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A VARIATION ON THE THEME OF THE MOST PREDICTABLE AND RELIABLE CRITERION

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In a recent paper (Momirovic, Bogdanovic, Wolf and Tenjovic, 1993) a simple method for the analysis of asymmetric relations between two sets of variates was proposed. This method consists of the maximization of covariance between linear composites of predictor and criterion variables and the maximization of reliability of the linear composite of critetion variables under the condition that linear composite of predictor variables is standardized, and the transformation vector for linear composite of criterion variables is of unit norm. In the present paper another method for the analysis of asymmetric relations between two sets of variates is proposed. The proposed method is defined so that the reliability of a function of criterion variates is maximized, and at the same time is maximized the covariance between this function and a function of predictor variates, under the conditions that the transformation vectors for both function are of unit norm. The present method does not give the maximized correlation between functions of predictor and criterion variates as gives the previous method, but only the maximized covariance between them; however, in the present method the regularity of covariance matrix of predictor variates is not a necessary condition. A new standardized measure of the relations between predictor and criterion variates is then derived, and a test of significance of this measure is proposed.

Key words: prediction, reliability, canonical covariance analysis, robust methods.

Introduction

The problem of relation between two sets of quantitative variables can be solved in many different, but mutually conected ways. The standard symmetric approach is canonical correlation analysis (Hotelling, 1935; 1936), and the standard asymmetric approach is multiple regression analysis (for some possibilities and variations of this approach see, for example, Anderson, 1984). A generalization of the first approach, canonical covariance analysis, was proposed ten years ago (Momirovic, Dobric and Karaman, 1983; Dobric, 1986; Momirovic, Radakovic and Dobric, 1988). Several different methods for the asymmetric analysis of relations between two sets of quantitative variables were proposed in the last fifteen years: redundancy analysis (Van Den Wollenberg, 1977), assymmetric canonical covariance analysis (Prot, Bosnar and Momirovic, 1983; Momirovic, Dobric and Karaman, 1984), and so-called stupid regression analysis (Stalec and Momirovic, 1983; Dobric, Stalec and Momirovic, 1984; Dobric and Momirovic, 1991).

In a recent paper an apparently new method for the analysis of asymmetric relations between two sets of quantitative variables was proposed (Momirovic, Bogdanovic, Wolf and Tenjovic, 1993). This method is defined so that the reliability of a function of criterion variables is maximized, and simultaneously is maximized the covariance between this function and a normalized functzion of predictor variables. It was shown that the function of criterion variables was just the first principal component of standardized criterion variables, and that the function of predictor variables can be obtained by the normalized projection of predictor variables to the eigenvector of correlation matrix of criterion variables related to maximal eigenvalue of this matrix. An asymmetric measure of the relations between predictor and criterion variables was derived, and it was demonstrated that this measure was nothing else but the multiple correlation between predictor variables and the most reliable linear combination of criterion variables.

Although simple, elegant and efficient, the method proposed by Momirovic, Bogdanovic, Wolf and Tenjovic has an undesirable property, at least from the point of view of social science and biological science researchers. The condition for the application of this method is the strict regularity of correlation matrix of predictor variables, and therefore the method is highly sensitive to nearly singular covariance matrices, and, indirectly, to the presence of outliers. The aim of the method proposed in the present paper is to overcome this drawback, retaining the main features of the method for the analysis of the relation between predictors and the most predictable and reliable criterion.

Method and Algorithm

Let be $V = (V_{pj};\ j = 1, ..., m_p)$ a set of quantitative variables, selected in accordance with some theoretical model from a universe U_p of independent, explanatory or predictor variables. Let be $V_c = (V_{cj};\ j = 1, ..., m_c)$ another set of quantitative variables, selected in some way from a universe U_c of dependent or criterion variables. Let be $E = (e_i;\ i = 1, ..., n)$ a set of objects, individuals or general entities, selected randomly from a homogenous population P.

