



Bending and Buckling Analysis of Antisymmetric Laminated Composite Plates Using a Refined Exponential Shear Deformation Theory

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Abstract

Introduction/Purpose: This study analyzes the bending response and buckling behavior of antisymmetric laminated composite plates using a refined exponential shear deformation theory (RESDT). The research aims to evaluate dimensionless deflections under transverse loading and critical buckling loads under uniaxial compression while investigating the influence of orthotropic ratio (E_1/E_2), length-to-thickness ratio (a/h), aspect ratio (a/b), and stacking sequence of the laminate plies. **Methods:** The RESDT formulation employs four independent variables to model transverse shear deformation through an exponential function that inherently satisfies zero shear stress conditions at the plate surfaces, eliminating empirical shear correction factors. Simply supported square and rectangular plates with cross-ply (0/90) and angle-ply ($\pm 45^\circ$) configurations were analyzed. Results were validated against classical plate theory (CLPT), first-order shear deformation theory (FSDT), higher-order shear deformation theory (HSDT), and exact three-dimensional elasticity solutions. **Results:** Numerical results demonstrate excellent agreement between RESDT predictions and reference solutions across varying thicknesses and orthotropy levels. For thick plates ($a/h = 2-10$), transverse shear effects significantly increase dimensionless deflections by up to 15% compared to thin-plate theories. Buckling loads rise with increasing orthotropy degree and a/h ratio, with RESDT matching exact solutions within 1% error for highly orthotropic laminates ($E_1 = 40E_2$). **Conclusion:** The four-variable RESDT provides an accurate, computationally efficient framework for predicting mechanical responses of antisymmetric laminates without shear correction factors. Transverse shear deformation critically influences thick-plate behavior, while laminate stiffness improves with higher orthotropy ratios and balanced angle-ply configurations, offering valuable insights for composite structural design optimization.

1. Introduction

Composite materials have revolutionized structural engineering across aerospace, automotive, marine, and civil infrastructure sectors due to their exceptional strength-to-weight ratios, tailorable anisotropic properties, and superior durability [1]. The increasing utilization of laminated composites in critical load-bearing applications necessitates robust theoretical frameworks capable of accurately predicting their mechanical response under diverse loading conditions. However, the pronounced anisotropy and relatively low transverse shear stiffness inherent in composite laminates pose significant challenges for classical structural theories.

Classical Plate Theory (CLPT), founded on Kirchhoff's assumptions that normals to the mid-plane remain straight and perpendicular after deformation, neglects transverse shear deformation effects [2]. While adequate for thin plates (length-to-thickness ratio $a/h > 50$), CLPT yields substantial errors for moderately thick and thick laminates where shear deformation becomes dominant. This limitation manifests as overestimation of buckling loads and natural frequencies and underestimation of deflections, compromising structural safety in practical design scenarios [3, 4].

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To address these shortcomings, First-order Shear Deformation Theory (**FSDT**), commonly associated with the Reissner-Mindlin formulation, incorporates constant transverse shear strains through the plate thickness [5-7]. Although **FSDT** improves accuracy for thick plates, it introduces a critical inconsistency: the assumption of uniform shear strain distribution contradicts the physical condition of zero shear stress at the plate's top and bottom surfaces. Consequently, empirical shear correction factors (typically 5/6) must be applied to adjust strain energy—a non-physical remedy lacking universality across material systems and lamination [8-11].

Higher-order Shear Deformation Theories (**HSDT**) emerged to eliminate reliance on shear correction factors by employing parabolic or higher-order polynomial functions to model shear strain distribution [12, 13]. Reddy's third-order theory, for instance, satisfies zero shear stress conditions at plate surfaces while capturing nonlinear shear effects. Nevertheless, these formulations typically require five or more displacement variables, increasing computational complexity without proportional gains in accuracy for many practical applications [12].

A significant advancement occurred with Shimpi's Refined Plate Theory (**RPT**), which reduced the number of unknown variables to four while maintaining accuracy comparable to higher-order models [14]. **RPT** achieves this efficiency by decoupling bending and shear components within the displacement field, thereby simplifying governing equations and boundary conditions without sacrificing fidelity. Subsequent adaptations by Thai and Kim [15, 16], Hadji et al. [17], Li et al. [18], Abdelhak et al. [19] and Han et al. [20] extended **RPT** to thermal buckling, functionally graded materials, and sandwich structures, demonstrating its versatility and computational advantages.

