A new solution for solving a multi-objective integer programming problem with probabilistic multi - objective optimization

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Abstract:

Introduction/purpose: In this paper, a new solution for solving a multi-objective integer programming problem with probabilistic multi – objective optimization is formulated. Furthermore, discretization by means of the good lattice point and sequential optimization are employed for a successive simplifying treatment and deep optimization.

Methods: In probabilistic multi – objective optimization, a new concept of preferable probability has been introduced to describe the preference degree of each performance utility of a candidate; each performance utility of a candidate contributes a partial preferable probability and the product of all partial preferable probabilities deduces the total preferable probability of a candidate; the total preferable probability thus transfers a multi-objective problem into a single-objective one. Discretization by means of the good lattice point is employed to conduct discrete sampling for a continuous objective function and sequential optimization is used to perform deep optimization. At first, the requirements of integers in the treatment could be given up so as to simply conduct above procedures. Finally, the optimal solutions of the input variables must be rounded to the nearest integers.

Results: This new scheme is used to deal with two production problems, i.e., maximizing profit while minimizing pollution and determining a purchasing plan for spending as little money as possible while getting as large amount of raw materials as possible. Promising results are obtained for the above two problems from the viewpoint of the probability theory for simultaneous optimization of multiple objectives.

Conclusion: This method properly considers simultaneous optimization of multiple objectives in multi-objective integer programming, which naturally reflects the essence of multi-objective programming, and opens a new way of solving multi-objective problems.

Key words: multi-objective optimization, integer programming problem, preferable probability, discrete sampling, sequential optimization.

Introduction

Multi-objective programming (GP) is an important branch of optimization theory. It is a mathematical method developed to solve multi-objective decision-making problems based on linear and nonlinear programmings. Since 1960s, it has been gradually developed and matured. It is widely used in economic management and planning, human resource management, government management, optimization of large - scale projects and other important areas.

The idea of multi-objective programming originated from the study of the utility theory in economics in 1776. In 1896, economist Pareto first put forward the multi-objective programming problem in the study of economic balance, and gave a simple idea which was later called the Pareto optimal solution. In 1947, von Neumann and Morgenster mentioned the multi-objective programming problem in their game theory work, which attracted more attention to this problem. In 1951, Koopmans put forward the multi-objective optimization problem in the analysis of production and sales activities, and first formed the concept of the Pareto optimal solution. In the same year, Kuhn and Tucker gave the concept of the Pareto optimal solution of the vector extremum problem from the angle of mathematical programming. The necessary and sufficient conditions for the existence of this solution are also studied. Debreu's discussion on evaluation balance in 1954 and Harwicz's research on multi-objective optimization in topological vector space in 1958 laid the foundation for the establishment of this discipline. In 1968, Johnsen published the first monograph on the multi-objective decision-making model. Until 1970s-1980s, the basic theory of multi-objective programming was finally established through the efforts of many scholars, making it a new branch of applied mathematics (Huang et al, 2017; Liu, 2014; Ying, 1988).

Up to now, there are the following general methods to solve multi-objective programming: one is to transfer multiple objectives into a single objective that is easier to solve, such as the main objective method, the linear weighting method, the ideal point method, etc.; the other method is called the hierarchical sequence method, i.e. a sequence is given according to the importance of the target, and the next target optimal solution is searched in the previous target optimal solution set every time until a common optimal solution is obtained; the third one is the main target method, which takes one $f_1(x)$ as the main target, and the other *P-1* as the non-main target. At this time, it is hoped that the main target will reach the maximum value, and other targets will meet certain conditions; the fourth one is the linear weighting method, which sets a series of weight coefficients ω_j for objective functions $f_j(x)$, and thus a new evaluation function $U(x) = \sum_{i=1}^{p} \omega_j \cdot f_{j(x)}$ is obtained by linear weighted summation, which makes the multi-objective problem become a single-objective problem. However, under the condition that the dimensions of the target are different, normalization is needed. For a multi-objective linear programming problem, decision makers hope to achieve these goals in turn under these constraints by minimizing the total deviation from the target value, which is the problem to be solved by goal programming (Huang et al, 2017; Liu, 2014; Ying, 1988).

