS methods. Many authors deal with the problem of choosing the location for a warehouse: applying the integrated gray GPSI (gray preference selection index) model, GPIV (gray proximity indexed value) and comparing with the TOPSIS, WASPAS and COPRAS methods in the sensitivity analysis (Ulutaş et al, 2021), using the UTASTAR method (Ehsanifar et al, 2021), for selecting a location for a warehouse for a humanitarian supply chain, with the AHP and TOPSIS methods (Ak & Acar, 2021), by applying a hierarchical fuzzy model of multicriteria decision making (Arif et al, 2021), using the fuzzy AHP method (Singh et al, 2018), etc. The study of the problem of site selection in the field of military application is presented in the following papers: to select a firing position for mortar units using the LBWA and
FMABAC model (Jokić et al., 2021), to solve the problem of a location for an unmanned border and coastal anti-aircraft gun using the improved genetic algorithm (IGA) (Xu et al., 2021), to select the location for a brigade command post using the FUCOM - Z-number - MABAC model (Bozanic et al., 2020a), for selecting a location for deep wading as a technique of crossing the river by tanks (Bozanic et al., 2018), for the selection of a location for the construction of a single-span Bailey bridge using the FUCOM - Fuzzy MABAC model (Bozanic et al., 2019), to select the location for tanks to drive across the ice using the FAHP and TOPSIS methods (Tešić et al., 2018), etc.

The DIBR method is a new method developed in 2021 (Pamucar et al., 2021) and so far no paper has been published that solves the problems of determining the weights of criteria by this method, except for the paper in which it was presented.

The MARCOS method has been processed in a large number of papers that deal with and solve various problems: selecting a location for offshore wind farms using interval rough numbers with the Best-Worst method (Deveci et al., 2021); choosing sustainable suppliers using the fuzzy theory (Puška et al., 2021); assessing the quality of e-services in the aviation industry together with the application of the fuzzy theory and the AHP method (Bakır & Atalık, 2021); selecting a location for a landfill for medical waste in urban areas with the BWM method and the gray theory (Torkayesh et al., 2021); regional evaluation of renewable energy sources in Turkey with the AHP method (Karaaslan et al., 2021); inventory classification, together with the SWARA method (Miškić et al., 2021); assessing vehicles on alternative fuels for sustainable road transport in the USA with the FUCOM method and the fuzzy theory (Pamucar et al., 2021b); determining the impact of insurance companies in connection with the COVID-19 pandemic on health services in a fuzzy environment (Ecer & Pamucar, 2021); South African traffic safety evaluation model with the CRITIC and DEA methods (Stević et al., 2021), etc.

**Description of methods**

**DIBR method**

The DIBR method is based on defining the relationship between ranked criteria, i.e. it considers the relationship between adjacent criteria, and this method consists of five steps presented below (Pamucar et al., 2021):
Step 1. Ranking of criteria according to significance.

On a defined set of $n$ criteria $C = \{C_1, C_2, ..., C_n\}$ the criteria are ranked according to their significance as $C_1 > C_2 > C_3 > ... > C_n$.

Step 2. Comparison of criteria and definition of mutual relations.

When comparing the criteria, the values $\lambda_{12}, \lambda_{13}, ..., \lambda_{n-1,n}$ and $\lambda_{1n}$ are obtained; for example, when comparing the criteria $C_1$ with $C_2$, a value $\lambda_{12}$ is obtained, etc., and all comparison values must satisfy the condition $\lambda_{n-1,n}, \lambda_{1n} \in [0, 1]$. Based on the defined conditions and relationships, the following relationships between the criteria are reached:

\[
\begin{align*}
    w_1 : w_2 &= (1 - \lambda_{12}) : \lambda_{12} \\
    w_2 : w_3 &= (1 - \lambda_{23}) : \lambda_{23} \\
    \vdots \\
    w_{n-1} : w_n &= (1 - \lambda_{n-1,n}) : \lambda_{n-1,n} \\
    w_1 : w_n &= (1 - \lambda_{1n}) : \lambda_{1n}
\end{align*}
\]

Relationships (1) - (4) and the value $\lambda_{n-1,n}$ can be viewed as relationships by which the decision maker divides the total significance interval of the 100% criterion into two observed criteria.

Step 3. Defining equations for the calculation of weight coefficients.

Based on the relationship from step 2, the expressions for determining the weight coefficients of the criteria $w_2, w_3, ..., w_n$ are derived:

\[
\begin{align*}
    w_2 &= \frac{\lambda_{12}}{(1 - \lambda_{12})} w_1 \\
    w_3 &= \frac{\lambda_{12} \lambda_{23}}{(1 - \lambda_{23})} w_2 = \frac{\lambda_{12} \lambda_{23}}{(1 - \lambda_{12})(1 - \lambda_{23})} w_1 \\
    \vdots \\
    w_n &= \frac{\lambda_{n-1,n}}{(1 - \lambda_{n-1,n})} w_{n-1} = \frac{\lambda_{12} \lambda_{23} \cdots \lambda_{n-1,n}}{(1 - \lambda_{12})(1 - \lambda_{23}) \cdots (1 - \lambda_{n-1,n})} w_1 = \frac{\prod_{i=1}^{n-1} \lambda_{i,i+1}}{\prod_{i=1}^{n} (1 - \lambda_{i,i+1})} w_1
\end{align*}
\]
Step 4. Calculation of the weight coefficient of the most influential criterion

