VOJNOTEHNIČKI GLASNIK / MILITARY TECHNICAL COURIER, 2024, Vol. 72, Issue 1

Mathematical modeling and simulation of a half-vehicle suspension system in the roll plane

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DOI: https://doi.org/10.5937/vojtehg72-47551

FIELD: mechanical engineering
ARTICLE TYPE: original scientific paper

Abstract:

Introduction/purpose: The study of vehicle suspension is a challenge for researchers in the field of vehicles regarding the impact of the suspension system on vehicle performances such as ride comfort, road holding, and working space. This paper presents the simulation of the Land Rover Defender 110 vehicle in the roll plane (half vehicle) in Simulink/MATLAB. The obtained results were compared with the results obtained in the ADAMS/CAR software package of the Land Rover Defender 110 simulation model previously experimentally validated. The Defender 110 vehicle has a dependent suspension system in both axles and a passive suspension type with four degrees of freedom (4 DOF).

Methods: The equations of the system can be solved mathematically with a scheme in Simulink/MATLAB while half-vehicle modeling has been done in ADAMS/CAR.

Results: The comparison of the vehicle characteristics obtained by the two simulation methods was done for three different scenarios, and it was noticed that there is a good correlation between them.

Conclusion: It was concluded that the Defender 110 vehicle simulation model in Simulink/MATLAB is validated. The validated model can be used to perform suspension system optimization in future work.

Key words: suspension system, Defender 110, ADAMS/CAR, Simulink/MATLAB.

Introduction

Automotive researchers pay attention to suspension because it ensures the comfort, stability and safety of passengers and keeps the wheels always in contact with the road whatever the nature of the road is.

Most of research activities during last decades have been directed to vibration control of vehicles which are influenced by the harmful effects of vibrations caused by road irregularities on driver's comfort (Mitra et al, 2013).

The primary function of the suspension is to minimize vibrations arising from irregularities in the road profile. Consequently, it is crucial to meticulously choose suspension characteristics such as spring stiffness and the damping coefficient to ensure optimal performance across diverse road profiles. When the vehicle is driven over an uneven road profile, there should not be too large-amplitude oscillations, and if they occur, they must be removed quickly (Turakhia & Modi, 2016).

One of the models used in the literature is a quarter vehicle model because of its modeling simplicity. This model can give sprung mass acceleration, road holding, and suspension working space. In (Mitra et al, 2018), an experiment was conducted on a quarter car test rig to obtain the ride comfort by varying different parameters. The same dimensions of the test rig were replicated to develop a quarter car simulation model in ADAMS/CAR.

Another model that has been carried out by researchers is a bicycle type model which is more complex than the quarter vehicle model. This model provides one more parameter than the quarter vehicle model, and that is the pitch angle. In (Žuraulis et al, 2014), there are the analyses of the impact of the road micro-profile on the duration and the type of the vehicle wheel contact with the road surface driving at different speed, and the selected vehicle bicycle model describes vertical displacements of front and rear wheels and their suspension as well as the impact of the vehicle body motion and longitudinal oscillation.

There are some other parameters which are essential in the study of vehicle suspension and among them is the roll angle. In order to be able to study this parameter, we have developed a mathematical model of the

Land Rover Defender 110 in the roll plane. The dynamic motions of the vehicle in the roll plane are generally described by a 4DOF model. The vehicle has a dependent type suspension, meaning that there is an axle which connects the left wheel to the right one and the suspensions are of a passive type.

The Land Rover Defender 110 model was made in ADAMS/CAR software and the simulation model was validated using the instrumented experimental vehicle for two scenarios, namely, the bump test and the double lane change maneuver (Khettou et al, 2016).

This article focuses on the validation of the mathematical model using Simulink/MATLAB with the ADAMS/CAR model of the vehicle in the roll plane of the Land Rover Defender 110.

In the analysis of suspension in general, there are parameters called design variables which are: stiffness and damping coefficients and the performances such as ride comfort, road holding, and suspension working space. These performances are dependent on design variables. The objective of this work is to validate and compare the mathematical model with the Adams model in order to optimize the performance characteristics of the suspension system in the future work.

Mathematical model for suspension

A Land Rover Defender 110 vehicle in the roll plane with four degrees of freedom (4 DOF) is presented in Figure 1: the vehicle body mass (sprung mass), the moment of the sprung mass relative to the vehicle x axis points forward and is parallel to the vehicle plane of symmetry, the unsprung mass and the moment of the unsprung mass relative to the vehicle x axis are identified by m_v, I_{vx}, m_a and I_{ax}, respectively.

The suspension stiffness k and the damper coefficient c are placed at a distance d from the center of gravity of the suspension system. The tire stiffness k_t and the damper coefficient of the tire c_t are placed at a distance b from the center of gravity of the suspension system.

The vertical displacement, the rolling angle of the sprung mass, the vertical displacement and the rolling angle of the unsprung mass are represented by z_v ,θ_v , $z_a$and θ_a, respectively.