In practical engineering systems, such as many nonlinear, multi-variable, multi-constraint and multi-objective optimization problems in power systems, the existing mathematical methods have limited ability to optimize these problems, and the solutions obtained are not satisfactory.

The above discussion shows that normalization and the introduction of subjective factors in the previous methods are indispensable processes in their "additive" algorithm, and the final result depends to a great extent on the normalization method adopted after the targets with different attributes are converted into "single" targets (Zheng et al, 2021). Different normalization methods may lead to completely different results. In addition, in some algorithms, the beneficial performance index and the unbeneficial performance index are treated unequally.

From the point of view of the set theory, the "additive" algorithm in the previous methods for multi-objective optimization corresponds to the form of "union". Therefore, the above algorithm can only be regarded as a semi-quantitative method in a sense.

Recently, a probabilistic multi - objective optimization (PMOO) method has been proposed to solve the inherent problems of subjective factors of the previous methods of multi - objective optimization (Zheng et al, 2021; Zheng et al, 2022a; Zheng et al, 2022b). A brand - new concept of preferable probability is put forward to reflect the preference degree of performance indicators in project management optimization. PMOO aims to deal with multi-objective simultaneous optimization from the perspective of the probability theory. In the new methodology of PMOO, the performance utility indicators of all candidates are preliminarily divided into the beneficial category and the unbeneficial category according to their roles and preferences in optimization; each performance utility index of the candidate quantitatively contributes to a partial preferable probability; the product of all partial preferable probabilities deduces the total preferable probability of a candidate; the total preferable probability thus transfers a multi-objective problem into a single-objective one. In the evaluation, the total preferable probability of a candidate is the unique and decisive index of the candidate.

In this article, by using probabilistic multi - objective optimization and the good lattice point to conduct discrete sampling and sequential optimization for successive deep optimization, a reasonable method of multi - objective programming is formulated, and the application details of this method are illustrated with two examples.

Solution for solving an integer programming problem by means of probabilistic multi - objective optimization

In this section, probabilistic multi - objective optimization, good lattice point (GLP) discretization and sequential optimization are organically combined, which establishes a rational method for solving a multi-objective programming problem. The probabilistic multi - objective optimization method is used to transfer a multi - objective optimization problem into a single - objective optimization one from the perspective of the probability theory; the discretization of GLP provides an effective discrete sampling to simplify mathematical processing, which is especially important for dealing with multi - objective programming problems with continuous objective functions; and sequential optimization is used for successive deep optimization.

The systematic implementation is demonstrated in the subsections A) and B).

A) A method based on the perspective of probabilistic multi - objective optimization

From the perspective of probabilistic multi - objective optimization, the whole event with multi - objective simultaneous optimization corresponds to the product of all single objectives (events). For multi - objective programming problems, each objective can be analogically seen as a single event (Zheng et al, 2021; Zheng et al, 2022a; Zheng et al, 2022b). All performance utility indexes of the candidate are preliminarily divided into two categories: beneficial and unbeneficial, according to their role and preference of a candidate in optimization, respectively.

Specifically, the assessment of the preferable probability P_{ij} of both beneficial indicators and unbeneficial indicators can be carried out according to the evaluation procedure in Figure 1 (Zheng et al, 2021; Zheng et al, 2022a; Zheng et al, 2022b).