Let be

$$\mathbf{Z}_p = E \otimes V_p \,\big|\, \mathbf{E}(\mathbf{Z}_p) = \mathbf{O},\ \mathrm{diag}(\mathbf{Z}_p^t \mathbf{Z}_p) = \mathbf{I}$$

a matrix, in the standard normal form, obtained by the description of E on V_p, and let be

$$\mathbf{Z}_c = E \otimes V_c \,\big|\, \mathbf{E}(\mathbf{Z}_c) = \mathbf{O},\ \mathrm{diag}(\mathbf{Z}_c^t \mathbf{Z}_c) = \mathbf{I}$$

another matrix, in standard normal form also, obtained by the description of E on V_c. Because the column vectors from $\mathbf{Z}_p$ and $\mathbf{Z}_c$ are normalized,

$$\mathbf{R}_{pp} = \mathbf{Z}_p^t \mathbf{Z}_p$$

and

$$\mathbf{R}_{cc} = \mathbf{Z}_c^t \mathbf{Z}_c$$

will be the intercorrelation matrices of variables from V_p and V_c, respectively, and

$$\mathbf{R}_{pc} = \mathbf{R}^{t}_{cp} = \mathbf{Z}^{t}_{c}\,\mathbf{Z}_{c}$$

will be the cross-correlation matrix between variables from V_p and V_c.

The proposed method for the analysis of asymmetric relations between V_p and V_c in R^n space defined by E consists in the solution of the following problem:

$$\begin{array}{l|l} \mathbf{Z}_c\,\mathbf{x} = \mathbf{k} & \sigma^2 = \mathbf{k}^t\mathbf{k} = \mathbf{x}^t\mathbf{R}_{cc}\,\mathbf{x} = \max \\ \mathbf{Z}_p\,\mathbf{y} = \mathbf{g} & c = \mathbf{g}^t\mathbf{k} = \mathbf{y}^t\mathbf{R}_{pc}\,\mathbf{x} = \max \\ & \mathbf{x}^t\mathbf{x} = 1 \\ & \mathbf{y}^t\mathbf{y} = 1 \end{array}$$

with $\mathbf{x}$ an unknown $\mathbf{m}_c$ - dimensional vector of unit norm, and y an unknown m_p - dimensional vector of unit norm also.

Maximization of σ^2, the variance of the linear combination **k** of variables from V_c is equivalent to the maximization of reliability of this linear combination, because the reliability of any linear combination of a set of standardized variables, obtained through a vector of unit length, is equal (see, for example, Cronbach, Glasser, Nanda nad Rajaratnam, 1972 or Kaiser and Caffrey, 1965) to

$$\alpha = (m / (m - 1))\,(1 - \sigma^{-2})$$

with m number of variables used to build the linear combination, in our case $m = m_c$.

Maximization of c, the covariance of linear combinations **k** and **g**, is actually the maximization of a natural measure of association between two composite variables (some notes on the importance of covariance between linear combinations of variables can be found in several papers concerning the canonical covariance analysis and related techniques, for example in Momirovic, Dobric and Karaman, 1983; Dobric, Stalec and Momirovic, 1984; Stalec and Momirovic, 1984; Prot, Bosnar and Momirovic, 1983; Momirovic, Dobric and Karaman, 1984; Dobric, 1986 etc.).

The problem of maximization of σ^2 and c can be easily solved by successive maximization of σ^2, and then of c.

The first function to be maximized is

$$\begin{aligned} f(x, \lambda) &= \sigma^2 - \lambda (x^t x - 1) \\ &= x^t \mathbf{R}_{cc} x - \lambda (x^t x - 1), \end{aligned}$$

with λ some unknown Lagrangeian multiplier.

Derivation of this function with respect to x gives

$$\partial f(x, \lambda) / \partial x = 2\mathbf{R}_{cc} x - 2x\lambda$$

From this

$$\mathbf{R}_{cc} x = x\lambda$$

so that the desired solution is simply the solution of eigenproblem

$$\mathbf{R}_{cc} x = x\lambda$$

or, stated in another way,

$$(\mathbf{R}_{cc} - \lambda \mathbf{I}) = \mathbf{O}$$

and

$$\sigma^2 = \lambda.$$

Therefore, as in the previous method (Momirovic, Bogdanovic, Wolf and Tenjovic, 1993) **k** is simply the first principal component of standardized criterion variables contained in V_c.

The derivation of second function gives, however, substantially different result. The second function to be maximized is

$$\begin{aligned} f(\mathbf{y}, \eta) &= c - \tfrac{1}{2} \eta (\mathbf{y}^t \mathbf{y} - 1) \\ &= \mathbf{y}^t \mathbf{R}_{pc} \mathbf{x} - \tfrac{1}{2} \eta (\mathbf{y}^t \mathbf{y} - 1) \end{aligned}$$

with $\frac{1}{2} \eta$ an unknown Lagrangeian multiplier.

