

An approach for solving transportation problem under Pythagorean fuzzy environment

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Abstract:

Introduction/purpose: In this paper, we introduce a new defuzzification function for trapezoidal Pythagorean fuzzy numbers (TrPFNs) and develop arithmetic operations associated with these numbers. Since the transportation problem is closely related to linear programming problems, we construct a transportation model in which all parameters are represented using TrPFNs. The purpose of this work is to address uncertainty and imprecision that often occur in real-life transportation planning.

Methods: A transportation model with trapezoidal Pythagorean fuzzy parameters is formulated. The proposed defuzzification function, along with the mathematical properties of TrPFNs, is used to transform the fuzzy transportation model into an equivalent deterministic model. The resulting deterministic problem is then solved using MATLAB.

Results: To demonstrate the effectiveness of the proposed approach, a case study involving the transportation of goods between factories and warehouses is presented. The example preserves the fuzzy nature of the parameters throughout the solution process. A sensitivity analysis is also conducted to examine the robustness of the model. Furthermore, the results obtained from the proposed method are compared with existing approaches to verify consistency.

Conclusion: The study shows that the proposed defuzzification function and the developed transportation model effectively handle uncertainty in transportation problems. The approach provides reliable and consis-

tent results, indicating its potential usefulness for solving real-world fuzzy transportation problems.

Key words: Pythagorean fuzzy numbers, Pythagorean fuzzy transportation problems, optimal solution, sensitivity analysis.

Introduction

The process of making decisions that are as optimal or efficient as possible is called optimization. It includes methods for maximizing or minimizing a function under particular circumstances. One kind of optimization problem that deals with one or more objectives is the transportation problem (TP). Since the expense of transporting raw materials and finished products plays a significant role, a decrease in this expense causes the commodity's final price to drop significantly. Thus, TPs are established in the decision-making domain to fulfill this need for price reduction. They offer a framework for addressing the problem of how to satisfy consumer demands while keeping costs down. A basic TP, first introduced by Hitchcock ([Hitchcock, 1941](#)), is a specialized form of linear programming that focuses on transporting a commodity from multiple sources to various destinations in the most efficient manner. However, since the input data may not always be deterministic, the problem does not always follow a standard approach. To address this uncertainty, different types of data representations are used, depending on the nature of the uncertainty.

In conventional decision-making situations, parameters are typically considered in crisp form. However, accurately determining the actual values of TP parameters such as transportation costs, availability, and demand can be challenging in practical applications. This uncertainty arises due to factors like incomplete data, inadequate statistical analysis, fluctuations in financial markets, road conditions, and other unpredictable influences. To address this, decision makers (DMs) are consulted to forecast parameter values. DMs frequently provide parameter values in linguistic terms, interval form, or another format. This is addressed by taking into account parameters in formats that can manage and depict ambiguity. To deal with such situations, Zadeh ([Zadeh, 1965](#)) introduced the fuzzy set theory (FST). For practical implications, fuzzy numbers and the operators that can be applied to the problems proposed, providing the mathematical basis for the use of FST. Fuzzy transportation problems (FTPs) are those in which the uncertainty of any or all parameters is represented in the form



of fuzzy numbers. Several authors have explored FTPs and their solution methods. Kaur and Kumar ([Kaur & Kumar, 2012](#)) proposed a ranking function-based approach for generalized trapezoidal fuzzy transportation problems with crisp supply and demand values but fuzzy costs. Ebrahimnejad ([Ebrahimnejad, 2015](#)) introduced a bounded dual simplex method to solve transportation problems with both fuzzy supply and demand. Baykasoğlu and Subulan ([Baykasoğlu and Subulan, 2019](#)) presented a direct approach in which all parameters and decision variables are treated as fuzzy numbers, solved using fuzzy ranking and arithmetic operations. In a similar direction, Kane et al. ([Kané et al., 2021](#)) proposed a two-step method that first transforms the fuzzy problem into interval problems and then converts them into crisp problems using midpoint values. Bisht and Dangwal ([Bisht and Dangwal, 2022](#)) addressed interval-valued transportation problems by transforming them into octagonal fuzzy problems, applying a ranking function to obtain a crisp model, and solving it using Vogel's approximation method and the modified distribution (MODI) method. More recently, Aroniadi and Beligiannis ([Aroniadi and Beligiannis, 2024](#)) applied particle swarm optimization with trigonometric acceleration coefficients to handle fuzzy transportation problems with uncertain costs, showing improved performance over conventional methods. Kokila and Deepa ([Kokila and Deepa, 2024](#)) considered a multi-objective FTP using triangular fuzzy numbers, converting it into a crisp model to simplify computations, while Akilbasha et al. ([Akilbasha et al., 2024](#)) addressed stochastic transportation problems with additional budget constraints through fuzzy goal programming combined with octagonal fuzzy numbers to effectively manage uncertainty. Collectively, these studies illustrate various techniques for modeling real-world transportation systems under fuzzy conditions.

Moreover, in many situations the degree of reluctance must be considered alongside the degrees of acceptance and rejection. Traditional fuzzy numbers cannot capture both doubt and hesitation simultaneously, whereas intuitionistic fuzzy numbers are well suited for such cases. Atanassov ([Atanassov, 1986](#)) extended the concept of fuzzy sets by introducing intuitionistic fuzzy sets (IFS), which incorporate a hesitation margin arising from incomplete or uncertain information. In an IFS framework, both acceptance and non-acceptance degrees are represented, making it highly relevant for practical applications. Consequently, intuitionistic fuzzy modelling has been widely adopted in various decision-making problems. In

contexts such as transportation systems, inventory planning, logistics, supply chain management, and optimal control, hesitation often emerges in the available data. Representing such parameters through intuitionistic fuzzy numbers (IFNs) is therefore appropriate, giving rise to intuitionistic fuzzy transportation problem (IFTP). Solving IFTPs leads to the movement of raw materials and finished goods at minimal cost, even under uncertainty. Recent research on TPs under intuitionistic fuzzy environments has advanced both modeling approaches and solution techniques. Early contributions include Gani and Abbas [Gani & Abbas \(2014\)](#), who incorporated intuitionistic fuzzy production and consumption values with variable transportation costs, and Antony et al. [Antony et al. \(2014\)](#), who handled uncertainty in supply, demand, and costs using trapezoidal IFNs. Singh and Yadav [Singh & Yadav \(2015, 2016\)](#) extended this line of work by developing type-1 and type-2 IFTPs and solution procedures based on accuracy-function-based ordering for obtaining both the initial basic feasible solution (IBFS) and the optimal solution. Ebrahimnejad and Verdegay [Ebrahimnejad & Verdegay \(2016\)](#) later enhanced computational efficiency by adapting classical transportation algorithms to intuitionistic fuzzy settings. More recent studies focus on fully intuitionistic fuzzy models and multi-objective formulations: a 2021 work [Ghosh et al. \(2021\)](#) formulated a multi-objective fully intuitionistic fuzzy fixed-charge solid transportation problem using (α, β) -cuts and an accuracy function to derive a deterministic equivalent model. Niroomand et al. [Niroomand et al. \(2024\)](#) proposed a hybrid multi-objective optimization framework for fully IFTPs, converting fuzzy models to crisp form without relying on ranking functions. Jyothi et al. [Jyothi et al. \(2024\)](#) introduced generalized intuitionistic trapezoidal fuzzy numbers for modeling TP parameters and presented an efficient optimization-based approach for determining minimum transportation cost under uncertainty.