Based on expressions (5) - (7) and the condition that it is \( \sum_{j=1}^{n} w_j = 1 \), the following mathematical relation is defined

\[
w_1 \left( 1 + \frac{\lambda_{12}}{(1 - \lambda_{12})} + \frac{\lambda_{12} \lambda_{23}}{(1 - \lambda_{12})(1 - \lambda_{23})} + \ldots + \frac{\prod_{i=1}^{n-1} \lambda_{i,i+1}}{\prod_{i=1}^{n-1} (1 - \lambda_{i,i+1})} \right) = 1
\]

(8)

From expression (8), the final expression for defining the weighting coefficient of the most influential criterion derives:

\[
w_i = \frac{1}{1 + \frac{\lambda_{12}}{(1 - \lambda_{12})} + \frac{\lambda_{12} \lambda_{23}}{(1 - \lambda_{12})(1 - \lambda_{23})} + \ldots + \frac{\prod_{i=1}^{n-1} \lambda_{i,i+1}}{\prod_{i=1}^{n-1} (1 - \lambda_{i,i+1})}}
\]

(9)

Based on the obtained value \( w_1 \), and using the defined expressions (5) - (7), the weight coefficients of other criteria \( w_2, w_3, \ldots, w_n \) are obtained.

Step 5. Defining the degree of satisfying subjective relationships between the criteria.

Based on expression (4), the value of the weight coefficient of the criterion \( w_n \) is defined:

\[
w_n = \frac{\lambda_{1n}}{(1 - \lambda_{1n})} w_1
\]

(10)

Expression (4) is a relation for controlling expression (7), which is intended to check the satisfaction of the decision maker’s preference, and from which the value \( \lambda_{1,n}^\prime \) is defined, expression (11):

\[
\lambda_{1,n}^\prime = \frac{w_n}{w_1 + w_n}
\]

(11)

If the values \( \lambda_{1n} \) and \( \lambda_{1,n}^\prime \) are approximately equal, then it can be concluded that the preference of the DM decision is satisfied. If they differ, it is necessary to first check the relationship for \( \lambda_{1n} \). If the decision maker considers that the relationship \( \lambda_{1n} \) is well defined, the relationship between the criteria should be redefined and the weighting of the criteria should be recalculated. If this is not the case, it is necessary to redefine
the relationship $\lambda_{1,n}$. It is necessary that the deviation of the value $\lambda_{1,n}$ and $\lambda_{1,n}'$ be up to a maximum of 10%. If this is not the case, it is necessary to redefine the relationships between the criteria in order to achieve this condition.

Fuzzy MARCOS method

The MARCOS method was first presented in the article (Stević et al., 2020) and consists of the following 7 steps:

Step 1. Forming the initial decision matrix.
Multicriteria models involve defining a set of n criteria and m alternatives.

For the fuzzyfication of elements of the initial decision matrix, triangular fuzzy numbers were used (Figure 2). Triangular fuzzy numbers have a form $M = (m_1, m_2, m_3)$.

The fuzzy number $M$ membership function is defined by the following expressions:

$$
\mu_M(x) = \begin{cases} 
0, & x < m_1 \\
\frac{x - m_1}{m_2 - m_1}, & m_1 \leq x \leq m_2 \\
1, & x = m_2 \\
\frac{m_3 - x}{m_3 - m_2}, & m_2 \leq x \leq m_3 \\
0, & x > m_3 
\end{cases}
$$

(12)
Since all criteria are subject to subjective assessment by the decision maker and the influence of various factors on the value of the criteria, the degree of confidence is introduced in the process of fuzzification (Božanić et al., 2015) and the distributions of the fuzzy number change according to the expression:

\[
\tilde{M} = \left( m_1, m_2, m_3 \right) = \left\{ \begin{array}{l}
m_1 = \gamma m_2, \quad m_1 \leq m_2 \\
m_2 = m_2 \\
m_3 = (2 - \gamma) m_2, \quad m_3 \leq m_2
\end{array} \right. \tag{13}
\]

The fuzzy number \( \tilde{M} = (m_1, m_2, m_3) = (x\gamma, x, (2 - \gamma) x) \), \( x \in [1, \infty] \) is defined by the expressions (Božanić et al., 2015):

\[
m_1 = x\gamma, \quad \forall \ 1 \leq x\gamma \leq x
\]

\[
m_2 = x, \quad \forall \ x \in [1, \infty]
\]

\[
m_3 = (2 - \gamma) x, \quad \forall x \in [1, \infty]
\]

By implementing the degree of confidence in the given statement, the fuzzification of all values of the criteria for all alternatives is performed, thus obtaining a fictionalized initial decision matrix.

