



Interior ballistics modelling of a rifled barrel with cumulative wear effects

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Abstract

Background/Aim: Barrel wear progressively changes the interior ballistics of a gun-projectile system during service. In conventional models, its effect is often represented by empirical corrections to chamber volume or resistance parameters, without explicitly accounting for wear accumulation over successive shots. This study develops an interior ballistics model that incorporates cumulative chamber wear as a function of shot count and evaluates its effect on projectile motion in a rifled barrel. **Methods:** An interior ballistics model was developed by coupling cumulative chamber wear with the governing equations of propellant combustion, gas generation, and projectile motion. Projectile motion was considered in two stages: from cartridge-case separation to complete engraving of the driving band into the rifling, and from engraving to muzzle exit. Wear accumulation was introduced through changes in free-travel distance, effective chamber volume, and gas flow area. The model was applied to a 5.56 mm rifled system using 5.56×45 mm ammunition over a simulated service life of up to 16,000 shots. **Results:** Increasing shot count resulted in a progressive decrease in peak pressure and muzzle velocity, accompanied by a shift of the pressure peak toward the muzzle. At 16,000 shots, peak pressure and muzzle velocity decreased by approximately 15.6% and 5.1%, respectively, compared with the new-barrel condition. Experimental muzzle-velocity measurements agreed well with the numerical results, with relative deviations of 1.6% - 3.3% over the firing cycle. The axial chamber wear length measured after 16,000 shots was also used to validate the wear parameter adopted in the model. **Conclusion:** Explicit consideration of cumulative chamber wear provides a more consistent representation of interior ballistics degradation over barrel service life. The proposed model captures the changes in pressure and projectile velocity associated with wear and can be used to assess barrel performance and service life.

1. Introduction

Interior ballistics is a scientific discipline concerned with the fundamental laws governing the firing process, including the motion of the projectile within the barrel, the combustion behaviour of the propellant, and the variation of propellant gas pressure. This process is inherently a complex thermodynamic and gas-dynamic phenomenon that occurs over a very short time interval (on the order of a few milliseconds), during which temperature and pressure reach incredibly high levels. The primary objective of interior ballistics research is to design the propellant charge and barrel configuration such that the projectile exits the barrel with the required muzzle velocity while ensuring that the maximum chamber pressure does not exceed the structural strength and safety limits of the gun-projectile system.

The development of interior ballistics theory has progressed through several stages, ranging from classical analytical models to modern computational approaches with increasing levels of detail [1]. Classical theories with

analytical solutions are presented in [2]. Semi-empirical models, such as the zero-order model proposed by Corner [2] and the first-order model developed by Coppock [2, 3], were subsequently introduced. More recent studies have focused on numerical models, among which lumped parameter models (LPMs) are widely used [2, 4], along with quasi-one-dimensional models [5-8] and multidimensional (2D/3D) models [9-13]. Each model class has its own advantages and limitations and is selected based on the research objectives and the desired level of physical detail.

In practical operation, wear of the chamber and barrel is a significant factor that directly affects the interior ballistics characteristics of a gun-projectile system. This effect has been partially accounted for in several computational models through adjustments to coefficients or input parameters; however, the extent and methodology of these treatments vary considerably. Two representative modelling approaches that address the influence of axial chamber wear are classical models and lumped parameter models (LPMs).

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In these models, particularly in LPMs, the effect of chamber wear is typically represented indirectly by adjusting the initial chamber volume, increasing the projectile's effective mass to account for mechanical losses, or adjusting friction-related coefficients to compensate for model discrepancies arising from barrel wear.

Nevertheless, such approaches remain largely empirical and do not fully capture the physical nature of the wear process. Existing models generally do not establish governing equations that directly describe the influence of chamber wear, especially during the initial stage of projectile motion - from the moment the projectile separates from the cartridge case to the point at which the driving band is engraved and begins to engage with the rifling. In most studies, the effect of wear during this stage is incorporated only through changes in the initial chamber volume or equivalent resistance parameters, without explicitly accounting for the projectile's kinematics and dynamics in an axially worn chamber. In addition, the progressive wear of the barrel over successive shots has rarely been coupled directly with the governing differential equations of interior ballistics.

Consequently, most existing interior ballistics models account for barrel wear primarily through indirect adjustments to equivalent parameters. In contrast, the cumulative nature of wear over successive shots and its effect on the initial projectile motion, particularly during the stage from projectile separation from the cartridge case to the complete engraving of the driving band into the rifling, have not been explicitly described. Therefore, the development of an interior ballistics model capable of directly representing the effects of cumulative chamber wear as a function of shot count is necessary. Motivated by this requirement, the present study focuses on developing an interior ballistics model for a rifled barrel that explicitly accounts for cumulative barrel wear over repeated firings. In this model, the projectile motion during the initial stage is described directly, thereby improving the accuracy of predictions for interior ballistics characteristics of gun-projectile systems that have undergone service operation.

2. Materials and Methods

The interior ballistics model for a rifled barrel, accounting for cumulative barrel wear as a function of shot count, is developed within the classical interior ballistics framework and extended to describe the influence of chamber wear on projectile motion directly. The fundamental difference between a new barrel and a worn barrel is reflected in the geometric modification of the chamber and in the shift of the location at which the driving band begins to engage with the rifling.

The interior ballistics model incorporating axial chamber wear is schematically illustrated in Figure 1. In this configuration, the primary distinction between a worn barrel and an unworn barrel lies in the axial displacement of the initial driving-band engraving position. For a worn barrel, the location at which the driving band begins to be engraved into the rifling is shifted by a distance $\delta\lambda_2$ relative

to that of an unworn barrel. This implies that, after separating from the cartridge case, the projectile travels an additional free distance before reaching the initial engraving position. Accordingly, the projectile motion in a worn barrel is divided into two stages:

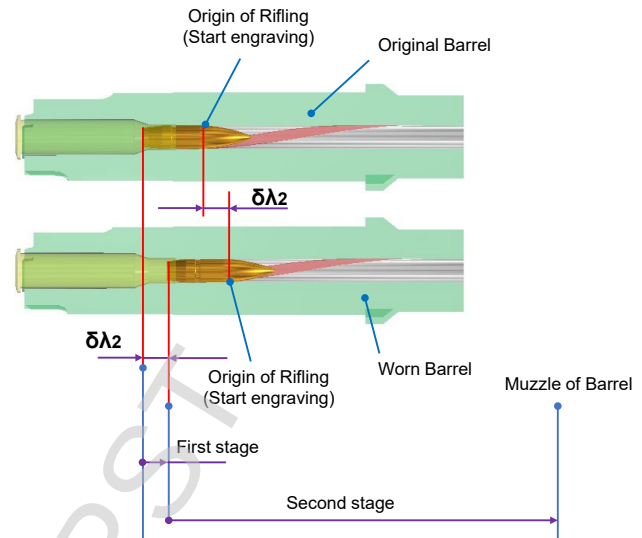


Figure 1. Stages of projectile motion within the barrel under barrel wear conditions

- Stage I: From the moment the projectile separates from the cartridge case to the complete engraving of the driving band into the rifling, including the free-travel segment induced by axial chamber wear.
- Stage II: From the end of Stage I until the projectile exits the muzzle.