In IFS, the sum of the membership degree (MD) and the non-membership degree (NMD) should be less than or equal to 1. However, the assignment of these values by decision-makers, who adhere to this requirement, is not always justifiable. IFS is unable to address the problem in such instances. On the other hand, there exist situations in which the total squares of MD and NMD are larger than one, but their sum is less than or equal to one. In order to address this, the Pythagorean fuzzy set (PFS) ([Yager & Abbasov, 2013](#)), a superset of IFS, was introduced. Several recent studies have extended TPs to Pythagorean fuzzy



settings to better manage uncertainty in costs, supply, and demand. Kumar and Edalatpanah ([Kumar et al., 2019](#)) examined Pythagorean fuzzy arithmetic and numerical conditions across three models, while Prabha ([Prabha, 2021](#)) proposed a geometric-mean and scoring function based approach for solving Pythagorean fuzzy TPs. Jeyalakshmi et al. ([Jeyalakshmi et al. \(2021\)](#)) formulated a balanced Pythagorean fuzzy TP incorporating fuzzy costs, availability, and demand to support decision-making under imprecision. Nagar et al. ([Nagar et al. \(2021\)](#)) introduced a Pythagorean fuzzy species transportation model with a new IFBFS method and a Pythagorean fuzzy MODI method, obtaining solutions directly in fuzzy form to avoid information loss. In the context of perishable goods, Ghosh et al. ([Ghosh et al. \(2022\)](#)) developed a multi-objective solid TP with preservation technology in a Pythagorean fuzzy environment and solved it using the ϵ -constraint method, neutrosophic linear programming, and fuzzy TOPSIS. Nagar et al. ([Nagar et al., 2022](#)) further proposed a new score function for Pythagorean fuzzy numbers, applying it to obtain the IBFS and optimal solutions using the MODI method. Chhibber et al. ([Chhibber et al. \(2022\)](#)) contributed two direct solution techniques—one for the initial feasible solution and another for optimality—along with a defuzzification procedure. Bhatia et al. ([Bhatia et al. \(2023\)](#)) identified inconsistencies in Nagar et al.'s approach and introduced the Mehar method as a corrected solution procedure. Hemalatha and Venkateswarlu ([Hemalatha and Venkateswarlu \(2023\)](#)) proposed a novel sorting technique and mean square strategy for managing uncertainty in Pythagorean fuzzy TPs. More recently, Gahlawat et al. ([Gahlawat et al. \(2024\)](#)) developed a score-function-based method for solving Pythagorean fuzzy TPs, demonstrating superior ranking and solution performance through comparative analyses.

While several studies have addressed transportation problems under fuzzy, intuitionistic fuzzy, or even Pythagorean fuzzy environments, none of them has incorporated the parameterization of uncertainty in the same structured form as proposed in this work. In particular, the existing literature does not explore transportation cost, availability, and demand simultaneously through trapezoidal Pythagorean fuzzy parameters controlled by the (α, β) -pair, nor does it analyze their impact through a systematic sensitivity framework. This gap forms the key motivation for our study. Thus, in this paper, we propose a new defuzzification function and arithmetic operations for TrPFNs, and formulate a transportation problem with all parameters ex-

pressed in TrPFNs to effectively capture uncertainty. Using the proposed defuzzification approach, we transform the fuzzy transportation model into an equivalent crisp model and solve it using MATLAB. Further, we present a case study and sensitivity analysis to demonstrate the practicality and robustness of the proposed method. The primary contributions of this paper are as follows:

1. Introduction of a solution method for Trapezoidal Pythagorean Fuzzy Transportation Problems (TrPFTP): A new and efficient solution methodology is developed specifically for solving transportation problems under a trapezoidal Pythagorean fuzzy environment. This approach effectively incorporates both the trapezoidal structure and Pythagorean fuzzy logic, which allows for a more accurate representation of uncertainty and hesitation in transportation parameters such as cost, supply, and demand. The method is designed to improve decision-making in complex scenarios where traditional crisp or basic fuzzy models may fall short.
2. Development of a new linear defuzzification method for TrPFNs: A linear defuzzification technique is proposed for converting TrPFNs into crisp values. This method provides a straightforward yet powerful tool for interpreting fuzzy quantities in a precise and meaningful way. By ensuring linearity, the defuzzification process maintains the proportional relationships within fuzzy data, thereby preserving its structural integrity while enabling computational tractability in optimization models.
3. Demonstration through a practical example of TrPFTP: To validate the efficiency and applicability of the proposed method, the paper includes an example of a TrPFTP. This example illustrates the complete modeling process — from fuzzy representation and defuzzification to the final solution — showcasing how the new methodology can be effectively implemented in solving real-world transportation problems under uncertainty.

The remainder of the paper is structured as follows: The next section outlines the motivation for employing TrPFNs in modeling transportation problems parameters. The Preliminaries section introduces the fundamental concepts and definitions related to TrPFNs, which are essential for modeling and solving the problem under uncertainty. The Trapezoidal Pythagorean fuzzy transportation problem section presents the mathemat-

ical formulation of the TrPFTP along with the proposed solution methodology, including a flowchart that outlines the computational steps. The Numerical example section illustrates the application of the proposed approach through a detailed numerical example, demonstrating its effectiveness in handling uncertain transportation parameters. The Sensitivity analysis section provides sensitivity analysis under varying α and β -levels to examine the robustness of the proposed model. The Comparative analysis section evaluates the performance of the proposed approach against existing methods in the literature. Finally, the conclusion section concludes the paper by summarizing the key findings and suggesting possible directions for future research.

Motivation for using TrPFNs

In classical transportation problems, the cost coefficients, supply availability, and demand at destinations are required to be crisp and precisely known. However, in many real-life decision-making environments, such as logistics, supply chain management, and freight distribution, these parameters are often imprecise, vague, or incomplete. To model such uncertainties effectively, fuzzy-based extensions of the transportation problem have been widely explored.

While FNs and IFNs effectively handle data uncertainty, they have certain limitations compared to TrPFNs. Triangular fuzzy numbers provide limited flexibility in representing uncertainty, as they capture only a single peak value and assume linear membership variation on both sides. This often fails to reflect situations where experts express uncertainty in terms of ranges or plateaus. Similarly, intuitionistic fuzzy numbers extend fuzzy sets by incorporating membership and non-membership degrees, but they impose the constraint that the sum of these degrees cannot exceed one. This hard restriction limits their expressive capability when describing higher-order hesitation or ambiguity in transportation parameters. However, TrPFNs overcome the above limitations through two major features. First, the trapezoidal shape allows for a flat region (core) in the membership function, which is more suitable for representing expert assessments of cost, supply, and demand that are not point-specific but lie within a range of equally plausible values. This makes TrPFNs more realistic for handling various parameters in different types of TPs. Second, the Pythagorean fuzzy environment relaxes the restrictive condition of intu-

itionistic fuzzy sets by allowing the squared sum of the membership degree and non-membership degree to be at most one. This additional degree of freedom enhances the ability to express hesitation, which is essential in transportation planning where uncertainty arises due to market volatility, unpredictable transit conditions, and unreliable data records.

The use of TrPFNs is highly relevant to transportation problems as they provide a more flexible and accurate way to model uncertainty in real-world decision-making. When the cost coefficients, supply quantities, and demand requirements are modeled as TrPFNs, the resulting transportation model can capture wider uncertainty intervals and reflect human judgment more accurately. The trapezoidal structure allows decision-makers to specify optimistic, most likely, and pessimistic scenarios for each parameter, whereas the Pythagorean framework quantifies the corresponding belief, disbelief, and hesitation in a more flexible manner. Consequently, the TrPFN-based formulation yields solutions that are more robust, realistic, and applicable to practical transportation systems facing ambiguous and fluctuating information.

In summary, TrPFNs provide a richer and more adaptable representation of uncertainty compared to triangular or intuitionistic fuzzy numbers, making them particularly suitable for modeling the cost, availability, and demand parameters of transportation problems.

Preliminaries

This section presents some key definitions pertaining to the proposed study.

