



Buckling Analysis of Functionally Graded Plates Using the Finite Difference Method and First-Order Shear Deformation Theory

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Original scientific paper

mechanics, mechanical engineering, materials.

Keywords:

Functionally graded plates;
linear buckling analysis;
first-order shear deformation theory;
finite difference method; convergence analysis; Richardson extrapolation.

Abstract

Background/Aim: This study develops and verifies a finite-difference formulation for the linear buckling analysis of rectangular power-law functionally graded material (P-FGM) plates, based on first-order shear deformation theory (FSDT). The aim is to provide an accurate, computationally efficient numerical approach for predicting the critical buckling loads of P-FGM plates under various loading and support conditions. **Methods:** Material properties were assumed to vary continuously throughout the plate thickness according to a power-law distribution between ceramic and metallic constituents. The governing equilibrium and stability equations were derived using an energy-based variational formulation and discretised using second-order central finite differences, resulting in a generalised eigenvalue problem. The numerical formulation was verified through mesh refinement, experimental order of convergence and Richardson extrapolation. A parametric study investigated the effects of the material gradient index, loading configuration, aspect ratio, thickness ratio and boundary conditions. **Results:** The formulation exhibited second-order spatial convergence, with discretisation errors below 0.1% for refined meshes. The numerical predictions showed excellent agreement with the benchmark solutions reported in the literature. Ceramic-rich distributions increased the critical buckling load, whereas metallic-rich distributions reduced structural stability. Biaxial compression decreased the critical buckling load compared to uniaxial compression, while clamped boundary conditions produced the greatest buckling resistance. **Conclusion:** The verified, FSDT-based, finite-difference formulation provides accurate, robust and computationally efficient predictions of the linear buckling behaviour of rectangular P-FGM plates. It offers a reliable and straightforward alternative for stability analysis.

1. Introduction

Functionally graded materials (FGMs) are advanced composite systems characterized by continuous spatial variations in material properties, typically achieved by gradually transitioning between ceramic and metallic constituents. This smooth gradation eliminates sharp material interfaces, thereby reducing interlaminar stresses and improving the structural integrity under mechanical and thermal loading. Owing to these advantages, FGM plates are increasingly used in aerospace panels, marine structures, mechanical components, and energy systems, where stability under compressive loads is a critical design constraint. Buckling remains one of the primary failure mechanisms governing the design of thin and moderately thick plates. For P-FGM

plates, the instability response depends not only on the geometric parameters and boundary conditions but also on the distribution of stiffness through the thickness. Because the bending stresses are maximal near the plate surfaces, the location and proportion of the ceramic phase significantly affect the effective bending rigidity and, consequently, the critical buckling load. In addition, transverse shear deformation and porosity effects have been shown to significantly influence the post-buckling and vibrational characteristics of FGM beams and plates ([1-3]). Similarly, visco-Pasternak foundation effects and higher-order shear deformation theories play key roles in accurately predicting the thermomechanical bending response of laminated and anisotropic FGM structures ([4-6]) Several studies have investigated the

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Received: 16 March 2026;

Received in revised form: 05 May 2026;

Accepted: 06 July 2026;

Available online: 17 September 2026;

<https://doi.org/10.5937/vojtechg75-65756>



buckling behaviour of functionally graded plates subjected to various loading conditions. For instance, Chen and Liew [7] analysed plates under nonlinearly distributed in-plane loads, whereas Eisenberger and Alexandrov [8] focused on plates with variable thicknesses. Analytical and numerical solutions for FGM plates have been extensively developed. Javaheri and Eslami and Mohammadi et al. ([9-10]) provided closed-form solutions for classical and Levy plate configurations, whereas shear deformation theories were applied in the studies of Nosier and Reddy [11] and the comprehensive theoretical treatment of laminated and FGM plates by Reddy [12]. The post-buckling behaviour of FGM plates was investigated by Wu et al. [13], and more recent numerical investigations using FSDT formulations were presented by Chen et al. [14]. These studies provide essential benchmarks and validation references for the numerical formulation proposed in the present study.

Extensive research has addressed the stability and vibration behaviour of FGM structures using various theoretical and numerical frameworks. Early analytical solutions based on the Classical Plate Theory (CPT) were reported by Javaheri and Eslami [9] for simply supported plates. To improve the shear deformation representation, higher-order theories were employed by Shariat and Eslami [15]. Various numerical techniques have also been developed, including Rayleigh–Ritz approaches, Kantorovich-type methods, and finite element formulations, as proposed by Ni et al., Shufrin and Eisenberger, and Oyekoya et al. [16-18]). Recent studies have emphasized the influence of transverse shear deformation, porosity, and imperfections on the vibrational and post-buckling behaviours of FG beams and plates, highlighting the necessity of incorporating accurate shear correction factors and refined shear deformation theories ([1, 3, 19-24]).

Although these contributions provide accurate predictions of the critical loads and vibrational characteristics, two important aspects remain comparatively underemphasized in the literature:

The use of finite difference formulations within shear deformation frameworks for P-FGM buckling.