2.1. Basic interior ballistics equations

The interior ballistics problem of a gun-projectile system with a worn barrel is formulated based on the fundamental interior ballistics equations, including the equations describing the propellant burning law, the gas generation law, the equation of state of the propellant gases, and the equation governing the motion of the projectile:

$$\begin{cases} \frac{dz}{dt} = \frac{p}{I_k} \\ \frac{d\psi}{dt} = \chi(1 + 2\lambda z + 3\mu z^2) \frac{dz}{dt} \\ Sp(l + l_\psi) = f\omega\psi - \frac{\theta}{2}\rho m v^2 \\ m \frac{d^2 l}{dt^2} = F(t) \end{cases} \quad (1)$$

Where: z - is the relative burned thickness of the propellant; I_k - is the total impulse of the propellant gas pressure; χ, λ, μ - denotes the shape characteristics of the propellant; S - is the barrel cross-sectional area; ψ - is the relative mass fraction of the burned propellant; m - is the projectile mass; v - is the projectile velocity; $F(t)$ - represents the forces acting on the projectile during its motion.

$$l_\psi = l_0 \left[1 - \frac{\Delta}{\delta} (1 - \psi) - \alpha \Delta \psi \right] \quad (2)$$

Where: $l_0 = \frac{W_0}{S} + \delta \lambda_1$ - is the equivalent length of the combustion chamber accounting for barrel wear; W_0 - is the combustion chamber volume; δ - is the propellant density.

The above system of equations is solved in two stages.

2.1.1. Projectile motion model in Stage I

Stage I begins when the propellant gas pressure is sufficient to separate the projectile from the cartridge case and ends when the driving band is entirely engraved into the rifling. Under conditions of axial barrel wear, the projectile first undergoes a free-travel distance $\delta \lambda_2$ before entering the engraving process.

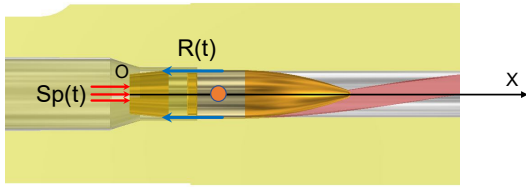


Figure 2. Model of projectile motion during the driving-band engraving process

During the engraving process, the projectile moves under the action of the propellant gas pressure and is subjected to the engraving resistance force, as illustrated in Figure 2.

The equation of motion of the projectile at this stage can be written as follows:

$$m \frac{d^2 l}{dt^2} = Sp(t) - R(t) \quad (3)$$

Where: m - is the projectile mass; $Sp(t)$ - is the force exerted by the propellant gases on the projectile; $R(t)$ - is the engraving resistance force; l - is the projectile displacement; $p(t)$ - is the pressure acting on the projectile base.

The model proposed by Tao Xue [13] is employed to determine the engraving resistance force. The force $R(t)$ is divided into two components: one associated with material friction F_R and the other related to the deformation of the driving band F_D . According to Ref. [13], the force $R(t)$ can be expressed as follows:

$$R(t) = F_R + F_D \quad (4)$$

$$\frac{d(\Delta d)}{dN} = A \cdot \frac{d \cdot C_E^2 \varphi (1 - \alpha \Delta)}{f C_q \left(1,74 + 2 \lg \left(\frac{d}{t_{rx}} \right) \right)^2} \cdot \frac{F(\chi, \Lambda_d)}{\left\{ 1 - thk_1 \left[\Delta t_n + \frac{T_{k0} - \Delta t_n}{1,74 + 4,1 \cdot d^{-0,36} \left(\frac{d^3}{\omega} \right)^{0,86}} \right] \right\}} \quad (8)$$

Where $F_R = D \frac{dl}{dt}$ $D = 2\xi m \omega_n$ are the resistance coefficients associated with material friction, and K is the hardness of the driving band, and $F_D = Kl$. Substituting these expressions and Equation (4) into Equation (3), the projectile equation of motion can be rewritten as follows:

$$m \frac{d^2 l}{dt^2} + D \frac{dl}{dt} + Kl = Sp(t) \quad (5)$$

For numerical implementation, Equation (5) can be rewritten as a system of first-order differential equations, with v denoting the projectile velocity:

$$\begin{cases} \frac{dl}{dt} = v \\ \frac{dv}{dt} = \frac{Sp - Kl - Dv}{m} \end{cases} \quad (6)$$

2.1.2. Projectile motion model in Stage II

Stage II extends from the end of Stage I until the projectile exits the barrel. The projectile motion during this stage is governed by the propellant gas pressure and the resisting forces acting on the projectile. The secondary work terms are assumed to be proportional to the work associated with the projectile translational motion and are accounted for through the secondary work coefficient φ . Accordingly, the following system of differential equations for the projectile motion is obtained:

$$\begin{cases} \frac{dl}{dt} = v \\ \frac{dv}{dt} = \frac{Sp}{\varphi m} \end{cases} \quad (7)$$

2.2. Modelling of cumulative barrel wear over successive shots

To describe the influence of cumulative barrel wear as a function of shot count, the barrel wear after each shot is determined based on the relationship between the specific energy of a shot and the wear resistance characteristics of the barrel material. This wear accumulates over successive shots and is converted into corresponding geometric changes in the barrel.

According to [14] and [15], the wear per shot is assumed to be proportional to the ratio of the specific energy to the wear resistance of the barrel material and is calculated as follows: :

where: N - is the number of shots; d - is the bore caliber; φ - is the secondary work coefficient; Δ - is the loading density; α - is the accumulation factor; f - is the propellant force; t_{rx} - is the rifling groove depth; T_{k0} - is the propellant gas combustion temperature; ω - is the propellant mass; $C_E = \frac{qv_0^2}{2gd^3}$ - is the firing power coefficient; q - is the projectile mass; v_0 - is the muzzle velocity; g - is the gravitational acceleration; A - is the fitting coefficient is determined according to the following expression:

$$A = \frac{K_r}{0,64K\rho K_s} \quad (9)$$

where: K_r - is the coefficient accounting for the barrel-wear increment during the subsequent action period of the propellant gases; K_ρ and K_s - is a coefficient determined experimentally from barrel-wear tests based on the processing of measured data; $C_q = \frac{q}{d^3}$ - is the projectile weight coefficient.