Derivation of this function with respect to **y**, because **x** is already known, gives

$$\partial f(\mathbf{y}, \eta) / \partial \mathbf{y} = \mathbf{R}_{pc} \mathbf{x} - \mathbf{y}\eta.$$

From this

$$\mathbf{R}_{pc}\,\mathbf{x} \;-\; \mathbf{y}\eta$$

so that unknown vector **y** is simply

$$\mathbf{y} \;=\; \eta^{-1}\,\mathbf{R}_{pc}\,\mathbf{x};$$

value of η can be derived from

$$1 \;=\; \mathbf{y}^{\,t}\mathbf{y} \;=\; \eta^{-1}\,\mathbf{x}^{\,t}\mathbf{R}_{cp}\,\mathbf{R}_{pc}\,\mathbf{x}\;\eta^{-1}$$

$$\Rightarrow \eta^{2} \;=\; \mathbf{x}^{\,t}\mathbf{R}_{cp}\,\mathbf{R}_{pc}\,\mathbf{x}.$$

Therefore, the solution for **g** is similar to the solution of problem of so-called stupid regression analysis (Stalec and Momirovic, 1983; Dobric, Stalec and Momirovic, 1984; Dobric and Momirovic, 1991), with the difference that criterion variable is, actually, the linear combination of a set of criterion variables with maximum reliability.

Now, the covariance between **k** and **g** is

$$\begin{aligned} c &= \mathbf{y}^{t}\mathbf{R}_{pc}\,\mathbf{x} \\ &= \eta^{-1}\,\mathbf{x}^{t}\mathbf{R}_{cp}\,\mathbf{R}_{pc}\,\mathbf{x} \\ &= \eta, \end{aligned}$$

equal to the double value of Lagrangeian multiplier for the maximization of the second function.

The variance of **k** is, of course,

$$\sigma^{2} \;=\; \mathbf{k}^{\,t}\mathbf{k} \;=\; \mathbf{x}^{\,t}\mathbf{R}_{cc}\,\mathbf{x} \;=\; \lambda$$

The variance of **g** is

$$\begin{aligned} v^{2} &= \mathbf{g}^{\,t}\mathbf{g} \;=\; \mathbf{y}^{\,t}\mathbf{R}_{pp}\,\mathbf{y} \\ &= \eta^{-1}\,\mathbf{x}^{t}\mathbf{R}_{cp}\,\mathbf{R}_{pp}\,\mathbf{R}_{pc}\,\mathbf{x}\;\eta^{-1} \end{aligned}$$

Note that inverse of $\mathbf{R}_{pp}$ das not exist in either solution.

A standardized measure of association between **k** and **g** is now

$$\phi = \eta\lambda^{-1/2} v^{-1}$$

and is analogous to the so-called stupid multiple coefficient of correlation (Stalec and Momirovic, 1983).

Finally, note that ϕ was not directly maximized; only λ and η were subject to maximization. Therefore, ϕ behaves as a simple product - moment coefficient of correlation, so that the test of hypothesis $H_0 : \phi^* = 0$ is

$$f = \phi^2 ((n-2)/(1-\phi^2));$$

As it is well known, f is, approximately, distributed as Fisher - Snedecor f function with $df_1 = 1$ and $df_2 = n - 2$ degrees of freedom.

Recognition of the content of variables **k** and **g** can be based on the elements of transformation vectors **x** and **y**, and on the basis of structure vectors

$$\mathbf{h}_K = \mathbf{Z}_c^t \mathbf{k} \lambda^{-1/2} = \mathbf{R}_{cc} x\lambda^{-1/2}$$

$$= x\lambda^{1/2}$$

and

$$\mathbf{h}_G = \mathbf{Z}_p^t \mathbf{g} v^{-1}$$

$$= \mathbf{R}_{pp}\ \mathbf{y} v^{-1}.$$

The proposed method can be applied, concurrently, with the method proposed in the paper of Momirovic, Bogdanovic, Wolf and Tenjovic (1993), or as a robust method for the analysis of relations between a set of predictor and a set of criterion variables in the case when all the requirements for the analysis of relations based on regular data sets are not fulfilled.

