MODELLING THE VOLATILITY OF THE GLOBAL GOLD PRICE BY APPLYING THE ARCH/GARCH MODELS

Abstract

The purpose of this article is to analyse the trend of global gold price yields and volatility with the aim of making effective financial decisions about investments in this precious metal, by applying and comparing the results of ARCH and GARCH competing models, using the lowest values of the chosen information criteria. ARCH and GARCH models are intended for the time series’ analysis, with the observed instability of the conditional variance. Gold usually has the function of a safe haven, also serving as a warrantor of monetary stability, especially in times of crisis. On the example of 1,151 daily observations on the gold global price and yield rates, the E-GARCH model was applied, the results of which suggest investors to be cautious in their decision-making since the impact of negative shocks (losses) on yield volatility has a strong long-memory effect, making this investments highly risky, especially in bad economic circumstances.

Key words: gold, volatility, heteroscedasticity, ARCH/GARCH models, safe haven, uncertainty.

JEL classification: C22, C52, G11, G15

МОДЕЛИРАЊЕ ВОЛАТИЛНОСТИ ГЛОБАЛНЕ ЦЕНЕ ЗЛАТА ПРИМЕНОМ ARCH/GARCH МОДЕЛА

Апстракт

Сврха овог рада је да изврши анализу тренда приноса и волатилности глобалне цене злата са циљем доношења ефективних финансијских одлука о улагањима у овај драгоцени метал, применом и поређењем резултата ARCH и GARCH конкурентних модела, коришћењем највиших вредности уобичајених информационих критеријума. Модели ауторегресионе условне хетероскедастичности (ARCH модели) и уопштене ауторегресионе условне хетероскедастичности (GARCH модели) су напрегнути анализи временских
Introduction and Theoretical Background

Global economic shocks such as financial and economic crisis, geostrategic turmoil, wars, environmental problems and climate change, and more recently the current pandemics and other global health issues faced many economic sectors with pronounced and challenging obstacles. One of them is the global financial sector, as a combination of the world markets of securities, money and gold, which reacted to recent global shocks in various ways in diverse parts of the world. In addition, the global commodity sector also experienced unique and so far unprecedented reactions to current global turbulences, stimulated by governmental fiscal and monetary policy measures, cautious behaviour of investors and consumers, anti-crisis regulations of central banks, as well as by general welfare policy measures. Unlike the global financial market, on which these adverse shocks have led to general uncertainty, mistrust and reduced investment, the global commodity market has seen a rapid rise in commodity prices, with the exception of the gold price that has recently experienced a stagnant and/or declining trend in short run (Sheth, Sushra, Kshirsagar, & Shah, 2022).

In the times of financial crises, the threat to the entire financial system and of defaults on debts, there is also a crisis in the functioning of financial institutions, which usually spreads from one to the other ones through the domino effect. In such circumstances, serious consequences usually occur, such as failures of paying liabilities, financial institutions’ bankruptcy waves, reduced liquidity due to reduced credit transfers, decreased money supply, reduced confidence in financial assets, and naturally negative effects on the economy itself. Therefore, it is more important than ever to explore ways in which investors could protect themselves from the risk of a possible financial market crash (Rosskopf, & Rutersten, 2020, p. 2). In such circumstances, investors themselves are obliged to change the structure of their financial portfolios, from shares and other securities to safer forms of investment such as gold. Therefore, determining the transmission channels of shocks and volatility from one market to another is an important issue for all market participants (Gencer, & Musoglu, 2014, p. 88). In such conditions, gold is usually perceived as the last land of resort and the safest form of investment, especially in times of crises and economic uncertainty. As a result, there is usually a marked increase in demand for gold. Gold still represents the most efficient form of commodity investment since it has retained certain attributes that set it apart
from other types of commodities. It is also perceived as a special, differentiated type of asset because it brings above-average returns in various time frames, playing the role of the store of value and unique diversifier in asset protection. At the same time, it is also characterized by lower price volatility and high liquidity. Therefore, even during the pandemic year 2021, the weight of gold in commodity indices gradually increased, while its price volatility remained stable, despite the stocks, bonds and other assets’ price instability (World Gold Council, 2021, p. 1).