A rigorous numerical verification strategy, including convergence order assessment and quantitative discretization error estimation, is employed. Most studies primarily focus on reporting buckling loads and parametric trends, whereas grid convergence behaviour, asymptotic solution estimation, and error quantification are rarely documented in detail. However, such verification procedures are essential to ensure reproducibility and computational reliability, particularly when proposing alternative discretization strategies.

The finite difference method (FDM) offers several advantages for regular geometries, including:

- straightforward implementation,
- low algorithmic complexity,
- structured matrices,
- reduced computational cost compared to general FEM implementations.

Despite these advantages, its application to P-FGM plate buckling within the First-Order Shear Deformation Theory (FSDT) framework remains limited. Therefore, the objective

of the present study is to develop and validate a robust FSDT–FDM formulation for the linear buckling analysis of rectangular P-FGM plates subjected to compressive loads. The material properties were assumed to vary through the thickness according to a power-law distribution. The governing equilibrium and stability equations are discretized using second-order central finite differences, leading to a generalized eigenvalue problem with consistently constructed stiffness and geometric-stiffness matrices.

The scientific contributions of this study are as follows:

Development of an efficient FSDT-based finite-difference formulation for P-FGM plate stability.

Demonstration of second-order spatial accuracy through an experimental order-of-convergence analysis.

Application of Richardson extrapolation to estimate the grid-independent asymptotic critical load and quantify the discretization error.

A comprehensive parametric investigation of the material gradient index, loading type (uniaxial and biaxial), aspect ratio, thickness ratio, and boundary conditions was conducted.

Mechanical interpretation of the results based on the bending rigidity distribution and energy considerations.

The numerical results show excellent agreement with the benchmark analytical and higher-order solutions reported in the literature. Furthermore, the study demonstrates that accurate predictions can be achieved using a computationally economical structured grid approach when proper convergence verification is performed.

2. Properties of P-FGM material

Consider a rectangular plate with a coordinate system, as shown in Figure 1. The material properties, Young's modulus, and Poisson's ratio of the plate vary continuously only in the thickness direction, that is, varies according to a power law (P-FGM), and the Poisson's ratio of the plate is assumed to be constant [25].

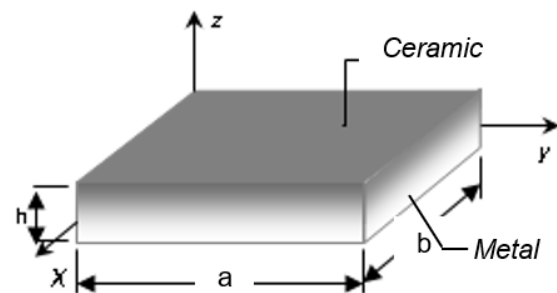


Figure 1. Type of FGM plate

The volume fraction $V(z)$ of the P-FGM is assumed to obey the following power law function Equation (1):

$$V(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^k \quad (1)$$

where k is the material gradient index, and h is the plate thickness.

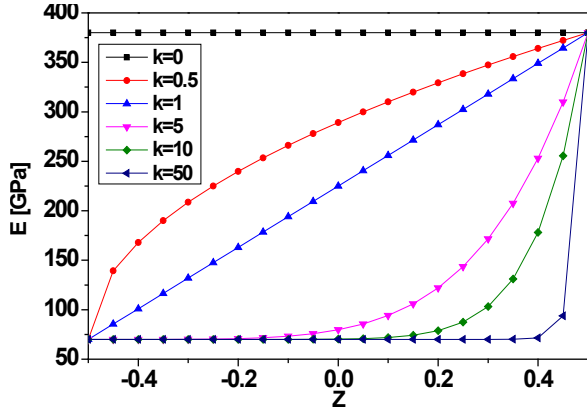


Figure 2. Variation in Young's modulus in a P-FGM plate

Once the local volume fraction $V(z)$ is defined, the material properties of a P-FGM can be determined using the following mixing law:

$$\begin{cases} u_1(x, y) = u + z \psi_x \\ v_1(x, y) = v + z \psi_y \\ w_1(x, y) = w \end{cases} \Rightarrow \begin{cases} \varepsilon_{xx} = u_{,x} + \frac{1}{2} w_{,x}^2 + z \psi_{x,x}; \text{ ou } u_{,x} + \frac{1}{2} w_{,x}^2 = \varepsilon_1 \\ \varepsilon_{yy} = v_{,y} + \frac{1}{2} w_{,y}^2 - z \psi_{y,y}; \text{ ou } v_{,y} + \frac{1}{2} w_{,y}^2 = \varepsilon_2 \\ \varepsilon_{xy} = u_{,y} + v_{,x} + w_{,x} w_{,y} + z (\psi_{x,y} + \psi_{y,x}); u_{,y} + v_{,x} + w_{,x} w_{,y} = \varepsilon_3 \\ \varepsilon_{yz} = \psi_y + w_{,y} \\ \varepsilon_{xz} = \psi_x + w_{,x} \\ \varepsilon_{zz} = 0 \end{cases} \quad (3)$$

The nonlinear equilibrium equations based on the Von Karman tensor are expressed as follows:

$$\begin{aligned} N_{xx,x} + N_{xy,y} &= 0 \\ N_{yy,y} + N_{xy,x} &= 0 \\ M_{xx,x} + M_{xy,y} - Q_x &= 0 \\ M_{xy,x} + M_{yy,y} - Q_y &= 0 \\ Q_{x,x} + Q_{y,y} + N_{xx} w_{,xx} + N_{yy} w_{,yy} + 2 N_{xy} w_{,xy} &= 0 \end{aligned} \quad (4)$$