$F(\chi, \Lambda_d)$ is the following approximate expression is used:

$$F(\chi, \Lambda_d) = \frac{1}{\Lambda_d^2 \chi_H^3} \left[\frac{1}{12} \chi_c^2 - \frac{\frac{1}{12} + \frac{1}{3} \chi_c \Lambda_d + \frac{1}{2} \chi_c^2 \Lambda_d^2}{\chi_c^2 \left(\frac{1}{\chi_c} + \Lambda_d \right)^4} \right] \quad (10)$$

where: $\Lambda_d = \frac{l_d}{l_0}$ - is the relative travel distance of the projectile;

$\chi_H = \frac{1}{\frac{1}{\chi} + \frac{0,75d}{l_0}}$; $\frac{1}{\chi_c} = \frac{1 + \frac{1}{\chi}}{2}$; χ - is the chamber expansion coefficient.

To investigate the influence of barrel wear on the interior ballistics parameters, the barrel wear after one shot Δd is converted into three quantities:

- the elongation of the combustion chamber of the worn barrel relative to the new barrel, δl_1 ;
- the increase in the projectile travel distance before the completion of the driving-band engraving into the rifling up to the full groove depth, δl_2 ;
- the maximum flow area available for the propellant gases δs .

The relationship among these quantities is illustrated in Figure 3.

From the schematic in Figure 3, it can be observed that, by neglecting the term δl and approximating the wear profile along the rifling land by the straight line AB, the following approximate relationship can be written:

$$\delta l_1 = \Delta d \cdot \cot \beta \cdot \left(1 - \frac{D_0 - d}{\Delta d} \right) \quad (11)$$

$$\delta l_2 = \Delta d \cdot \cot \beta \quad (12)$$

$$\delta s = \frac{\pi d}{2} \cdot \Delta d \cdot \left(1 - \frac{D_0 - d}{\Delta d} \right) \quad (13)$$

Where: D_0 - is the diameter of the driving band; $\cot \beta$ - is the fitting coefficient, which can be determined by processing wear data from barrels of similar calibre, power level, and structural configuration.

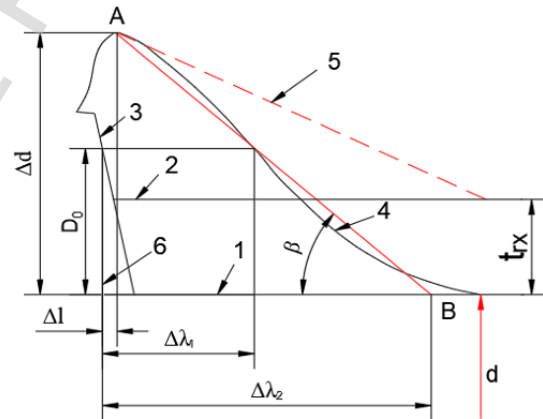


Figure 3. Schematic of a new barrel and a worn barrel 1 - Rifling land in a new barrel; 2 - Rifling groove bottom in a new barrel; 3 - Front chamfer surface in a new barrel; 4 - Rifling land in a worn barrel; 5 - Rifling groove bottom in a worn barrel; 6 - Undeformed projectile driving band.

2.3. General system of equations

By combining the equations describing propellant combustion, gas generation, the propellant-gas equation of state, and the projectile motion in the two stages, together with the equation modelling cumulative barrel wear over successive shots, the following general system of differential equations for the interior ballistics problem of a worn rifled barrel is obtained:

$$\begin{cases}
 \frac{dz}{dt} = s_1 \cdot \frac{p}{I_k} \\
 \frac{d\psi}{dt} = s_1 \cdot \chi(1 + 2\lambda z + 3\mu z^2) \frac{dz}{dt} \\
 Sp(l + l_\psi) = f\omega\psi - \frac{\theta}{2}\phi mv^2 \\
 \frac{dl}{dt} = v \\
 \frac{dv}{dt} = \frac{Sp - s_2 \cdot Kl - s_2 \cdot Dv}{s_2 \cdot m + (1 - s_2)\phi m} \\
 \frac{d(\Delta d)}{dN} = A \cdot \frac{d \cdot C_E^2 \phi(1 - \alpha\Delta)}{fC_q \left(1,74 + 2\lg\left(\frac{d}{t_{rx}}\right)\right)^2} \cdot \frac{F(\chi, \Lambda_d)}{\left\{1 - thk_1 \left[\frac{\Delta t_n + \frac{T_{k0} - \Delta t_n}{1,74 + 4,1 \cdot d^{-0,36} \left(\frac{d^3}{\omega}\right)^{0,86}}}{\left(\frac{d^3}{\omega}\right)^{0,86}} \right] \right\}}
 \end{cases} \quad (14)$$

where: s_1, s_2 - is the control coefficient, in Stage I and in the first period $s_1 = 1$, while in the second period $s_1 = 0$; when the driving-band engraving occurs, $s_2 = 1$.

The initial conditions are specified at the ignition of the propellant, namely: when $t = 0$ thì $v = 0, l = 0$, the projectile is at rest, $p = p_{moi}$ (the initial pressure is equal to the primer pressure), $z = 0$, the propellant combustion

$$\text{starts, } \psi_0 = \frac{\frac{1}{\Delta} - \frac{1}{\delta}}{\frac{f_{moi}}{p_{moi}} + \alpha - \frac{1}{\delta}}$$

Table 1. Input parameters

No.	Parameter name	Symbol	Unit	Value
1	Total impulse of propellant gas pressure	I_k	N·s/m ²	156,906
2	Propellant mass	ω_0	kg	0.0018
3	Propellant density	ρ	kg/m ³	1600
4	Propellant shape characteristics	χ	-	1.6
		λ	-	-0.6
		η	-	0
5	Propellant temperature	T	K	288.15
6	Propellant ignition temperature	T_v	K	2900
7	Ambient temperature	T_{mi}	K	288.15
8	Propellant force	f	J/kg	706,079
9	Adiabatic exponent of propellant gases	κ	-	1.26
10	Covolume	α	m ³ /kg	1.0×10^{-3}
11	Primer pressure	p_m	MPa	2.94
12	Projectile mass	m_d	kg	0.004
13	Shot-start pressure	p_0	MPa	29.42
14	Driving-band hardness	K	N/m	1.83×10^6
15	Material friction resistance coefficient	D	N·s/m	29.42

with f_{moi} - denoting the propellant force of the primer charge.

3. Results

3.1. Input parameters and development of the computational flowchart

A theoretical computational model is employed to investigate the interior ballistics of a rifled barrel, considering cumulative barrel wear as a function of shot count for a 5.56x45mm projectile system. The input parameters used for solving the problem are presented in Table 1.

No.	Parameter name	Symbol	Unit	Value
16	Initial chamber volume	W_0	m ³	1.85×10^{-6}
17	Bore caliber	d	m	5.56×10^{-3}
18	Barrel cross-sectional area	S	m ²	2.43×10^{-5}
19	Barrel length	l_d	m	0.361
20	Rifling twist angle	α	degree	6
21	Rifling groove depth	t_{rx}	m	3.0×10^{-4}
22	Driving-band diameter	D_0	m	5.86×10^{-3}
23	Driving-band width	H	m	7.0×10^{-3}
24	Friction coefficient	f_{ms}	-	0.25
25	Barrel wear coefficient	$\cot B$	-	1.5

To solve the general system of differential equations (14), a computational flowchart is developed, as shown in Figure 4. The calculations are performed using MATLAB, where the shot count N is gradually increased to simulate the evolution of barrel wear and its effects on the interior ballistics characteristics. For each value of N , the wear-dependent geometric parameters are updated before solving the differential equations, thereby capturing the cumulative

nature of wear over repeated firing cycles.