Due to its favourable physical and chemical properties, durability and relative rarity, gold has taken a special place in the international commodity exchange and figures as a unique measure of value. In addition to its market functions, gold also serves to ensure the international liquidity of contemporary countries and represents an indispensable part of their monetary reserves. The gold reserves ensure the external stability of currencies and maintaining the liquidity of countries, while these functions are usually achieved through its sale on the international gold market, either through auctions, classical sales or through its pledge to international financial institutions with the aim of obtaining loans (Jakšić, 2006, pp. 897-898). Gold reserves secure the stability of the monetary system, while its stability stems from the fact that, unlike paper money, it cannot be printed in unlimited quantities. Its value also cannot be affected by various movements such as changes in interest rates, changes in prices of securities and other turbulences in the financial and commodity markets (Božić, 2019). From the point of view of the central bank as an institutional investor, gold usually serves as a form of protection against inflation effects in the long term, while in the structure of foreign exchange reserves it reduces the risk of interest rates and contributes to preserving the value of investments (Brakočević, 2022). Apart from monetary functions, gold also serves as a means of private accumulation of wealth in conditions of currency, financial and general economic instability.

Many studies indicate that gold has a function of safe haven that protects investors from various risks in times of financial crisis. In that case, gold occurs as a form of property that protects them from sudden changes in securities’ prices and bond yields, but at the same time, it is not endangered by the eventual market crash. Investigating constant and time-varying relationships among returns on stocks, bonds and gold in the United States (US), Great Britain and Germany, Dirk G. Baur and Brian M. Lucey (2009) concluded that gold represents a hedge and a safe haven in extreme stock market conditions, within a short period of up to 15 trading days. This function of gold generally affects the stability and resilience of financial markets since it reduces the risks and losses of investors in the short term, in times of general uncertainty. However, they conclude that unlike investing in stocks and other securities, gold is still not a good alternative to safer bond investments. At the same time, investors usually buy gold on days of extremely negative returns, while selling it in the market after the volatility has decreased and its price has stabilized.

Juan C. Reboredo (2013, p. 2665) also found a positive, statistically significant and symmetric interdependence between gold and the depreciation of US dollar, indicating that gold can act as a hedge against the US dollar exchange rate volatility, but also as an effective and safe resort for all investors. In addition, its utility in investment portfolios is reflected in the advantages of asset diversification and reduction of investment risk. However, unlike previous research, there are also opposite findings (Coudert, & Raymond,
2010, p. 4) that analysed the relationship among the price of gold, the price of shares and the main stock market indices in the G7 countries. They state that in the short term, during periods of recession, there is no correlation between the price of gold and stocks, which may qualify it as a weak safe haven. On the other hand, in the somewhat longer term, negative relationships between the price of gold and some stock market indicators are visible, which does not allow for the construction of a fully protected portfolio that would be resistant to all financial and economic crises. It further points to the riskiness of investing in gold, even during the period of crises. Finally, there are also studies that point to certain disadvantages of gold in the role of investment commodity. First, its real intrinsic value is difficult to estimate given the fact that the market price of gold is generally not determined by its use value. Second, gold, unlike classic securities such as stocks and bonds, does not have the ability to pay out concrete returns. Thirdly, gold is also characterized by relative inelasticity of supply because its research, discoveries and extracting is a time consuming processes. Fourth, the demand for gold is countercyclical, which all points to its role as a reliable store of value, even in times of crisis and general uncertainty (Rosskopf, & Rutersten, 2020, p. 3).