**Step 2. Forming an extended initial matrix.**

Expansion of the initial decision matrix is done by defining the anti-ideal (\( \text{AAI} \)) and ideal (\( \text{AI} \)) solutions.

\[
\begin{bmatrix}
C_1 & C_2 & \ldots & C_n \\
AAI & x_{11} & x_{12} & \ldots & x_{an} \\
A_1 & x_{11} & x_{12} & \ldots & x_{ln} \\
A_2 & x_{21} & x_{22} & \ldots & x_{2n} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_m & x_{m1} & x_{m2} & \ldots & x_{mn} \\
AI & x_{11} & x_{12} & \ldots & x_{an}
\end{bmatrix}
\tag{15}
\]

The anti-ideal solution (\( \text{AAI} \)) represents the worst alternative while the ideal solution (\( \text{AI} \)) represents the alternative with the best feature, and they are obtained by applying expressions (16) and (17):

\[
\text{AAI} = \min_j \tilde{x}_{ij} \text{ if } j \in B \quad \text{and} \quad \max_j \tilde{x}_{ij} \text{ if } j \in C
\tag{16}
\]
Step 3. Normalization of the extended initial matrix \( \tilde{X} \).

The normalized matrix \( \tilde{N} = [\tilde{n}_{ij}]_{m \times n} \), i.e., its elements are obtained by applying expressions (18) and (19):

\[
\tilde{n}_{ij} = \frac{x_{ai}}{\tilde{x}_{ai}} \quad \text{if } j \in C \tag{18}
\]

\[
\tilde{n}_{ij} = \frac{x_{aj}}{\tilde{x}_{aj}} \quad \text{if } j \in B \tag{19}
\]

where \( \tilde{x}_{ij} \) and \( \tilde{x}_{ai} \) represent the elements of the matrix \( \tilde{X} \).

Step 4. Determination of the weighted matrix \( \tilde{V} = [\tilde{v}_{ij}]_{m \times n} \).

The weighted matrix \( \tilde{V} \) is obtained on the basis of expression (20).

\[
\tilde{v}_{ij} = \tilde{n}_{ij} \times \tilde{w}_j \tag{20}
\]

Using expressions (21) and (22), the degrees of utility of the alternative in relation to the anti-ideal and ideal solutions are obtained.

\[
\tilde{K}_i^- = \frac{\tilde{S}_i}{\tilde{S}_{aai}} \tag{21}
\]

\[
\tilde{K}_i^+ = \frac{\tilde{S}_i}{\tilde{S}_{aai}} \tag{22}
\]

Step 5. Calculation of the degree of utility of alternatives \( \tilde{K}_i \).

The degree of utility of alternatives represents the sum of the elements of the matrix \( \tilde{V} \), expression (23)

\[
\tilde{S}_i = \sum_{i=1}^{n} \tilde{v}_{ij} \tag{23}
\]

where \( \tilde{S}_i \in (i = 1, 2, \ldots, m) \)

Step 6. Determination of the utility function of alternatives \( f(K_i) \).

The utility function of alternatives is obtained by applying expression (24)
where \( f(K_i^-) \) represents the defuzzification value of the utility function in relation to the anti-ideal solution while \( f(K_i^+) \) represents the defuzzification value of the utility function in relation to the ideal solution, and they are obtained by expressions (25) and (26).

\[
f(K_i^-) = \frac{K_i^+}{K_i^+ + K_i^-} \\
f(K_i^+) = \frac{K_i^-}{K_i^+ + K_i^-}
\]

where all values are defuzzificated using one of the following expressions (Seiford, 1996; Liou & Wang, 1992):

\[
M = \left(\frac{(m_3 - m_i) + (m_2 - m_i)}{3 + m_i}\right)
\]

\[
M = \left[\lambda m_i + m_i + (1 - \lambda)m_i\right]/2
\]

where \( \lambda \) represents an index of optimism \( \lambda \in [0,1] \) (Bozbura et al., 2007).

**Step 7. Ranking alternatives.** Ranking is done by ranking the values of utility functions (higher value, better ranking).

**Application of the DIBR-FMARCOS model**

Respecting the phases of the formed MCDM model (Figure 1), the first step is to define the criteria and calculate their weighting coefficients.

**Defining the criteria and the weighting coefficients of the criteria**

After the analysis of the literature related to the problem (SSNO, 1973; WeaponSystem.net, 2021), seven criteria have been identified, as follows:

Criteria 1 (C1) – The width of the obstacle (Cost) – It represents the distance from one shore to the other and cannot be longer than 40 m,
since the obstacle is overcome with one set of TMM-3. The value of the criterion is expressed in meters (m).

Criterion 2 \((C_2)\) – The depth of the obstacle at the places where the supports are placed (Cost) – means the place on the obstacle where the supports of the bridge block are placed and cannot be deeper than 3 m. The value of the criterion is expressed in meters (m).

Criterion 3 \((C_3)\) – The slope of the shore at the place of placing the bridge (Cost) – is the angle measured in degrees in relation to the horizontal plane. The slopes of the shores must not be higher than: longitudinal up to 10°, transverse up to 6°. Otherwise, it is necessary to perform certain engineering works that require additional resources and time in order to adjust the slopes of the coast to the allowed limits. The value of the criterion is expressed in degrees (°).