Definition 1 (Fuzzy Number). *A fuzzy subset $\tilde{A}$ of the real line $\mathbb{R}$ is said to be a fuzzy number if its membership function $\mu_{\tilde{A}}(\Lambda) : \mathbb{R} \rightarrow [0, 1]$ satisfies the following properties:*

1. $\mu_{\tilde{A}}(\Lambda)$ is piecewise continuous in its domain.
2. $\tilde{A}$ is normal, i.e., there exists some $\Lambda_0 \in \mathbb{R}$ such that $\mu_{\tilde{A}}(\Lambda_0) = 1$.
3. $\tilde{A}$ is convex, i.e., $\mu_{\tilde{A}}(\lambda\Lambda_1 + (1 - \lambda)\Lambda_2) \geq \min \{\mu_{\tilde{A}}(\Lambda_1), \mu_{\tilde{A}}(\Lambda_2)\}$ for every $\Lambda_1, \Lambda_2 \in \mathbb{R}$ and $\lambda \in [0, 1]$.
4. The support of $\tilde{A}$, i.e., $\text{Supp}(\tilde{A}) = \{\Lambda \in \mathbb{R} : \mu_{\tilde{A}}(\Lambda) > 0\}$ is bounded.

Definition 2 (Trapezoidal Fuzzy Number). *A fuzzy number $\tilde{\chi}$ is said to be a trapezoidal fuzzy number denoted by $\tilde{\chi} = (\varsigma_1, \varsigma_2, a_3, \varsigma_4)$, where $\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4 \in$*

$\mathbb{R}$ such that $\varsigma_1 < \varsigma_2 < \varsigma_3 < \varsigma_4$ and its membership function $\mu_{\tilde{\chi}}(\Lambda)$ is given by:

$$\mu_{\tilde{A}}(\Lambda) = \begin{cases} \frac{\Lambda - \varsigma_1}{\varsigma_2 - \varsigma_1}, & \text{if } \varsigma_1 \leq \Lambda \leq \varsigma_2, \\ 1, & \text{if } \varsigma_2 < \Lambda < \varsigma_3, \\ \frac{\varsigma_4 - \Lambda}{\varsigma_4 - \varsigma_3}, & \text{if } \varsigma_3 \leq \Lambda \leq \varsigma_4, \\ 0, & \text{otherwise.} \end{cases}$$

Definition 3 (Intuitionistic Fuzzy Number). An intuitionistic fuzzy subset $\tilde{\chi}^I = \{ \langle \Lambda, \mu_{\tilde{\chi}^I}(\Lambda), \nu_{\tilde{\chi}^I}(\Lambda) \rangle : \Lambda \in \mathbb{R} \}$ of the real line $\mathbb{R}$ is said to be an IFN if:

1. $\tilde{\chi}^I$ is intuitionistic fuzzy convex and intuitionistic fuzzy normal.
2. $\mu_{\tilde{\chi}^I}(\Lambda)$ is upper semi-continuous and $\nu_{\tilde{\chi}^I}(\Lambda)$ is lower semi-continuous.
3. The support of $\tilde{\chi}^I$, i.e., $\text{Supp}(\tilde{\chi}^I) = \{ \Lambda \in \mathbb{R} : \nu_{\tilde{\chi}^I}(\Lambda) < 1 \}$ is bounded.

Definition 4 (Trapezoidal Intuitionistic Fuzzy Number (TrIFN)). A TrIFN in $\mathbb{R}$ is denoted by $\chi^I = \{ [\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4]; [\varsigma'_1, \varsigma'_2, \varsigma'_3, \varsigma'_4]; \gamma_{\chi^I}(\Lambda), \lambda_{\chi^I}(\Lambda) \}$, with $\xi'_1 < \xi_1 < \xi_2 < \varsigma_3 < \varsigma_4 < \varsigma'_4$, where $0 \leq \gamma_{\chi^I}(\Lambda) \leq 1$, $0 \leq \lambda_{\chi^I}(\Lambda) \leq 1$, and $0 \leq \gamma_{\chi^I}(\Lambda) + \lambda_{\chi^I}(\Lambda) \leq 1$. For TrIFNs, the non-membership function (NMF) $\lambda_{\chi^I}(\Lambda)$ and the membership function (MF) $\gamma_{\chi^I}(\Lambda)$ are of the form:

$$\gamma_{A^I}(\Lambda) = \begin{cases} \frac{\Lambda - \varsigma_1}{\varsigma_2 - \varsigma_1}, & \text{if } \varsigma_1 < \Lambda < \varsigma_2; \\ 1, & \text{if } \varsigma_2 \leq \Lambda \leq \varsigma_3; \\ \frac{\varsigma_4 - \Lambda}{\varsigma_4 - \varsigma_3}, & \text{if } \varsigma_3 < \Lambda < \varsigma_4. \end{cases} \quad \lambda_{A^I}(\Lambda) = \begin{cases} \frac{\varsigma_2 - \Lambda}{\varsigma_2 - \varsigma_1}, & \text{if } \varsigma_1 < \Lambda < \varsigma_2; \\ 0, & \text{if } \varsigma_2 < \Lambda < \varsigma_3; \\ \frac{\Lambda - \varsigma_3}{\varsigma_4 - \varsigma_3}, & \text{if } \varsigma_3 < \Lambda < \varsigma_4. \end{cases}$$

Definition 5 (Trapezoidal Pythagorean Fuzzy Number). A TrPFN on the real axis is defined as $\chi^P = \{ (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4); \gamma_{\chi^P}(\Lambda), \lambda_{\chi^P}(\Lambda) \} : \Lambda \in X$, where $0 \leq \gamma_{\chi^P}(\Lambda) \leq 1$, $0 \leq \lambda_{\chi^P}(\Lambda) \leq 1$, $\gamma_{\chi^P}^2(\Lambda) + \lambda_{\chi^P}^2(\Lambda) \leq 1$, $0 \leq \varsigma_1 \leq \varsigma_2 \leq \varsigma_3 \leq \varsigma_4$, and $(\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ is a trapezoidal fuzzy number. Here, $\gamma_{\chi^P}(\Lambda)$ and $\lambda_{\chi^P}(\Lambda)$ are the MF and NMF, respectively, defined as:

$$\gamma_{\chi^P}(\Lambda) = \begin{cases} \frac{\Lambda - \varsigma_1}{\varsigma_2 - \varsigma_1}, & \text{if } \varsigma_1 < \Lambda < \varsigma_2; \\ 1, & \text{if } \varsigma_2 \leq \Lambda \leq \varsigma_3; \\ \frac{\varsigma_4 - \Lambda}{\varsigma_4 - \varsigma_3}, & \text{if } \varsigma_3 < \Lambda < \varsigma_4. \end{cases}$$

$$\lambda_{\chi^P}(\Lambda) = \begin{cases} \sqrt{\left(1 - \left(\frac{\Lambda - \varsigma_1}{\varsigma_2 - \varsigma_1}\right)^2\right)}, & \text{if } \varsigma_1 < \Lambda < \varsigma_2; \\ 0, & \text{if } \varsigma_2 \leq \Lambda \leq \varsigma_3; \\ \sqrt{\left(1 - \left(\frac{\varsigma_4 - \Lambda}{\varsigma_4 - \varsigma_3}\right)^2\right)}, & \text{if } \varsigma_3 < \Lambda < \varsigma_4. \end{cases}$$

Definition 6. [Arithmetic Operations on TrPFNs] Let the two positive TrPFNs be $\chi^P = (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ and $\varpi^P = (\psi_1, \psi_2, \psi_3, \psi_4)$, and let δ be a scalar. Then the following arithmetic operations are defined for TrPFNs:

- **Addition:**

$$\chi^P \oplus \varpi^P = (\varsigma_1 + \psi_1, \varsigma_2 + \psi_2, \varsigma_3 + \psi_3, \varsigma_4 + \psi_4).$$