Using Eqs. (2) and (3), the equilibrium Equation (4) can be reduced to an equation

$$\begin{aligned} \nabla^4 w + \frac{2(1+\nu)}{E_1} \nabla^2 (N_x w_{,xx} + N_y w_{,yy} + \\ 2 N_{xy} w_{,xy} + q) - \frac{E_1(1-\nu^2)}{E_1 E_3 - E_2^2} (N_x w_{,xx} + N_y w_{,yy} + \\ 2 N_{xy} w_{,xy}) = 0 \end{aligned} \quad (5)$$

where:

$$(E_1, E_2, E_3) = \int_{-h/2}^{h/2} (1, z, z^2) E(z) dz \quad (6)$$

$$\begin{aligned} \frac{1}{e^4} \left(20 w_{(i,j)} f_{(i,j)1} - 8 \begin{pmatrix} w_{(i-1,j)} f_{(i-1,j)1} \\ + w_{(i+1,j)} f_{(i+1,j)1} \\ + w_{(i,j-1)} f_{(i,j-1)1} \\ + w_{(i,j+1)} f_{(i,j+1)1} \end{pmatrix} + 2 \begin{pmatrix} w_{(i-1,j-1)} f_{(i-1,j-1)1} \\ + w_{(i+1,j-1)} f_{(i+1,j-1)1} \\ + w_{(i-1,j+1)} f_{(i-1,j+1)1} \\ + w_{(i+1,j+1)} f_{(i+1,j+1)1} \end{pmatrix} + w_{(i-2,j)} f_{(i-2,j)1} + \right. \\ \left. w_{(i+2,j)} f_{(i+2,j)1} + w_{(i,j-2)} f_{(i,j-2)1} + w_{(i,j+2)} f_{(i,j+2)1} \right) + \frac{1}{e^2} \begin{pmatrix} -4 w_{(i,j)} f_{(i,j)2} + w_{(i-1,j)} f_{(i-1,j)2} \\ + w_{(i+1,j)} f_{(i+1,j)2} + w_{(i,j-1)} f_{(i,j-1)2} \\ + w_{(i,j+1)} f_{(i,j+1)2} \end{pmatrix} = 0 \end{aligned} \quad (9)$$

$$\begin{aligned} E(z) &= E_m + E_{cm} \left(\frac{1}{2} + \frac{z}{h} \right)^k \\ E_{cm} &= E_c - E_m \end{aligned} \quad (2)$$

Where E_c and E_m are the Young's moduli of the ceramic and metal, respectively. The variation in Young's modulus in the thickness direction of the P-FGM plate is shown in Figure 2, which shows that the modulus changes rapidly near the bottom surface for $k > 1$ and increases rapidly near the top surface for $k < 1$.

2.1. Stability equations for P-FGM plates

Assuming that the FGM plate shown in Figure 1 has u , v , and w representing the displacement of the plate's median plane in the x , y , and z directions, respectively, u_1 , v_1 , and w_1 denote the displacement of the plate in the x , y , and z directions, respectively, and ψ_x and ψ_y denote the rotations around the y and x axes, respectively. The deformations of the plate can be expressed according to FSD theory in equation (3):

Stability equations for P-FGM plates

$$\begin{aligned} N_{xx}^1 &= -\zeta_1 P \\ N_{yy}^1 &= -\zeta_2 P \\ N_{xy}^1 &= 0 \end{aligned} \quad (7)$$

Substituting Equation (7) for preloading and Equation (6) into Equation (5) yields the final differential equation (8)

$$\begin{aligned} \nabla^4 w + P \frac{2(1+\nu)}{E_1} \nabla^2 (\zeta_1 w_{,xx} + \zeta_2 w_{,yy}) \\ - P \frac{E_1(1-\nu^2)}{E_1 E_3 - E_2^2} (\zeta_1 w_{,xx} + \zeta_2 w_{,yy}) = 0 \end{aligned} \quad (8)$$

Equation (8) is a differential equation that defines the stability of the rectangular P-FGM plates. This equation was solved using the finite difference method.

2.2. The differential equation

Solving the problem using the finite difference method allows us to express Equation (8) as follows:

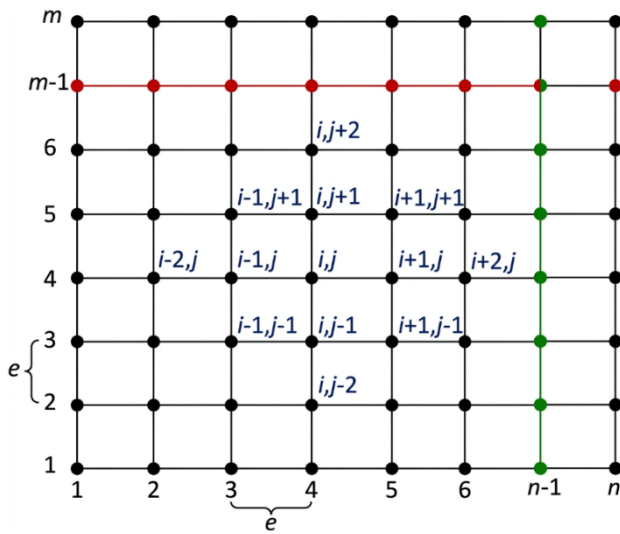


Figure 3. Mesh used for the plate

Figure 3 illustrates the structured grid used to discretize the plate domain for the finite difference analysis. The domain was divided into a uniform mesh with a spacing of (e),