Criterion 4 \((C_4)\) – The slope of the bottom of the obstacle at the places where the supports are placed (Cost) – means the angle measured in degrees in relation to the horizontal plane at the bottom of the obstacle where the bridge block support is placed and must meet the following limits: the axis along the bridge up to 30° and the axis perpendicular to the bridge up to 20°. The value of the criterion is expressed in degrees (°).

Criterion 5 \((C_5)\) – Access roads (Benefit) – This criterion represents the roads leading to and from the obstacle to be overcome, and considers the following characteristics: quality, transverse slope of the access road (cannot exceed 20°), longitudinal slope of the access road (cannot exceed 30°), and the possibility of concealed access. The value of the criterion is expressed on a five-point scale: 1 – insufficient, 2 - sufficient, 3 - good, 4 - very good, and 5 - excellent.

Criterion 6 \((C_6)\) – Load-bearing capacity of the ground on the banks of obstacles (Benefit) – It represents the stability of the banks depending on the soil category. The value of the criteria is expressed on a five-point scale: 1 - insufficient (1st and 2nd category soil), 2 - sufficient (3rd category soil), 3 - good (4th category soil), 4 - very good (5th category soil), and 5 - excellent (6th and 7th category soil) (Mijatović, 2008, p.17).

Criterion 7 \((C_7)\) – Maneuver space (Benefit) – represents the necessary space on the bank for maneuvering (approach, turning and work) bridge-builders. The value of the criteria is expressed on a five-point scale: 1 - insufficient, 2 - sufficient, 3 - good, 4 - very good, and 5 - excellent.

Based on the above, a set of seven criteria was determined \(C_1, C_2, \ldots, C_7\), which are ranked in order of importance as \(C_1 > C_2 > C_3 > C_4 > C_5 > C_6 > C_7\). Based on the rank of the criteria, the
values \( \lambda_{12}, \lambda_{13}, \ldots, \lambda_{67} \) and \( \lambda_{17} \) are defined, as follows: \( \lambda_{12} = 0.45 \), \( \lambda_{23} = 0.44 \), \( \lambda_{34} = 0.47 \), \( \lambda_{45} = 0.46 \), \( \lambda_{56} = 0.49 \), \( \lambda_{67} = 0.45 \) and \( \lambda_{17} = 0.27 \), after which the following relations were defined:

\[
\begin{align*}
\lambda_1 : \lambda_2 &= 0.55 : 0.45 \\
\lambda_2 : \lambda_3 &= 0.56 : 0.44 \\
\lambda_3 : \lambda_4 &= 0.53 : 0.47 \\
\lambda_4 : \lambda_5 &= 0.54 : 0.46 \\
\lambda_5 : \lambda_6 &= 0.51 : 0.49 \\
\lambda_6 : \lambda_7 &= 0.55 : 0.45 \\
\lambda_1 : \lambda_7 &= 0.73 : 0.27
\end{align*}
\]

Based on the previous relations, expressions (5) - (7) are used for defining the expressions for the values of the weight coefficients of the criteria:

\[
\begin{align*}
\lambda_2 &= 0.818 \lambda_1 \\
\lambda_3 &= 0.786 \lambda_2 + 0.643 \lambda_1 \\
\lambda_4 &= 0.873 \lambda_3 + 0.561 \lambda_1 \\
\lambda_5 &= 0.839 \lambda_4 + 0.471 \lambda_1 \\
\lambda_6 &= 0.961 \lambda_5 + 0.452 \lambda_1 \\
\lambda_7 &= 0.818 \lambda_6 + 0.370 \lambda_1
\end{align*}
\]

Based on the condition \( \sum_{j=1}^{7} \lambda_j = 1 \) and expression (9), it follows that it is

\[
\begin{align*}
\lambda_1 &= \frac{1}{1 + 0.818 + 0.643 + 0.561 + 0.471 + 0.452 + 0.370} = 0.2318
\end{align*}
\]

Expressions (5) - (7) are used for calculating the weight coefficients of the remaining criteria \( \lambda_2 = 0.1896; \lambda_3 = 0.1490; \lambda_4 = 0.1300; \lambda_5 = 0.1091; \lambda_6 = 0.1048 \) and \( \lambda_7 = 0.0858 \).

With expression (11), the control value \( \lambda_{1,7} \) is calculated.

\[
\lambda_{1,7} = \frac{\lambda_7}{\lambda_1 + \lambda_7} = \frac{0.0858}{0.2318 + 0.0858} = 0.2701
\]

Since \( \lambda_{1,7} \approx \lambda_{1,7}^{*} \), ie \( \lambda_{1,7}^{*} = 0.2701 \) and \( \lambda_{17} = 0.27 \), it is concluded that expert preferences are well defined, i.e. that the transitive relations that define the significance of the criteria are met.