- **Subtraction:**

$$\chi^P \ominus \varpi^P = (\varsigma_1 - \psi_4, \varsigma_2 - \psi_3, \varsigma_3 - \psi_2, \varsigma_4 - \psi_1).$$

- **Scalar Multiplication:**

$$\delta \chi^P = \begin{cases} (\delta \varsigma_1, \delta \varsigma_2, \delta \varsigma_3, \delta \varsigma_4), & \text{if } \delta \geq 0; \\ (\delta \varsigma_4, \delta \varsigma_3, \delta \varsigma_2, \delta \varsigma_1), & \text{if } \delta < 0. \end{cases}$$

- **Multiplication:**

$$\chi^P \otimes \varpi^P = (\varsigma_1 \psi_1, \varsigma_2 \psi_2, \varsigma_3 \psi_3, \varsigma_4 \psi_4).$$

Definition 7. [α -cut of Trapezoidal Pythagorean Fuzzy Number] The α -cut of $\chi^P = (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ is $\chi_\alpha^P = \{\Lambda : \gamma_{\chi^P}(\Lambda) \geq \alpha; \Lambda \in N\}$, where $0 \leq \alpha \leq 1$. Thus, the α -cut of $\chi^P = (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ is

$$[\chi_{\alpha,L}^P, \chi_{\alpha,U}^P] = [\alpha(\varsigma_2 - \varsigma_1) + \varsigma_1, \varsigma_4 - \alpha(\varsigma_4 - \varsigma_3)].$$

Definition 8. [β -cut of Trapezoidal Pythagorean Fuzzy Number] The β -cut of $\chi^P = (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ is $\chi_\beta^P = \{\Lambda : \lambda_{\chi^P}(\Lambda) \leq \beta; \Lambda \in N\}$, where $0 \leq \beta \leq 1$. Thus, the β -cut of $\chi^P = (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ is

$$[\chi_{\beta,L}^P, \chi_{\beta,U}^P] = \left[\varsigma_1 + (\varsigma_2 - \varsigma_1)\sqrt{1 - \beta^2}, \varsigma_4(1 - \sqrt{1 - \beta^2}) + \varsigma_3\sqrt{1 - \beta^2} \right].$$

Definition 9 ((α, β)-cut of Trapezoidal Pythagorean Fuzzy Number). The (α, β)-cut of a TrPFN $\chi^P = (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ is the set of all Λ whose NMD $\lambda_{\chi^P}(\Lambda)$ is less than or equal to β and MD $\gamma_{\chi^P}(\Lambda)$ is greater than or equal to α , i.e.,

$$\chi_{\alpha,\beta}^P = \{\Lambda : \gamma_{\chi^P}(\Lambda) \geq \alpha; \lambda_{\chi^P}(\Lambda) \leq \beta; 0 \leq \alpha^2 + \beta^2 \leq 1; \Lambda \in \mathbb{R}\}.$$

Definition 10 (Defuzzification Function for Trapezoidal Pythagorean Fuzzy Number). Let $\chi^P = (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ be a TrPFN. Then the defuzzification function for χ^P based on (α, β)-cut is defined as:

$$\mathfrak{D}(\chi^P) = \frac{\chi_{\alpha,L}^P + \chi_{\alpha,U}^P + \chi_{\beta,L}^P + \chi_{\beta,U}^P}{4}.$$

Using Definitions 7 and 8, we obtain

$$\mathfrak{D}(\chi^P) = \frac{(\alpha + \sqrt{1 - \beta^2})(\varsigma_2 - \varsigma_1 - \varsigma_4 + \varsigma_3) + 2(\varsigma_1 + \varsigma_4)}{4}.$$

Definition 11 (Positive Trapezoidal Pythagorean Fuzzy Number). A trapezoidal Pythagorean fuzzy number $\zeta^P = (\kappa_1, \kappa_2, \kappa_3, \kappa_4)$ is positive if $\kappa_1 \geq 0, \kappa_2 - \kappa_1 \geq 0, \kappa_3 - \kappa_2 \geq 0$ and $\kappa_4 - \kappa_3 \geq 0$.

Definition 12 (Equality of Two Trapezoidal Pythagorean Fuzzy Numbers). Let $\zeta^P = (\kappa_1, \kappa_2, \kappa_3, \kappa_4)$ and $\varpi^P = (\varepsilon_1, \varepsilon_2, \varepsilon_3, \varepsilon_4)$ be any two trapezoidal Pythagorean fuzzy numbers. Then $\zeta^P \cong \varpi^P$ if and only if $\kappa_1 = \varepsilon_1, \kappa_2 = \varepsilon_2, \kappa_3 = \varepsilon_3$ and $\kappa_4 = \varepsilon_4$.

Theorem 1. If $\chi^P = (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ and $\varpi^P = (\psi_1, \psi_2, \psi_3, \psi_4)$ are any two TrPFNs, and ϱ is any real number, then the defuzzification function $\mathfrak{D}$ is linear, i.e., $\mathfrak{D}(\chi^P + \varrho\varpi^P) = \mathfrak{D}(\chi^P) + \varrho\mathfrak{D}(\varpi^P)$.

Proof. We have $\chi^P = (\varsigma_1, \varsigma_2, \varsigma_3, \varsigma_4)$ and $\varpi^P = (\psi_1, \psi_2, \psi_3, \psi_4)$.
 Case I: When $\varrho \geq 0$. Using Definition 6,

$$\chi^P + \varrho \varpi^P = (\varsigma_1 + \varrho \psi_1, \varsigma_2 + \varrho \psi_2, \varsigma_3 + \varrho \psi_3, \varsigma_4 + \varrho \psi_4).$$

Using the defuzzification function,

$$\begin{aligned} & \mathfrak{D}(\chi^P + \varrho \varpi^P) \\ &= \frac{1}{4} \left[(\alpha + \sqrt{1 - \beta^2}) \{(\varsigma_2 + \varrho \psi_2) - (\varsigma_1 + \varrho \psi_1) - (\varsigma_4 + \varrho \psi_4) \right. \\ & \quad \left. + (\varsigma_3 + \varrho \psi_3)\} + 2 \{(\varsigma_1 + \varrho \psi_1) + (\varsigma_4 + \varrho \psi_4)\} \right] \\ &= \frac{1}{4} \left[(\alpha + \sqrt{1 - \beta^2}) \{(\varsigma_2 - \varsigma_1 - \varsigma_4 + \varsigma_3) + \varrho(\psi_2 - \psi_1 - \psi_4 \right. \\ & \quad \left. + \psi_3)\} + (2\varsigma_1 + 2\varsigma_4) + \varrho(2\psi_1 + 2\psi_4) \right] \\ &= \frac{1}{4} \left[(\alpha + \sqrt{1 - \beta^2})(\varsigma_2 - \varsigma_1 - \varsigma_4 + \varsigma_3) + 2(\varsigma_1 + \varsigma_4) \right] \\ & \quad + \frac{1}{4} \varrho \left[(\alpha + \sqrt{1 - \beta^2})(\psi_2 - \psi_1 - \psi_4 + \psi_3) + 2(\psi_1 + \psi_4) \right] \\ &= \mathfrak{D}(\chi^P) + \varrho \mathfrak{D}(\varpi^P) \end{aligned}$$

Case II: When $\varrho < 0$. Using Definition 6,

$$\chi^P + \varrho \varpi^P = (\varsigma_1 + \varrho \psi_4, \varsigma_2 + \varrho \psi_3, \varsigma_3 + \varrho \psi_2, \varsigma_4 + \varrho \psi_1).$$

