

Ryszard Szewczyk¹
*Andrzej Frycz Modrzewski Krakow University,
Krakow, Poland*

PRELIMINARY COMMUNICATION
doi:10.5937/ekonomika1701111S
Received: January, 20, 2017
Accepted: February, 15, 2017

CERTAIN FLAWS OF GERARD DEBREU'S THEORY OF VALUE AND IT'S IMPLICATIONS²

Abstract

Since its advent in 1959 the theory of value formulated by Gerard Debreu in his work titled "Theory of Value. An Axiomatic Analysis of Economic Equilibrium", that gave him the Prize of the Bank of Sweden in Economics in 1983, mathematical economics which pretends to be considered the purest of scientific theories of economic equilibrium, gained a new incentive to expand. The aim of the article is to demonstrate some weak points of the Debreu model which allow to challenge its economic validity due to misinterpreting the economic content of variables used in his model of economy. After a critical analysis of inconsistencies of the arguments and of semantic errors of the applied terms, conclusions concerning the theory of equilibrium are presented.

Key words: service, commodity, Gerard Debreu, theory of economic equilibrium

JEL classification: E50, E52.

НЕКОЛИКО НЕДОСТАКА ТЕОРИЈЕ ВРЕДНОСТИ ЖЕРАРА ДЕБРЕА И ЊИХОВЕ ИМПЛИКАЦИЈЕ

Апстракт

Од појава теорије вредности 1959. у раду Жерара Дебреа под насловом „Теорија вредности: аксиоматска анализа економске равнотеже“, који му је донела награду Шведске банке за економију 1983., математичка економија, која претендује да се сматра најчистијом научном теоријом економске равнотеже, добила је нови подстицај за ширење. Циљ овог рада је да покаже неке недостатке модела Дебреовог који омогућавају да се оспори његова економска валидност, због погрешног тумачења економског садржаја варијабли које се користе у Дебреовом моделу економије. Након критичке анализе неконзистентности аргумената и семантичких грешака које се тичу коришћених термина, презентовани су закључци о теорије економске равнотеже.

Кључне речи: услуге, роба, Жерар Дебре, теорија економске равнотеже.

¹ ryszardszewczyk@gmail.com

² This is a revised version of the paper published in Polish, titled O pewnych niekonsekwencjach modelu G. Debreu i ich skutkach. In: Państwo demokratyczne, prawne i socjalne. Studia społeczne, polityczne i ekonomiczne. Krakowska Akademia im. A.F. Modrzewskiego, Kraków 2014

Introduction

No commodity can emerge until a man makes an item or an action useful to somebody. Therefore, if a good or a service is offered in the market, there must be a producer of the good or a person who renders the service. In both cases we are facing a person who is to be called producer of the commodity. Commodities can only be a *result of producers' actions*.

Consumption of goods is an *ultimate end* of all economic processes. In the narrow sense consumption means a single *act* of using goods created in the production process in order to meet human needs and wants. In wider sense the term means a *process* of using an item for a certain period of time. Regardless of the scope of the term, there can be no doubt, however, that *no product may result from the process of consumption*; consumption is the *final end* of consumers' actions.

Definitions of terms used in the Debreu model

Out of the complete set of definitions, explanations and axioms as well as limitations which constitute the formal model of economy created by Debreu, only few are needed for our considerations. We use quotations in order to avoid possible misinterpretation (*italics original*).

“The first example of an economic *service* will be human labor. Its description is that of the task performed; thus one has the labor of a coal miner, of a truck driver, of a member of some category of teachers, of engineers, of draftsmen, of executives, etc. (all including any further specification necessary for a complete description). When one adds date and location one has again a well-defined *commodity*.” (Debreu, 1987, p.31)

“(…) a commodity is a good or a service completely specified physically, temporally, and spatially. (...) For any economic *agent* a complete plan of action (made now for the whole future), or more briefly an *action*, is a specification for each commodity of the quantity that he will make available or that will be made available to him, i.e., a complete listing of the quantities of his inputs and of his outputs. With one of the sign conventions of [subchapter] 2.3 an action is therefore represented by point *a* of *R'*.” (ibidem, p. 32)

“In the study of production, when one abstracts from legal forms of organization (...) and types of activity (Agriculture, Mining, Construction, Manufacturing, Transportation, Services,...) one obtains the concept of a *producer*, i.e., an economic agent whose role is to choose (and carry out) a production plan.” (ibidem, p.37)

“*Given a price system p, the jth producer chooses his production in his production set Y_j so as to maximize his profit. The resulting action is called an equilibrium production of the jth producer relative to p.*” (ibidem, p. 43).