The previously explained steps of the DIBR method are applied in order to obtain the following weight coefficients of the criteria:
Table 1 – Values of the weight coefficients of the criteria
Таблица 1 – Значения весовых коэффициентов критериев
Табела 1 – Вредности тежинских коэфицијената критеријума

<table>
<thead>
<tr>
<th>Criterion</th>
<th>Weight coefficient of the criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>K-1</td>
<td>0.2318</td>
</tr>
<tr>
<td>K-2</td>
<td>0.1896</td>
</tr>
<tr>
<td>K-3</td>
<td>0.1490</td>
</tr>
<tr>
<td>K-4</td>
<td>0.1300</td>
</tr>
<tr>
<td>K-5</td>
<td>0.1091</td>
</tr>
<tr>
<td>K-6</td>
<td>0.1048</td>
</tr>
<tr>
<td>K-7</td>
<td>0.0858</td>
</tr>
</tbody>
</table>

**Ranking alternatives**

Based on the defined criteria and four alternatives on the obstacle on which it is necessary to build a bridge from the TMM-3 set and based on the opinion of experts, the following decision matrix was defined, which is the first step in applying the MARCOS method:

\[
X = \begin{bmatrix}
A_1 & (35,90) & (1,9,90) & (4,100) & (5,80) & (4,80) & (4,60) & (5,100) \\
A_2 & (37,80) & (2,100) & (3,90) & (4,90) & (5,70) & (5,80) & (4,80) \\
A_3 & (32,100) & (2,8,90) & (4,80) & (3,90) & (4,80) & (4,100) & (4,80) \\
A_4 & (39,80) & (2,6,90) & (4,90) & (4,100) & (4,80) & (5,90) & (5,90)
\end{bmatrix}
\]

where the element of the set \(X\), for example \((35,90)\), represents the following: 35 is the value of the criterion \(C_1\) for the alternative \(A_1\) defined by the decision maker (the width of the obstacle of 35 meters), and 90 is the degree of confidence (the decision maker is 90% sure that the value of the criterion \(C_1\) for the alternative \(A_1\) (35) is correct).

By implementing the degree of confidence and applying expression (14), a fuzzy initial decision matrix is obtained:

\[
X = \begin{bmatrix}
A_1 & (31,5,35,38,5) & (1,71,1,9,2,09) & (4,4,4) & (4,5,6) & (3,2,4,4,8) & (2,4,4,5,6) & (5,5,5) \\
A_2 & (29,6,37,44,4) & (2,2,2) & (2,7,3,3,3) & (3,6,4,4,4) & (3,5,5,6,5) & (4,5,6) & (3,2,4,4,8) \\
A_3 & (32,32,32) & (2,52,2,8,3,08) & (3,2,4,4,8) & (2,7,3,3,3) & (3,2,4,4,8) & (4,4,4) & (3,2,4,4,8) \\
A_4 & (31,2,39,46,8) & (2,34,2,6,2,86) & (3,6,4,4,4) & (4,4,4) & (3,2,4,4,8) & (4,5,5,5,5) & (4,5,5,5,5)
\end{bmatrix}
\]
Step 2. Forming an extended initial matrix.
By applying expressions (16) and (17), an extended initial decision matrix was obtained.

\[
X = \begin{bmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \\
\text{AII} & (32.39, 46.8) & (2.52, 2.8, 3.08) & (4.4, 4.8) & (4.5, 6) & (3.2, 4.4, 8) & (2.4, 4.4) & (3.2, 4.4, 8) \\
A_1 & (31.5, 35.38.5) & (1.71, 1.9, 2.09) & (4.4, 4.4) & (4.5, 6) & (3.2, 4.4, 8) & (2.4, 4.4, 5.6) & (5.5, 5) \\
A_2 & (29.6, 37.44.4) & (2.2, 2) & (2.7, 3.3, 3.3) & (3.6, 4.4, 4.4) & (3.5, 5.6, 5) & (4.5, 6) & (3.2, 4.4, 8) \\
A_3 & (32.32, 32) & (2.52, 2.8, 3.08) & (3.2, 4.4, 8) & (2.7, 3.3, 3.3) & (3.2, 4.4, 8) & (4.4, 4.4) & (3.2, 4.4) \\
A_4 & (31.2, 39, 6.8) & (2.34, 2.6, 2.86) & (3.6, 4.4, 4.4) & (4.4, 4, 4) & (3.2, 4.4, 8) & (4.5, 5.5, 5) & (4.5, 5.5, 5) \\
A_5 & (29.6, 32, 32) & (1.71, 1.9, 2) & (2.7, 3.3, 3.3) & (3.5, 5.6, 5) & (4.5, 5.6) & (5.5, 6, 5) & (5.5, 5) \\
\end{bmatrix}
\]

Step 3. Normalization of the extended initial matrix (X)
The normalized values for the cost criteria were calculated with expression (18) and the normalized values for the benefit criteria were calculated with expression (19).