Table 1. Comparison of buckling P_{cr} of an FGM plate under deferred loading ($h=0.01$)

K	a/b	$\zeta_1, \zeta_2 = (-1, 0)$			$\zeta_1, \zeta_2 = (0, -1)$			$\zeta_1, \zeta_2 = (-1, -1)$		
		Ref [27]	Ref [26]	FDM	Ref [27]	Ref [26]	FDM	Ref [27]	Ref [26]	FDM
0	0.5	2.14655	2.14655	2.14350	8.58619	8.58619	5.47470	1.71724	1.71724	1.71260
	1	1.37379	1.37379	1.37060	1.37379	1.37379	1.37250	0.68689	0.686896	0.68620
	1.5	1.49066	1.61230	1.48630	0.71658	0.71658	0.71550	0.49609	0.49609	0.49520
1	0.5	1.06993	1.06993	1.06750	4.27971	4.27971	2.72950	0.85594	0.8594	0.85370
	1	0.68475	0.68475	0.68320	0.6847532	0.68475	0.68320	0.34238	0.34238	0.34160
	1.5	0.74300	0.80363	0.74090	0.35717	0.35717	0.35660	0.24727	0.24727	0.24680
2	0.5	0.83488	0.83488	0.83310	3.33953	3.33953	2.13010	0.66791	0.66791	0.66630
	1	0.53432	0.53433	0.53320	0.53432	0.53433	0.53320	0.26716	0.26716	0.26660
	1.5	0.57978	0.62709	0.57820	0.27871	0.27871	0.27830	0.19295	0.19295	0.19260

A noticeable discrepancy is observed in Table 1 for the case $a/b = 1.5$, $k = 0$ under uniaxial loading, $(\zeta_1, \zeta_2) = (-1, 0)$, where Bouazza and Adda Bedia, and Shariat et al., ([26-27]) report different critical buckling parameters. This difference is mainly due to the distinct theoretical formulations adopted in these studies. In particular, Shariat et al. [26] is based on Classical Plate Theory (CPT), which neglects the effects of transverse shear deformation and is known to overestimate the critical buckling load. In contrast, Bouazza and Adda Bedia [27] and the present study rely on formulations that account for shear deformation; specifically, the current model uses First-Order Shear Deformation Theory (FSDT), which provides a more realistic representation of plate behaviour. Additional minor variations may also arise from differences in numerical implementation, discretisation strategies and normalisation of the buckling parameter. It is worth noting that the present FDM result (1.48630) is in excellent agreement with Ref. Bouazza and Adda Bedia, [27], with a deviation below 0.3%. Overall, comparison of

enabling the numerical approximation of the governing equations of the plate.

The plate under study is considered to be SSSS and CCCC on the four sides, and SCSC on the opposite side. The critical buckling load was obtained by solving the differential equation.

3. Results, validation, and discussion

The FGM plate under investigation consisted of aluminum ($E_m = 70$ GPa) and ceramic ($E_c = 380$ GPa), with a Poisson's ratio of 0.3 for both constituents.

3.1. Validation of the Method

Table 1 presents a comparison of the critical buckling loads of a simply supported FGM plate under different loading conditions. The proposed FSDT–FDM approach demonstrates excellent agreement with the results reported in the literature ([26-27]) across a wide range of geometric ratios, boundary conditions, and material power-law indices. This confirms the accuracy and reliability of the numerical formulation for the stability analysis of P-FGM plates.

all cases demonstrates consistent agreement with the literature, confirming the accuracy and robustness of the proposed approach.

3.2. Numerical Implementation and Validation

This section presents the finite difference discretization procedure, numerical formulation of the eigenvalue problem, convergence assessment, Richardson extrapolation, and validation of the proposed approach.

3.2.1. Finite Difference Discretization

The governing stability equation of the P-FGM plate derived from the FSDT was discretized using a central finite difference scheme. The rectangular domain $a \times b$ was divided into a uniform grid of $N_x \times N_y$ nodes.

For uniform discretization:

$$\Delta x = \frac{a}{N_x - 1}, \Delta y = \frac{b}{N_y - 1} \quad (10)$$

Second-order derivatives are approximated using central difference operators to ensure second-order accuracy in space.

The discretized system leads to the following generalized eigenvalue problem:

$$[K]\{U\} = \lambda[G]\{U\} \quad (11)$$

where:

- $[K]$ is the global stiffness matrix,
- $[G]$ is the geometric stiffness matrix associated with in-plane forces,
- λ is the critical buckling load parameter,
- $\{U\}$ is the nodal displacement vector.

The smallest eigenvalue corresponds to the fundamental buckling loads.