B) Discrete sampling by means of the good lattice point and successive sequential optimization

In multi - objective programming problems, the objective function is usually continuous. In order to simplify mathematical processing, the discrete sampling by means of the good lattice point (GLP) can be used. As described in literature (Hua & Wang, 1981; Fang & Wang, 1994; Fang et al, 2018), the methods of good lattice point and uniform experimental design (UED) make discrete sampling possible and practical. The GLP method and UED are based on the number theory, and it can obtain an effective approximate value for a definite integral or an extreme value problem with a limited number of sampling points (Hua & Wang, 1981; Fang & Wang, 1994; Fang et al, 2018). Such a limited number of sampling points is uniformly distributed in the super space with low discrepancy. The characteristic of the uniform point set makes its convergence speed much faster than that of the Monte Carlo sampling method (Hua & Wang, 1981; Fang & Wang, 1994; Fang et al, 2018), so it is considered as an efficient approximation named quasi-Monte Carlo method. In order to use this uniformly distributed point set appropriately, Professor Fang specially developed uniform design and uniform design tables (Fang, 1994; Fang et al, 2018).

As to the successive sequential optimization of multi - objective optimization problems, a sequential optimization algorithm (SNTO) can be employed for deep optimization (Zheng et al, 2022c; Zheng et al, 2023).

Moreover, by combining probabilistic multi - objective optimization, discrete sampling, and sequential optimization, the multi - objective programming problem can be solved rationally.

At first, the requirements of integers could be given up so as to simply conduct the above procedures. Finally, the optimum solutions of the input variables must be rounded to the nearest integers, which must be withstanding the constraint conditions as well.

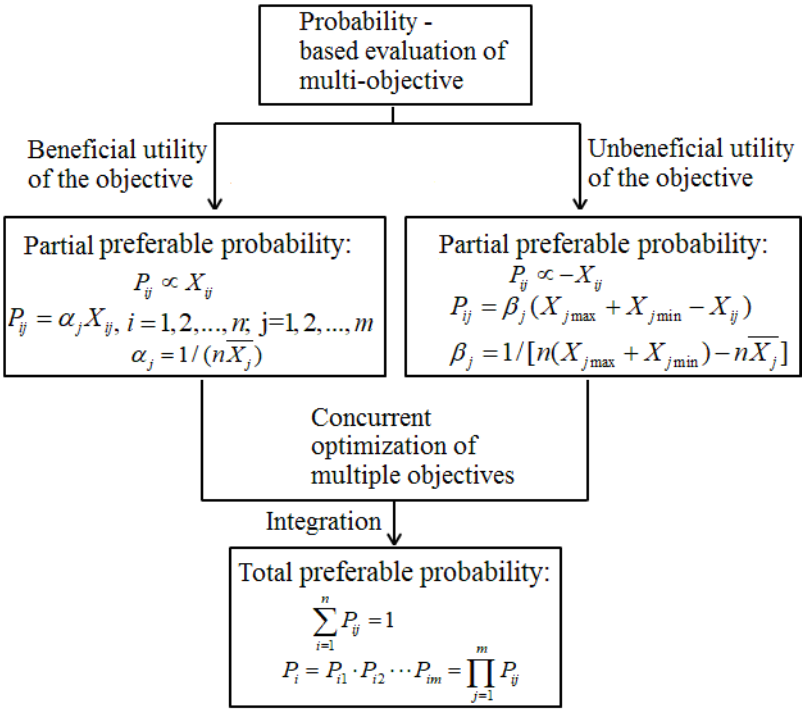

Figure 1 – Evaluation procedure of the PMOO method
Рис. 1 – Процедура оценки метода PMOO
Слика 1 – Поступак евалуације метода PMOO

Applications

In this section, two examples are employed to illustrate the use of the above methods in solving multi - objective integer programming problems.

I) An integer programming problem that maximizes profits and minimizes pollution

A factory plans to produce two products, *PV cell 1* and *PV cell 2*. During production, it causes certain polluting gas release into the air (Huang et al, 2017). So, in the production plan, the goals are to get maximum profit with minimum pollution at the same time. Profit, unit pollution of each product, mechanical ability, manpower resource and resource limits are shown in Table 1 (Huang et al, 2017). Therefore, the problem is how to organise production which maximizes profits and causes the least pollution.