The vertical excitation in the left and right wheels is represented by z_{0l} and z_{0r}, respectively, and Figure 1 shows the details of the vehicle half in the roll plane. The specifications of the suspension system are given in Table 1.

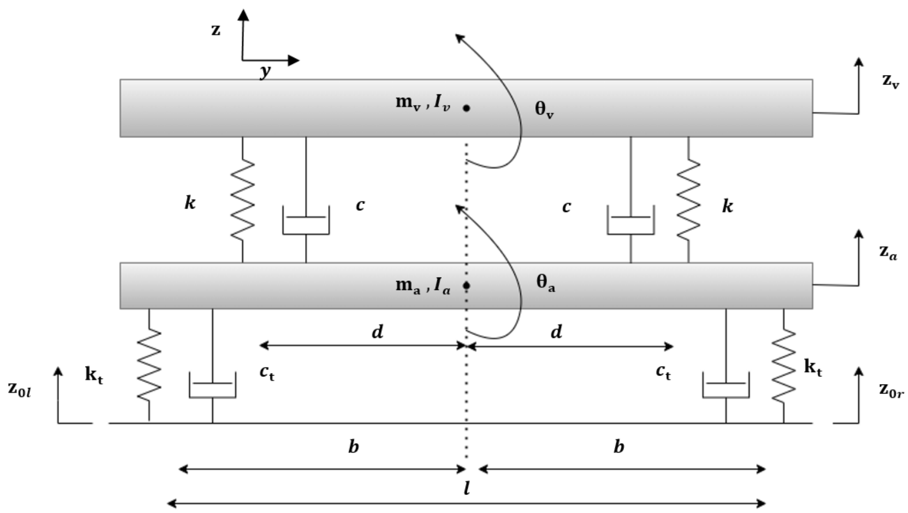

Figure 1 – Suspension system of the half vehicle in the roll plan

Table 1 – Suspension system specifications

Entity	Value	Entity	Value
m_v	$2125\ kg$	m_a	$232\ kg$
I_{vx}	$744\ kgm^2$	I_{ax}	$45.08\ kgm^2$
k	$55000\ N/m$	k_t	$200000\ N/m$
c	$5700\ Ns/m$	c_t	$0\ Ns/m$

The equations of motion of the passive model are given in Eqs. (1), (2), (3), and (4):

$$m_v\,\ddot{z}_v + 2c\dot{z}_v + 2kz_v - 2c\dot{z}_a - 2kz_a = 0 \quad (1)$$

$$m_a\,\ddot{z}_a + 2c\dot{z}_a + 2kz_a - 2c\dot{z}_v - 2kz_v + c_t(\dot{z}_{0l} + \dot{z}_{0r}) + k_t(z_{0l} + z_{0r}) = 0 \quad (2)$$

$$I_{vx}\ddot{\theta}_v + 2cd^2\dot{\theta}_v + 2kd^2\theta_v - 2kd^2\dot{\theta}_a - 2kd^2\theta_a = 0 \quad (3)$$

$$I_{ax}\ddot{\theta}_a + 2cd^2\dot{\theta}_a + 2kd^2\theta_a - 2cd^2\dot{\theta}_v - 2kd^2\theta_v + c_t b^2(\dot{z}_{0l} - \dot{z}_{0r}) + k_t b^2(z_{0l} - z_{0r}) = 0 \quad (4)$$

The performance characteristics are described as follows:

1. The vertical acceleration of the vehicle body is a measure of the comfortability of ride and is expressed in Eq.(5):

$$f_1 = |\ddot{z_v}| \tag{5}$$

2. The dynamic tire load is a measure of the road holding and is expressed in Eq.(6):

$$f_2 = |z_a - z_0| \tag{6}$$

3. The suspension working space is expressed in Eq.(7):

$$f_3 = |z_v - z_a| \tag{7}$$

4. The roll angle is expressed in Eq.(8):

$$f_4 = |\theta_v| \tag{8}$$

Development of a simulation model

Two methods are used for the simulation of the half vehicle model to make the comparison between them. The first method is to perform the simulation using Simulink/MATLAB Software and the second method is to use ADAMS/CAR Software.

Simulink model

Figure 2 shows the Simulink model which is made using Eqs. (1), (2), (3), and (4). In the solver, we chose step fix equal to 0.001, and automatic solver selection.

Following the input excitation and from the graphic representation of the model, we obtain essential information such as: displacement, velocity, acceleration along the z axis and the angular displacement, angular velocity, and angular acceleration along the x axis for the two masses (sprung and unsprung).