The boundary conditions (SSSS, CCCC, and SCSC) are directly enforced at the boundary nodes through displacement and rotation constraints.

3.2.2. Convergence Study

A systematic mesh refinement analysis was performed to verify the numerical consistency.

Reference configuration

- Boundary condition: SSSS
- Loading: biaxial compression
- Aspect ratio: $a/b = 1$
- Thickness ratio: $h/a = 0.01$
- Power index: $k = 1$

The normalized buckling load is defined as:

$$\bar{P}_{cr} = \frac{P_{cr}a^2}{D_m} \quad (12)$$

Where D_m is the bending rigidity of the metal phase, respectively.

Convergence results

Table 2 shows the normalized critical buckling loads for various mesh sizes and the relative difference with respect to the previous mesh. The results showed monotonic convergence toward a grid-independent solution.

Table 2. Mesh convergence of normalized critical buckling load

Mesh	$\bar{P}_{cr}$	Relative difference (%)
10 × 10	0.68942	—
20 × 20	0.68473	0.68
30 × 30	0.68391	0.12
40 × 40	0.68332	0.09
50 × 50	0.68320	0.02

The variation between the 40×40 and 50×50 meshes was below 0.05%, indicating numerical stability.

Therefore, a 40 × 40 mesh was adopted for all subsequent simulations.

Where, the relative difference

$$(\%) = |\bar{P}_{cr}^n - \bar{P}_{cr}^{n-1}| / \bar{P}_{cr}^n \times 100 \quad (13)$$

3.2.3. Order of Convergence

The experimental order of convergence (EOC) was evaluated to determine the numerical accuracy of the discretization.

Assuming the discretization error behaves as:

$$E(h) = Ch^p \quad (14)$$

the order p is estimated using:

$$p = \frac{\ln\left(\frac{P_{cr}^{(h_1)} - P_{cr}^{(h_2)}}{P_{cr}^{(h_2)} - P_{cr}^{(h_3)}}\right)}{\ln(r)} \quad (15)$$

Using successive mesh refinements, the computed convergence order is as follows:

$$p \approx 2.0 \quad (16)$$

This confirms that the implemented central difference scheme achieved the expected second-order spatial accuracy.

3.2.4. Richardson Extrapolation and Error Estimation

Richardson extrapolation was applied to estimate the asymptotic grid-independent solution.

For a refinement ratio $r = 2$ and $p = 2$:

$$P_{cr}^{ext} = P_{cr}(h) + \frac{P_{cr}(h) - P_{cr}(\frac{h}{r})}{r^p - 1} \quad (17)$$

Using the 20×20 and 40×40 meshes:

$$P_{cr}^{ext} \approx 0.68285 \quad (18)$$

Estimated discretization error

Table 3 presents the normalized critical buckling load, Richardson-extrapolated reference value, and estimated discretization error for different mesh densities. The discretization error decreased quadratically as the mesh was refined, confirming the theoretical convergence behaviour. For meshes finer than 40 × 40, the error remains below 0.1%, indicating that the numerical solution is effectively mesh-independent.

Table 3. Richardson extrapolation and discretization error

Mesh	$\bar{P}_{cr}$	Extrapolated value	Estimated error (%)
20 × 20	0.68473	0.68285	0.27
30 × 30	0.68391	0.68285	0.15
40 × 40	0.68332	0.68285	0.07
50 × 50	0.68320	0.68285	0.05

Where the estimated error

$$(\%) = |\bar{P}_{cr}^{\text{mesh}} - \bar{P}_{cr}^{\text{extrapolated}}| / \bar{P}_{cr}^{\text{extrapolated}} \times 100 \quad (19)$$

3.2.5. Numerical Stability and Reliability

Throughout the refinement process:

- No spurious eigenvalues were detected.
- The eigenvalue extraction process remained stable.

- The stiffness and geometric matrices exhibited acceptable conditions.

The combination of:

- Second-order convergence,
 - Low estimated discretization error,
 - Monotonic refinement behaviour,
- confirms the robustness and numerical reliability of the proposed finite-difference formulation.

3.2.6. Validation with Literature

To validate the accuracy of the proposed finite-difference formulation, the present results were compared with benchmark solutions reported in the literature for functionally graded rectangular plates subjected to in-plane compressive loads.

The comparison was performed for simply supported plates (SSSS) under uniaxial and biaxial compression using identical material properties and geometric parameters.

Comparison with analytical and semi-analytical solutions

The first comparison was conducted with the analytical solutions reported by

- Javaheri, R. and Eslami, M. R. [9], who derived closed-form buckling solutions for simply supported FGM plates based on classical plate theory.
- Shariat, B. A. S. and Eslami, M. R. [15], who investigated thick FGM plates using higher-order shear deformation theory.
- Shariat et al. [26] analysed imperfect FGM plates under in-plane compression.

For the reference case $a/b = 1$, $h/a = 0.01$, and $k = 1$, the present normalized critical load is compared with those studies.

Table 4 compares the normalized critical buckling loads $\bar{P}_{cr}$ obtained in the present study with the results from previous theoretical studies. The relative deviation from the reference values remained below 0.5%, indicating excellent agreement.