3.2. Results of Stage I (preliminary period)

Using the input parameters and the computational flowchart described above, and with the assistance of MATLAB, the variations in pressure and projectile velocity are obtained, as shown in Figures 5 and 6.

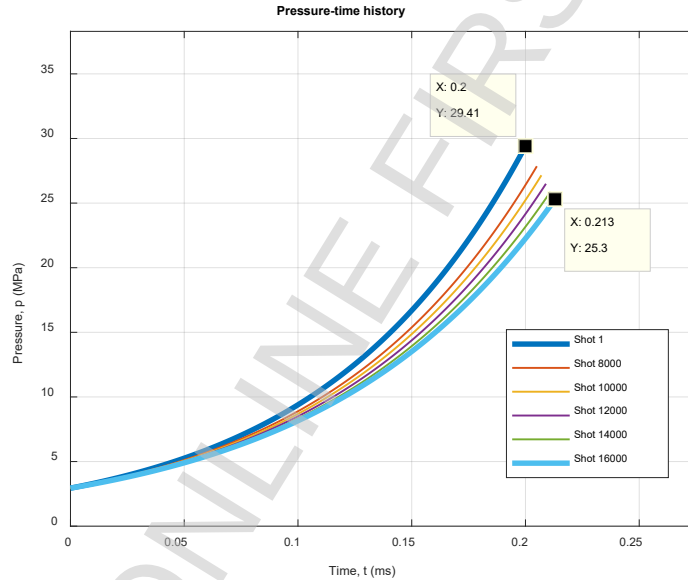


Figure 5. Chamber pressure histories in Stage I of firing for representative shots (1, 8000, 10000, 12000, 14000, and 16000) under bore wear effects

Figure 5 presents the pressure-time histories during Stage I of the firing process for representative shots (Shot 1, 8000, 10000, 12000, 14000, and 16000), accounting for the cumulative effects of barrel wear. The pressure values are analyzed at the end of Stage I, when the projectile has begun stable motion within the barrel. The results indicate that, for Shot 1, the pressure at the end of Stage I reaches approximately 29.4 MPa at 0.20 ms. As the number of shots increases to Shot 16000, the corresponding pressure decreases to about 25.3 MPa at 0.213 ms. Thus, after 16,000 shots, the pressure at the end of Stage I decreases by approximately 14% compared with the initial condition.

In addition to the reduction in amplitude, a shift of the end time of Stage I toward later times can be observed as the shot count increases. This time increases from approximately 0.20 ms for Shot 1 to about 0.213 ms for Shot

16000. This behaviour reflects changes in in-barrel dynamic conditions caused by wear, which reduce the propellant gases' ability to accumulate pressure during the early stage of firing.

Figure 6 illustrates the time histories of projectile velocity during Stage I for the representative shots. The velocity at the end of Stage I is used to assess the influence of barrel wear on the interior ballistics performance. For Shot 1, the projectile velocity at the end of Stage I reaches approximately 11.65 m/s at 0.20 ms. As the shot count increases to Shot 16000, the corresponding velocity decreases to about 11.26 m/s at 0.213 ms, corresponding to a reduction of 3.3%. This reduction indicates that barrel wear reduces the projectile's effective acceleration during the early stage of the firing process.

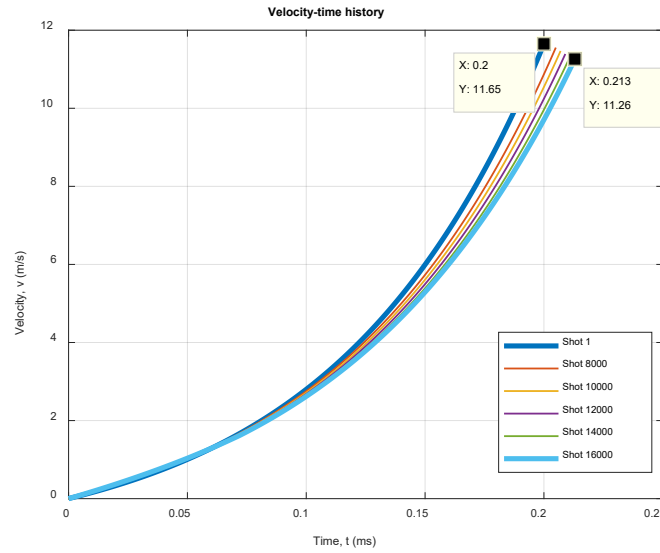


Figure 6. Projectile velocity histories in Stage I of firing for selected shots, showing the influence of bore wear accumulation

The results indicate that, during Stage I, barrel wear leads to a pronounced reduction in chamber pressure and projectile velocity at the end of this stage, while simultaneously extending the duration of Stage I. These changes reflect a degradation in the efficiency of interior ballistics due to cumulative barrel wear and provide an essential basis for evaluating barrel service life and the stability of ballistic performance during operation.

3.3. Results over the entire projectile travel (Stages I and II)

Figures 7 and 8 present the pressure–time and velocity–time histories over the entire projectile travel through the barrel, accounting for the cumulative effects of barrel wear with increasing shot count. The resulting curves form continuous bands, reflecting the systematic and gradual evolution of interior ballistics parameters as the number of fired projectiles increases. This behaviour is a direct consequence of the adopted fine-simulation resolution, with an increment of 100 fired projectiles between successive cases, leading to closely spaced curves rather than abrupt changes. The peak-pressure region (Figure 7) and the final stage of projectile motion (Figure 8) are of particular importance; therefore, additional zoomed-in views are provided to highlight the subtle but systematic differences between successive firing conditions.

From Figure 7, it can be observed that, as the shot count increases, the pressure amplitude over the entire firing process tends to decrease. This trend is clearly evident in the reduction of peak pressure and the overall lowering of the post-peak decay branch. At the same time, the pressure band shifts downward with increasing shot count, reflecting a reduction in the pressure-generation efficiency of the propellant gases within the barrel as cumulative wear develops.

Figure 8 shows that the projectile velocity throughout its travel is significantly affected by barrel wear, as evidenced by the downward shift in the velocity band with increasing shot count. Although the overall shape of the velocity curves is preserved, the velocity attained at any given time decreases progressively as the number of shots increases.