Table 1 – Resource consumption, profit and pollution of each product
Таблица 1 – Потребление ресурсов, прибыль и загрязнение каждого продукта
Табела 1 – Потрошња ресурса, профит и загађивање сваког производа

Content	Product		Limit unit
	Cell 1	*Cell 2*	
Resource exhaust unit per product	1	5	72
Mechanical ability exhaust per product	0.5	0.25	8
Manpower resource exhaust per product	0.2	0.2	4
Profit per product (¥RMB)	1	3	
Pollution unit per product	1.5	1	

Solution

Assuming that the output of the products *Cell 1* and *Cell 2* is x_1 and x_2, respectively, the mathematical model and the constraint conditions of this problem are as follows,

Max $f_1(x) = x_1 + 3x_2$,

Min $f_2(x) = 1.5x_1 + x_2$,

s. t. $0.5x_1 + 0.25x_2 \leq 8$ (Mechanical ability),

$0.2x_1 + 0.2x_2 \leq 4$ (Manpower resource),

$x_1 + 5x_2 \leq 72$ (Resource limit), and

$x_1, x_2 > 0$.

Because there are two input variables x_1 and x_2 in this problem, at least 17 evenly distributed sampling points are needed for the discretization in the working domain according to literature (Zheng et al, 2022c). Here, we try to use the uniform design table $U^*_{24}(24^9)$ to implement the discretization (Fang, 1994; Fang et al, 2018), and the results are shown in Table 2. As it can be seen from Table 2, five sampling points were excluded due to the limitation of the constraint conditions, and the remaining 19 sampling points were within the working area, which meets the basic requirement of at least 17 uniformly distributed sampling points within the working zone.

Additionally, in this problem, the objective function $f_1(x)$ is a beneficial indicator while the objective function $f_2(x)$ is an unbeneficial indicator. Table 3 shows the evaluation results of the partial preferable probabilities P_{f1} and P_{f2} of the objective functions $f_1(x)$ and $f_2(x)$ at the corresponding discrete sampling point, respectively; P_t represents the total/overall preferable probabilities of each sampling point.

As it can be seen from Table 3, sampling point No 2 shows the maximum value of the total preferable probability. Therefore, around sampling point No 2 of Table 2, sequential uniform design is adopted for successive deep optimization.

Table 2 – Evaluation results of discrete sampling with $U^*_{24}(24^9)$

Таблица 2 – Результаты оценки дискретной выборки с $U^*_{24}(24^9)$

Табела 2 – Резултати евалуације дискретног узорковања са $U^*_{24}(24^9)$

No	Input variable		Objective		Note
	x1	x2	f_1	f_2	
1	0.3333	6.3	19.2333	6.8	
2	**1**	**12.9**	**39.7**	**14.4**	
3	1.6667	4.5	15.1667	7	
4	2.3333	11.1	35.6333	14.6	
5	3	2.7	11.1	7.2	
6	3.6667	9.3	31.5667	14.8	
7	4.3333	0.9	7.0333	7.4	
8	5	7.5	27.5	15	
9	5.6667	14.1			Excl.
10	6.3333	5.7	23.4333	15.2	
11	7	12.3	43.9	22.8	
12	7.6667	3.9	19.3667	15.4	
13	8.3333	10.5	39.8333	23	
14	9	2.1	15.3	15.6	
15	9.6667	8.7	35.7667	23.2	
16	10.3333	0.3	11.2333	15.8	
17	11	6.9	31.7	23.4	
18	11.6667	13.5			Excl.
19	12.3333	5.1	27.6333	23.6	
20	13	11.7			Excl.
21	13.6667	3.3	23.5667	23.8	
22	14.3333	9.9			Excl.
23	15	1.5	19.5	24	
24	15.6667	8.1			Excl.