Program

In order to facilitate the application of the proposed method for a wide set of users, a program is written in GENSTAT, Version 4.4 B. On the basis of this program and, of course, of the algorithm described in the previous section, another program can be easily written in any other programming language.

```
´MACRO´  MWTB $
"        WRITTEN BY
                 K. MOMIROVIC AND B. WOLF
         AT
                 NOVEMBAR 7, 1992

IMPLEMENTED BY
                 L. TENJOVIC

FUNCTION
             ROBUST ANALYSIS OF ASYMMETRIC RELATIONS
             BETWEEN TWO SETS OF QUANTITATIVE VARIABLES
             UNDER THE MODEL OF THE MOST PREDICTABLE AND
             RELIABLE CRITERION.

DOCUMENTATION
             K. MOMIROVIC, B. WOLF, L. TENJOVIC, M. BOGDANOVIC,
             (1993):
             A VARIATION ON THE THEME OF THE MOST
             PREDICTABLE AND RELIABLE CRITERION.
             TECHNICAL REPORT, FACULTY OF ARTS, UNIVERSITY
             OF BELGRADE.

REQUIREMENTS
             REFERENCE OR SOME OTHER MACRO PROGRAM
             MUST TRANSFER TO THIS MACRO:
             (1) TWO VARIATE STRUCTURES, UNDER THE NAMES V1
                 AND V2, WITH DATA. PREDICTOR SET OF VARIATES
                 MUST BE IN THE STRUCTURE V1, AND CRITERION
                 SET OF VARIATES IN THE STRUCTURE V2.
             (2) TWO POINTER STRUCTURES, UNDER THE NAMES P1
                 AND P2, WITH VARIATE NAMES. THE NAMES OF
                 PREDICTOR VARIATES MUST BE IN THE
                 STRUCTURE P1, AND THE NAMES OF CRITERION
                 VARIATES IN THE STRUCTURE P2.
             (3) A SCALAR, N, WITH THE NUMBER OF ENTITIES.

WARNING
             THE DATA STRUCTURES MUST BE CONFORMED.
```

```
´START´
"
        SECTION O.
              DECLARATION OF DATA STRUSTURES.
"
 ´LOCAL   C, R, R11, R22, R12, M, X, Y, G, H, ETA, RHO, LAMBDA,
 ´        SIGMA, F, DF, Q, V, P, H1, H2, H3, H4, H5, H6, M1, M2,
          ALPHA, W
 ´VARI´   V = V1, V2,
 ´POIN´   P = P1, P2
 ´DSSP´   C $ V
 ´SCAL´   ETA, RHO, SIGMA, F, DF, Q, M1, M2, ALPHA, W
 ´MATR´   R12 $ P1, P2
 ´MATR´   X, G $ P1, 1
 ´MATR´   Y, H $ P2,1
 ´SYMM´   R11 $ P1
 ´SYMM´   R22, M $ P2
 ´DIAG´   LAMBDA $ 1
 ´HEAD´   H1 = "CORRELATIONS OF PREDICTOR VARIABLES"
 ´HEAD´   H2 = "CORRELATIONS OF CRITERION VARIABLES"
 ´HEAD´   H3 = "CROSS CORRELATIONS BETWEEN
          PREDICTOR AND CRITERION VARIABLES"
 ´HEAD´   H4 = "PSEUDOCANONICAL CORRELATION
          BETWEEN PREDICTOR AND CRITERION VARIABLES
          (RHO), STANDARD ERROR OF PREDICTION (SIGMA),
          THE SIGNIFICANCE OF RHO (Q) AND THE
          RELIABILITY OF CRITERION (ALPHA)"
 ´HEAD´   H5 = "VECTOR OF WEIGHTS (X) AND THE
          STRUCTURE VECTOR (G) OF THE PREDICTOR
          VARIABLES"
 ´HEAD´   H6 = "VECTOR OF WEIGHTS (Y) AND THE
          STRUCTURE VECTOR (H) OF THE CRITERION
          VARIABLES"
"
        SECTION 1.
                ALGORITHM.
"
 ´CALC´   M1, M2 = NVAL (P1, P2)
 ´CALC´   DF = N - 2.0
 ´SSP´    C
 ´CALC´   R = CORMAT (C)
 ´CALC´   R11, R12, R22 = SUBMAT (R)
 ´CALC´   M = TPDT (R12; R12)
 ´LRV´    R22; Y, LAMBDA, SIGMA
 ´DEVA´   SIGMA
```

```
 ´CALC´     ALPHA  =  (M2/ (M2 - 1.0) ) * (1.0 - 1.0/LAMBDA)
 ´CALC´     LAMBDA  ' SQRT (LAMBDA)
 ´CALC´     ETA  = TPDT (Y; PDT (M;Y))
 ´CALC´     ETA =  SQRT (ETA)
 ´CALC´     X  =  PDT (R12;Y)
 ´CALC´     X  =  X/ETA
 ´CALC´     W  =  TPDT (X; PDT (R11;X))
 ´CALC´     W  =  1.0/SQRT (W)
 ´CALC´     G  =  PDT (R11; X)
 ´CALC´     G  =  G*W
 ´CALC´     H  =  Y*LAMBDA
 ´CALC´     RHO  =  ETA*W/LAMBDA
 ´CALC´     F  =  (RHO*RHO) * (DF/M1)
 ´CALC´     Q  =  PRBF (F; 1.0;DF)
 ´CALC´     SIGMA  =  SQRT (1.0 - RHO*RHO)
"
        SECTION 2.
                PRINTED OUTPUT.
"

 ´PAGE´
 ´LINE´     20
 ´CAPT´     "
            ROBUST ANALYSIS OF ASYMMETRIC RELATIONS
            BETWEEN TWO SETS OF VARIATES UNDER THE
            MODEL OF THE MOST PREDICTABLE AND RELIABLE
            CRITERION
 ´LINE´     6
 ´CAPT´     " NUMBER OF ENTITIES "
 ´PRIN´     N $ 16.0
 ´LINE´     4
 ´CAPT´     " NUMBER OF VARIABLES IN THE PREDICTOR SET "
 ´PRIN´     M1  $  16.0
 ´LINE´     4
 ´CAPT´     " NUMBER OF VARIABLES IN THE CRITERION SET "
 ´PRIN´     M2  $  16.0
 ´PAGE´
 ´LINE´     2
 ´PRIN´     H1
 ´LINE´     2
 ´PRIN/P    R11  $  9.3
 ´
 ´PAGE´
 ´LINE´     2
```

```
´PRIN´     H2
´LINE´     2
´PRIN/P    R22 $ 9.3
´
´PAGE´
´LINE´     2
´PRIN´     H3
´LINE´     2
´PRIN/P    R12 $ 9.3
´
´PAGE´
´LINE´     20
´PRIN´     H4
´LINE´     2
´PRIN/S´   RHO, SIGMA, Q, ALPHA $ 12.3
´PAGE´
´LINE´     2
´PRIN´     H5
´LINE´     2
´PRIN/P    X, G  $  12.3
´
´PAGE´
´LINE´     2
´PRIN´     H6
´LINE´     2
´PRIN´     Y, H  $  12.3
´PAGE´
                     END OF MACRO MWTB
"
´ENDMACRO/LOCAL  =  DESTROY´
```