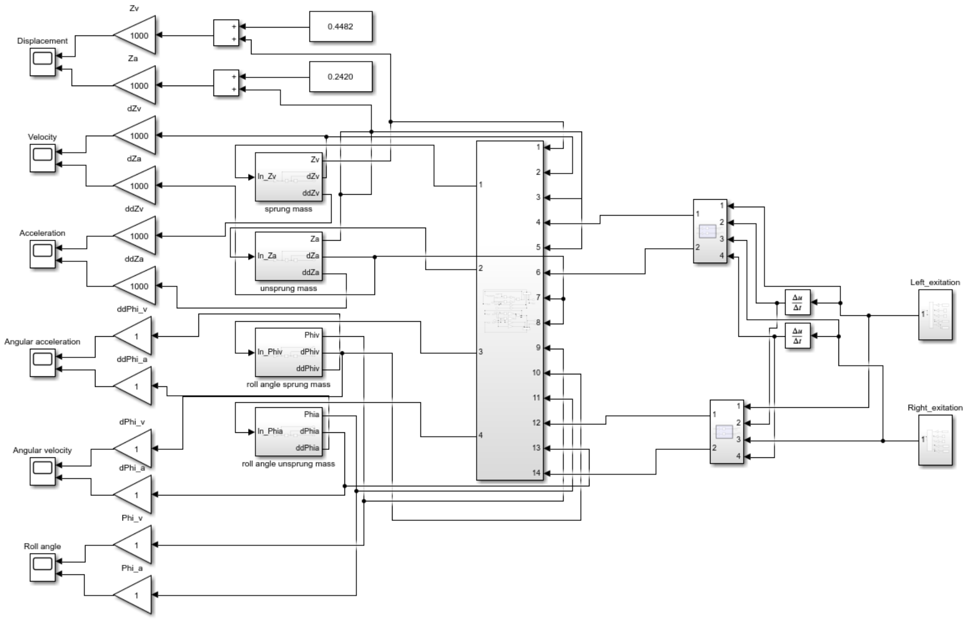

Figure 2 – Simulink model for the vehicle in the roll plan

Adams/Car model

The Land Rover Defender 110 suspension model was initially developed by (Khettou et al, 2016) and, based on it, some modifications were made, shown in Figure 3 representing the vehicle in the roll plane in ADAMS/CAR. The model is represented by two masses connected by a suspension system. One of the masses is called sprung mass and it represents the body which carries the same characteristics of the vehicle while the other is called unsprung mass and it represents the axle with two wheels which have the same characteristics as real vehicle wheels.

After preparing the model and specifying input excitations of the wheels, the simulation was performed using the same test parameters used in the original model and in Simulink model which are cited in Table 1. We chose the resolution parameters in the solver: integrator: GSTIFF, Formulation: I3, Corrector: Modified, and at the end we read the simulation results in the postprocessing windows.

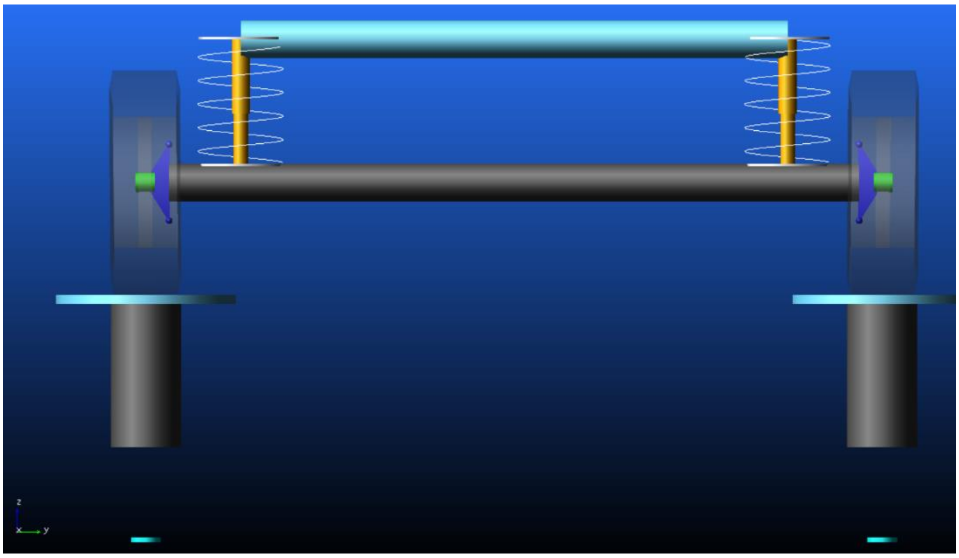

Figure 3 – Representation of the vehicle Defender 110 in the roll plan in Adams/CAR

Results and discussion

In this study, we focus much more on the performance characteristics mentioned above, in addition to vertical displacement, vertical velocity, and angular velocity of the vehicle in three scenarios:

- Sinusoidal profile;
- Obstacle test; and
- Double bump test.

Scenario 1: sinusoidal profile

The profile of (Baumal et al, 1998) is used and shown in Figure 4.

The profile of the road is of a sinusoidal shape with the amplitude $h_l = 0.102m$, $h_r = 2h_l$ and the wavelength $\lambda = 24.4\,m$.