The World Bank states that after the emergence of the Ukrainian crisis, in the first quarter of 2022 there was a growth of its Precious Metals Index by 4% due to the growth of global inflation, the increase in investment demand and the increased desire of market participants towards safe investments. In the same period, the global price of gold increased by 4.3%, along with the growth of inflation, nominal interest rates and persistent geopolitical tensions. However, from the beginning of the second quarter, gold prices and yields began to decrease, affecting the reduction of investment flows as a key driver of the price of this precious metal. The War in Ukraine undoubtedly affected the global gold market, considering that Russia is the world’s second largest producer and exporter of this metal, accounting for about 10% of its global product. At the same time, Russia is also one of the world’s largest buyers of gold, which is supported by the fact that the Central Bank of Russia has not stopped buying it from domestic producers with the aim of increasing its monetary reserves. The World Bank expects a further modest increase in the price of gold during 2022, as well as its decline during the next year, 2023, by as much as 10% due to the expected measures of stricter monetary policy in developed countries and a further increase in interest rates (World Bank, 2022, p. 47). Finally, in its latest report the World Gold Council states that unlike the volatility of various forms of assets and goods, during 2020, as the year of great health uncertainties, gold still maintained its price stability. This stems from its role of a unique asset diversifier in turbulent and uncertain markets, its economic importance and liquidity, as well as the volatility and vulnerability of other commodities such as silver, copper, platinum and other metals, livestock, timber and crude oil (World Gold Council, 2021, pp. 2-3).

The purpose of this paper is to analyse the trend of the gold yields and price volatility with the aim of making effective financial decisions about investments in this precious metal, by applying and comparing the results of Autoregressive Conditional Heteroscedasticity (ARCH) and the Generalized Autoregressive Conditional Heteroscedasticity (GARCH) competing models, using the lowest values of chosen information criteria. The article starts from the null hypothesis that gold represents an efficient safe haven for all investors, especially in times of economic turbulence and general uncertainty. The next part of the article refers to the description of the data
used and the research methodology, followed by a section on the discussion of the results obtained by testing different ARCH/GARCH models and the evaluation of the proposed specification. The fifth part of the article is devoted to the evaluation of the statistical quality of the proposed E-GARCH(1,1) model with GED (Generalized Error) distribution, while its last part summarizes the presented research results and gives specific recommendations to global investors.

**Data, Research Design and Methodology**

In the conducted research, we used a closing price of gold since it reflects all buying and selling activities during one trading day. We collected daily data on gold prices per ounce in US dollars from the Internet portal *Investing.com*. The research covered the period from December 30 2017 to June 1 2022, which makes a total of 1,151 daily gold price observations and 1,150 daily yield/loss rates. Statistical and econometric data processing was carried out using EViews8 software (Copyright © 1994–2013 IHS Global Inc.). Quantitative modelling of yield oscillations was conducted using ARCH and three variants of GARCH competing models (two of which are asymmetric GARCH models: T-GARCH and E-GARCH). Figure 1 indicates the trend of the gold price expressed in US dollars in the observed period.

*Figure 1: Trend of the gold price in US dollars in the observed period*

![Figure 1: Trend of the gold price in US dollars in the observed period](image)

Testing the stationarity of the time series was carried out using the Augmented Dickey-Fuller (ADF) test, while its non-stationarity was removed by transforming the data by differentiating of the first order of their logarithmic values. An increase in the price of gold represents a capital gain, while its decrease is a capital loss. After this step, the testing of the hypothesis about the normal distribution of the empirical data of the thus obtained stationary variable was approached.
Figure 2: Trend of the logarithmic value of the gold return/loss in the observed period

Source: authors’ calculation

Figure 2 clearly shows the high oscillations in the trend of the daily gold yield rate, while the presence of a large number of extremely positive and extremely negative values was observed, which made it necessary to test the hypothesis about the presence of a normal distribution in the empirical data distribution. Excess kurtosis coefficient (leptokurtosis) explains the observed extreme values from Figure 2. Testing for a normal distribution was carried out using the Jarque-Bera test, based on which we rejected the null hypothesis about the existence of a normal distribution and accepted the alternative hypothesis that the time series of the gold yield rate was not normally distributed. In addition, the constructed Q-Q plot confirmed the deviation of the empirical values from the normal Gaussian distribution, with the conclusion that the presence of extreme values is conditioned by the observed leptokurtosis. After this step, the classical ordinary least squares (OLS) regression was performed to test the clustering of volatility in the residuals, that is, to confirm the presence of their heteroscedasticity.