Recent developments have focused on exponential shear deformation functions that inherently satisfy traction-free boundary conditions at plate surfaces [21, 22]. Unlike polynomial distributions, exponential functions provide a more physically consistent representation of shear stress variation through the thickness. Sehoul et al. [21] successfully applied exponential formulations via finite element methods to free vibration analysis of antisymmetric laminates, while Bendahane et al. [22] demonstrated excellent agreement with three-dimensional elasticity solutions using only four independent variables.

Despite these advances, a comprehensive investigation of bending and buckling responses in antisymmetric laminates using a unified exponential framework remains limited in the literature. Antisymmetric configurations—both cross-ply (0/90) and angle-ply ($\pm 45^\circ$)—exhibit complex coupling effects between bending and extension that significantly influence structural performance [7]. Furthermore, the combined effects of orthotropy ratio (E_1/E_2), length-to-thickness ratio (a/h), aspect ratio (a/b), and stacking sequence on critical buckling loads and deflection patterns warrant systematic examination, particularly for highly orthotropic materials ($E_1 = 40E_2$) where shear deformation effects become pronounced [23, 24].

This study addresses these gaps by developing and validating a Refined Exponential Shear Deformation Theory (**RESDT**) for antisymmetric laminated composite plates. The proposed formulation employs an exponential function to model transverse shear deformation, inherently satisfying zero shear stress conditions at the plate surfaces without empirical correction factors. Utilizing only four displacement variables—reducing computational cost relative to **FSDT** and **HSDT**—the theory maintains high accuracy across the full spectrum of plate thicknesses. The research objectives are threefold:

- to derive equilibrium equations for bending and buckling using Hamilton's principle within the **RESDT** framework [25, 26];
- to evaluate dimensionless deflections under sinusoidal transverse loading and critical buckling loads under uniaxial compression for various antisymmetric configurations;
- to quantify the influence of orthotropy ratio, geometric parameters, and lamination schemes through rigorous comparison with **CLPT**, **FSDT**, **HSDT**, refined plate theories, and exact 3D elasticity solutions [23, 24].

The novelty of this work lies in the unified treatment of bending and buckling phenomena using a computationally efficient four-variable exponential model that eliminates shear correction factors while delivering accuracy comparable to exact solutions. The findings provide valuable insights for optimizing laminate design in engineering applications where weight reduction and structural stability are paramount.

2. Mathematical problem analysis

Consider a laminated composite plate with a total thickness h , a length a , and a width b . This plate is composed of n orthotropic elastic layers. The coordinates x and y lie in the plane $(\overline{O}_x, \overline{O}_y)$, and the $\overline{O}_z$ axis is oriented along the thickness direction. The upper and lower surfaces of the plate are located at $z = \pm h/2$, and the edges of the plate are parallel to the $\overline{O}_x$ and $\overline{O}_y$ axes, as illustrated in Figure 1.

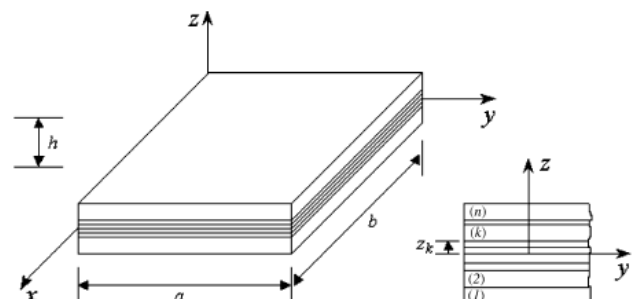


Figure 1. Representation of the composite laminated plate and coordinate systems [21].

2.1. Refined shear deformation theory

The present theory describes the displacement field as follows:

$$\begin{cases} u(x, y, z, t) = u_0(x, y, t) - z \left[\frac{\partial}{\partial x} w_b(x, y, t) + \left(1 - e^{-2\left(\frac{z}{h}\right)^2} \right) \frac{\partial}{\partial x} w_s(x, y, t) \right] \\ v(x, y, z, t) = v_0(x, y, t) - z \left[\frac{\partial}{\partial y} w_b(x, y, t) + \left(1 - e^{-2\left(\frac{z}{h}\right)^2} \right) \frac{\partial}{\partial y} w_s(x, y, t) \right] \\ w(x, y, z, t) = w_b(x, y, t) + w_s(x, y, t) \end{cases} \quad (1)$$

Where:

u, v, w : Displacements in x, y , and z directions, respectively

u_0, v_0 : Extensional deformation in mid-plane.

w_b, w_s : Bending and shear components.