References

Anderson, T.W. (1984): *An Introduction to Multivariate Statistical Analysis*. (2^0 edition). Wiley, New York.

Cronbach, l.j.; Glasser, G.C.; Nanda, A.; Rajaratnam, N. (1972): *The Dependability of Behavioral Measurements: Theory of Generalizability for Scores and Profiles*. Wiley, New York.

Dobric, V.; Stalec; J.; Momirovic, K. (1984): Note on Relationships between Least Squares and Robust Linear Regression Esstimates. *Proceedings of 6th International Symposium "Computer at the University",* 507:1-7.

Dobric, V. (1986): On a Class of Robust Methods for Multivariate Data Analysis. COMPSTAT 86, *Proceedings in Computational Statistics*, Physica - Verlag, Heidelberg, 211-216.

Dobric, V.; Momirovic, K. (1991): Some Relations between Least Squares and Robust Linear Regression Estimates. *Information technology interfaces*, 5:1-5.

Hotelling, H. (1935:The Most Predictable Criterion. *Journal of Educational Psychology*, 26:139-142.

Hotelling, H. (1936): Relations between Two Sets of VAriates. *Biometrika*, 28:321-377.

Kaiser, H.F.; Caffrey, J. (1965): Alpha Factor Analysis. *Psychometrika*, 30:1-14.

Momirovic, K.; Dobric, v.; Karaman, Z. (1983): Canonical Covariance Analysis. *Proceedings of 5th International Symposium "Computer at the University",* 5, 1:1-11.