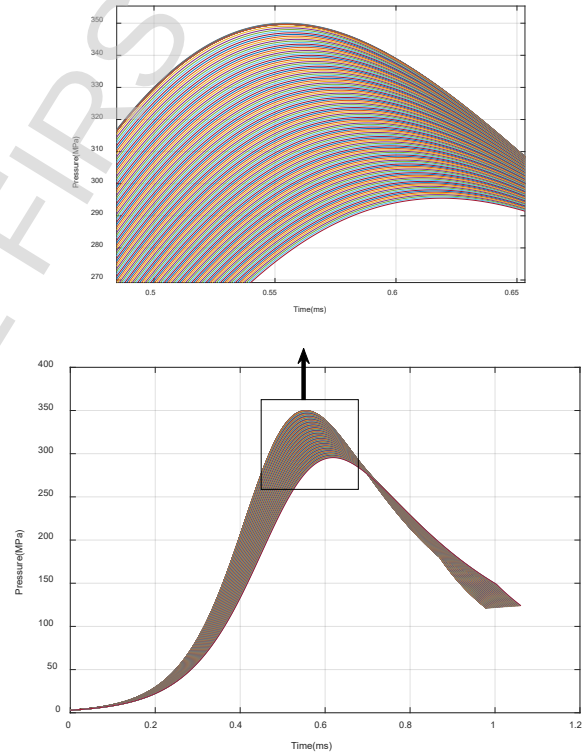


Figure 7. Pressure–time histories for successive firing conditions, calculated with a resolution of 100 fired projectiles between cases.

This trend is further clarified in Figure 9, which presents a contour map of the peak pressure as a function of shot count and the barrel wear parameter. The results show that the peak pressure decreases almost monotonically as the number of shots increases. At the same time, the influence of the wear parameter is evident in the variation in the pressure-reduction rate. This indicates that barrel wear does not act only locally at a specific instant, but affects the entire pressure evolution throughout the projectile's travel.

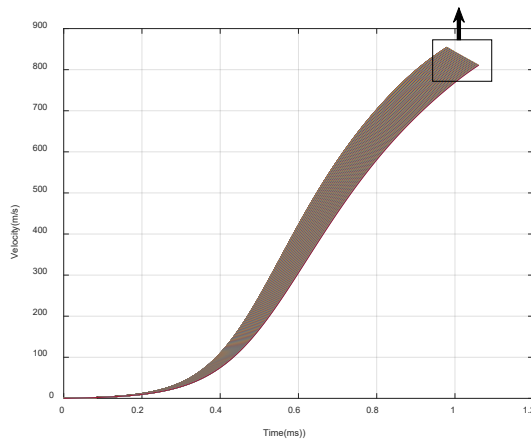
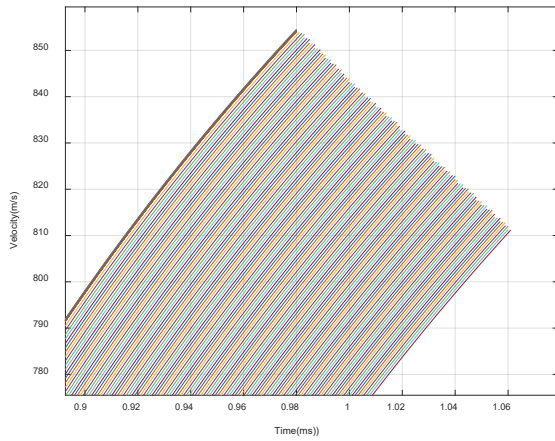


Figure 8. Velocity–time for successive firing conditions, calculated with a resolution of 100 fired projectiles between cases

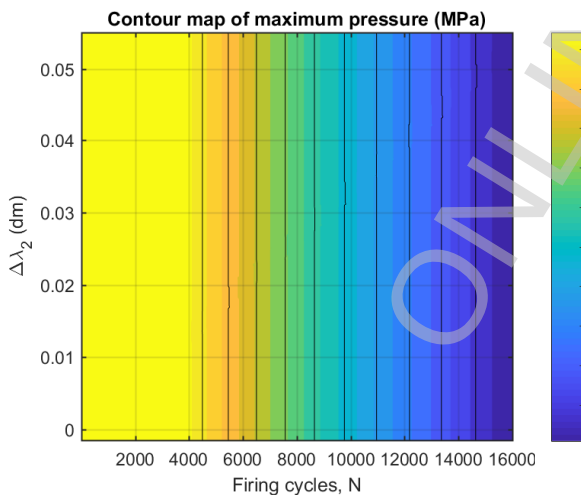


Figure 9. Contour map of the maximum chamber pressure versus firing cycles and bore wear parameter

This trend is summarised quantitatively in Figure 10, which presents a contour map of muzzle velocity as a function of shot count and the barrel wear parameter. Compared with the peak pressure, the muzzle velocity decreases more slowly, indicating that velocity is less sensitive to barrel wear. This behaviour is consistent with the physical nature of the interior ballistics process.

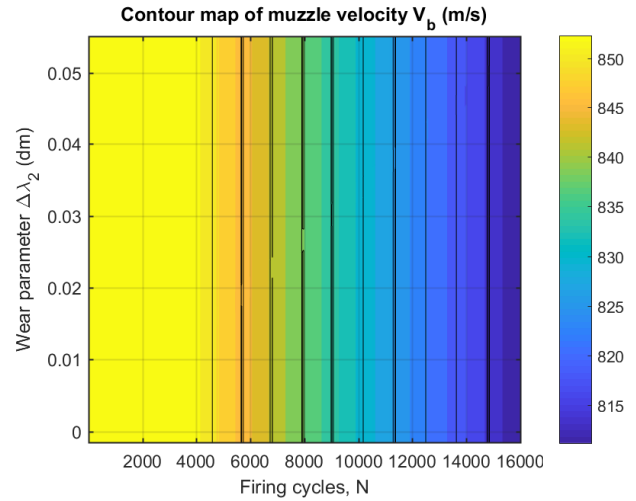


Figure 10. Contour map of the muzzle velocity versus firing cycles and bore wear parameter

The plots in Figures 7-10 provide a comprehensive and consistent view of the influence of barrel wear on the entire interior ballistics process, through variations in in-bore pressure and projectile velocity. Presenting the results as time histories combined with contour maps enables simultaneous assessment of both the dynamic evolution and the degradation trend associated with barrel service life. Figure 11 presents the pressure-time histories over the entire projectile travel for representative shots (Shot 1, 8000, 10000, 12000, 14000, and 16000), accounting for barrel wear. The pressure curves retain the typical shape of the interior ballistics process, including a rapid rise, attainment of a peak, and subsequent decay as the projectile approaches the muzzle.

For Shot 1, the peak pressure reaches approximately 350 MPa at 0.556 ms. As the shot count increases, the peak pressure decreases progressively, reaching about 295.5 MPa for Shot 16000, corresponding to a reduction of approximately 15.6%. At the same time, the time at which the peak pressure occurs tends to shift toward later values, as evidenced by the fact that the peak pressure time for Shot 16000 (at 0.617 ms) is later than that for Shot 1.

This pressure reduction trend is consistent with the general results presented in the previous section. It reflects the cumulative effect of barrel wear, which reduces the propellant gas's pressure-generation efficiency throughout the projectile's travel.