The vehicle velocity v is assumed to be $24.4m/s$.

As a function of time, the road conditions are given by Eqs.(9) and (10):

$$z_{ol} = \frac{h}{2}(cos(\omega t) - 1) \quad (9)$$

$$z_{or} = \boldsymbol{h}(cos(\omega t) - 1) \quad (10)$$

where z_{0l}, z_{0r} are the road profiles in the left and right wheel, respectively, represented in Figure 4, while ω is the forcing frequency and is given by Eq.(11):

$$\omega = \frac{2\pi V}{\lambda} \tag{11}$$

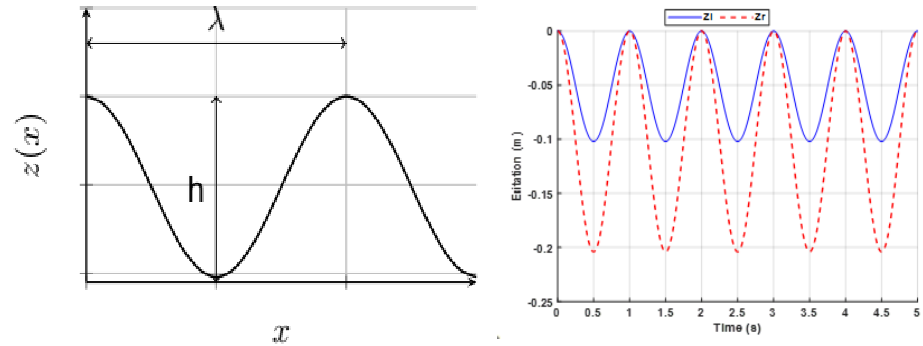

Figure 4 – Sinusoidal profile of the road (scenario1)

Figure 5 shows the comparison between the Adams and Simulink results of the sprung mass for scenario 1, while Table 2 shows the comparison of the performance characteristic values between the Adams and Simulink models for scenario 1.

Table 2 – Comparison of the performance characteristic values between Adams and Simulink for Scenario 1

Performance characteristic	$Max\|\ddot{z}_v\|$ (m/s^2)	$\text{Max}\|(z_a - z_{0l})\|$ (m)	$\text{Max}\|(z_a - z_{0r})\|$ (m)	$Max\|z_v - z_a\|$ (m)	$Max\|\theta_v\|$ (rad)
Adams	6.96	0.083	0.058	0.113	0.101
Simulink	7.20	0.085	0.057	0.117	0.101
Deviation (%)	3.34	2.67	1.81	3.65	0