Using the defuzzification function,

$$\begin{aligned} & \mathfrak{D}(\chi^P + \varrho \varpi^P) \\ &= \frac{1}{4} \left[(\alpha + \sqrt{1 - \beta^2}) \{(\varsigma_2 + \varrho \psi_3) - (\varsigma_1 + \varrho \psi_4) - (\varsigma_4 + \varrho \psi_1) \right. \\ & \quad \left. + (\varsigma_3 + \varrho \psi_2)\} + 2 \{(\varsigma_1 + \varrho \psi_4) + (\varsigma_4 + \varrho \psi_1)\} \right] \\ &= \frac{1}{4} \left[(\alpha + \sqrt{1 - \beta^2}) \{(\varsigma_2 - \varsigma_1 - \varsigma_4 + \varsigma_3) + \varrho(\psi_3 - \psi_4 - \psi_1 \right. \\ & \quad \left. + \psi_2)\} + 2[(\varsigma_1 + \varsigma_4) + \varrho(\psi_4 + \psi_1)] \right] \\ &= \frac{1}{4} \left\{ [(\alpha + \sqrt{1 - \beta^2})(\varsigma_2 - \varsigma_1 - \varsigma_4 + \varsigma_3) + 2(\varsigma_1 + \varsigma_4)] \right\} \\ & \quad + \varrho \left\{ \frac{1}{4} [(\alpha + \sqrt{1 - \beta^2})(\psi_3 - \psi_4 - \psi_1 + \psi_2) + 2(\psi_4 + \psi_1)] \right\} \\ &= \mathfrak{D}(\chi^P) + \varrho \mathfrak{D}(\varpi^P). \end{aligned}$$

Hence, proved. □

Trapezoidal Pythagorean fuzzy transportation problem

The primary goal of a TP is to determine the optimal way to distribute a product or item from several sources (origins) to various destinations, aiming to minimize the total transportation cost. This scenario is common in logistics and supply chain management, where efficient allocation of resources is essential to meet demand while minimizing operational expenses. The most common real-world example involves a company with multiple factories (serving as origins) and warehouses (acting as destinations).

Each factory has a specified supply limit, indicating the maximum quantity that can be shipped, while each warehouse has a certain demand, representing the amount required to satisfy customer needs. The objective is to determine a transportation plan that satisfies both supply and demand constraints while minimizing the overall transportation cost.

In real-world scenarios, however, many uncertainties affect the input parameters of the transportation model. These uncertainties may stem from fluctuating supply availability, inconsistent demand patterns, or variable transportation costs. Traditional models using crisp values may fall short in capturing such complexities. To better represent this inherent vagueness, the TrPFTP framework is employed, where the parameters—namely, supply, demand, and transportation costs—are expressed as TrPFNs.

Pythagorean fuzzy sets extend intuitionistic fuzzy sets by allowing the sum of squares of membership and non-membership degrees to be less than or equal to one, thereby offering a broader scope to model uncertainty. When these are further defined by a trapezoidal structure, they allow the representation of a range of possible values rather than single-point estimates, accommodating uncertainty in a more flexible and realistic way.

By integrating trapezoidal Pythagorean fuzzy numbers into the transportation model, the TrPFTP formulation captures a richer degree of imprecision and ambiguity in the data. This enhances the model's adaptability to real-world logistics, enabling the generation of transportation plans that are not only cost-effective but also robust under varying conditions. Such models equip decision-makers with deeper insight into potential fluctuations and prepare them for multiple scenarios, making TrPFTP a powerful tool in uncertain supply chain environments.

The notations and assumptions used for formulating the TrPFTP are as follows:

Notations and assumptions

The following notations and assumptions have been considered in formulation of the mathematical model:

- $\Delta_{ab}^P = (\delta_{ab_1}, \delta_{ab_2}, \delta_{ab_3}, \delta_{ab_4})$: Trapezoidal Pythagorean fuzzy number of units to be transported from the a^{th} origin to the b^{th} destination.
- $C_{ab}^P = (c_{ab_1}, c_{ab_2}, c_{ab_3}, c_{ab_4})$: Trapezoidal Pythagorean fuzzy transportation cost for a unit item/product from the a^{th} origin to the b^{th} destination.
- $S_a^P = (s_{a_1}, s_{a_2}, s_{a_3}, s_{a_4})$: Trapezoidal Pythagorean fuzzy supply at the a^{th} origin.
- $D_b^P = (d_{b_1}, d_{b_2}, d_{b_3}, d_{b_4})$: Trapezoidal Pythagorean fuzzy demand at the b^{th} destination.
- r : Number of sources ($a = 1, 2, 3, \dots, r$).
- s : Number of destinations ($b = 1, 2, 3, \dots, s$).
- $S_a^P > 0, D_b^P > 0, C_{ab}^P > 0, \Delta_{ab}^P > 0 \forall a \text{ and } b$.

Model formulation

- **Objective Function:** The primary objective is to determine the quantities of goods Δ_{ab}^P to be transported from the a^{th} origin to the b^{th} destination in such a way that the demand at each destination is fulfilled, and the transportation cost is minimized. The total transportation cost is expressed through the following objective: minimizing the overall cost incurred during transportation.

$$\text{Minimize } Z^P = \sum_{a=1}^r \sum_{b=1}^s \{C_{ab}^P \otimes \Delta_{ab}^P\}.$$

This objective function seeks to minimize the total transportation cost, which is defined as the aggregated sum of the products of trapezoidal Pythagorean fuzzy per-unit transportation costs and the corresponding quantities transported, over all origins and destinations.

- **Supply Constraints:** The supply constraints ensure that the total quantity of goods shipped from each origin does not exceed its available supply capacity. These constraints are formulated to restrict the sum of all



shipments from a particular source to all destinations. For a transportation model involving r sources, the supply constraints can be mathematically expressed as follows:

$$\sum_{b=1}^s \Delta_{ab}^P = S_a^P \text{ for } a = 1, 2, \dots, r.$$

- **Demand Constraints:** The demand constraints ensure that each destination receives a total quantity of goods that satisfies its specified demand. These constraints are formulated to regulate the cumulative shipments directed to a given destination from all sources. In the proposed model comprising s destinations, the demand constraints can be mathematically expressed as follows:

$$\sum_{a=1}^r \Delta_{ab}^P = D_b^P \text{ for } b = 1, 2, \dots, s.$$

- **Non-Negativity Constraints:** The non-negativity constraints ensure that the quantities of products transported from each source to each destination are zero or positive. These constraints are fundamental to the formulation of transportation models, as they reflect the practical reality that negative shipments are not physically meaningful or permissible in real-world distribution systems. In the proposed model with r sources and s destinations, the non-negativity constraints can be represented as follows:

$$\Delta_{ab}^P \geq 0 \text{ for } a = 1, 2, \dots, r \text{ and } b = 1, 2, \dots, s.$$

Therefore, by incorporating the defined objective function along with the associated constraints, the formulated model for the TrPFTP can be represented as follows:

Model 1: Trapezoidal Pythagorean fuzzy transportation problem

Based on the notations and assumptions described above, and considering all the parameters of the problem as trapezoidal Pythagorean fuzzy numbers in the initial stage, the mathematical model of the TrPFTP is formulated as follows:

$$\text{Min } Z^P = \sum_{a=1}^r \sum_{b=1}^s \{C_{ab}^P \otimes \Delta_{ab}^P\}$$

$$\begin{aligned}
 \text{subject to } \sum_{b=1}^s \Delta_{ab}^P &= S_a^P \text{ for } a = 1, 2, \dots, r, \\
 \sum_{a=1}^r \Delta_{ab}^P &= D_b^P \text{ for } b = 1, 2, \dots, s, \\
 \Delta_{ab}^P &\geq 0 \text{ for } a = 1, 2, \dots, r \text{ and } b = 1, 2, \dots, s.
 \end{aligned}$$

With the previously introduced notation, Model 1 is rewritten in the form of Model 2.