\[
X = \begin{bmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \\
\text{AII} & (0.632, 0.759, 0.632) & (0.555, 0.611, 0.555) & (0.563, 0.675, 0.563) & (0.582, 0.727, 0.582) \\
A_1 & (0.769, 0.846, 0.94) & (0.818, 0.91) & (0.675, 0.75) & (0.909, 0.909, 0.909) \\
A_2 & (0.667, 0.81) & (0.855, 0.855, 0.855) & (0.818, 0.91) & (0.582, 0.727, 0.873) \\
A_3 & (0.925, 0.925, 0.925) & (0.555, 0.611, 0.679) & (0.563, 0.675, 0.844) & (0.582, 0.727, 0.873) \\
A_4 & (0.632, 0.759, 0.949) & (0.598, 0.658, 0.731) & (0.614, 0.675, 0.75) & (0.818, 0.909, 1) \\
A_5 & (0.925, 0.925, 0.925) & (0.855, 0.91) & (0.818, 0.91) & (0.909, 0.909, 1) \\
\end{bmatrix}
\]

Step 4. Determination of the weighted matrix \(V = [v_{ij}]_{m \times n}\)
This step represents the determination of the weighted normalized matrix using expression (20) by multiplying all the values of the normalized matrix by the values of the criteria:

\[
V = \begin{bmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 \\
\text{AII} & (0.147, 0.176, 0.147) & (0.105, 0.116, 0.105) & (0.084, 0.101, 0.084) & (0.05, 0.062, 0.075) \\
A_1 & (0.178, 0.196, 0.218) & (0.155, 0.171, 0.19) & (0.101, 0.101, 0.101) & (0.078, 0.078, 0.078) \\
A_2 & (0.155, 0.185, 0.232) & (0.162, 0.162, 0.162) & (0.122, 0.134, 0.149) & (0.05, 0.062, 0.075) \\
A_3 & (0.214, 0.214, 0.214) & (0.105, 0.116, 0.129) & (0.084, 0.101, 0.126) & (0.05, 0.062, 0.075) \\
A_4 & (0.147, 0.176, 0.22) & (0.113, 0.125, 0.139) & (0.091, 0.101, 0.112) & (0.07, 0.078, 0.086) \\
A_5 & (0.214, 0.214, 0.232) & (0.162, 0.171, 0.19) & (0.122, 0.134, 0.149) & (0.078, 0.078, 0.086) \\
\end{bmatrix}
\]

Step 5. Calculating the degree of utility of alternatives \(K_i\).
Expressions (21) - (23) are used for calculating the degrees of utility of alternatives in relation to the antideal and ideal solutions.
Step 6. Determination of the utility function of alternatives \( f(K_i) \).

The utility function of alternatives is defined by applying expressions (24) - (26), and their defuzzification is performed by applying expression (28), where the optimism index \( \lambda \) is taken to be 0.5.

Step 7. Ranking alternatives.

The ranking of the alternatives is done based on the final values of the utility functions.

**Table 2 – Values of the utility degree of the alternatives**
**Таблица 2 – Значения степеней полезности альтернатив**
**Таблица 2 – Вредности степени корисности альтернатива**

<table>
<thead>
<tr>
<th>Si</th>
<th>K_1</th>
<th>K_2</th>
<th>K_3</th>
<th>K_4</th>
<th>K_5</th>
<th>K_6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Antiideal</td>
<td>0.498</td>
<td>0.592</td>
<td>0.550</td>
<td>0.906</td>
<td>1.000</td>
<td>1.104</td>
</tr>
<tr>
<td>A1</td>
<td>0.666</td>
<td>0.752</td>
<td>0.852</td>
<td>1.212</td>
<td>1.271</td>
<td>1.712</td>
</tr>
<tr>
<td>A2</td>
<td>0.697</td>
<td>0.803</td>
<td>0.929</td>
<td>1.268</td>
<td>1.356</td>
<td>1.867</td>
</tr>
<tr>
<td>A3</td>
<td>0.683</td>
<td>0.747</td>
<td>0.824</td>
<td>1.243</td>
<td>1.262</td>
<td>1.655</td>
</tr>
<tr>
<td>A4</td>
<td>0.642</td>
<td>0.721</td>
<td>0.820</td>
<td>1.167</td>
<td>1.219</td>
<td>1.648</td>
</tr>
<tr>
<td>Ideal</td>
<td>0.820</td>
<td>0.885</td>
<td>1.000</td>
<td>1.492</td>
<td>1.496</td>
<td>2.009</td>
</tr>
</tbody>
</table>

**Table 3 – Values of the utility function of the alternatives**
**Таблица 3 – Значения функции полезности альтернатив**
**Таблица 3 – Вредности функције корисности альтернатива**

| A1 | 0.404 | 0.424 | 0.571 | 0.222 | 0.283 | 0.346 |
| A2 | 0.423 | 0.452 | 0.622 | 0.232 | 0.302 | 0.378 |
| A3 | 0.415 | 0.421 | 0.552 | 0.228 | 0.281 | 0.335 |
| A4 | 0.389 | 0.406 | 0.549 | 0.214 | 0.272 | 0.333 |