ϵ_x, ϵ_y and $\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$: Normal and shear strains:

$$\begin{Bmatrix} \epsilon_x \\ \epsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u_0}{\partial x} \\ \frac{\partial v_0}{\partial x} \\ \frac{\partial u_0}{\partial y} + \frac{\partial v_0}{\partial x} \end{Bmatrix} - z \begin{Bmatrix} \frac{\partial^2 w_b}{\partial x^2} \\ \frac{\partial^2 w_b}{\partial y^2} \\ 2 \frac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix} - f(z) \begin{Bmatrix} \frac{\partial^2 w_s}{\partial x^2} \\ \frac{\partial^2 w_s}{\partial y^2} \\ 2 \frac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}; \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \frac{\partial w_s}{\partial y} \\ \frac{\partial w_s}{\partial x} \end{Bmatrix}$$

2.2. Formulation of equilibrium equations

The principle of Hamilton is employed to derive the static equilibrium equations. This principle can be stated in its analytical form [25, 26]. The equation of motion for a laminated plate is given by:

$$\int_{t_1}^{t_2} (\delta U - \delta K) dt = 0 \quad (2)$$

The variation of the strain energy for the laminated plate can be expressed as:

$$\delta U = \iint \left[N_x \delta \epsilon_x^0 + N_y \delta \epsilon_y^0 + N_{xy} \delta \gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta \gamma_{yz}^s + S_{xz}^s \delta \gamma_{xz}^s \right] dx dy \quad (3)$$

Where:

N, M and S : Normal force, moment and shear force respectively.

$$\begin{Bmatrix} N_x & N_y & N_{xy} \end{Bmatrix} = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} (\sigma_{xx}, \sigma_{yy}, \tau_{xy}) z dz$$

$$\begin{Bmatrix} M_x^b & M_y^b & M_{xy}^b \\ M_x^s & M_y^s & M_{xy}^s \end{Bmatrix} = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} (\sigma_{xx}, \sigma_{yy}, \tau_{xy}) \begin{Bmatrix} z \\ f(z) \end{Bmatrix} dz$$

$$\begin{Bmatrix} S_{yz}^s & S_{xz}^s \end{Bmatrix} = \sum_{k=1}^n \int_{z_k}^{z_{k+1}} (\tau_{yz}, \tau_{xz}) g(z) dz$$

The static equilibrium equations for an antisymmetric laminated plate are then deduced as:

$$\begin{aligned} \delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} \\ \delta v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} \\ \delta w_b : \frac{\partial^2 M_x^b}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q + N(w) = 0 \\ \delta w_s : \frac{\partial^2 M_x^s}{\partial x^2} + 2 \frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y} + q + N(w) = 0 \end{aligned} \quad (4)$$

Where:

- $N(w)=0$ for bending

- $q=0$ for buckling

2.3. Displacement model

In the examples presented, a shear correction factor of 5/6 is adopted for the first-order shear deformation theory (FSDT). All layers are assumed to have identical thickness, mass density, and orthotropic material properties aligned with the principal material axes. The displacement models considered are summarized in Table 1.

Table 1. Displacement Model.

Models	Theory	Variables
CLPT	Classical Plate Theory [2]	3
FSDT	First-order Shear Deformation Theory [5]	5
HSDT	Higher-order Shear Deformation Theory [12]	5
Ren	Higher-order Shear Deformation Theory [13]	7
RPT1	Refined Plate Theory 1 [27]	4
RPT2	Refined Plate Theory 2 [27]	5
Present	Refined Exponential Shear Deformation Theory (RESDT)	4

The properties of the laminate used are given as follows:

Material 1 [23]:

$$E_1 = 25E_2, \quad G_{12} = G_{13} = 0.5E_2, \quad G_{23} = 0.2E_2$$

and $\nu_{12} = 0.25$.

Material 2 [13]:

$$E_1 = 40E_2, \quad G_{12} = G_{13} = 0.5E_2, \quad G_{23} = 0.6E_2$$

and $\nu_{12} = 0.25$.

Material 3 [24]:

$$E_1 = 40E_2, \quad G_{12} = G_{13} = 0.6E_2, \quad G_{23} = 0.5E_2$$

and $\nu_{12} = 0.25$.

The influences of parameters such as the Young's modulus ratio, the length-to-thickness ratio, the width-

to-length ratio, and the number of layers on the bending and buckling behavior of laminated plates are investigated through several applications.

The results obtained using the present model are compared with results obtained from various plate theories and exact solutions from 3D elasticity theory.