Momirovic, K.; Dobric, V.; Karaman, Z. (1984): Algorithm and Program for Multivariate Selection with Consistent Linear Constraints. In *Genetics of Psychomotor Traits in man*, Polish Academy of sciences, Warsaw, pp. 283-293.Momirovic, K.; Radakovic, j.;

Dobric, V. (1988): An Ecpert System for the Interpretation of Results of Canonical Covariance Analysis. COMPSTAT 88, *Proceedings in Computational Statistics*, Physica - Verlag, Heidelberg, 135-141.

Momirovic, K.; Bogdanovic, M.; Wolf, B.; Tenjovic, L. (1993): The Most Predictable and Reliable Criterion. *Technical report*, Faculty of Arts, Belgrade.

Prot, F.; Bosnar, K.; Momirovic, K. (1983): An Algorithm for Asymmetric Redundancy Analysis without Orthogonality Constraints. *Proceedings of 5th International Symposium "Computer at the University",* 5, 2:1-9.

Van Den Wollenberg, A.L. (1977): Redundancy Analysis - an Alternative Model for canonical Correlation Analysis. *Psychometrika*; 42:207-219

Stalec, J.; Momirovic, K. (1983): Some Properties of a Very Simple Model for Robust Regression Analysis. *Proceedings of 5th International Symposium "Computer at the University",* 453-461.

VARIJACIJA NA TEMU NAJPREDIKTIVNIJEG I NAJPOUZDANIJEG KRITERIJUMA

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U jednom novijem radu (Momirović, Bogdanović, Wolf i Tenjović, 1993.) predložen je noviji metod za analizu asimetričkog odnosa između dva seta varijata. Ovaj metod se sastoji od maksimizovanja kovarijance između lineranih kompozita prediktivne i kriterijum varijable i maksimizovanja pouzdanosti linearnih kompozita kriterijum varijable, pod uslovom da je linearni kompozit prediktivne varijable standardizovan i da je transformacioni vektor za linearni kompozit kriterijum varijable jedinično normiran. U ovom radu je predložen još jedan metod za analizu asimetrične relacije izmedju dva seta varijata. Predložen metod je definisan tako da se istovremeno maksimizuje pouzdanost funkcije kriterijumskih varijata i kovarijanca između te funkcije i funkcije prediktivnih varijata, pod uslovom da su transformacijski vektori za obe funkcije jedinično normirani. Ovaj metod, za razliku od prethodnog, ne daje maksimizovanu korelaciju između funkcija prediktivnih i kriterijum varijata. On daje samo maksimizovanu kovarijancu između njih; međutim, u ovom metodu pravilnost kovarijantne matrice prediktivnih varijata nije neophodni uslov. Zatim se izvodi nova standarizovana mera odnosa između prediktivnih i kriterijum varijata i predložen je test za proveru značajnosti ove mere.

Ključne reči: predikcija, pouzdanost, kanonična analiza kovarijance, grubi metodi.

ВАРИАЦИЯ НАД ТЕМОЙ САМОГО ПРЕДИКТАБИЛЬНОГО И САМОГО НАДЕЖНОГО КРИТЕРИЯ

Константин Момирович, Борис Вольф,
Лазар Теньович, Миливое Богданович

В одной из более новых работ (Момирович, Богданович, Вольф и Теньович, 1993-ьго года) предложен более новый метод для анализа асимметрического отношения между двумя комплектами вариатов. Этот метод состоит из максимирования ковариансы между линейными композитами предикативной вариаблы и вариаблы критерия, а также максимирования надежности линейных композитов вариаблы критерия, при условии что линейный композит предиктивной вариаблы стандартизирован и что трансформационный вектор для линейного композита вариаблы критерия является единично нормированым. В этой работе предложен еще один метод анализа асимметрического отношения между двумя комплектами вариатов. Предложеный метод определен таким способом, что одновременно максимируется надежность функции вариатов критерия и ковариaнс между той функцией и функцией предиктивных вариатов, при условии что трансформационные векторы для обеих функций являются единично нормированым. Этот метод, в различие от предыдущего, не дает максимированую корреляцию между функциями предиктивных вариатов и вариатов критерия. Он дает только максимированую ковариансу между ними; между тем, в этом методе привильность ковариантной матрицы предиктивных вариатов не явхляется необходимым условием. Потом выводится новая стандартизированая мера отношений между предиктивными вариатами и вариатами критерия, а также предложен тест для проверки значительности этих мер.

Ключевые слова: предикция, надежность, канонический анализ ковариансы, грубые методы.