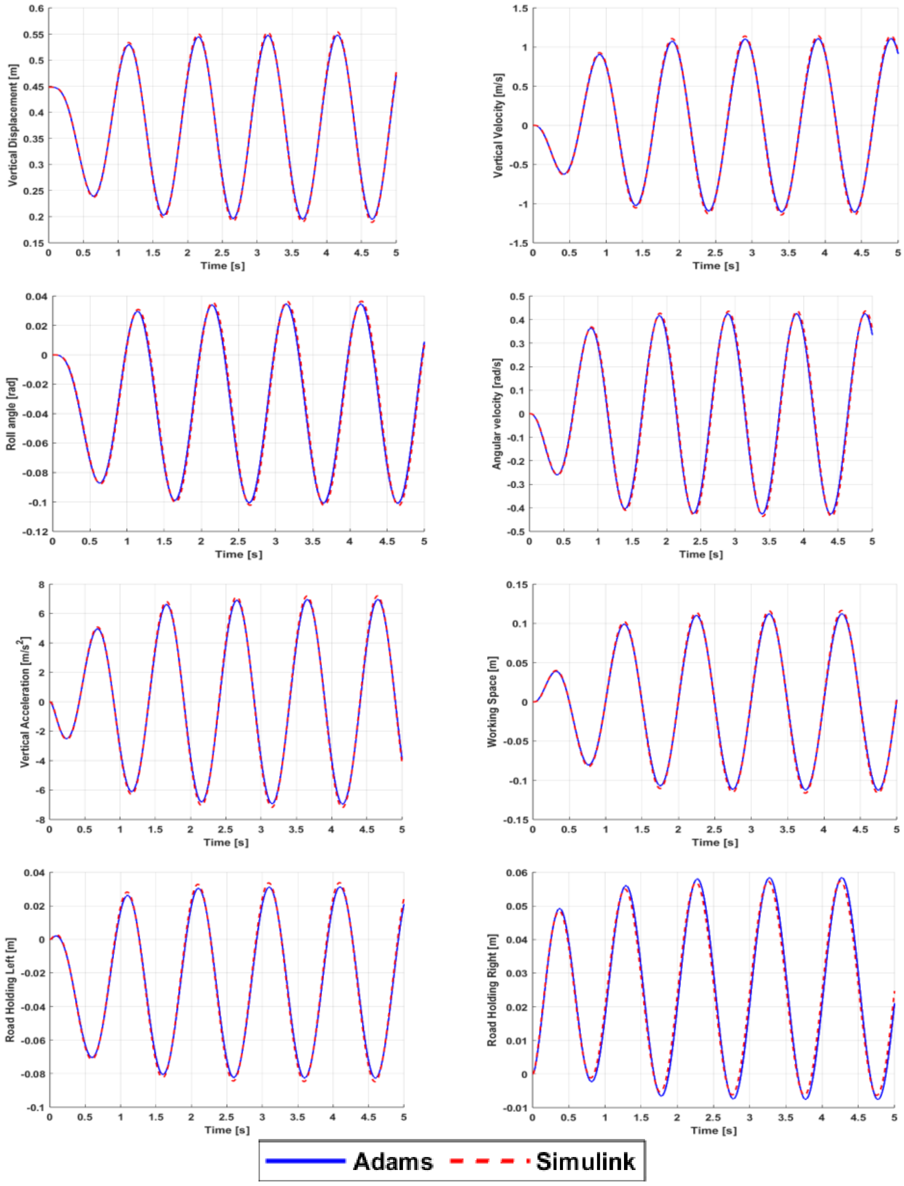

Figure 5 – Simulation results in Adams and Simulink of a sinusoidal road profile

Scenario 2: Obstacle test

In this test, the vehicle must cross a discrete obstacle with one wheel (left) and then its response is determined.

The same discrete obstacle with the same speed, $v = 8.33\ m/s$, was used as in (Khettou et al, 2016).

The characteristics of the discrete obstacle are presented in Figure 6:

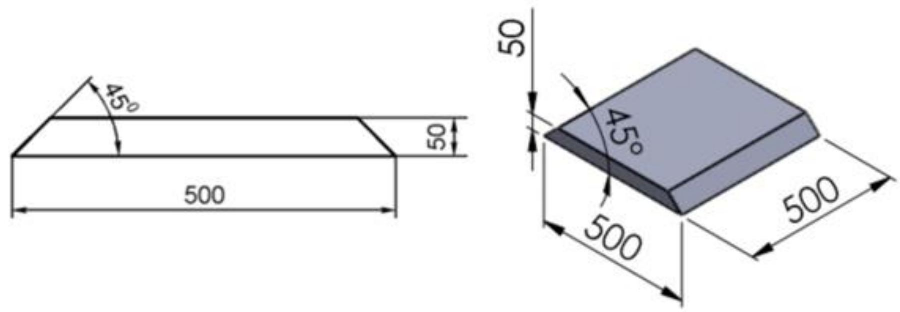

Figure 6 – Dimensions and the profile of the discrete obstacle (scenario 2)

Figure 7 shows the comparison between the Adams and Simulink results of the sprung mass for scenario 2, while Table 3 shows the comparison of the performance characteristic values between the Adams and Simulink models for scenario 2.

Table 3 – Comparison of the performance characteristic values between Adams and Simulink for Scenario 2

Performance characteristic	$Max\|\ddot{z}_v\|$ (m/s^2)	$\text{Max}\|(z_a - z_{0l})\|$ (m)	$\text{Max}\|(z_a - z_{0r})\|$ (m)	$Max\|z_v - z_a\|$ (m)	$Max\|\theta_v\|$ (rad)
Adams	2.79	0.049	0.021	0.0172	0.011
Simulink	2.91	0.050	0.023	0.0191	0.0109
Deviation (%)	4.3	2.27	8.53	10.6	0.91