For practical reasons and to standardize the presentation of our results with those in the literature, the following dimensionless deflection $\bar{w}$ and the dimensionless buckling load $\bar{N}$ forms are used.

$$\bar{w} = \frac{100E_2 \cdot h^3}{q \cdot a^4} w \left(\frac{a}{2}, \frac{b}{2} \right) \quad (5)$$

$$\bar{N} = \frac{a^2}{E_2 \cdot h^3} N_{cr} \quad (6)$$

3. Results and discussions

3.1. Bending problem

3.1.1. Application 1

Consider a square-shaped antisymmetric cross-ply laminated plate (0/90). This plate is simply supported and subjected to a sinusoidal bending load as shown in Figure 2.

To study material 1, we investigated the effect of the length-to-thickness ratio a/h on the dimensionless deflection $\bar{w}$. The numerical results are presented and compared to those obtained by different plate theories and those of Pagano [23] for exact solutions in Table 2.

Table 2 demonstrates good agreement between the shear deformation theories and the present theory. It can be observed that the results calculated by HSDT, RPT1, and the present theory are very close.

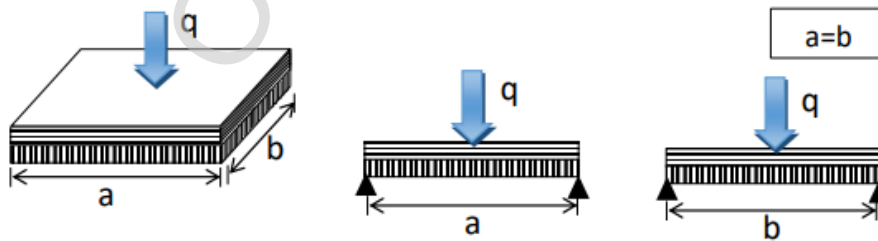


Figure 2. Square plate in antisymmetric cross-ply laminate (0/90), simply supported and subjected to a sinusoidal bending load.

Table 2. Dimensionless deflection of a square antisymmetric cross-ply laminated plate (0/90)

a/h	$\bar{w}$					
	Exact [23]	FSDT	HSDT	RPT1	RPT2	Present Theory
2	4,9362	5,4103	4,5619	4,5619	4,8677	4,5620
5	1,7287	1,7627	1,6670	1,6670	1,6852	1,6670
10	1,2318	1,2416	1,2161	1,2161	1,2197	1,2162
20	1,2318	1,1113	1,1018	1,1018	1,1027	1,1019
100	1,0742	1,0653	1,0651	1,0651	1,0651	1,0651

The influence of the a/h ratio of material 1 on the dimensionless deflection of a rectangular plate with high orthotropy $E_1=25E_2$ in a two-layer (0/90) antisymmetric cross-ply laminate, simply supported and subjected to a sinusoidal bending load, is demonstrated in Figure 3.

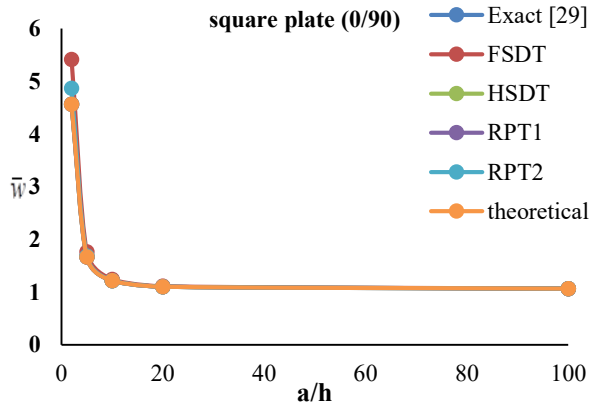


Figure 3 - The effect of the a/h ratio of material 1 on the dimensionless deflection of a rectangular plate with high orthotropy $E_1=25E_2$ in a two-layer (0/90) antisymmetric cross-ply laminate, simply supported and subjected to a sinusoidal bending load.

The analysis of the variation of dimensionless deflection as a function of the a/h ratio; for five plate types: very thick $a=2h$, thick $a=5h$, moderately thick $a=10h$, nearly thin $a=20h$, and very thin $a=100h$ shows that increasing the a/h ratio increases the dimensionless deflection $\bar{w}$ due to the transverse shear effect.