Table 3 – Evaluation results of discrete PMOO using $U^*_{24}(24^9)$
Таблица 3 – Результаты оценки дискретного PMOO с помощью $U^*_{24}(24^9)$
Табела 3 – Резултати евалуације дискретне PMOO помоћу $U^*_{24}(24^9)$

No	Preferable probability		
	Partial		Total
	P_{f1}	P_{f2}	$P_t \times 10^3$
1	0.0402	0.0877	3.5268
2	**0.0830**	**0.0601**	**4.9936**
3	0.0317	0.0870	2.7581
4	0.0745	0.0594	4.4280
5	0.0232	0.0862	2.0018
6	0.0660	0.0587	3.8749
7	0.0147	0.0855	1.2577
8	0.0575	0.0580	3.3340
10	0.0490	0.0572	2.8055
11	0.0918	0.0297	2.7277
12	0.0405	0.0565	2.2892
13	0.0833	0.0290	2.4146
14	0.0320	0.0558	1.7854
15	0.0748	0.0283	2.1139
16	0.0235	0.0551	1.2938
17	0.0663	0.0275	1.8255
19	0.0578	0.0268	1.5494
21	0.0493	0.0261	1.2857
23	0.0408	0.0254	1.0343

Table 4 shows the evaluation results of the successive deep optimization using sequential uniform optimization, in which $c(t)$ = (Max $P_t^{(i-1)}$ – Max $P_t^{(i)}$)/Max $P_t^{(i-1)}$ represents the relative error of the maximum total preferable probability of the *i*-th sequential step. If we assume that the pre-assignment of $c^{(t)}$ = 2%, the successive deep optimization can be terminated in step 3. At this time, the final optimal results of this multi-objective programming optimization problem are $f_{1Opt.}$ = 42.7625 ¥RMB, $f_{2Opt.}$ = 14.4 unit, while the instant input variables of the successive deep optimization in step 3 are x_1 = 0.125 and x_2 = 14.2125, respectively. Since this is an integer programming problem, the solution for x_1 and x_2 must be rounded to the nearest integers, so the values of x_1 and x_2 are 0 and 14, respectively, and the optimal values of objective functions are thus $f_{1Opt.}$ = 42 ¥RMB yuan and $f_{2Opt.}$ = 14 unit, individually. This result is much better than that given by Huang with a linear weighting algorithm (Huang et al, 2017).

Table 4 – Evaluation results of sequential optimization with $U^*_{24}(24^9)$ discrete sampling
Таблица 4 – Результаты оценки последовательной оптимизации с дискретной выборкой $U^*_{24}(24^9)$
Табела 4 – Резултати евалуације секвенцијалне оптимизације помоћу дискретног узорковања $U^*_{24}(24^9)$

Step	Range	Instant input variable		Optimal objective		Max $P_t \times 10^3$	$c^{(t)}$
		x_1^*	x_2^*	$f_{1Opt.}$	$f_{2Opt.}$		
0	[0, 16] x [0, 14.4]	1	12.9	39.7	14.4	4.9936	
1	[0,8] x [7.2, 14.4]	0.5	13.65	41.45	14.4	4.4152	
2	[0, 4] x [10.8, 14.4]	0.25	14.025	42.325	14.4	4.2855	0.0294
3	[0, 2] x [12.6, 14.4]	0.125	14.2125	42.7625	14.4	4.2034	0.0191

II) Purchasing raw material for production

A factory needs to purchase certain raw material for production. There are two kinds of raw materials in the market, *A* and *B*, with unit prices of 2 ¥RMB yuan / kg and 1.5 ¥RMB yuan /kg, respectively. It is required that the total cost now should not exceed 300 ¥RMB yuan, and the raw material *A* should not be less than 60 kg. How to determine the best purchasing plan, spend the least money and purchase the largest amount of raw materials? The smallest weight unit is 1 kg.

Assuming that the two raw materials, *A* and *B*, are purchased in x_1 and x_2 kg, respectively, then the total cost is:

$f_1(x) = 2x_1 + 1.5x_2$;

The total amount of the purchased raw materials is:

$f_2(x) = x_1 + x_2$.

Then the goal of our solution is to spend the least money to buy the most raw materials, i.e. to minimize $f_1(x)$ while maximizing $f_2(x)$.