Analyzing the terms

Despite numerous examples given by the Author in order to precisely explain the meaning of the terms used, neither the definition of the *producer* nor this of the *commodity* quoted above is clear. Although we know that commodities are all possible goods and all possible services, including labor services, well specified physically, temporally and spatially, we cannot be sure whose action is the source of the commodities and thus cannot unambiguously distinguish between a producer and a consumer as *economic* entities.

What we only know from the Debreu's definition is that *producer* is *an agent who chooses and implements a production plan*, where some commodities become inputs of the plan, and the others – outputs of the plan, with his end being profit maximization. Inputs are here represented by negative numbers whereas outputs – by positive numbers. Certain subjective and objective restrictions of the model are also discussed, but the Author avoids unambiguously labeling the result of actions undertaken by producers. Such a clear statement, that *commodities are the result of producers' action*, has never been declared. The reason becomes not clear until we get deeper into the definition and axioms concerning the term *consumer* and the nature of his activities.

Alike producer, the second economic agent, a *consumer*, is defined by the *role* he plays in the Debreu's economy. It may be a person or a group of people, i.e. a household, a corporate or any entity pursuing common aims. His role is to select and carry out such a consumption plan from the set of a priori possibly plans which best fits his preferences. There are also inputs of the consumption plans, that are represented this time by positive numbers, and the only one element of output, labeled as '*various kinds of labor performed*' represented by negative number. Again, however, the Author avoids unambiguously expressing what the inputs and outputs are. He writes: "Typically, the inputs of a consumption plan are various goods and services (related to food, clothing, housing, ..., dated and located), its only outputs are various kinds of labor performed (dated and located) (Debreu 1987, p. 51)."

There is of course no doubt that the inputs of each consumption plan are commodities (goods and services), but a fact, that Author has labeled the only element of output of all consumption plans as '*various types of labor performed*' proofs that he wanted to avoid using here the unambiguous defined term '*labor service*'. The reason is that he previously counted human labor among services, and services – among commodities. In the place under discussion, however, the term '*labor service*' did not really suit him.

Otherwise at least two serious problems would arise at once. Let us then consider them carefully.

The first problem concerns the meaning of the term *output*. Let us quote the original definition (Debreu 1987, p.30, italics original): "What is made available *to* an economic agent is called an *input* for him, what is made available *by* an economic agent is an *output* for him." The definition repeats only what everybody means while using the term *output*; it is *a result of a process*, which – eventually – may be made available to others. Conform to the definition then *various types of labor performed* which are made available *to* producers must be regarded as *a result of a process of consumption*. It should be interpreted then, that *consumption is the means* that makes people able to reach their *final end* which is *labor services* rendered to producers – conclusion that seems hardly to be defended.

More important, however, seems the second problem. If ‘*various types of labor performed*’ in a consumption plan were unambiguously labeled *labor services* and thus counted as *commodities*, then the question would arise as to why had providers of the commodities been not treated the same way as all other service providers classified in the Debreu economy to producers. Such a question becomes legitimate the more so that all services have been unconditionally counted as commodities, regardless the kind of entity who makes them available to buyers. Therefore, if every well-defined labor service was regarded as a commodity (as it really is), then all specific labor services should be always treated as an *output of the production plan* of the entity who renders the service, and – consequently – *all employees should be counted among producers*. Such conclusions, however, would undermine the theory built by Debreu. That is probably why he had chosen to get around the problems the way he did it.