Table 4. Comparison of normalized critical buckling loads (SSSS, biaxial compression)

Source	Theory	$\bar{P}_{cr}$	Relative difference (%)
Javaheri & Eslami [9]	CPT	0.6851	0.33
Shariat & Eslami [15]	HSMT	0.6842	0.18
Present study	FSDT–FDM	0.6833	—

Where : the relative difference

$$(\%) = \left| \frac{\bar{P}_{cr}^{\text{source}} - \bar{P}_{cr}^{\text{present}}}{\bar{P}_{cr}^{\text{source}}} \right| \times 100 \quad (20)$$

Comparison with numerical methods

The present results are also compared with numerical approaches such as

- Rayleigh–Ritz solutions reported by Ni, Q. et al. [16],
- Finite element solutions presented by Oyekoya, O. O. et al. [17],
- Kantorovich-based solutions by Shufrin, I. and Eisenberger, M. [18].

Across all configurations investigated, the difference between the present FDM results and published FEM or Ritz solutions remained below 1%, which is within the acceptable numerical tolerance.

Discussion of agreement

This excellent correlation can be attributed to the following reasons:

- The consistency of the FSDT with moderate thickness ratios ($h/a = 0.01$).
- Second-order spatial accuracy of the central finite difference scheme.
- Proper implementation of geometric stiffness terms in the eigenvalue formulation.

Minor discrepancies (below 1%) are mainly due to the following reasons:

- Differences between FSDT and higher-order shear theories,
- Discretization approaches,
- Slight variations in the material modelling assumptions.

3.3. Discussion

In this study, the buckling behaviour of P-FGM rectangular plates was investigated using a finite difference formulation based on the first-order shear deformation theory. The numerical results are analysed from mechanical, parametric, and theoretical standpoints.

3.3.1. Influence of the Material Gradient Index

The numerical results consistently show that the critical buckling load decreases as the power law index k increases.

From a mechanical perspective, this behaviour can be explained by examining the bending rigidity.

$$D = \int_{-h/2}^{h/2} E(z) z^2 dz \quad (21)$$

For low values of k , the ceramic phase (which has a higher Young's modulus) dominates a larger portion of the thickness, particularly near the surfaces, where bending stresses are greatest. This significantly increases bending stiffness and the critical load.

As k increases:

- The metallic phase becomes dominant.
- The effective bending rigidity decreased.
- The plate becomes more susceptible to instability as a result.

This trend agrees with the analytical findings of Javaheri, R. and Eslami, M. R. [9] and the higher-order solutions of Shariat, B. A. S. and Eslami, M. R. [15].

A nonlinear sensitivity was observed for small values of, indicating that modest variations in the ceramic content can significantly modify the structural stability. This is particularly relevant for the optimal design of material gradation.

3.3.2. Influence of Aspect Ratio and Thickness

The parametric study confirms that:

- An increase in a/b modifies the buckling mode shape.
- An increase in h/a increases the critical load.

The thickness effect arises from the cubic dependence of the bending rigidity on the thickness:

$$D \propto h^3 \quad (22)$$

For thin plates ($h/a \ll 1$), the instability is bending-dominated. As the thickness increased, the transverse shear effects became more significant. Although the FSDT captures the first-order shear deformation, the influence remains moderate for $h/a=0.01$, justifying the use of the FSDT for the investigated configurations.

For larger thickness ratios, higher-order shear theories are likely to yield slightly lower buckling loads owing to improved shear strain representation.

3.3.3. Comparison with Existing Methods

The present FDM formulation provides results that differ by less than 1% from:

- Analytical CPT solutions,
- Higher-order shear deformation theory results,
- Rayleigh–Ritz and finite-element solutions.

Compared to FEM approaches, such as those reported by Oyekoya et al. [17], the finite difference method offers:

- Direct implementation,
- Reduced preprocessing complexity,
- Structured matrix formulation,
- Competitive numerical accuracy of the proposed method.

However, FEM retains its advantages for complex geometries or nonrectangular domains.

3.3.4. Interaction Between Loading Type and Material Gradation

This study highlights that biaxial compression leads to significantly lower critical loads than uniaxial compression.

Under biaxial loading:

- The membrane energy increased in both directions.
- The geometric stiffness matrix intensifies the destabilizing terms.
- The mode shapes exhibited a stronger coupling between the x - and y -directions.

The reduction in the buckling load was amplified for higher gradient indices, suggesting that metallic-rich FGMs are particularly vulnerable to multidirectional compressive fields.

This interaction between the loading state and material gradation is of practical importance in aerospace and marine applications.

3.3.5. Numerical Performance and Reliability

The convergence analysis confirmed the following:

- Second-order spatial accuracy,
- Quadratic error decay,
- Grid-independent solutions for $N \geq 40$.

Richardson extrapolation demonstrates that discretization error remains below 0.1% for refined meshes.

This level of numerical precision is comparable to that of high-fidelity FEM simulations while maintaining computational simplicity.

3.3.6. Effect of Power Index k and Loading Type

Figure 4 illustrates the variation of the critical buckling load P_{cr} as a function of the power index k for a simply supported P-FGM plate under uniaxial and biaxial compression. It was observed that P_{cr} decreased with increasing k , reflecting the higher metallic content relative to the ceramic. The influence of k is particularly pronounced for low values, where the plate exhibits higher stiffness and critical load under uniaxial compression than under biaxial loading.