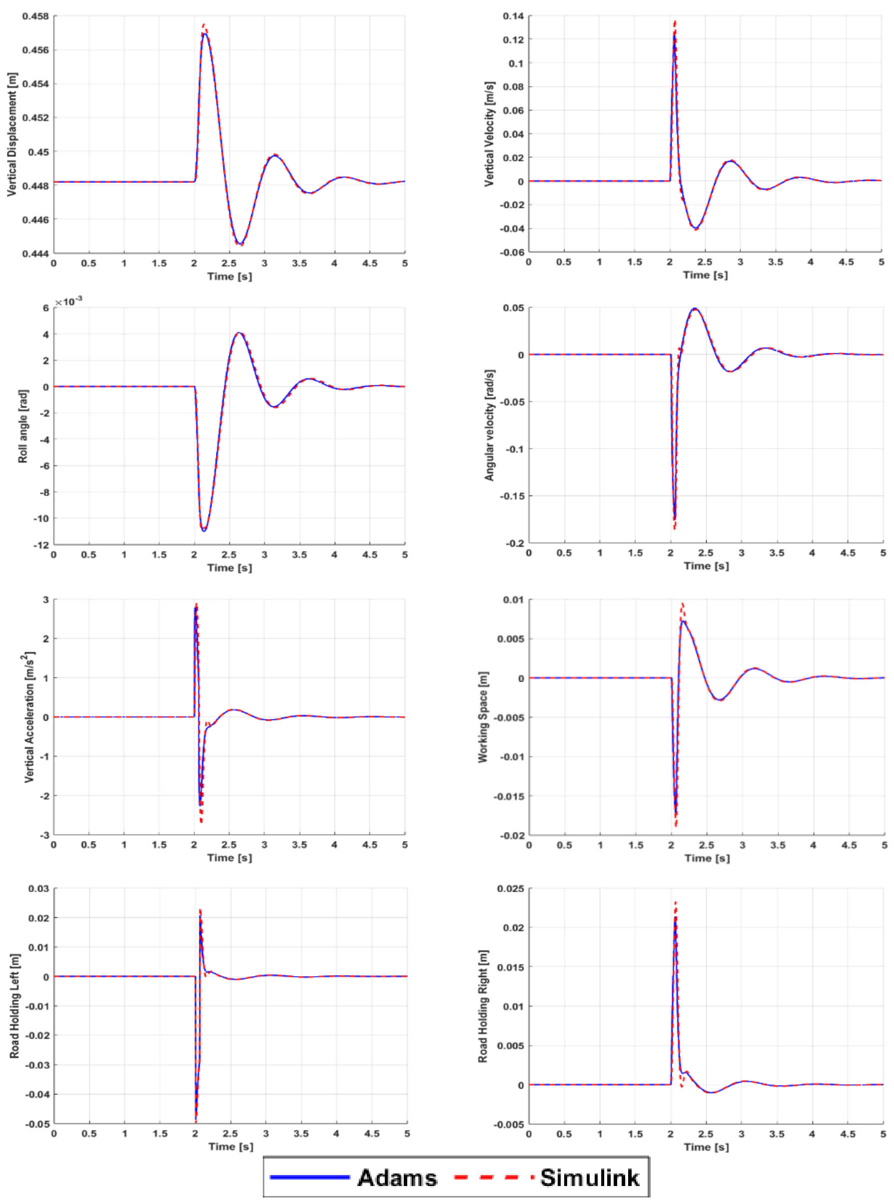

Figure 7 – Simulation results in Adams and Simulink of the discrete obstacle

Scenario 3: Double bump test

In this test, the vehicle is driven over a double bump shape shown in Figure 8. The wheel on the right-hand side is late in time t_0 in comparison to the wheel on the left-hand side. The vehicle has a speed of $v = 8.33\ m/s$ while the bump width is $L = 0.5m$ and its amplitude is $h = 0.1m$.

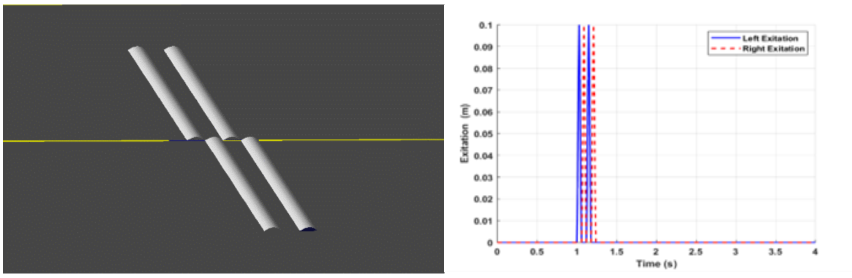

Figure 8 – The double bump profile scenario 3)

As a function of time, the road conditions are given by Eqs. (12) and (13):

$$z_{ol}(t) = \begin{cases} \frac{h}{2}\Big(1 - cos\big(\omega(t - t_1)\big)\Big)\ if\ t_1 < t < t_2 \\ \frac{h}{2}\Big(1 - cos\big(\omega(t - t_3)\big)\Big)\ if\ t_3 < t < t_4 \\ \mathbf{0} \qquad\qquad elsewhere \end{cases} \tag{12}$$

$$z_{or}(t) = \begin{cases} \frac{h}{2}\Big(1 - cos\big(\omega(t - t_2)\big)\Big)\ if\ t_2 < t < t_3 \\ \frac{h}{2}\Big(1 - cos\big(\omega(t - t_4)\big)\Big)\ if\ t_4 < t < t_5 \\ \mathbf{0} \qquad\qquad elsewhere \end{cases} \tag{13}$$

And the time steps are given in Eq.(14):

$$t_1 = 1s$$
$$T = \frac{L}{V}$$
$$t_i = t_1 + nT \quad n = 2, \dots, 5 \qquad t_0 = T \tag{14}$$

Figure 9 shows the comparison between the Adams and Simulink results of the sprung mass for scenario 3, while Table 4 shows the comparison of the performance characteristic values between the Adams and Simulink models for scenario 3.