Figure 3: Residual trend of logarithmic values of gold return/loss in the observed period

Source: authors’ calculation

Figure 3 confirms the presence of volatility clusters, i.e. time-varying volatility (heteroscedasticity), when the volatility of the time series is grouped in time. Small
volatility fluctuations are often followed by periods of low fluctuations, while periods of large volatility fluctuations are often followed by periods of high fluctuations. The applied test confirmed the presence of the ARCH effect, as well as the observable volatility clusters. Therefore, the statistical and graphical confirmation of the presence of heteroscedasticity justified the need for testing of this problem with some of the models from the ARCH family.

**E-GARCH(1,1) Model**

Nelson (1991, pp. 347-370) presented the exponential or E-GARCH model that allows for asymmetric shocks in volatility. More specifically, he proposed the E-GARCH model with the aim of overcoming the GARCH model shortcomings in handling of financial time series, and especially with the goal of acknowledging and enabling asymmetric effects among positive and negative financial instrument returns. Nelson extended the standard GARCH model by integrating the leverage term, when negative past shocks have a greater impact on future volatility than positive past shocks of the same intensity. In that manner, the asymmetric E-GARCH (1,1) model provides evidence of a leverage effect.

Due to the mentioned asymmetry, the model includes the leverage effect, which is usually interpreted as a negative correlation between negative returns on lag and volatility. Empirical analyses of the leverage effect encouraged the development of a model with asymmetric volatility, which arises due to positive and negative shocks. The leverage effect was first noticed by Black (1976, pp. 177-181), and it is best shown by the news impact curve, which was later analysed in more detail by Pagan and Schwert (1990, pp. 267-290). This curve shows how future volatility reacts to good or poor news – in asymmetric GARCH models, the curve appears to be asymmetric. This leads to negative shocks that affect volatility more strongly than positive ones of the same strength (Engle, & Ng, 1993, pp. 1749-1778). It has also been empirically established that the error distribution affects the estimate of the asymmetry parameter $\gamma$, but that it is not the case with the volatility of the other parameters (Rodríguez, & Ruiz, 2012, p. 661). This model can be represented by the following mathematical expression:

$$E\text{-GARCH}(1,1): \log(\sigma_t^2) = \omega + \beta \log(\sigma_{t-1}^2) + \gamma \frac{\epsilon_{t-1}}{\sigma_{t-1}} + \alpha \left[ \frac{|\epsilon_{t-1}|}{\sigma_{t-1}} - \sqrt{\pi} \right]$$  \hspace{1cm} (1)

where

- $\sigma_t^2$ is the conditional standard deviation, i.e. the volatility at time $t$,
- $\omega$ is the constant term being evaluated,
- $\beta$ is the parameter of the GARCH component model,
- $\alpha$ is the parameter of the ARCH component model,
- $[\epsilon_t]$ are the standardized residuals: $[\epsilon_t] \sim $ i.i.d.; $E[\epsilon_t]=0$ and $\text{VAR}[\epsilon_t]=1$, while
- $\gamma$ is the leverage coefficient, i.e. the asymmetric component of the model.

The coefficients $\omega$, $\alpha$ and $\beta$ of the conditional variability equation must satisfy the following requests: $\alpha + \beta < 1$, as well as $\omega > 0$, $\alpha \geq 0$ and $\beta \geq 0$. These requests ensure the firmness of the unconditional variance.
The parameter $\gamma$ measures the asymmetry of leverage effect, so the E-GARCH model allows testing for asymmetry. When $\gamma = 0$, the model is symmetric, that is, positive and negative shocks affect the volatility of the series of asset returns in an equal way. If the leverage coefficient is negative ($\gamma < 0$), than positive (affirmative) news from the market creates less volatility than negative shocks. On the other hand, if the leverage coefficient is positive ($\gamma > 0$), than positive shocks have a greater impact than the negative ones. At the same time, the direction of the yields’ change also influences volatility. The sum of parameters $\alpha$ and $\gamma$ shows the effect of positive shocks on the series of asset returns. In the case of negative value of leverage coefficient $\gamma$, this sum will be less than the value of $\alpha$, and vice versa, if the value of $\gamma$ is positive, this sum will be higher than the value of parameter $\alpha$ (Kovačević, 2016, pp. 26-30).

### Testing for Different ARCH/GARCH Models and Evaluation of the Proposed Specification

Using the lowest value of criteria such as AIC (Akaike information criterion), SIC (Schwartz information criterion) and HQC (Hannan–Quinn information criterion) (Table 1), E-GARCH(1,1) model with the Generalized error (GED) distribution was selected as the optimal model with the lowest values of the mentioned information criteria.