At the same time, it is necessary to meet the requirements that the total cost should not exceed 300 ¥RMB yuan, and the raw materials *A* should not be less than 60 kg, so the constraint conditions are as follows:

$2x_1 + 1.5x_2 \leq 300$;

$x_1 \geq 60$, $x_2 \geq 0$.

Solution

Based on the above analysis, the following optimal mathematical model is given:

Min $f_1(x) = 2x_1 + 1.5x_2$;

Max $f_2(x) = x_1 + x_2$;

s. t.

$2x_1 + 1.5x_2 \leq 300$;

$x_1 \geq 60$, $x_2 \geq 0$.

Because this problem has two input variables x_1 and x_2, similarly, at least 17 evenly distributed sampling points in the working domain are needed (Zheng et al, 2022c; Zheng et al, 2023). Here, we try to use the uniform test table $U_{37}(37^{12})$ conduct the discrete sampling (Fang, 1994; Fang et al, 2018), and the results are shown in Table 5. It can be seen from Table 5 that, due to the limitation of the constraint conditions, 18 sampling points are excluded, and, luckily, 19 sampling points are within the scope of the constraint conditions, which meets the requirement of at least 17 uniformly distributed sampling points within the scope of s. t. condition. In this problem, the objective function $f_1(x)$ is the unbeneficial indicator, and $f_2(x)$ is the beneficial indicator.

Table 6 shows the evaluation results of the partial preferable probabilities of the functions f_1 and f_2 at the discrete sampling points, P_{f1} and P_{f2}, respectively; P_t represents the total/overall preferable probability of each sampling point. As it can be seen from Table 6, sampling point No 2 shows the maximum value of the total preferable probability. Therefore, around the 2nd sampling point in Table 6, sequential optimization is adopted for successive deep optimization. Table 7 shows the evaluation results of the sequential optimization using the uniform design table $U_{37}(37^{12})$. Similarly, if a pre-specified value of 0.7% is set for $c^{(t)}$, then the deep optimization can be terminated in step 3. At this point, the final optimal results of this multi-objective programming optimization problem are $f_{1Opt.}$ = 298.7635 ¥RMB yuan and $f_{2Opt.}$ = 179.0270 kg, while the instant input variables of the successive deep optimization at step 3 are x_1 = 60.4460 kg and x_2 = 118.5810 kg. Similarly, since this is an integer programming problem, the solution for x_1 and x_2 must be rounded to the nearest integers, so the values of x_1 and x_2 are 60 kg and 119 kg, respectively, and the optimal values of objective functions are thus $f_{1Opt.}$ = 298.5 ¥RMB and $f_{2Opt.}$ = 179 kg, individually.

Table 5 – Results of discretization with $U_{37}(37^{12})$
Таблица 5 – Результаты дискретизации с $U_{37}(37^{12})$
Табела 5 – Резултати дискретизације са $U_{37}(37^{12})$

No	Input variable		Objective		Note
	x_1	x_2	f_1	f_2	
1	61.2162	53.5135	202.7027	114.7297	
2	**63.6487**	**108.6490**	**290.2703**	**172.2970**	
3	66.0811	43.7838	197.8378	109.8649	
4	68.5135	98.9189	285.4054	167.4324	
5	70.9459	34.0541	192.9730	105	
6	73.3784	89.1892	280.5405	162.5676	
7	75.8108	24.3243	188.1081	100.1351	
8	78.2432	79.4595	275.6757	157.7027	
9	80.6757	14.5946	183.2432	95.2703	
10	83.1081	69.7297	270.8108	152.8378	
11	85.5405	4.8649	178.3784	90.4054	
12	87.9730	60	265.9459	147.9730	
13	90.4054	115.1351			Excl.
14	92.8378	50.2703	261.0811	143.1081	
15	95.2703	105.4054			Excl.
16	97.7027	40.5405	256.2162	138.2432	
17	100.1351	95.6757			Excl.
18	102.5676	30.8108	251.3514	133.3784	
19	105	85.9460			Excl.
20	107.4324	21.0811	246.4865	128.5135	
21	109.8649	76.2162			Excl.
22	112.2973	11.3514	241.6216	123.6486	
23	114.7297	66.4865			Excl.
24	117.1622	1.6216	236.7568	118.7838	
25	119.5946	56.7568			Excl.
26	122.0270	111.8919			Excl.
27	124.4595	47.0270			Excl.
28	126.8919	102.1622			Excl.
29	129.3243	37.2973			Excl.
30	131.7568	92.4324			Excl.
31	134.1892	27.5676			Excl.
32	136.6216	82.7027			Excl.
33	139.0541	17.8378			Excl.
34	141.4865	72.9730			Excl.
35	143.9189	8.1081	300	152.027	
36	146.3514	63.2432			Excl.
37	148.7838	118.3784			Excl.