In such a way, however, the Author did not manage to avoid falling into another trap. The trap constitutes the lack of possibility to distinguish *labor services* rendered by economic entities counted by him among producers from ‘*various kinds of labor*’ made available to others by consumers. Indeed, one could hardly discover for example a difference between mowing a lawn in a production factory or doing bookkeeping respectively by workers of the factory and the same activities performed and made available to the producer by one-men cleaning company vs. one-man accounting services firm. In either case the service is rendered by a person, brings the same result and counts as input, i.e. cost, of the production plan of this producer. Despite this, we face in the Debreu economy in the first case consumers whose services are treated as *outputs of consumption plans* of those persons, while in the second case the services become *outputs of production plans* of those persons respectively, since they are counted as producers this time. Due to the lack of criterion, decides the will of an arbiter. Such examples could be listed endlessly, with consequences discussed below.

Only the will of Debreu then decided that all but one services are a result of *a process of production* and thus belong to commodities, with the one being a result of a process of consumption, which thus belongs to ...what? Is there any product of consumption process that might be made available to somebody else?

There is also next reason to undermine relevance of the Debreu’s theory. As soon as we start considering production plans, we find that all services, without any exception, are treated by the Author as commodities, since each of them is represented in the model by a point in the commodity space R^l , in which both production and consumption plans are chosen and carried out by economic entities of the model (Debreu 1987, p. 32). It is confirmed once more, when the Author writes (Debreu 1987, p. 38): “Generally, inputs and outputs together contain only a relatively *small number of commodities*, in other words most coordinates of y_j are null; this corresponds to the fact that Y_j is, in general, contained in a coordinate subspace of R^l with a relatively small number of dimensions.” No doubt therefore may arise that also ‘*various kinds of labor performed*’ belong to commodities. Despite this as long as consumption plans are considered, labor services as an output of consumers’ actions do not count among commodities (they are labeled ‘*various kinds of labor performed*’), but as soon as we go on to production plans, they become commodities. We face thus rather strange situation where something is and at the same time is not what it is, and it depends only on the point of view of a theorist. This is thus another sign that there is something wrong with the system of Debreu’s economy.

I do not know whether and to what extent Debreu was aware of the problems discussed so far, but three features of terms (variables) used in his theory of value remain indisputable; contradiction with logic, arbitrariness of classification used by him in his model of economy, and the loose connection of the variables with their economic content.

Attempt to interpret the model

Seeking to clarify the growing doubt we reach the place in his work where Debreu explains the duality of roles an economic agent can play in the system. He uses there an example of a person who buys a house, a car, etc. for his own needs. According to the explanation such person should be treated as a producer, who buys that house, that car, etc. in order to sell its services to himself as a consumer (Debreu 1987, p. 51).

Letting aside the issue of common sense, which forces to ask how would then look the problem of profit maximization of such a ‘producer’, let us try to apply the same approach to consumers who work for a living, and to consider their actions from such dual perspective; as producers of their services, who sell them to other producers, with well-defined production plans from the one side, and as consumers with their well-defined consumption plans, from the other. If it really worked, doubts could be dispelled. We will do it considering an example like Debreu did it. Let our consumer-producer be for example a turner, who works as a regular employee with a lathe in a manufacturing company.

Our turner *as a producer* of turning services would carry out his production plan where quantities of inputs of the plan (conform to the convention with the negative sign) would be his outlays on what he needs in the role he plays, and the sole output of the plan (with a positive sign) – a well-defined turning service, dated and located. From the other side, while playing the role of the *consumer* our turner would carry out his consumption plan, in which inputs (with positive signs) would be goods and services necessary for him to live, and the sole output (with the negative sign) would be his labor services on the lathe.

Could our turner be regarded as a producer of turning services sold to his actual employer? The answer might be positive on the condition that we correctly specify the coordinates (inputs and outputs) of the vector of his production plan. We already know that the only element of output in this case would be a well-defined turning service (dated and located). The question is, however, what would an input of his production plan consist of, or – to use the normal language of economics – what could be counted as his production outlays?

None of consumption goods could be counted here, despite the fact that at least some of them – let’s call them food in general – provide our turner with energy required for the ‘production’ of turning services. The reason is that – in line with convention of the Debreu theory – they have to appear as inputs in our turner’s consumption plan. Putting them also in the production plan would cause a double counting of the same goods, what cannot be taken into account.