Figure 12 shows the time histories of projectile velocity for representative shots over the entire firing process. The velocity increases monotonically from the onset of projectile motion until muzzle exit, with the overall curve shape remaining relatively stable among different shots.

According to Table 2 and Figure 12, the muzzle velocity for Shot 1 reaches approximately 854.56 m/s, while this value decreases to about 811.19 m/s for Shot 16000, corresponding to a reduction of approximately 5.1%. Compared with the reduction in peak pressure, the decrease in muzzle velocity occurs more slowly. In addition, as with the pressure behaviour, the time to reach muzzle velocity shifts later as the shot count increases, reflecting a reduction in the projectile's effective acceleration during its motion due to changes in in-barrel operating conditions.

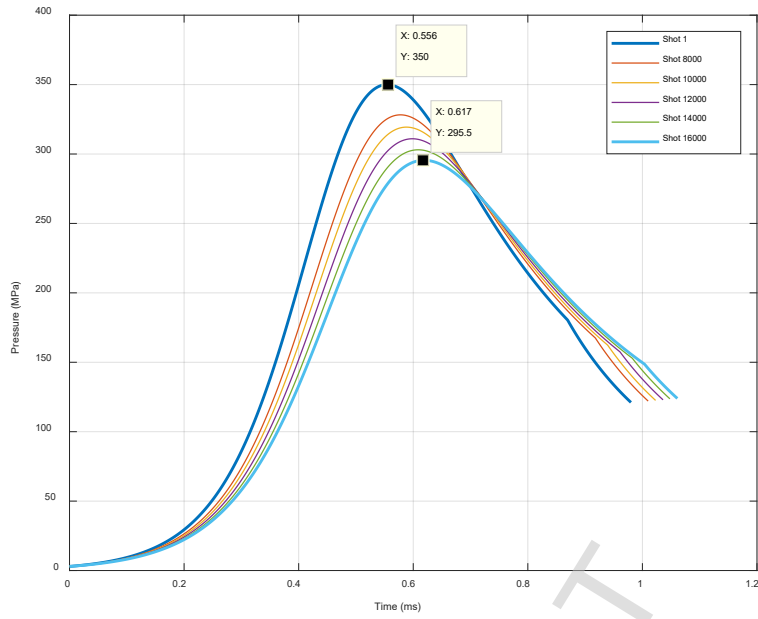


Figure 11. Pressure–time histories over the entire projectile travel for selected firing shots under bore wear effects

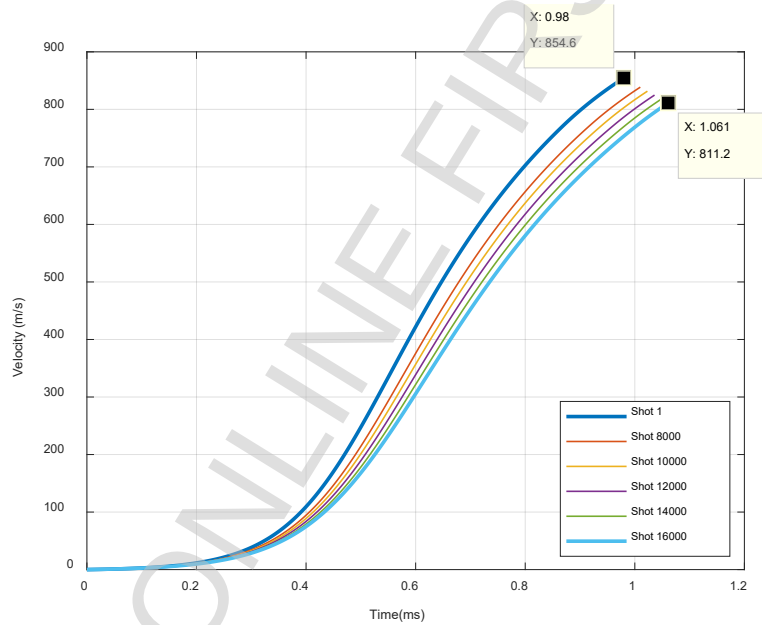


Figure 12. Velocity–time histories over the entire projectile travel for representative firing shots

Table 2. Results of the interior ballistics analysis for a rifled barrel, considering the effects of chamber wear

Shot number	1	8000	10000	12000	14000	16000
Peak pressure (MPa)	350.0	328.2	319.5	311.1	303.0	295.5
Muzzle velocity (m/s)	854.56	838.62	831.61	824.81	817.60	811.19
Additional free-travel length $\delta\lambda_1$ (cm)	0	0.14780	0.21476	0.28171	0.34866	0.41561
Total projectile travel increase $\delta\lambda_2$ (cm)	$3.35 \cdot 10^{-5}$	0.26780	0.33476	0.40171	0.46866	0.53561
Maximum gas flow area δs (cm ²)	0	0.0323	0.0469	0.0615	0.0769	0.0907

Table 2 summarises the main interior ballistics parameters of the representative shots, including the peak pressure, muzzle velocity, and the quantities characterizing chamber and barrel wear. The results show that, as wear-related parameters such as Δd , $\delta\lambda_1$, $\delta\lambda_2$, and δS increase progressively with the shot count, both the peak pressure and muzzle velocity decrease in a systematic manner.

Specifically, while the peak pressure decreases by approximately 15-16%, the muzzle velocity is reduced by only about 5%, reflecting the nonlinear relationship between these two quantities. This behaviour is consistent with the theoretical basis of interior ballistics.

From the above analyses, it can be concluded that barrel wear leads to a pronounced reduction in both peak pressure and muzzle velocity, while simultaneously altering the temporal characteristics of pressure and velocity development throughout the projectile's travel. The selected representative shots clearly capture the variation trends associated with barrel service life and provide a solid quantitative basis for evaluating the influence of barrel wear on the primary interior ballistics characteristics.

These results also indicate that using a limited number of representative shots is appropriate for describing the general behaviour of the firing process under barrel-wear conditions, while avoiding the unnecessary complexity of treating each shot separately.

4. Discussion

To evaluate the proposed model, after completing 16,000 firing cycles, the chamber length was measured using a coordinate measuring machine (CMM XO87). The measurement was carried out along the chamber axis, as illustrated in Figure 13. The initial chamber length of the new barrel was 49.20 mm, while the chamber length of the worn barrel after 16,000 shots reached 54.62 mm. The difference between these two values was defined as the axial chamber wear length, $\delta\lambda_2$, which increased by 5.42 mm. This experimentally measured $\delta\lambda_2$ was subsequently used to quantify the increase in projectile free-travel length induced by chamber wear and to support the ballistic performance analysis. The measured value $\delta\lambda_2$ provides direct experimental evidence for the axial chamber wear assumed in the numerical model.