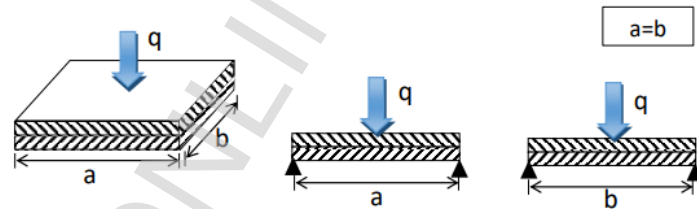


Figure 4-a. Square plate in composite material symmetrically balanced antisymmetric laminate ($-45^\circ/45^\circ$), simply supported and subjected to a sinusoidal bending load.

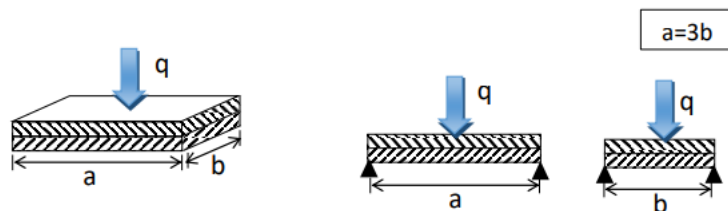


Figure 4-b. Rectangular plate in composite material symmetrically balanced antisymmetric laminate ($-45^\circ/45^\circ$), simply supported and subjected to a sinusoidal bending load.

The dimensionless deflection of a square and rectangular plate in a symmetrically balanced antisymmetric laminate calculated by various plate theories are presented in Table 3-a. Excellent agreement is observed between the results obtained by RESDT and RPT1.

3.1.2. Application 2

Consider two symmetrically balanced antisymmetric laminated plates ($-45^\circ/45^\circ$), square-shaped (Figure 4-a) and rectangular (Figure 4-b). These plates are simply supported and subjected to a sinusoidal bending load as shown below.

To examine material 2, we studied the effect of the length-to-thickness ratio a/h on the dimensionless deflection $\bar{w}$. The numerical results are presented in Tables 3-a and 3-b.

Table 3-a. Dimensionless deflection of a square symmetrically balanced antisymmetric laminated plate ($-45^\circ/45^\circ$) with $E_1=40E_2$.

a/h	$\bar{w}$					
h	Ren [28]	FSDT	HSDT	RPT1	RPT2	Present
4	1,4471	1,1576	1,0203	1,0203	1,0633	1,0203
20	0,6427	0,5773	0,5581	0,5581	0,5628	0,5582
100	0,4685	0,4678	0,4676	0,4676	0,4677	0,4676

Table 3-b. Dimensionless deflection of a rectangular plate in composite material symmetrically balanced antisymmetric laminate ($-45^\circ/45^\circ$) with $E_1=40E_2$.

a/h	$\bar{w}$					
h	Ren [28]	FSDT	HSDT	RPT1	RPT2	Present
4	3,9653	3,3814	3,1560	3,0971	3,1503	3,0972
20	2,3953	2,2784	2,2784	2,2325	2,2395	2,2325
100	2,0686	2,0674	2,0671	2,0670	2,0670	2,0670

The effect of the a/h ratio of material 2 on the dimensionless deflection of a square and rectangular plate with high orthotropy $E_1=40E_2$ in a two-layer symmetrically balanced antisymmetric laminate ($-45^\circ/45^\circ$), simply supported and subjected to a sinusoidal bending load, is shown in Figures 5.

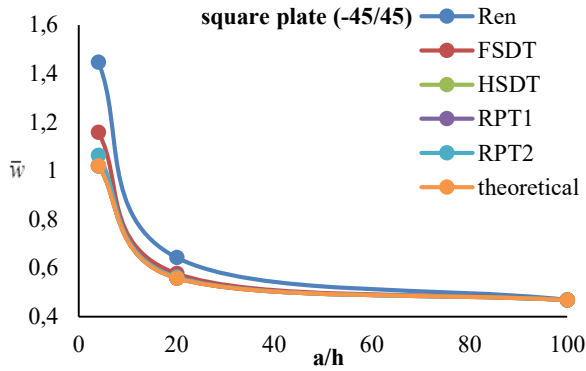


Figure 05-a. The effect of the a/h ratio of material 1 on the dimensionless deflection of a square plate with high orthotropy $E_1=40E_2$ in a two-layer symmetrically balanced antisymmetric laminate $(-45^\circ/45^\circ)$, simply supported and subjected to a sinusoidal bending load.