Model 2: Auxiliary transformation model

Directly solving the TrPFTP is difficult because it contains trapezoidal Pythagorean fuzzy variables. Therefore, Model 1 must first be converted into an equivalent deterministic form. To achieve this, we rewrite Model 1 by substituting the formal notations defined for all its parameters, which serves as an intermediate step before obtaining the final crisp model.

$$\text{Min } Z = \sum_{a=1}^r \sum_{b=1}^s (c_{ab_1}, c_{ab_2}, c_{ab_3}, c_{ab_4}) \otimes (\delta_{ab_1}, \delta_{ab_2}, \delta_{ab_3}, \delta_{ab_4})$$

subject to

$$\begin{aligned}
 \sum_{b=1}^s (\delta_{ab_1}, \delta_{ab_2}, \delta_{ab_3}, \delta_{ab_4}) &= (s_{a_1}, s_{a_2}, s_{a_3}, s_{a_4}) \text{ for } a = 1, 2, \dots, r, \\
 \sum_{a=1}^r (\delta_{ab_1}, \delta_{ab_2}, \delta_{ab_3}, \delta_{ab_4}) &= (d_{b_1}, d_{b_2}, d_{b_3}, d_{b_4}) \text{ for } b = 1, 2, \dots, s, \\
 (\delta_{ab_1}, \delta_{ab_2}, \delta_{ab_3}, \delta_{ab_4}) &\geq 0 \text{ for } a = 1, 2, \dots, r \text{ and } b = 1, 2, \dots, s.
 \end{aligned}$$

By applying the defuzzification and comparison techniques outlined in Definitions 10, 11, and 12, Model 2, which incorporates trapezoidal Pythagorean fuzzy parameters, is systematically converted into its equivalent crisp linear form, referred to as Model 3. Definition 10 provides the defuzzification function used to transform the fuzzy objective function into a crisp objective, while Definitions 11 and 12 are employed to convert the fuzzy inequality and equality constraints, respectively, into their deterministic counterparts. As a result, the fully crisp version of the transportation problem is obtained and presented as follows:

Model 3: Equivalent deterministic model

$$\text{Min } Z = \sum_{a=1}^r \sum_{b=1}^s \frac{1}{4} \left[\left(\alpha + \sqrt{1 - \beta^2} \right) (c_{ab_2} \delta_{ab_2} - c_{ab_1} \delta_{ab_1} - c_{ab_4} \delta_{ab_4} + c_{ab_3} \delta_{ab_3}) \right. \\ \left. + 2 (c_{ab_1} \delta_{ab_1} + c_{ab_4} \delta_{ab_4}) \right]$$

subject to

$$\sum_{b=1}^s \delta_{ab_1} = s_{a_1}, \quad \sum_{b=1}^s \delta_{ab_2} = s_{a_2}, \quad \sum_{b=1}^s \delta_{ab_3} = s_{a_3}, \\ \sum_{b=1}^s \delta_{ab_4} = s_{a_4} \text{ for } a = 1, 2, \dots, r, \\ \sum_{a=1}^r \delta_{ab_1} = d_{b_1}, \quad \sum_{a=1}^r \delta_{ab_2} = d_{b_2}, \quad \sum_{a=1}^r \delta_{ab_3} = d_{b_3}, \\ \sum_{a=1}^r \delta_{ab_4} = d_{b_4} \text{ for } b = 1, 2, \dots, s, \\ \delta_{ab_1} \geq 0, \quad \delta_{ab_2} - \delta_{ab_1} \geq 0, \quad \delta_{ab_3} - \delta_{ab_2} \geq 0, \\ \delta_{ab_4} - \delta_{ab_3} \geq 0 \text{ for } a = 1, 2, \dots, r \text{ and } b = 1, 2, \dots, s.$$

Ultimately, the optimal solution to this linear programming problem is derived using MATLAB. Fig. 1 illustrates the flowchart of the proposed solution approach for TrPFTP.

Numerical example

To illustrate the applicability of the proposed methodology, we consider a TrPFTP. In this example, there are three factories—Factory A, Factory B, and Factory C—which serve as sources, supplying products to three destinations—Warehouse X, Warehouse Y, and Warehouse Z. The objective is to determine an optimal transportation plan that minimizes the total transportation cost while satisfying the supply capacities of the factories and the demand requirements of the warehouses.

In real-world logistics, transportation costs, supply capacities, and demand levels are often subject to uncertainty due to various factors such as fuel price fluctuations, changing road or weather conditions, and variability in labor costs. To effectively model and manage this inherent uncertainty,

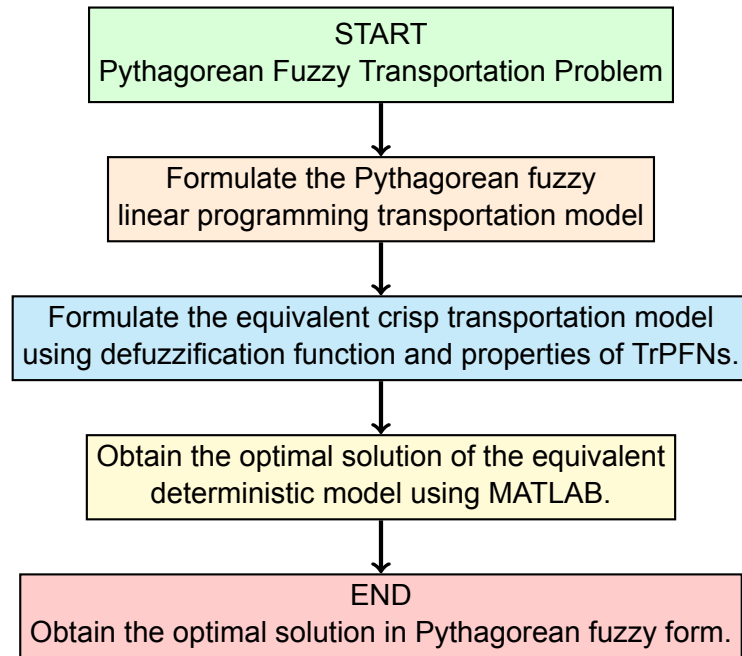


Figure 1 – Flowchart for the proposed solution procedure

all parameters in the problem, namely, transportation costs, supply, and demand, are represented using TrPFNs.

The fuzzy per-unit transportation costs (in dollars) from each factory to each warehouse are presented in Table 1. The fuzzy demand values (in tonnes) for Warehouses X, Y, and Z are given as (180, 190, 220, 250), (190, 270, 280, 320), and (250, 290, 310, 330), respectively. Similarly, the fuzzy supply capacities (in tonnes) of Factory A, Factory B, and Factory C are given as (100, 150, 170, 200), (210, 250, 270, 300), and (310, 350, 370, 400), respectively.

The goal is to determine the optimal transportation plan that minimizes the overall fuzzy transportation cost, taking into account the vagueness in the system parameters. The subsequent sections will detail the defuzzification process and solution procedure, ultimately leading to the computation of the crisp optimal transportation cost.