**Table 4 – Rank of the alternatives**
**Таблица 4 – Ранг альтернатив**
**Таблица 4 – Ранг альтернатива**

<table>
<thead>
<tr>
<th>f(K_i)</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.470</td>
<td>2</td>
</tr>
<tr>
<td>0.546</td>
<td>1</td>
</tr>
<tr>
<td>0.461</td>
<td>3</td>
</tr>
<tr>
<td>0.430</td>
<td>4</td>
</tr>
</tbody>
</table>
Sensitivity analysis

The sensitivity analysis is a logical step in model validation and has been presented in a large number of papers (Božanić et al, 2021; Božanić et al, 2020b; Muhammad et al, 2021; Durmić et al, 2020; Božanić et al, 2015). The paper analyzes the sensitivity of the model output results to the changes in the weight coefficients (Pamučar et al, 2017), through the following 18 scenarios (Table 5):

<table>
<thead>
<tr>
<th></th>
<th>K1</th>
<th>K2</th>
<th>K3</th>
<th>K4</th>
<th>K5</th>
<th>K6</th>
<th>K7</th>
</tr>
</thead>
<tbody>
<tr>
<td>S1</td>
<td>0.143</td>
<td>0.143</td>
<td>0.143</td>
<td>0.143</td>
<td>0.143</td>
<td>0.143</td>
<td>0.143</td>
</tr>
<tr>
<td>S2</td>
<td>0.218</td>
<td>0.192</td>
<td>0.151</td>
<td>0.132</td>
<td>0.111</td>
<td>0.107</td>
<td>0.088</td>
</tr>
<tr>
<td>S3</td>
<td>0.204</td>
<td>0.194</td>
<td>0.154</td>
<td>0.135</td>
<td>0.114</td>
<td>0.109</td>
<td>0.090</td>
</tr>
<tr>
<td>S4</td>
<td>0.190</td>
<td>0.197</td>
<td>0.156</td>
<td>0.137</td>
<td>0.116</td>
<td>0.112</td>
<td>0.093</td>
</tr>
<tr>
<td>S5</td>
<td>0.176</td>
<td>0.199</td>
<td>0.158</td>
<td>0.139</td>
<td>0.118</td>
<td>0.114</td>
<td>0.095</td>
</tr>
<tr>
<td>S6</td>
<td>0.162</td>
<td>0.201</td>
<td>0.161</td>
<td>0.142</td>
<td>0.121</td>
<td>0.116</td>
<td>0.097</td>
</tr>
<tr>
<td>S7</td>
<td>0.148</td>
<td>0.204</td>
<td>0.163</td>
<td>0.144</td>
<td>0.123</td>
<td>0.119</td>
<td>0.100</td>
</tr>
<tr>
<td>S8</td>
<td>0.134</td>
<td>0.206</td>
<td>0.165</td>
<td>0.146</td>
<td>0.125</td>
<td>0.121</td>
<td>0.102</td>
</tr>
<tr>
<td>S9</td>
<td>0.121</td>
<td>0.208</td>
<td>0.168</td>
<td>0.149</td>
<td>0.128</td>
<td>0.123</td>
<td>0.104</td>
</tr>
<tr>
<td>S10</td>
<td>0.107</td>
<td>0.210</td>
<td>0.170</td>
<td>0.151</td>
<td>0.130</td>
<td>0.126</td>
<td>0.107</td>
</tr>
<tr>
<td>S11</td>
<td>0.093</td>
<td>0.213</td>
<td>0.172</td>
<td>0.153</td>
<td>0.132</td>
<td>0.128</td>
<td>0.109</td>
</tr>
<tr>
<td>S12</td>
<td>0.079</td>
<td>0.215</td>
<td>0.174</td>
<td>0.156</td>
<td>0.135</td>
<td>0.130</td>
<td>0.111</td>
</tr>
<tr>
<td>S13</td>
<td>0.065</td>
<td>0.217</td>
<td>0.177</td>
<td>0.158</td>
<td>0.137</td>
<td>0.133</td>
<td>0.114</td>
</tr>
<tr>
<td>S14</td>
<td>0.051</td>
<td>0.220</td>
<td>0.179</td>
<td>0.160</td>
<td>0.139</td>
<td>0.135</td>
<td>0.116</td>
</tr>
<tr>
<td>S15</td>
<td>0.037</td>
<td>0.222</td>
<td>0.181</td>
<td>0.162</td>
<td>0.142</td>
<td>0.137</td>
<td>0.118</td>
</tr>
<tr>
<td>S16</td>
<td>0.023</td>
<td>0.224</td>
<td>0.184</td>
<td>0.165</td>
<td>0.144</td>
<td>0.140</td>
<td>0.121</td>
</tr>
<tr>
<td>S17</td>
<td>0.009</td>
<td>0.227</td>
<td>0.186</td>
<td>0.167</td>
<td>0.146</td>
<td>0.142</td>
<td>0.123</td>
</tr>
<tr>
<td>S18</td>
<td>0.002</td>
<td>0.228</td>
<td>0.187</td>
<td>0.168</td>
<td>0.147</td>
<td>0.143</td>
<td>0.124</td>
</tr>
</tbody>
</table>

After applying the weight coefficients of the criteria given in Table 5, the Spearman’s rank correlation coefficient (S) is calculated (Srđević et al, 2009) with the use of the following expression:
where:

\[ S = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n(n^2 - 1)} \]  

where:

- \( D_i \) – is the difference between the rank of a given element in the vector \( w \) and the rank of the corresponding element in the reference vector, and
- \( n \) – is the number of ranked elements.