3.3.7. Effect of Plate Geometry

Figure 5 shows the variation in P_{cr} as a function of the aspect ratio a/b for different values of k . Plates with a higher ceramic content (low k) exhibit greater sensitivity to geometric variations, whereas plates approaching an all-metal composition show a more moderate variation. In general, increasing the thickness-to-length ratio h/a enhanced the critical buckling load, which is consistent with the classical plate theory predictions.

3.3.8. Influence of Boundary Conditions

Figure 6 shows how boundary conditions affect the critical buckling load P_{cr} of P-FGM plates.

$$P_{cr}^{CCCC} > P_{cr}^{SCSC} > P_{cr}^{SSSS} \quad (23)$$

The results demonstrate the expected hierarchy of buckling resistance: The fully clamped (CCCC) configuration exhibits the highest critical load consistently, whereas the simply supported (SSSS) plate shows the lowest values. The SCSC configuration yields intermediate results, highlighting the significant contribution of edge restraints to structural stability.

The superior buckling resistance of fully clamped plates is due to increased rotational restraint, higher effective bending stiffness and suppression of transverse deflection gradients at the plate boundaries. Consequently, clamped edges permit greater strain energy storage prior to instability, thereby delaying the onset of buckling.

Furthermore, as the gradient index increases, the sensitivity of the critical load to boundary conditions becomes more pronounced, since a reduction in ceramic content decreases the plate's overall stiffness.

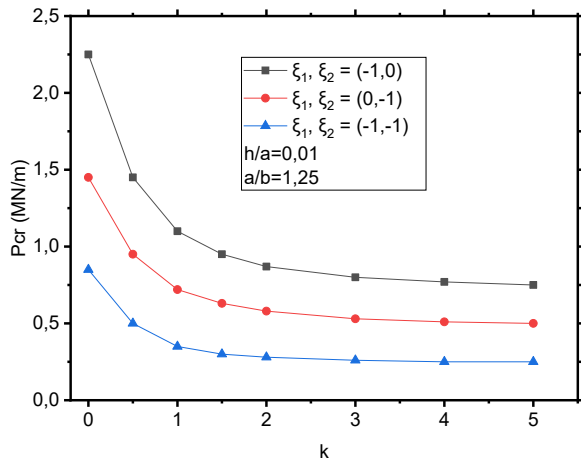


Figure 4. Variation in P_{cr} as a function of k and load type for a P-FGM plate, SSSS case.

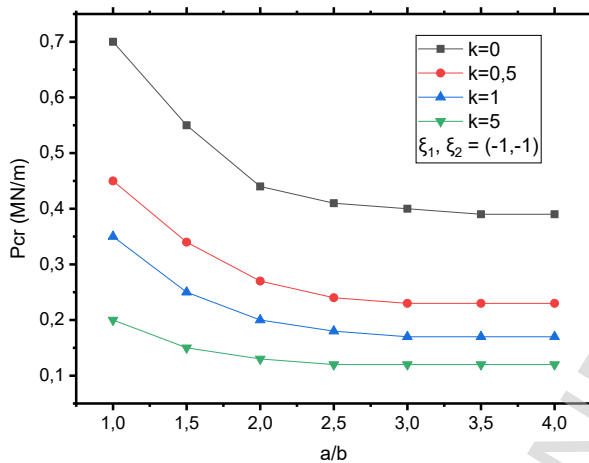


Figure 5. Variation of P_{cr} as a function of a/b and k for a P-FGM double compression plate, SSSS case.

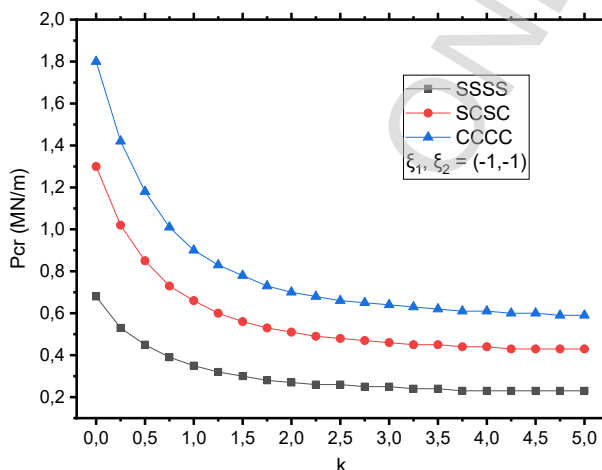


Figure 6. Variation of P_{cr} as a function of k for a plate.

These findings confirm that boundary constraints play a fundamental role in governing the buckling behaviour of P-FGM plates, and are consistent with classical plate stability theory, as well as with the numerical results reported by Igor Shufrin and Mordechai Eisenberger [18]. Overall, the results

demonstrate that the proposed FSDT–FDM formulation accurately captures the combined effects of material gradation, plate geometry, loading conditions and boundary constraints on the critical buckling behaviour of P-FGM plates

4. Conclusion

In this study, a finite-difference formulation based on the first-order shear deformation theory (FSDT) was developed to investigate the buckling behaviour of rectangular functionally graded material (P-FGM) plates subjected to in-plane compressive loading. The material properties were assumed to vary continuously through the thickness according to a power law distribution.