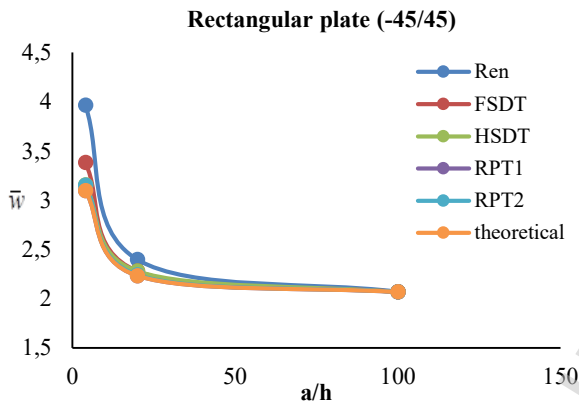


Figure 05-b. The effect of the a/h ratio of material 1 on the dimensionless deflection of a rectangular plate with high orthotropy $E_1=40E_2$ in a two-layer symmetrically balanced antisymmetric laminate $(-45^\circ/45^\circ)$, simply supported and subjected to a sinusoidal bending load.

- A first observation drawn from reading this figure is that this influence is considerable for thick plates where the effect of transverse shear deformation is more important.

- A second observation is that increasing the a/h ratio implies an increase in the dimensionless deflection. It can be said that square plates are stiffer than rectangular plates.

3.2. Buckling Problem

3.2.1. Application 1

Consider a thick square plate with a length-to-

Table 4. Values of the dimensionless buckling load of a square plate $a=10h$ in cross-ply antisymmetric laminate $(0/90)_n$ simply supported and subjected to a uniaxial compressive load on the sides $x=0$ and $x=a$ with $E_1=40E_2$.

a/h	$\bar{N}$						
	Exact [24]	CLPT	FSDT	HSDT	RPT1	RPT2	Present
2	21,2796	30.3591	22,8060	21,2796	22,5790	22,5700	22,5792
3	23,6689	33.5817	24,5777	24,4596	24,4596	24,4581	24,4597
5	24,9636	35.2316	25,4500	25,4225	25,4225	25,4225	25,4225

thickness ratio $a=10h$ in a cross-ply antisymmetric laminate $(0/90)_n$, with an orthotropy ratio $E_1=40E_2$. This plate is simply supported and subjected to a uniaxial compressive load on the sides $x=0$ and $x=a$ as shown in Figure 6.

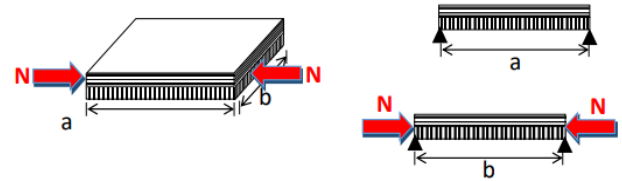


Figure 6. Thick square plate in cross-ply antisymmetric laminate $(0/90)_n$, simply supported and subjected to a uniaxial compressive load on the sides $x=0$ and $x=a$ with $E_1=40E_2$.

We study material 3 for different values of the degree of orthotropy $n=2, 3, 5$ on the buckling load $\bar{N}$. The dimensionless buckling loads predicted using various plate models are presented in Table 4.

Table 4 shows that the results obtained by the present model are close to the values obtained by the HSDT, RPT1, and RPT2 models.

The influence of the degree of orthotropy n of material 3 on the dimensionless buckling load of a thick square plate $a=10h$ and highly orthotropic $E_1=40E_2$ in a cross-ply antisymmetric laminate $(0/90)_n$ simply supported and subjected to a uniaxial compressive load on the sides $x=0$ and $x=a$ is demonstrated in Figure 7.

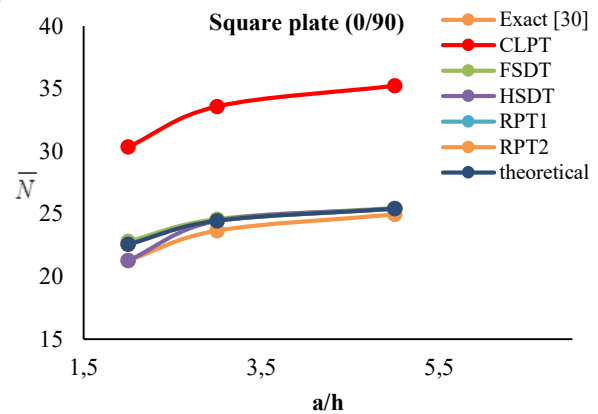


Figure 7: The effect of the degree of orthotropy n of material 3 on the dimensionless buckling load of a thick square plate $a=10h$ and highly orthotropic $E_1=40E_2$ in a cross-ply antisymmetric laminate $(0/90)_n$ simply supported and subjected to a uniaxial compressive load on the sides $x=0$ and $x=a$.