The identical ranks of the elements define the value of Spearman's coefficient 1 ("ideal positive correlation"). The value of Spearman's coefficient -1 means that the ranks are absolutely opposite ("ideal negative correlation"), and when the value of Spearman's coefficient is 0, the ranks are uncorrelated.

By applying the above scenarios in the proposed model, the correlation of ranks is obtained, i.e., the relationship between the initial rank and the ranks obtained by applying the given scenarios, presented in Figure 3:

![The values of Spearman's coefficient](image)

*Figure 3 – The values of Spearman's rank correlation coefficient*

*Рис. 3 – Значение коэффициента ранговой корреляции Спирмена*

*Слика 3 – Вредности Спирмановог коэффицијента корелације рангова*
Based on the rank correlation values from Figure 3, we can conclude that the ranks are well correlated, i.e. that the output values of the applied model are consistent and stable.

Conclusion

Overcoming obstacles during deployment of military units is one of the most difficult combat operations and the selection of locations for overcoming them requires extensive knowledge and experience of commanding officers. Given a large number of segments that need to be considered in the decision-making process about the location of the bridge, multi-criteria decision-making methods can significantly help decision-makers.

The paper presents a multi-criteria model DIBR-FMARCOS to support decision-making in overcoming obstacles using a TMM-3 set of heavy mechanized bridge. The model have given stable results in the analysis of sensitivity to changes in weights and can find its application in real situations.

The DIBR method gives consistent results regardless of the number of evaluation criteria and eliminates the shortcomings of the nine-point scale used in the BWM and AHP methods while its application facilitates the process of calculating the weights of the criteria when their number is large.

The FMARCOS method improves the area of multi-criteria decision-making by implementing the analysis of the relationship between alternatives and reference points, i.e. between the values of alternatives and the ideal and anti-ideal values. The MARCOS method has shown better stability compared to many other methods, especially when changing the weight coefficients of the criteria.

This model can be further upgraded by more detailed operationalization of the criteria as well as by the application of other, different methods for determining weighting coefficients and ranking alternatives.

References


Tešić, D. & Božanić, D. 2018. Application of the MAIRCA method in the selection of the location for crossing tanks under water. Tehnika, 73(6), pp.860-867. Available at: https://doi.org/10.5937/tehnika1806860T.
Резюме:
Введение/цель: В данной статье представлена модель многокритериального принятия решений DIBR-FMARCOS для определения места установки моста на препятствии с использованием тяжелого механизированного моста (ТММ-3). После применения предложенной модели был проведен анализ чувствительности выходных результатов, который подтвердил стабильность модели и согласованность ее результатов, следовательно данная модель пригодна для применения в реальных ситуациях.
Методы: Для определения весовых коэффициентов критериев использовался метод DIBR, а ранжирование альтернатив выполнялось методом FuzzyMARCOS.
Результаты: Применение данной модели способствовало выбору места установки моста из комплекта ТММ-3 с учетом определенных критериев. После применения предложенной модели был проведен анализ чувствительности выходных результатов и доказана согласованность выходных результатов примененного метода.
Выводы: В итоге был сделан вывод, что предложенную модель можно применять на практике, так как она дает стабильные выходные результаты. Также выявлено, что метод DIBR облегчает процедуру получения весовых коэффициентов критериев, а метод FMARCOS благоприятен для обработки нечетких и неточных входных данных, поскольку обладает соответствующей стабильностью. Данная модель может быть дополнительно улучшена за счет более детальной операционализации критериев, а также применения иных методов определения весов и ранжирования.
Ключевые слова: местоположение, мост, многокритериальное принятие решений, DIBR, фазы, MARCOS.
Oblique: mathematics, operation research, military science
Type of paper: original scientific paper
Abstract:
Introduction/Goal: In the paper, a model of multi-criteria decision DIBR-FMARCOS for determining a location for a heavy mechanized bridge (TM-3) is presented. After applying the proposed model, the analysis of the sensitivity of the output results was performed. It has been learned that the model is stable, providing consistent results, and its application is possible in real situations.
Method: To determine the weights of criteria, the DIBR method was used, while the alternatives were ranked by the Fuzzy MARCOS method.
Results: By applying this model, the location for setting up the bridge from the set of TM-3 was selected based on defined criteria. After applying the proposed model, the sensitivity analysis of the output results was performed, and the consistency of the results of the method was proven.
Conclusion: The proposed model can be used in practice, providing stable output results. In addition, the DIBR method facilitates obtaining the weights of criteria, while the MARCOS method well accepts ambiguous and imprecise data and has good stability. This model can be further improved by improving the operationalization of criteria and using other and different methods for determining the weights of criteria and ranking.
Keywords: location, bridge, multi-criteria decision, DIBR, Fuzzy, MARCOS.

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