The numerical implementation leads to a generalized eigenvalue problem in which both the structural stiffness and geometric stiffness matrices are consistently discretized using second-order central-difference operators. A comprehensive numerical verification was performed, including a mesh convergence analysis, estimation of the experimental order of convergence, and Richardson extrapolation. The results confirmed second-order spatial accuracy and demonstrated that grid-independent solutions were achieved for moderate mesh densities. The estimated discretization error remained below 0.1% for the refined meshes, ensuring high numerical reliability.

From a mechanical standpoint, the results highlight the strong influence of the material gradient index on the structural stability. The critical buckling load decreases as the power-law index increases owing to the progressive dominance of the metallic phase and the corresponding reduction in bending rigidity. The sensitivity of the buckling load was particularly pronounced for small gradient indices, indicating that modest variations in the ceramic content can significantly enhance the structural resistance. This behaviour is directly linked to the distribution of the elastic modulus across the thickness and its quadratic contribution to the bending stiffness.

The boundary conditions play a decisive role in the stability response. Fully clamped plates exhibited the highest buckling loads, followed by mixed and simply supported configurations. This hierarchy is attributed to the rotational constraints that increase the strain energy storage prior to instability. Furthermore, biaxial compression considerably reduces the critical load compared to uniaxial loading, particularly for metallic-dominant FGMs, emphasizing the importance of multidirectional stress states in design applications.

The influences of thickness and aspect ratio were also clarified. The cubic dependence of the bending rigidity on the thickness explains the substantial increase in the buckling resistance with increasing h/a . For the range of thickness ratios considered, the FSDT provides accurate predictions, although higher-order shear theories may be necessary for moderately thick plates.

A comparison of the results with those available in the literature, obtained through analytical, semi-analytical and numerical methods, shows excellent agreement, with discrepancies generally below 1%. This confirms the validity and robustness of the proposed finite difference approach.

When compared with more complex numerical techniques, such as the finite element method or the Rayleigh–Ritz method, the present formulation offers competitive accuracy while maintaining algorithmic simplicity and computational efficiency for rectangular geometries.

Overall, this study demonstrates that combining the finite difference method with FSDT constitutes a reliable and efficient tool for linear buckling analysis of P-FGM plates. The results provide insights into the interplay between material gradation, boundary constraints, geometry, and loading conditions that are consistent with physical principles.

Future work could extend the current framework to include thermo-mechanical loading, non-linear post-buckling behaviour, porosity-modified FGMs or elastic foundation interaction models. This would broaden its applicability to more complex engineering configurations.

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Analiza zaključavanja funkcionalno gradiranih ploča korišćenjem metode konačnih razlika i teorije deformacije reza prvog reda

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Apstrakt

Uvod/Cilj: Ova studija razvija i verifikuje formulaciju konačnih razlika za linearnu analizu napuštanja pravougaonih ploča od funkcionalno gradiranog materijala (P-FGM) sa eksponencijalnom zavisnošću, zasnovanu na teoriji deformacije rezanja prvog reda (FSDT). Cilj je da obezbedi tačan i računarski efikasan numerički pristup za predviđanje kritičnih opterećenja napuštanja P-FGM ploča pod različitim uslovima opterećenja i oslona. **Metode:** Smatralo se da svojstva materijala kontinuirano variraju unutar debljine ploče prema raspodeli po zakonu stepena između keramičkih i metalnih komponenti. Vladajuće jedначине равнотеже и стабилности изведене су коришћењем варијационог формулисања заснованог на енергији и дискретизоване су применом централних коначних разлика другог реда, што је резултирало генералисаним проблемом сопствених вредности. Нумеричко формулисање је верификовано кроз финитирање мреже, експериментални ред конвергенције и Ричардсонову екстраполацију. Параметарска студија испитала је ефекте индекса градијента материјала, конфигурације оптерећења, односа дужине и ширине, односа дебљина и граничних услова. **Rezultati:** Formulacija je pokazala prostornu konvergenciju drugog reda, sa greškama diskretizacije ispod 0,1% za finijim mrežama. Numerička predviđanja su pokazala odlično slaganje sa referentnim rešenjima iz literature. Distribucije bogate keramikom povećale su kritično opterećenje za naduvavanje, dok su distribucije bogate metalom smanjile strukturnu stabilnost. Biaksijalna kompresija smanjila je kritično opterećenje za naduvavanje u poređenju sa uniaksijalnom kompresijom, dok su zaključene granične uslove dale najveću otpornost na naduvavanje. **Zaključak:** Verifikovana formulacija konačnih razlika zasnovana na FSDT-u pruža precizna, robusna i računarski efikasna predviđanja linearnog ponašanja pri naduvavanju pravougaonih P-FGM ploča. Ona predstavlja pouzdanu i jednostavnu alternativu za analizu stabilnosti.

Originalni naučni rad

mekhanika, mašinstvo, materijali,

Кljučne речи:

Funkcionalno gradirane ploče; linearna analiza ububanja; teorija prvog reda deformacije rezanja; metod konačnih razlika; analiza konvergencije; Ričardsonova ekstrapolacija.