The analysis of the variation of the dimensionless buckling load as a function of the degree of orthotropy $n=1, 3, 5$ with a length-to-thickness ratio $a=10h$ and an orthotropy ratio $E_1=40E_2$ shows that increasing the degree of orthotropy n increases the buckling load $\bar{N}$.

3.2.2. Application 2

Consider a symmetrically balanced antisymmetric laminated plate of two layers ($45^\circ/-45^\circ$), square-shaped.

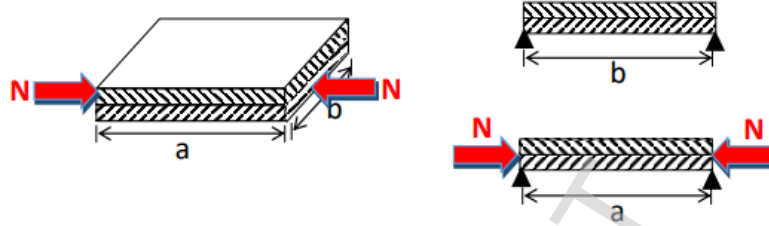


Figure 8. Square plate in symmetrically balanced antisymmetric laminate ($45^\circ/-45^\circ$), simply supported and subjected to a uniaxial compressive load on the sides $x=0$ and $x=a$, with $E_1=40E_2$.

Table 5. Dimensionless buckling load of a square plate in composite material symmetrically balanced antisymmetric laminate ($45^\circ/-45^\circ$) highly orthotropic, simply supported and subjected to a uniaxial compressive load on the sides $x=0$ and $x=a$.

a/h	$\bar{N}$					
	Ren [28]	FSDT	HSDT	RPT1	RPT2	Present
4	9,8200	6,7858	8,2377	8,3963	7,7898	8,3969
10	16,4558	17,5522	18,1544	18,1544	18,0031	18,1547
100	21,6384	21,6576	21,6663	21,6663	21,6643	21,6663

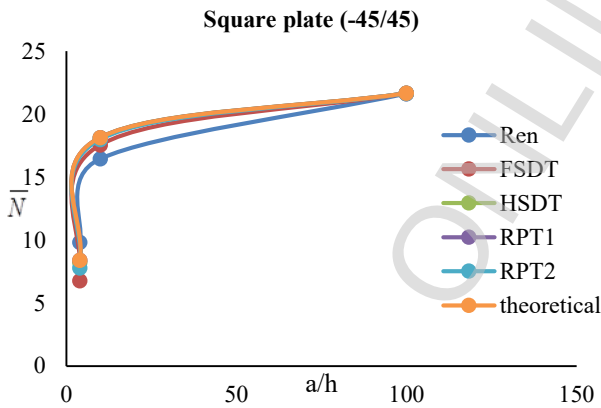


Figure 9. The effect of the length-to-thickness ratio on the dimensionless buckling load of material 3 for a square plate in a symmetrically balanced antisymmetric laminate ($45^\circ/-45^\circ$) simply supported and subjected to a uniaxial compressive load on the sides $x=0$ and $x=a$, $E_1=40E_2$.

The results obtained by **RESDT** for thick laminated plates with higher orthotropy ratio $E_1=40E_2$ are identical to those obtained by **RPT1** and **HSDT** [28]. The unknown variables in Reddy's higher-order shear deformation theory are seven, and for Ren's and **FSDT**'s shear deformation theories, the number of variables is five. We can

This plate is simply supported and subjected to a uniaxial compressive load on the sides $x=0$ and $x=a$, as shown in Figure 8.

We examined material 3 for different values of the length-to-thickness ratio a/h , with an orthotropy ratio $E_1=40E_2$, on the dimensionless buckling load $\bar{N}$. The dimensionless buckling loads calculated using various plate models are presented in Table 5.

conclude that the **RESDT** theory, containing four variables, is not only accurate but also simple for predicting the critical buckling loads of laminated plates.

The influence of the length-to-thickness ratio a/h of material 3 on the dimensionless buckling load of a square plate in a two-layer symmetrically balanced antisymmetric laminate ($45^\circ/-45^\circ$) simply supported with $E_1=40E_2$, this plate is simply supported and subjected to a uniaxial compressive load on the sides $x=0$ and $x=a$ is given in Figure 9.