It is also not possible that the production plan of our turner had a positive output coordinate, in our case a value of turning service, and all coordinates of inputs were equal zero, because it stays in contradiction with the assumption of the *impossibility of*

free production (Debreu, 1987, p. 40). Each of cases considered thus far evidence then that our turner *cannot be treated from the dual perspective*; as a producer of turning services and a consumer at the same time.

Taking this into account we have to consider only one more case of treating economic entity from the dual perspective similar to this proposed by the Author (Debreu, 1987, p. 51), when a person buys a house to sell housing services to himself. Let us consider then what would happen if we assumed that our turner-producer buys the turning service from himself as a consumer to sell it to the factory where he works. In such a case, however, there is no doubt that this would be *the same service* that had to be counted once with negative sign as an input of the production plan of the turner-as-the-producer, and once with positive sign, as an output of the consumption plan of the turner-as-the-consumer; its value would thus cancel out to zero, and the whole procedure of treating the turner from the dual perspective would have not any sense.

The cases analyzed above prove that the dual perspective of treating economic agents in the model of Debreu economy may not be applied to employees selling labor services. Such a perspective is there permitted by Debreu only to those consumers who buy *tangible goods* like houses, cars, etc. in order to sell ‘services’ of the goods *to themselves*. Let common sense of the approach be still aside.

Objections to the model

Despite our efforts to find such interpretation within the assumed axioms of the Debreu economy that could be accepted without falling into contradiction with the rules of logic, doubts concerning the nature of labor services in the system did not dissipate. We have to conclude therefore that the model assumes implicitly two types of services. First of them are services rendered by producers to other producers. They fully conform to the axioms assumed for commodities in the Debreu economy, and so they are treated there. The second type of services are those rendered to producers by consumers. Despite the fact that their economic content does not differ at all from the former ones, they are not treated in the model as commodities like all other results of people’s activities, but – against all logic – as a result of the process of consumption. Nevertheless, they become commodities as soon as they enter as inputs into production plans.

It has to be stated then, that we face a dual perspective in treating economic agents twice in the Debreu economy. Once, declared explicitly vis-à-vis some consumers, and once – implicitly for one specific type of commodities, namely for labor services. In some cases labor services are counted as commodities, and in the others – not. The most important, however, is that decision of whether or not such a dual perspective is allowed depends not on any objective features of the service but only on *arbitrary decision* of an external subject who includes it either in the one or in the other *set* of services. Assessing it from purely formal side, the division of services made in such a way does not meet any of the three logical rules of correctness, i.e. *condition of explicitness*, *condition of mutual exclusivity* and *condition of exhaustiveness*. For these reasons, it cannot be considered as proper and consistent.

There is also another argument that calls for the fallacy of the Debreu economic model – the fact that the system allows *free manipulation* with the results. As already

mentioned above, the model allows a dual perspective of treatment of economic entities, once as producers who buy houses, cars, etc. in order to sell their services, and at the same time as consumers who buy those services from themselves. Debreu mentioned only houses and cars *as examples* of such a dual treatment, without considering possible effects of such approach. However, conform to the model, every case of treating an *additional* entity in such a dual perspective causes – *ceteris paribus* – a growth of *total production* of that economy *by the net worth* of the account of this entity. This raises the logically legitimate question: why should such a dual treatment of entities be limited only to consumers of durables like cars, houses, etc.? Bringing it to the absurd, we could extend it – in line with the model – to toothbrushes, pots, shoelaces and any other items that people could buy as producers in order to sell the ‘services’ of these items to themselves as consumers. Who and where will put a borderline? And why there if indicators of economic efficiency could be improved in such a way?

Beyond the model

It is not only economic flaws that contains the model of Debreu. There are also some interesting findings in it. The most important, and very relevant to economic theory is the definition of commodity. Let us quote it again (Debreu 1987, p. 32): “A commodity is a good or a service completely specified physically, temporally and spatially.” It is not only physical features of goods and services, but also place and time they are made available to buyers that make different commodities. The last two features cause that the same good offered at another place and/or time makes different commodities. This can be a strong argument among other things against Marxian ideology.