Table 6 – Evaluation results of PMOO discrete sampling with $U_{37}(37^{12})$
Таблица 6 – Результаты оценки дискретной выборки PMOO с $U_{37}(37^{12})$
Табела 6 – Резултати евалуације дискретног узорковања PMOO са $U_{37}(37^{12})$

No	Preferable probability		
	Partial		Total
	P_{f1}	P_{f2}	$P_t \times 10^3$
1	0.0615	0.0456	2.8059
2	**0.0420**	**0.0685**	**2.8754**
3	0.0626	0.0437	2.7344
4	0.0430	0.0666	2.8664
5	0.0637	0.0418	2.6586
6	0.0441	0.0647	2.8533
7	0.0647	0.0398	2.5787
8	0.0452	0.0627	2.8360
9	0.0658	0.0379	2.4945
10	0.0463	0.0608	2.8145
11	0.0669	0.0360	2.4061
12	0.0474	0.0589	2.7887
14	0.0485	0.0569	2.7588
16	0.0495	0.0550	2.7247
18	0.0506	0.0531	2.6864
20	0.0517	0.0511	2.6439
22	0.0528	0.0492	2.5971
24	0.0539	0.0473	2.5462
35	0.0398	0.0605	2.4058

Table 7 – Evaluation results of sequential optimization with $U_{37}(37^{12})$ discrete sampling
Таблица 7 – Результаты оценки последовательной оптимизации с дискретной выборкой $U_{37}(37^{12})$
Табела 7 – Резултати евалуације секвенцијалне оптимизације помоћу дискретног узорковања $U_{37}(37^{12})$

Step	Range	Instant input variable		Objective		Max $P_t \times 10^3$	$c^{(t)}$
		x_1^*	x_2^*	$f_{1Opt.}$	$f_{2Opt.}$		
0	[60, 150] × [0, 120]	63.6487	108.6490	290.2703	172.2970	2.8754	
1	[60, 105]× [60, 120]	61.8243	114.3240	295.1351	176.1490	2.8481	0.0095
2	[60, 82]× [90, 120]	60.8919	117.1620	297.5270	178.0540	2.8102	0.0133
3	[60, 71] × [105, 120]	60.4460	118.5810	298.7635	179.0270	2.7909	0.0069

Discussion

When solving multi - objective programming problems, the approaches employed in the previous work of other methods include the linear weighting method (Zheng et al, 2022c), i.e. the "additive" algorithm, which transfers multi – objective problems into single objective ones. However, from the perspective of the probability theory, this essentially means a "union", and some methods even take several objectives as constraint conditions to solve multi-objective programming problems (Zheng et al, 2022c), which is not realized to the intrinsic meaning of multi - objective programming problems, while probabilistic multi-objective optimization tries to deal with simultaneous optimization of multiple objectives from the perspective of the probability theory, which is a rational method of multi - objective optimization (Zheng et al, 2022c). Therefore, the results obtained by other methods cannot be compared with the results of probabilistic multi - objective optimization.

Conclusion

By using the combination of probabilistic multi - objective optimization, discrete sampling by means of the good lattice points, and successive sequential optimization to solve the multi - objective integer programming problem, we establish a reasonable scheme for solving the multi - objective programming problem. This method properly considers simultaneous optimization of many objectives in the problem, which rationally reflects the essence of simultaneous optimization of multiple objectives, and opens a new approach to the relevant problem.