Table 1 – Transportation cost per unit item for the considered numerical example

Factory ↓ Warehouse →	X	Y	Z
A	(5, 10, 15, 20)	(10, 11, 12, 13)	(6, 7, 8, 9)
B	(8, 9, 10, 11)	(20, 25, 30, 35)	(2, 3, 4, 5)
C	(10, 20, 30, 40)	(13, 14, 15, 16)	(21, 25, 28, 30)

Solution

In this problem, we consider $\alpha = 0.9$ and $\beta = 0.3$, such that $\alpha + \beta > 1$ but $\alpha^2 + \beta^2 \leq 1$, and obtain the deterministic model as follows:

$$\begin{aligned} \text{Max} = & \frac{1}{4} [1.8539 (10\delta_{112} - 5\delta_{111} - 20\delta_{114} + 15\delta_{113} + 11\delta_{122} - 10\delta_{121} \\ & - 13\delta_{124} + 12\delta_{123} + 7\delta_{132} - 6\delta_{131} - 9\delta_{134} + 8\delta_{133} + 9\delta_{212} - 8\delta_{211} \\ & - 11\delta_{214} + 10\delta_{213} + 25\delta_{222} - 20\delta_{221} - 35\delta_{224} + 30\delta_{223} + 3\delta_{232} \\ & - 2\delta_{231} - 5\delta_{234} + 4\delta_{233} + 20\delta_{312} - 10\delta_{311} - 40\delta_{314} + 30\delta_{313} + 14\delta_{322} \\ & - 13\delta_{321} - 16\delta_{324} + 15\delta_{323} + 25\delta_{332} - 21\delta_{331} - 30\delta_{334} + 28\delta_{333}) \\ & + 2 (5\delta_{111} + 20\delta_{114} + 10\delta_{121} + 13\delta_{124} + 6\delta_{131} + 9\delta_{134} + 8\delta_{211} \\ & + 11\delta_{214} + 20\delta_{221} + 35\delta_{224} + 2\delta_{231} + 5\delta_{234} + 10\delta_{311} + 40\delta_{314} \\ & + 13\delta_{321} + 16\delta_{324} + 21\delta_{331} + 30\delta_{334})] \end{aligned}$$

subject to

$$\begin{aligned} \delta_{111} + \delta_{121} + \delta_{131} &= 100, \delta_{112} + \delta_{122} + \delta_{132} = 150, \\ \delta_{113} + \delta_{123} + \delta_{133} &= 170, \delta_{114} + \delta_{124} + \delta_{134} = 200, \\ \delta_{211} + \delta_{221} + \delta_{231} &= 210, \delta_{212} + \delta_{222} + \delta_{232} = 250, \\ \delta_{213} + \delta_{223} + \delta_{233} &= 270, \delta_{214} + \delta_{224} + \delta_{234} = 300, \\ \delta_{311} + \delta_{321} + \delta_{331} &= 310, \delta_{312} + \delta_{322} + \delta_{332} = 350, \\ \delta_{313} + \delta_{323} + \delta_{333} &= 370, \delta_{314} + \delta_{324} + \delta_{334} = 400, \\ \delta_{111} + \delta_{211} + \delta_{311} &= 180, \delta_{112} + \delta_{212} + \delta_{312} = 190, \\ \delta_{113} + \delta_{213} + \delta_{313} &= 220, \delta_{114} + \delta_{214} + \delta_{314} = 250, \\ \delta_{121} + \delta_{221} + \delta_{321} &= 190, \delta_{122} + \delta_{222} + \delta_{322} = 270, \\ \delta_{123} + \delta_{223} + \delta_{323} &= 280, \delta_{124} + \delta_{224} + \delta_{324} = 320, \\ \delta_{131} + \delta_{231} + \delta_{331} &= 250, \delta_{132} + \delta_{232} + \delta_{332} = 290, \end{aligned}$$

$$\begin{aligned}
 &\delta_{133} + \delta_{233} + \delta_{333} = 310, \delta_{134} + \delta_{234} + \delta_{334} = 330, \\
 &\delta_{111} \geq 0, \delta_{112} - \delta_{111} \geq 0, \delta_{113} - \delta_{112} \geq 0, \delta_{114} - \delta_{113} \geq 0, \\
 &\delta_{121} \geq 0, \delta_{122} - \delta_{121} \geq 0, \delta_{123} - \delta_{122} \geq 0, \delta_{124} - \delta_{123} \geq 0, \\
 &\delta_{131} \geq 0, \delta_{132} - \delta_{131} \geq 0, \delta_{133} - \delta_{132} \geq 0, \delta_{134} - \delta_{133} \geq 0, \\
 &\delta_{211} \geq 0, \delta_{212} - \delta_{211} \geq 0, \delta_{213} - \delta_{212} \geq 0, \delta_{214} - \delta_{213} \geq 0, \\
 &\delta_{221} \geq 0, \delta_{222} - \delta_{221} \geq 0, \delta_{223} - \delta_{222} \geq 0, \delta_{224} - \delta_{223} \geq 0, \\
 &\delta_{231} \geq 0, \delta_{232} - \delta_{231} \geq 0, \delta_{233} - \delta_{232} \geq 0, \delta_{234} - \delta_{233} \geq 0, \\
 &\delta_{311} \geq 0, \delta_{312} - \delta_{311} \geq 0, \delta_{313} - \delta_{312} \geq 0, \delta_{314} - \delta_{313} \geq 0, \\
 &\delta_{321} \geq 0, \delta_{322} - \delta_{321} \geq 0, \delta_{323} - \delta_{322} \geq 0, \delta_{324} - \delta_{323} \geq 0, \\
 &\delta_{331} \geq 0, \delta_{332} - \delta_{331} \geq 0, \delta_{333} - \delta_{332} \geq 0, \delta_{334} - \delta_{333} \geq 0.
 \end{aligned}$$

Using MATLAB, the optimal solution obtained is:

$$\begin{aligned}
 \Delta_{11}^P &= (60, 70, 70, 70), \quad \Delta_{12}^P = (0, 40, 40, 50), \quad \Delta_{13}^P = (40, 40, 60, 80), \\
 \Delta_{21}^P &= (0, 0, 20, 50), \quad \Delta_{23}^P = (210, 250, 250, 250), \\
 \Delta_{31}^P &= (120, 120, 130, 130), \quad \Delta_{32}^P = (190, 230, 240, 270),
 \end{aligned}$$

and the rest of the variables are trapezoidal Pythagorean fuzzy zero. Thus, the optimal transportation cost is given by:

$$\begin{aligned}
 Z_{ab}^P &= C_{ab}^P \otimes \Delta_{ab}^P \\
 &= (5, 10, 15, 20) \otimes (60, 70, 70, 70) \oplus (10, 11, 12, 13) \otimes (0, 40, 40, 50) \\
 &\quad \oplus (6, 7, 8, 9) \otimes (40, 40, 60, 80) \oplus (8, 9, 10, 11) \otimes (0, 0, 20, 50) \\
 &\quad \oplus (20, 25, 30, 35) \otimes (0, 0, 0, 0) \oplus (2, 3, 4, 5) \otimes (210, 250, 250, 250) \\
 &\quad \oplus (10, 20, 30, 40) \otimes (120, 120, 130, 130) \oplus (13, 14, 15, 16) \\
 &\quad \otimes (190, 230, 240, 270) \oplus (21, 25, 28, 30) \otimes (0, 0, 0, 0) \\
 &= \$ (4630, 7790, 10710, 14090).
 \end{aligned}$$

The crisp value of the optimal transportation cost by the proposed approach is \$9258 (see Figure 2(ii)).

Sensitivity analysis with varying α and β

In real-world transportation problems, the parameters represented as TrPFNs inherently carry uncertainty. To investigate the robustness of the



proposed model, we perform a sensitivity analysis by varying the parameters α and β within the feasible range, i.e., $\alpha^2 + \beta^2 \leq 1$. For each pair (α, β) , we solve the deterministic equivalent of the TrPFTP and record the corresponding objective function value.

From Figure 2, we can see that the value of the objective function exhibits a smooth and monotonic variation with respect to both parameters. As α increases and β decreases, the objective value gradually decreases, indicating that higher degrees of membership tend to favor lower total transportation cost in the crisp model. Conversely, larger values of β (with lower α) increase the objective value, reflecting the influence of stronger non-membership components of trapezoidal Pythagorean fuzzy numbers.

The surface plot does not contain abrupt jumps or oscillations, which confirms that the proposed crisp transformation method is numerically stable and the resulting transportation model responds continuously and predictably to variations in fuzzy uncertainty. This regularity is particularly important in decision-making environments where the selection of (α, β) reflects managerial preference, reliability of data, or uncertainty tolerance. The sensitivity structure also indicates that no region of the feasible (α, β) domain leads to infeasible or unstable solutions, further supporting the soundness of the proposed modeling approach.