Table 4 – Comparison of the performance characteristic values between Adams and Simulink for Scenario 3

Performance characteristic	$Max\lvert\ddot{Z}_v\rvert$ (m/s^2)	$\text{Max}\lvert(Z_a - Z_{0l})\rvert$ (m)	$\text{Max}\lvert(Z_a - Z_{0r})\rvert$ (m)	$Max\lvert Z_v - Z_a\rvert$ (m)	$Max\lvert\theta_v\rvert$ (rad)
Adams	5.34	0.086	0.077	0.024	$9.62\ 10^{-3}$
Simulink	5.21	0.092	0.077	0.027	$1.05\ 10^{-2}$
Deviation (%)	2.43	7.37	0	12.5	9.14

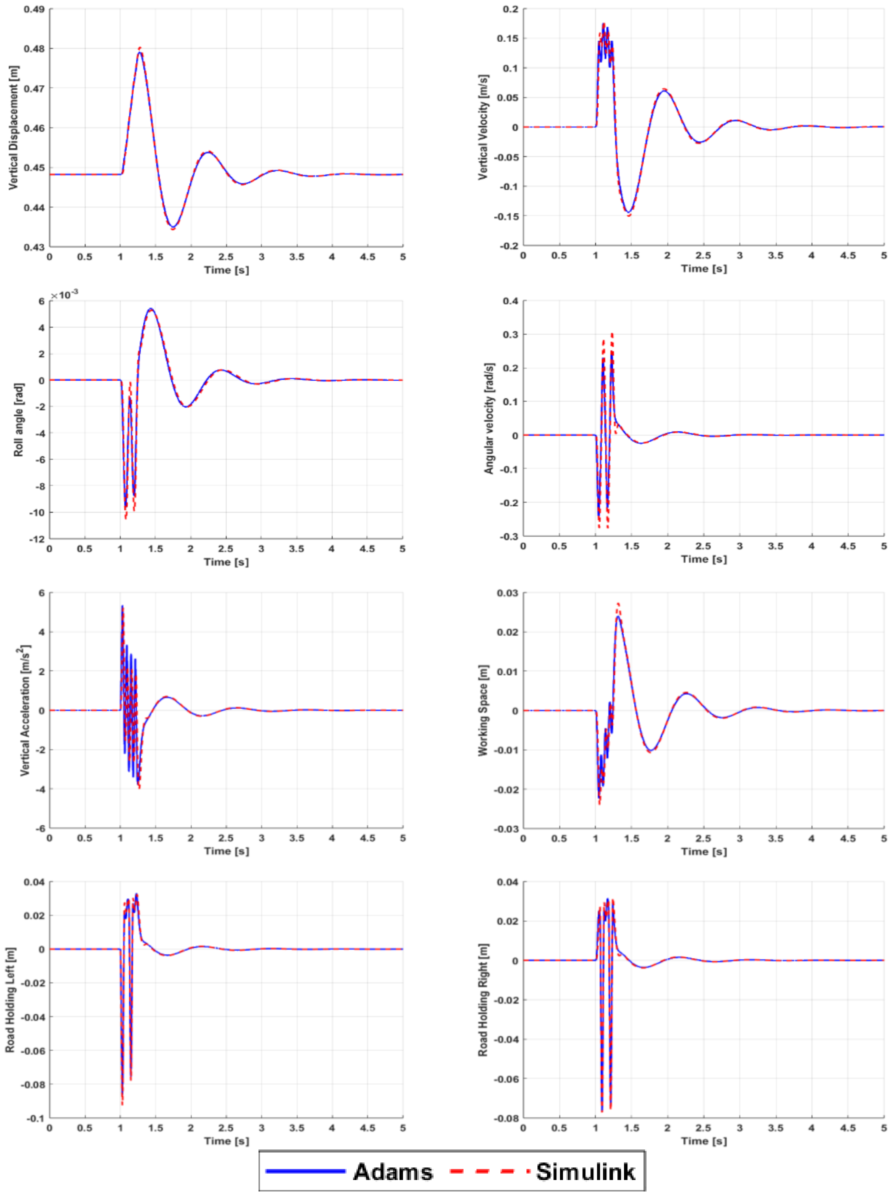

Figure 9 – Simulation results in Adams and Simulink of the double bump profile

Conclusion

In this paper, we have presented the Simulink model of the half vehicle of the Defender 110 and performed its validation with the ADAMS/CAR model for three scenarios. After the detailed analyses, we noticed that the simulation results obtained by Simulink are in good agreement with the simulation results of ADAMS. The extreme values of Simulink are greater than those of Adams which can be explained by the fact that the modeling in Adams is more detailed compared to the mathematical model. Also, another reason is that the numerical resolution method used in Adams is different from the methods used in Simulink, and the calculated percentage error between the two simulations for the three scenarios for all performance characteristics does not exceed 15%. Therefore, we can say that the model is validated. It can be, therefore, concluded that by using the validated model, the system can be further optimized to study the relationship and sensitivity of different design variables for optimal ride comfort, road holding, and suspension working space.