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Новое решение для многоцелевых задач целочисленного программирования с помощью вероятностной многоцелевой оптимизации

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ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: В данной статье представлено новое решение для многоцелевых задач целочисленного программирования с помощью вероятностной многоцелевой оптимизации. Кроме того, в целях сукцессивного упрощения обработки и глубокой оптимизации *используются дискретизация с помощью соответствующих узлов решетки и последовательная оптимизация.*

Методы: В вероятностную многоцелевую оптимизацию введена новая концепция предпочтительной вероятности для описания степени предпочтения полезности каждого кандидата. Каждая полезность характеристик кандидата вносит частичную предпочтительную вероятность, а произведение всех частичных предпочтительных вероятностей составляет общую предпочтительную вероятность кандидата. Таким образом, общая предпочтительная вероятность переводит многоцелевую проблему в одноцелевую. Дискретизация по методу узлов идеальной решетки применяется для дискретной выборки, а последовательная оптимизация — для глубокой оптимизации. Также в целях упрощения данной процедуры можно отказаться от целочисленных требований. В конце процедуры оптимальные решения введенных переменных необходимо округлить до ближайшего целого числа.

Результаты: Данный подход используется для решения двух производственных задач, а именно: максимизации прибыли при минимизации загрязнения и составления плана закупок как можно большего количества сырья при наименьших затратах. С помощью теории вероятностей вышеуказанные задачи показали многообещающие результаты одновременной оптимизации нескольких целей.

Выводы: Данное решение учитывает одновременную оптимизацию нескольких целей при многокритериальном целочисленном программировании, что, естественно, отражает суть многокритериального программирования и тем самым открывает новые возможности к решению многокритериальных задач.

Ключевые слова: многоцелевая оптимизация, задача целочисленного программирования, предпочтительная вероятность, дискретная выборка, последовательная оптимизация.

Ново решење проблема вишекритеријумског целобројног програмирања помоћу вишекритеријумске оптимизације засноване на вероватноћи

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Сажетак:

Увод/циљ: У раду се формулише ново решење проблема вишекритеријумског целобројног програмирања помоћу пробабилистичке вишекритеријумске оптимизације. Такође, користи се дискретизација помоћу добрих тачака решетке, као и секвенцијална оптимизација ради сукцесивног поједностављивања и дубинске оптимизације.

Методе: У пробабилистичку вишекритеријумску оптимизацију уведен је нови концепт пожељне вероватноће како би се описао степен пожељности сваке појединачне корисности перформансе неког кандидата. Свака појединачна корисност перформансе кандидата доприноси парцијалној пожељној вероватноћи, а производ свих тих вероватноћа чини укупну пожељну вероватноћу кандидата. На тај начин укупна пожељна вероватноћа преводи вишекритеријумски проблем у једнокритеријумски. Дискретизацијом помоћу метода добрих тачака решетке врши се дискретно узорковање за континуалну функцију циља, а секвенцијалном оптимизацијом дубинска оптимизација. Такође, може се одустати од захтева целих бројева ради поједностављивања наведеног поступка. На крају се оптимална решења унетих варијабли морају заокружити на најближи цели број.

Резултати: Овај приступ се користи за решавање два проблема у производњи: за максимизацију прихода уз најмање могуће загађење и за креирање плана за набавку највеће количине репроматеријала по најмањој цени. Обећавајући резултати су добијени за два наведена проблема помоћу теорије вероватноће за истовремену оптимизацију више циљева.

Закључак: Ово решење узима у обзир истовремену оптимизацију више циљева при вишекритеријумском целобројном програмирању, што природно одсликава суштину вишекритеријумског програмирања и тиме отвара нове путеве ка решавању вишекритеријумских проблема.

Кључне речи: вишекритеријумска оптимизација, проблем целобројног програмирања, пожељна вероватноћа, дискретно узорковање, секвенцијална оптимизација.

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