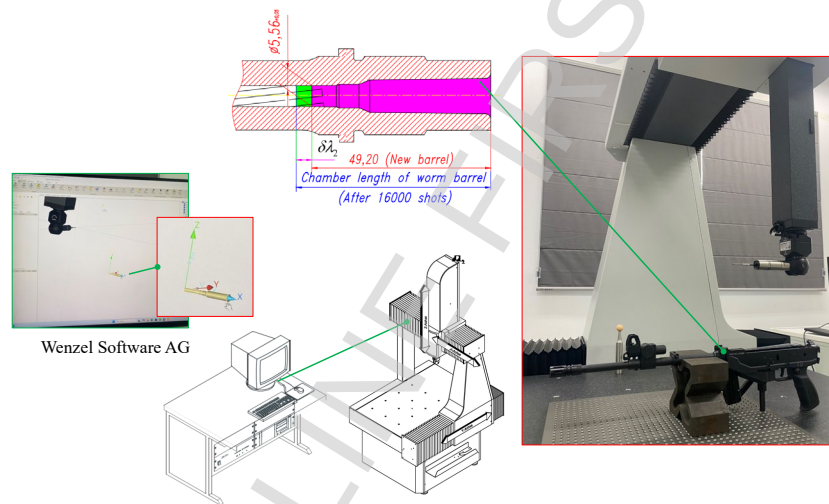


Figure 13. Determination of axial chamber wear length after 16,000 shots using a coordinate measuring machine (CMM XO87)

Experimental measurements of projectile muzzle velocity were conducted using a 5.56 mm system with a barrel length of 361 mm, firing 5.56x45 mm ammunition. The experiments were carried out by measuring the projectile velocity in several firing groups, including shots 1-10, shots 7,990-8,000, shots 9,990-10,000, shots 11,990-12,000, shots 13,990-14,000, and shots 15,990-16,000. The average value of the ten shots in each firing group was then calculated.

A schematic of the measurement setup is shown in Figure 14. The projectile velocity measurement system employed was an optical chronograph (Chronograph Model

36). The projectile velocity was measured at a distance of 5 m from the muzzle and subsequently converted to the muzzle velocity. The measured results and their comparison with the theoretical predictions for different shot counts are presented in Table 3 and Figures 15, 16 and 17.

To quantitatively evaluate the agreement between numerical predictions and experimental measurements, the relative deviation of muzzle velocity was calculated as:

$$\Delta v(\%) = \frac{|v_{num} - v_{exp}|}{v_{exp}} \times 100\% \quad (15)$$

Table 3. Comparison between experimental results and theoretical predictions

Firing cycles	Numerical velocity (m/s)	Experiment velocity (m/s)	Relative deviation (%)
1	854.56	840.6	1.66
8000	838.62	821.3	2.11
10000	831.61	805.8	3.2
12000	824.81	802.5	2.78
14000	817.597	800.3	2.16
16000	811.19	785.8	3.23

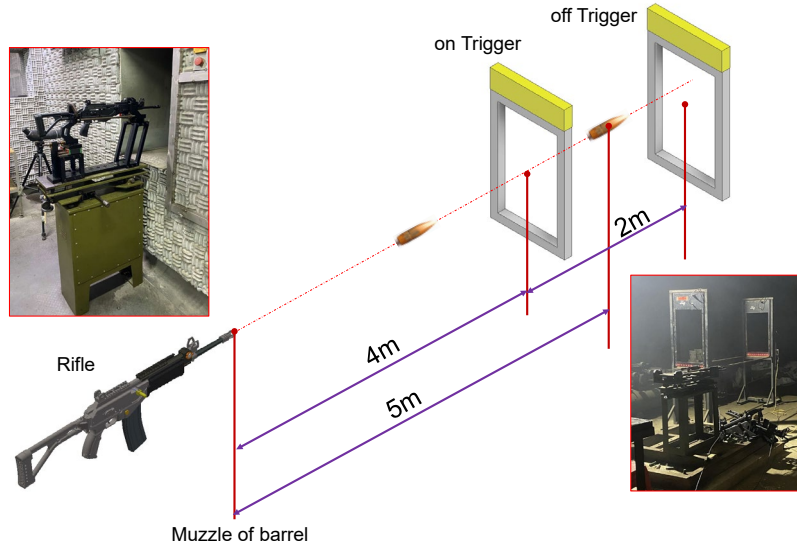


Figure 14. Experimental setup for projectile velocity measurement

Figure 15, Figure 16 and Figure 17, together with Table 3, demonstrate a good level of agreement between the numerical predictions and the experimental measurements of muzzle velocity over the entire range of shot counts considered. Both datasets exhibit a decreasing trend of muzzle velocity with increasing shot count, reflecting the cumulative effect of barrel wear on the interior ballistics process.

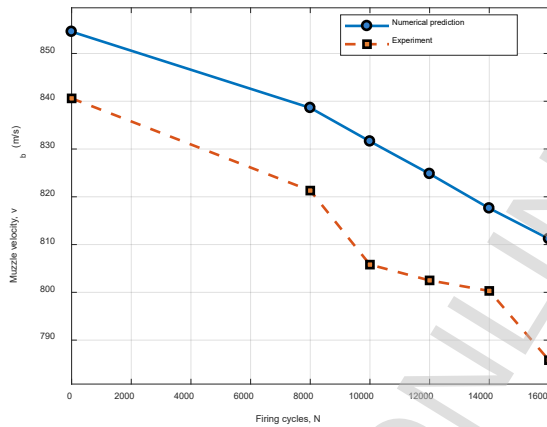


Figure 15. Numerical and experimental muzzle velocities versus firing cycles

From Figure 15, it can be observed that the calculated velocity values closely follow the measured data at all selected shot counts. The numerical results tend to be slightly higher than the experimental values; however, the discrepancies are small and consistent, with no abnormal deviations observed. This trend is consistent with the characteristics of one-dimensional interior ballistics models, which do not fully represent certain real losses and measurement uncertainties.

Figure 16 shows that the data points are clustered near the ideal line $y = x$, and all points lie within a $\pm 5\%$ deviation band, indicating good agreement between the numerical and experimental results. The absence of systematic deviations suggests that the model effectively captures the physical nature of the firing process within the scope of the present study.