Despite his efforts Debreu did not also succeed to evidence that different kinds of human labor can be separated from labor services. However if we reject classical approach that people sell their *labor* to their employers, and if we accept that what is sold in such cases are different *labor services* i.e. well defined commodities, specified physically, temporally and spatially, things start looking different way. We get then a system where there is indeed only *one type* of economic agents *independent on each other*. The agents are *people* who play the role of *producers of commodities* in order to take part in market transactions. Some of them make and sell *goods*, the others – all possible kinds of *services*, specific labor services included. All those agents do what they do in order to reach *their own end*, common to all of them. *The end is satisfying their needs with goods and services obtained through the market exchange*.

Exactly due to the fact that there are only independent people in the real economy who offer on the market their own commodities (goods and services) to each other in order to satisfy their needs, there is no point in searching for a *state of equilibrium* in the sense given to the term in classical economics, i.e. the state in which value of final goods produced in a given period is equal to incomes of the so-called production factors involved in this process. This is proven indirectly by the inconsistency of the Debreu model of economy which has been evidenced in this paper. If Debreu had followed consequently his definition of commodity, he might have come to conclusions mentioned above.

In such circumstances there is only one state of equilibrium possible and always present; a state in which a value of goods and services sold by their producers within

some period is exactly equal to the value of goods and services purchased in the same period by their customers – the other producers. It is a Say's equilibrium.

Conclusion

It is reasonable, therefore, to ask the question as to what actually proved Debreu in his Theory of value? We get the right answer only if we stop penetrating *economic content* and meaning of the axioms of the system, that is if we focus only on the syntax and formalisms of the so-called Debreu economy. Then we observe what cannot be seen when looking for the content, i.e. when semantics comes into consideration.

Looking at the Debreu economy system from the purely formal side and *disregarding all conditions and economic interpretations*, we can see that at the beginning we face two *independent* of each other classes (sets) of sets of coordinates of points belonging to the space R . Sets belonging to the first class, the set x , are marked with the index $i = 1, \dots, m$, it is therefore the set x of sets x_i . And those sets that belong to the second class, the set y , are marked with the index $j = 1, \dots, n$; We then get the set y of sets y_j .

Assumptions prescribe that each point x_i of the space R had a number of positive co-ordinates and some non-positive. Another assumptions restrict vectors of co-ordinates of any point y_j ; there cannot be positive co-ordinates, if there is not at least one negative. Each of the points x_i found itself in the set x because out of all possible sets marked with the same index (belonging to the set X_i) there is no other one that would satisfy certain preferences concerning vector p . On the other hand each vector of co-ordinates of points y_j of the space R found itself in set y because the scalar product of its elements by the respective elements of the vector p is the highest among all other possible sets belonging to the set Y_j .

Each co-ordinate of the space R , indexed by $h=1 \dots l$ is represented by a real number p_h , therefore we got vector p consisting of certain numbers, the third set of elements of the Debreu system that plays a key role in it. The last set of the model, set ω , consists of a priori given real numbers – co-ordinates of point ω in the space R .

As a supplement to the above formal constraints for sets x and y , Debreu system contains a number of other conditions and properties which are relevant only from the axiomatic point of view. It is to be strongly stressed, that all sets listed above are independent on each other.

For the four sets G . Debreu has proved unambiguously and in line with all formal conditions of mathematical reasoning that there exists a set p such that every x_i minimizes scalar product $p * x_i$ and every y_j maximizes scalar product $p * y_j$, and the sum $x-y-\omega = 0$. Although formally absolutely correct, the proof and the entire model of economy proposed by Debreu *has nothing common with economics* due to misinterpreting the *economic content* of variables used.

There is no doubt that mathematics is the queen of sciences. This does not mean, however, that any use of it is reasonable. Where we face causal relationships and where content and meaning of terms matters, syntax must give way to semantics. And this is exactly the case of economic considerations. Otherwise one can go astray.

References

Debreu, G. (1987). *Theory of Value. An Axiomatic Analysis of Economic Equilibrium*. New Haven and London, Yale University Press.

Endnotes

1. The idea that *the agents are people who play the role of producers of commodities in order to take part in market transactions* is one of the axioms which personalist economics is founded upon, a new discipline which has been developed by the author of this paper.