**Table 1: Information criteria of different ARCH/GARCH models with different distributions obtained by EViews software**

<table>
<thead>
<tr>
<th></th>
<th>ARCH</th>
<th>GARCH(1,1)</th>
<th>T-GARCH</th>
<th>E-GARCH(1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Distribution:</strong> Normal Gauss distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>2.454620</td>
<td>2.440517</td>
<td>2.428690</td>
<td>2.429448</td>
</tr>
<tr>
<td>SIC</td>
<td>2.485344</td>
<td>2.458074</td>
<td>2.450636</td>
<td>2.451393</td>
</tr>
<tr>
<td>HQC</td>
<td>2.466218</td>
<td>2.447144</td>
<td>2.436975</td>
<td>2.437732</td>
</tr>
<tr>
<td><strong>Distribution:</strong> Student’s t distribution</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>2.392737</td>
<td>2.369304</td>
<td>2.364119</td>
<td>2.36596</td>
</tr>
<tr>
<td>SIC</td>
<td>2.427850</td>
<td>2.391249</td>
<td>2.390454</td>
<td>2.391931</td>
</tr>
<tr>
<td>HQC</td>
<td>2.405991</td>
<td>2.377588</td>
<td>2.374060</td>
<td>2.375537</td>
</tr>
<tr>
<td><strong>Distribution:</strong> GED – Generalized Error</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>2.385924</td>
<td>2.367477</td>
<td>2.363024</td>
<td><strong>2.362086</strong></td>
</tr>
<tr>
<td>SIC</td>
<td>2.421037</td>
<td>2.389422</td>
<td>2.389359</td>
<td><strong>2.388421</strong></td>
</tr>
<tr>
<td>HQC</td>
<td>2.399178</td>
<td>2.375761</td>
<td>2.380965</td>
<td><strong>2.372027</strong></td>
</tr>
<tr>
<td><strong>Distribution:</strong> Student’s with fixed df</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>2.406552</td>
<td>2.382837</td>
<td>2.376763</td>
<td>2.378109</td>
</tr>
<tr>
<td>SIC</td>
<td>2.437276</td>
<td>2.400394</td>
<td>2.398709</td>
<td>2.400055</td>
</tr>
<tr>
<td>HQC</td>
<td>2.418150</td>
<td>2.389465</td>
<td>2.385047</td>
<td>2.386393</td>
</tr>
<tr>
<td><strong>Distribution:</strong> GED with fixed parameter</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AIC</td>
<td>2.397482</td>
<td>2.379308</td>
<td>2.371065</td>
<td>2.372070</td>
</tr>
<tr>
<td>SIC</td>
<td>2.428206</td>
<td>2.396864</td>
<td>2.393011</td>
<td>2.394016</td>
</tr>
<tr>
<td>HQC</td>
<td>2.409079</td>
<td>2.385935</td>
<td>2.379349</td>
<td>2.380354</td>
</tr>
</tbody>
</table>

*Source: authors’ calculation*
After this step, the authors approached to the parameterization of the chosen E-GARCH(1,1)-GED model. When estimating the parameters to maximize the likelihood function, 18 iterations were required for the process to converge. The value of coefficient $C(4)$ ($C(4) = -0.050626$), that indicated the existence of a Leverage effect, was negative and statistically significant ($p = 0.0094 < 0.05$). When the Leverage effect is negative, there is a correlation between past returns and future return volatility. In other words, a decrease in yields leads to an increase in volatility. Positive shocks have a smaller impact on the conditional variance than negative news or shocks. Good news, positive shocks or errors generate less volatility (variance) than bad news (for example, a decrease in GDP, inflation, unemployment, pessimistic forecasts, etc.).

**Evaluation of the Statistical Quality of the Proposed Model**

Testing for the statistical quality of the proposed model was performed by testing heteroscedasticity, serial correlation and normal distribution of residuals.