Thus, the crisp transportation model derived from trapezoidal Pythagorean fuzzy numbers is both robust and well-behaved under varying levels of membership and non-membership information. The objective value changes smoothly across the feasible (α, β) region, confirming that the model is not overly sensitive to small perturbations in fuzzy parameters. This behavior ensures reliability in practical applications where uncertainties in transportation cost, supply, or demand may influence the selection of (α, β) . Overall, the analysis validates that the proposed transformation technique yields stable and interpretable solutions, strengthening the applicability of the model in real-world logistics and decision-making scenarios.

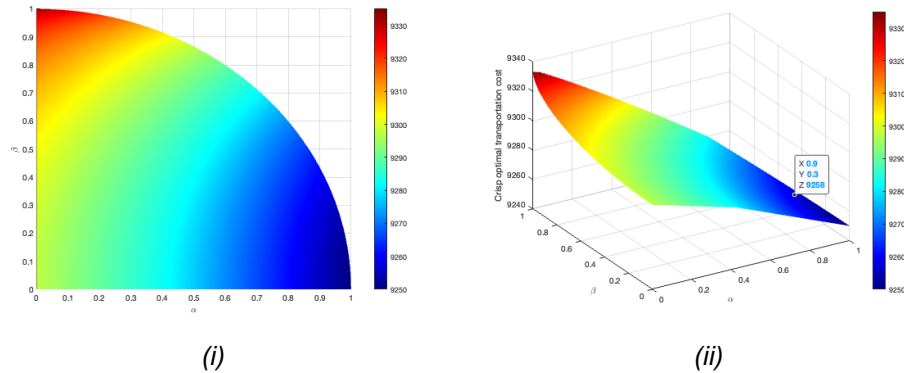


Figure 2 – Sensitivity analysis with varying α and β (i) feasible region of Pythagorean fuzzy numbers represented over the (α, β) -plane; (ii) corresponding variation of the objective value of the transformed transportation model

Comparative analysis for the proposed approach

1. To assess the validity and robustness of the proposed Pythagorean fuzzy transportation model and its solution method, we construct a TP (Table 2) whose structure is general enough to be compatible with the modelling assumptions used in several existing studies. It can be directly aligned with the forms employed in the literature through slight modifications in their respective frameworks.

Table 2 – Transportation problem for comparative analysis

Source	A	B	C	Availability
X	(3, 5, 8, 12)	(2, 5, 7, 10)	(3, 4, 6, 9)	50
Y	(12, 15, 18, 20)	(3, 6, 9, 12)	(4, 7, 9, 13)	80
P	(5, 7, 10, 12)	(4, 8, 10, 15)	(5, 9, 12, 16)	70
Demand	100	55	50	50

We solve the TP considered in Table 2 using different methods (Aggarwal & Gupta (2015); Beaula & Priyadharsini (2015); Ebrahimnejad & Verdegay (2018); Hunwisai et al. (2019)) and observe that all approaches lead to identical value of objective function ($\text{Min } Z = (770, 1230, 1780, 2370)$) and same allocations ($\Delta_{11}^P = (30, 30, 30, 30)$, $\Delta_{13}^P = (20, 20, 20, 20)$, $\Delta_{22}^P = (50, 50, 50, 50)$, $\Delta_{23}^P = (30, 30, 30, 30)$, $\Delta_{31}^P = (70, 70, 70, 70)$ and rest of the variables are trapezoidal Pythagorean fuzzy zero). This agreement across multiple methodologies confirms that our model is both mathematically

sound and computationally reliable. Moreover, it demonstrates that the proposed method integrates smoothly with established solution techniques, strengthening confidence in the correctness and applicability of the proposed formulation.

2. Several studies in the literature, for example, [Kumar et al. \(2019\)](#); [Prabha \(2021\)](#); [Jeyalakshmi et al. \(2021\)](#); [Nagar et al. \(2021, 2022\)](#); [Hemalatha and Venkateswarlu \(2023\)](#); [Bhatia et al. \(2023\)](#); [Gahlawat et al. \(2024\)](#) used Pythagorean fuzzy cost parameters expressed directly as membership–non-membership pairs such as $[0.8, 0.3]$. However, the interpretation of such pairs as actual transportation costs is not well-defined, and their operational significance within the transportation framework is unclear. Our formulation addresses this limitation by adopting a more structured and meaningful fuzzy parameter representation.

Conclusion

In this paper, we present a simple and complete framework for solving transportation problems using TrPFNs. We introduce basic arithmetic operations for TrPFNs and propose a new defuzzification function. To include the uncertain nature of real-world data, we model all cost values, supply, and demand as TrPFNs. We then convert this fuzzy transportation problem into a deterministic model using the proposed defuzzification function, which makes use of the special mathematical features of TrPFNs. This transformation keeps the uncertainty information but allows the problem to be solved using standard optimization methods. The final crisp model is solved in MATLAB. To demonstrate its usefulness, we provide a numerical example involving the transportation of goods between warehouses and factories. Throughout the process, we maintain the fuzzy nature of the data to show how well the model handles uncertainty and imprecision in real logistics situations. We also carry out a sensitivity analysis to study how the objective value behaves when the α and β values change within their feasible region. The results show that the objective value varies smoothly, meaning the model is stable and not heavily affected by small changes in fuzzy parameters. This confirms that the proposed approach remains reliable even when cost, supply, or demand values are uncertain. In addition, a comparative analysis with existing methods shows that all approaches lead to the same objective value and similar allocations once transformed through our defuzzification process. This agreement across methods fur-

ther supports that the model is consistent, dependable, and suitable for real-world decision-making.

For future work, we suggest extending this approach to unbalanced transportation problems, improving the defuzzification method further, and extending the work to other optimization problems such as assignment problems, supply chain systems, and multi-objective decision-making involving TrPFNs.

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Pristup rešavanju transportnog problema u Pitagorinom fazi okruženju

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OBLAST: matematika

KATEGORIJA (TIP) ČLANKA: originalni naučni rad

Sažetak:

Uvod/cilj: U ovom radu se uvodi nova funkcija defazifikacije za trapezoidne Pitagorine fazi brojeve (eng. TrPFN) i razvijaju se aritmetičke operacije koje se na njih odnose. S obzirom na blisku povezanost transportnog problema sa problemom linearnog programiranja, konstruisan je transportni model u kom su svi parametri predstavljeni pomoću trapezoidnih Pitagorinih fazi brojeva. Cilj rada je da obuhvati neizvesnost i nepreciznost koje se često javljaju u realnim problemima planiranja transporta.

Metode: Formulisan je transportni model sa trapezoidnim Pitagorinim fazi parametrima. Predložena funkcija defazifikacije i matematička svojstva trapezoidnih Pitagorinih fazi brojeva se koriste za transformaciju fazi transportnog modela u ekvivalentni deterministički model koji se potom rešava upotrebom programskog jezika MATLAB.

Rezultati: U cilju demonstracije efikasnosti predloženog pristupa, predstavljena je studija slučaja koja se odnosi na transport robe između fabrika i skladišta. U primeru je očuvana fazi prirodna parametara tokom čitavog postupka rešavanja, a sprovedena je i analiza osetljivosti kako bi se ispitala robusnost predloženog modela. Rezultati dobijeni predloženim metodom upoređeni su sa postojećim pristupima radi provere konzistentnosti.

Zaključak: Rezultati istraživanja pokazuju da predložena funkcija defazifikacije i razvijeni transportni model efikasno upravljaju nezvesnošću u transportnim problemima. Predloženi pristup daje pouzdane i konzistentne rezultate i može biti koristan za rešavanje realnih fazi transportnih problema.

Ključne reči: Pitagorini fazi brojevi, Pitagorini fazi transportni problem, optimalno rešenje, analiza osetljivosti.

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