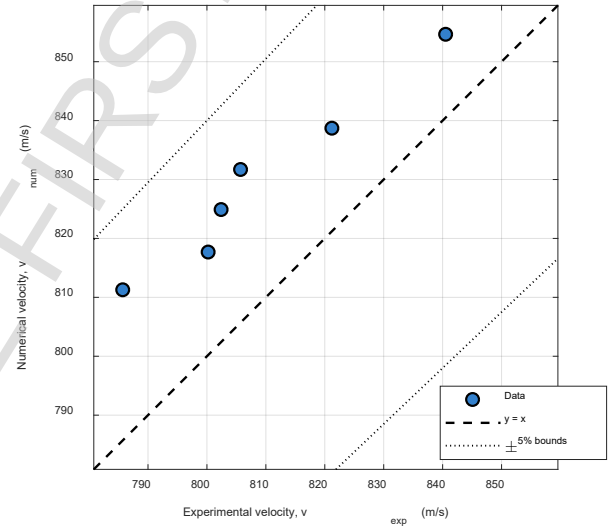


Figure 16. Parity plot of numerical versus experimental muzzle velocities ($\pm 5\%$ bounds)

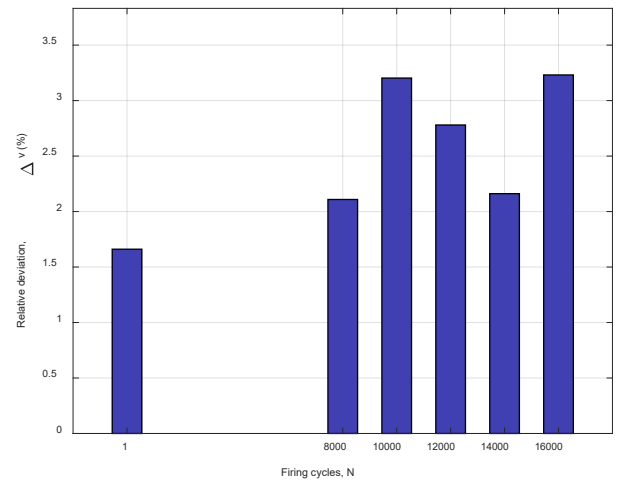


Figure 17. Relative deviation between numerical and experimental muzzle velocities

The quantitative results presented in Figure 17 and Table 3 indicate that the relative deviation between the numerical predictions and the experimental measurements ranges from approximately 1.6% to 3.3%, with an average value of about 2-3%. This level of deviation is considered acceptable for interior ballistics analyses, particularly when accounting for velocity measurement uncertainties, atmospheric conditions, the conversion of measured velocity to muzzle velocity, and the simplifying assumptions adopted in the model.

Overall, the comparison results indicate that the computational model accurately predicts both the trend and the magnitude of muzzle velocity as a function of shot count, thereby confirming its validity and reliability for evaluating the influence of barrel wear on interior ballistics characteristics.

5. Conclusion

This paper has developed an interior ballistics model for a rifled barrel that accounts for the cumulative effects of chamber wear as a function of shot count. Unlike conventional models, which typically account for barrel wear indirectly through adjustments of equivalent parameters, the proposed model explicitly describes the geometric changes in the combustion chamber and rifling caused by barrel wear after each shot and directly integrates these changes into the governing differential equations of interior ballistics.

A key distinction of the proposed model lies in its detailed description of projectile motion in two stages. The initial stage, from projectile separation from the cartridge case to the complete engraving of the driving band into the rifling, is modelled explicitly, including the free-travel distance induced by axial chamber wear. This approach enables a more accurate representation of projectile dynamics under worn-barrel conditions, particularly during the early phase of firing, which significantly influences pressure development and the initial projectile velocity.

The numerical results indicate that cumulative barrel wear over successive shots leads to a pronounced degradation of the primary interior ballistics characteristics. When the shot count increases to 16,000, the peak in-bore pressure decreases by approximately 15-16%, and the muzzle velocity decreases by about 5% compared with the new-barrel condition. Additionally, the times at which peak pressure and muzzle velocity occur tend to shift later. The slower reduction in muzzle velocity relative to peak pressure reflects the nonlinear relationship between the two and is consistent with the theoretical foundations of interior ballistics. The experimental measurement of the axial chamber wear length after 16,000 shots confirms the magnitude of cumulative chamber wear assumed in the model and supports the interpretation of the increased projectile free-travel length.

The theoretical model has been validated through experimental measurements of projectile muzzle velocity conducted on a 5.56 mm system, with the relative deviation between numerical predictions and experimental results ranging from 1.6% to 3.3% over the entire range of shot counts considered. This level of agreement demonstrates that the model is capable of reliably predicting both the trends and magnitudes of interior ballistics characteristics as a function

of barrel service life, while effectively capturing the underlying physical mechanisms of the firing process within the scope of the present study.

With its approach to modelling cumulative barrel wear over successive shots, the proposed model can serve as an effective tool for evaluating the influence of barrel wear on interior ballistics characteristics, supporting barrel design, operational assessment, performance adjustment, and predicting barrel service life. In future studies, the model may be extended to simultaneously account for radial wear, heat transfer through the barrel wall, and the effects of different firing regimes, thereby improving its accuracy and applicability.

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Modeliranje unutrašnje balistike olučene cevi sa kumulativnim efektima habanja

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Apstrakt

Uvod/Cilj: Habanje cevi progresivno menja unutrašnju balistiku sistema top-projektil tokom upotrebe. U konvencionalnim modelima, njegov efekat je često predstavljen empirijskim korekcijama zapremine komore ili parametara otpora, bez eksplicitnog uzimanja u obzir akumulacije habanja tokom uzastopnih pucnjeva. Ova studija razvija model unutrašnje balistike koji uključuje kumulativno habanje komore kao funkciju broja pucnjeva i procenjuje njegov uticaj na kretanje projektila u olučenoj cevi. **Metode:** Razvijen je model unutrašnje balistike povezivanjem kumulativnog habanja komore sa upravljajućim jednačinama sagorevanja pogonskog goriva, stvaranja gasa i kretanja projektila. Kretanje projektila je razmatrano u dve faze: od odvajanja čaure do potpunog urezivanja pogonskog traka u ožljeb i od urezivanja do izlaska iz cevi. Akumulacija habanja je uvedena kroz promene u slobodnom hodu, efektivnoj zapremini komore i površini protoka gasa. Model je primenjen na sistem sa ožljebom kalibra 5,56 mm koji koristi municiju 5,56×45 mm tokom simuliranog veka trajanja do 16.000 hitaca. **Rezultati:** Povećanje broja hitaca rezultiralo je progresivnim smanjenjem vršnog pritiska i brzine na ustima cevi, praćeno pomeranjem vrha pritiska ka ustima cevi. Pri 16.000 hitaca, vršni pritisak i brzina na ustima cevi su se smanjili za približno 15,6% i 5,1%, respektivno, u poređenju sa stanjem nove cevi. Eksperimentalna merenja brzine na ustima cevi su se dobro slagala sa numeričkim rezultatima, sa relativnim odstupanjima od 1,6% - 3,3% tokom ciklusa opaljivanja. Aksijalna dužina habanja komore merena nakon 16.000 hitaca je takođe korišćena za validaciju parametra habanja usvojenog u modelu. **Zaključak:** Eksplicitno razmatranje kumulativnog habanja komore pruža dosledniji prikaz degradacije unutrašnje balistike tokom veka trajanja cevi. Predloženi model obuhvata promene pritiska i brzine projektila povezane sa habanjem i može se koristiti za procenu performansi cevi i veka trajanja.

Originalni naučni rad,
mašinstvo

Ključne reči:

Unutrašnja balistika;
Habanje cevi;
Kumulativno habanje;
Olučena cev;
Kretanje projektila;
Vek trajanja.