*Table 2: Heteroscedasticity test results of the residuals of the proposed E-GARCH(1,1) model with GED distribution, obtained by EViews software*

<table>
<thead>
<tr>
<th>F-statistic</th>
<th>0.131672</th>
<th>Prob. F(1,1147)</th>
<th>0.7168</th>
</tr>
</thead>
<tbody>
<tr>
<td>Obs*R-squared</td>
<td>0.131887</td>
<td>Prob. Chi-Square(1)</td>
<td>0.7165</td>
</tr>
</tbody>
</table>

*Source: authors’ calculation*

The value of Prob. Chi-Square(1) was 0.7165 and was greater than 5%, so the $H_0$ that there was no ARCH effect in the residuals of the proposed model was accepted.

The conducted analysis of the ordinary and partial correlogram (AC and PAC) of the residuals showed that p-values of the Q-statistics on all lags were greater than 5%, based on which the $H_0$ that there was no autocorrelation in the residuals of the model was accepted. Based on the received estimates of this coefficient, we conclude that the residuals did not have an autoregressive structure of variability, that is, that there was no serial correlation in the residuals. The value of the Ljung-Box Q-statistics (Ljung, & Box, 1979, pp. 265-270) for the order of the correlation coefficient of 30 shown on the correlogram was 18.890, with a value of Prob* ($p = 0.942$), which was greater than 5%, as a result of which the null hypothesis that in the residuals of the observed time series was no serial autocorrelation was accepted. The tabular value of the Ljung-Box Q-test for the order of the correlation coefficient of 30 amounted to $\chi^2_{30,0.05} = 43.773$ and it was higher than the obtained value (18.890). Based on mentioned results of the conducted Q-test, we concluded that the null hypothesis that in the model time series of residuals there was no autocorrelation of order 30 could not be rejected.

Testing the hypothesis about the normal distribution of the residuals of the proposed model was performed using the Jarque-Bera statistical test. The value of JB statistics was 199.0835, while its p-value of $p = 0.0000$ was less than 5%, which is why the null hypothesis was rejected and the $H_1$ that the residuals of the model were not normally distributed was accepted.
The absence of heteroscedasticity and serial correlation of model residuals constitute two of the three statistical quality requirements of the proposed model. The only shortcoming of the model that prevents the conclusion that the model is completely statistically valid and qualitative is the absence of a normal distribution of residuals. The obtained results nevertheless point to the conclusion that the model can be accepted as valid and applicable when modelling the trend of the dependent variable, because it fulfils two of the three criteria of statistical quality.

Concluding Remarks

The paper confirmed the presence of variation in the conditional variance of the gold price returns’ time series in the world market. It was determined that volatilities are not constant over time, and that the analysed time series has the property of heteroscedasticity, that is, it possesses time-varying volatility that contributes to the excess kurtosis of the yield distribution (leptokurtosis of the empirical distribution). Investors are exposed to the risk of the gold price volatility on the world market, and they must integrate the observed instability of the variance into models that require the assessment of the dynamics of the variance by using models from the ARCH/GARCH family.

By quantitatively modelling gold yields’ volatility using ARCH and three variants of GARCH models with different distributions (two of which are asymmetric GARCH models: T-GARCH and E-GARCH(1,1)), and based on three information criteria, E-GARCH(1,1)-GED model was chosen as the optimal one since it realized the lowest information criteria values. The model is statistically valid and as such applicable when modelling the volatility of the price of gold on the world market. In addition, the proposed asymmetric E-GARCH(1,1)-GED model confirms the assumption that the impact of negative shocks (losses) is stronger than the impact of positive shocks (returns), and includes the Leverage effect in situations when negative information (losses) condition higher volatility in the following time interval in relation to positive information (yields).

In accordance with the obtained results, investors are recommended to act with special caution when making investment decisions, because the impact of negative shocks (losses) on yield volatility has a strong long-memory effect. Based on the obtained results, it can be said that due to the observed asymmetric volatility of gold yields, gold can rather be considered highly risky, especially in bad economic circumstances, than a safe alternative investment destination. Accordingly, based on the research results, we can reject the initial hypothesis that gold represents an efficient and safe investment alternative, especially in times of economic instability. Therefore, consideration of some other, less volatile and more reliable forms of investment could be the subject of further empirical research.

References