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Modelado matemático y simulación de un sistema de suspensión de medio vehículo en el plano de balanceo

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CAMPO: ingeniería mecánica
TIPO DE ARTÍCULO: artículo científico original

Resumen:

Introducción/objetivo: El estudio de la suspensión de vehículos es un desafío para investigadores en el campo de los vehículos sobre el impacto del sistema de suspensión en el rendimiento del vehículo, tales como la comodidad de marcha, el agarre en carretera y el funcionamiento del espacio. Este artículo presenta la simulación del vehículo Land Rover Defender 110 en el plano de balanceo (medio vehículo) en Simulink/MATLAB. Los resultados obtenidos se compararon con los resultados obtenidos en el paquete de software ADAMS/CAR del modelo de simulación Land Rover Defender 110 previamente validado experimentalmente. El vehículo Defender 110 cuenta con un sistema de suspensión dependiente en ambos ejes y un tipo de suspensión pasiva con cuatro grados de libertad (4 DOF).

Métodos: Las ecuaciones del sistema se pueden resolver matemáticamente con un esquema en Simulink/MATLAB mientras se realiza el modelado de medio vehículo en ADAMS/CAR.

Resultados: La comparación de las características del vehículo obtenidas por los dos métodos de simulación se realizó para tres escenarios diferentes, y se observó que existe una buena correlación entre ellos.

Conclusión: Se concluyó que el modelo de simulación del vehículo Defender 110 en Simulink/MATLAB está validado. El modelo validado se puede utilizar para realizar la optimización del sistema de suspensión en trabajos futuros.

Palabras claves: sistema de suspensión, Defender 110, ADAMS/CAR, Simulink/MATLAB.

Математическое моделирование системы подвески автомобиля в поперечной плоскости

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РУБРИКА ГРНТИ: 78.25.09 Военная автомобильная техника
ВИД СТАТЬИ: оригинальная научная статья

Резюме:

Введение/цель: Изучение систем подвески транспортных средств является непростой задачей для исследователей, изучающих влияние систем подвески на характеристики автомобиля: удобство при вождении, устойчивость автомобиля, управляемость автомобиля и т. д. В данной статье представлено моделирование работы системы подвески автомобиля Land Rover Defender 110 в поперечной плоскости. Имитационная модель разработана в программном пакете Simulink/MATTLAB. Полученные результаты были сопоставлены с результатами ранее экспериментально проверенной имитационной модели Land Rover Defender 110. Автомобиль Land Rover Defender 110 имеет зависимую систему подвески на обеих осях и подвеску пассивного типа с четырьмя степенями свободы (4 DOF).

Методы: Уравнения системы могут быть решены математически по схеме в Simulink/MATLAB, а моделирование транспортных средств в поперечной плоскости выполняется в ADAMS/CAR.

Результаты: Сравнение характеристик автомобиля, полученных двумя методами моделирования, было выполнено по трем различным сценариям. Сравнительный анализ подтвердил удовлетворительное совпадение полученных результатов.

Выводы: На основании результатов исследования сделан вывод, что имитационная модель автомобиля Defender 110, разработанная в Simulink/MATLAB дает удовлетворительно точные результаты. Следовательно, можно считать, что верификация выполнена. Валидированная модель может быть использована в оптимизации системы подвески в будущей эксплуатации.

Ключевые слова: система подвески, Defender 110, ADAMS/CAR, Simulink/MATLAB.

Математичко моделовање и симулација система за ослањање возила у попречној равни

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ОБЛАСТ: машинство
КАТЕГОРИЈА (ТИП) ЧЛАНКА: оригинални научни рад

Сажетак:

Увод/циљ: Проучавање система за ослањање возила представља изазов за истраживаче који сагледавају утицај система за ослањање на перформансе возила: удобност вожње, стабилност и управљивост возила, итд. У раду је представљена симулација рада система за ослањање возила у попречној равни, за возило Land Rover Defender 110, која је развијена у програмском пакету Simulink/MATLAB. Добијени резултати поређени су са резултатима добијеним у програмском пакету ADAMS/CAR за симулациони модел возила Land Rover Defender 110 који је верификован са експерименталним резулатима. Возило Land Rover Defender 110 има систем зависног ослањања на обе осовине и пасивни тип ослањања са четири степена слободе (4 DOF).

Методе: Једначине система могу се математички решити шемом у Simulink/MATLAB-у, а моделирање возила у попречној равни урађено је у пакету ADAMS/CAR.

Резултати: Поређење перформанси возила добијених помоћу две методе симулација извршено је за три различита сценарија; уочено је да постоји задовољавајуће подударање добијених резултата.

Закључак: Закључено је да симулациони модел возила Defender 110 у Simulink/MATLAB-у даје задовољавајуће тачне резултате, чиме се може сматрати да је извршена његова верификација . Верификовани симулациони модел може се користити са великом тачношћу за оптимизацију система за ослањање у будућем раду.

Кључне речи: систем за ослањање, Defender 110, ADAMS/CAR, Simulink/MATLAB.

Paper received on: 06.11.2023.
Manuscript corrections submitted on: 03.03.2024.
Paper accepted for publishing on: 04.03.2024.