The analysis of the variation of the dimensionless buckling load as a function of the length-to-thickness ratio with an orthotropy ratio $E_1=40E_2$ for two layers shows that increasing the length-to-thickness ratio a/h increases the dimensionless buckling load. It is noted that the dimensionless buckling loads are much higher for thin plates than for thick plates, and this is due to the transverse shear effect, which is more significant for thick plates. It can be said that thick plates are stiffer than thin plates.

4. Conclusion

This study establishes the Refined Exponential Shear Deformation Theory (**RESDT**) as a robust framework for analyzing the bending and buckling responses of antisymmetric laminated composite plates. By employing only

four independent variables and an exponential shear function, the proposed model inherently satisfies traction-free boundary conditions, thereby eliminating the need for empirical shear correction factors while maintaining computational efficiency. Numerical validations demonstrate exceptional agreement with exact three-dimensional elasticity solutions and higher-order theories, with errors remaining within 1% for highly orthotropic laminates.

Parametric investigations reveal that critical buckling loads and structural stiffness are profoundly enhanced by higher orthotropy ratios and balanced angle-ply configurations compared to cross-ply laminates. Furthermore, the length-to-thickness ratio plays a pivotal role, with transverse shear effects increasing dimensionless deflections by up to 15% in thick plates. These findings underscore the necessity of accurate shear deformation modeling in modern composite design, particularly where weight reduction and stability are paramount. Consequently, the **RESDT** offers a simplified yet precise alternative for optimizing lightweight structural components in aerospace and civil engineering. This work aligns with contemporary research trends towards computationally efficient, high-fidelity structural theories, providing valuable insights for the next generation of advanced composite material applications. Future extensions may explore thermal environments and dynamic loading scenarios.

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Analiza savijanja i izvijanja antisimetričnih laminatnih kompozitnih ploča koristeći rafinovanu eksponencijalnu teoriju smičućih deformacija

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Apstrakt

Uvod/Cilj: Ova studija analizuje odgovor na savijanje i ponašanje pri izvijanju antisimetričnih laminatnih kompozitnih ploča koristeći rafinovanu eksponencijalnu teoriju smičućih deformacija (RESDT). Istraživanje ima za cilj da evaluiira bezdimenzionalne otklone pod poprečnim opterećenjem i kritična opterećenja izvijanja pod jednoosnim pritiskom, istražujući uticaj odnosa ortotropije (E_1/E_2), odnosa dužine i debljine (a/h), odnosa stranica (a/b) i sekvence slaganja slojeva laminate. **Metode:** RESDT formulacija koristi četiri nezavisne promenljive za modeliranje transverzalne smičuće deformacije putem eksponencijalne funkcije koja inherentno zadovoljava uslove nultog smičućeg napona na površinama ploče, eliminišući empirijske faktore korekcije smicanja. Analizirane su jednostavno oslonjene kvadratne i pravougaone ploče sa cross-ply (0/90) i angle-ply ($\pm 45^\circ$) konfiguracijama. Rezultati su validirani u odnosu na klasičnu teoriju ploča (CLPT), teoriju smičućih deformacija prvog reda (FSDT), teoriju smičućih deformacija višeg reda (HSDT) i egzaktna trodimenzionalna rešenja elastičnosti. **Rezultati:** Numerički rezultati pokazuju odlično slaganje između RESDT predviđanja i referentnih rešenja kroz različite debljine i nivoe ortotropije. Za debele ploče ($a/h = 2-10$), efekti transverzalnog smicanja značajno povećavaju bezdimenzionalne otklone do 15% u poređenju sa teorijama tankih ploča. Opterećenja izvijanja rastu sa povećanjem stepena ortotropije i odnosa a/h , pri čemu se RESDT poklapa sa egzaktnim rešenjima unutar greške od 1% za visoko ortotropne laminate ($E_1 = 40E_2$). **Zaključak:** Četvoroparametarska RESDT pruža tačan, računarski efikasan okvir za predviđanje mehaničkih odgovora antisimetričnih laminata bez faktora korekcije smicanja. Transverzalna smičuća deformacija kritično utiče na ponašanje debelih ploča, dok se krutost laminate poboljšava sa višim odnosima ortotropije i balansiranim angle-ply konfiguracijama, nudeći vredne uvide za optimizaciju dizajna kompozitnih struktura.

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Ključne reči:

laminatne ploče;
bezdimenzionalni otklon;
jednoosno izvijanje;
transverzalno smicanje;
stepen ortotropije;
odnos ortotropije