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Okun's Law and the Chaotic Economic Growth Model: EU

Окунов закон и хаотични модел економског раста: ЕУ

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Сажетак: Основни циљ овог рада јесте приказивање релативно једноставног модела економског раста, који може генерисати стабилну равнотежу, циклусе или хаос, у зависности од вредности параметара. Оцењен хаотичан модел економског раста показује стабилне и монотоно растуће вредности реалног бруто домаћег производа у ЕУ у периоду 1990–2014. године.
Кључне речи: економски раст, модел, Окунов закон, хаос.

Abstract: The basic aim of this paper is to provide a relatively simple chaotic economic growth model that is capable of generating stable equilibria, cycles, or chaos depending on parameter values. The estimated chaotic economic rate growth model shows stable but monotonically increasing values of the real gross domestic product in EU in the period 1990–2014.
Keywords: economic growth, model, Okun's Law, chaos.

Introduction

Deterministic chaos refers to irregular or chaotic motion that is generated by nonlinear systems evolving according to dynamical laws that uniquely determine the state of the system at all times from a knowledge of the system's previous history. Chaos embodies three important principles: (i) extreme sensitivity to initial conditions ; (ii) cause and effect are not proportional; and (iii) nonlinearity. Chaotic systems exhibit a sensitive dependence on initial conditions: seemingly insignificant changes in the initial conditions produce large differences in outcomes. This is very different from stable dynamic systems in which a small change in one variable produces a small and easily quantifiable systematic change.

Chaos theory started with Lorenz's (1963) discovery of complex dynamics arising from three nonlinear differential equations leading to turbulence in the weather system. Li and Yorke (1975) discovered that the simple logistic curve can exhibit very complex behaviour. Further, May (1976) described chaos in population biology. Chaos theory has been applied in economics by Benhabib and Day (1981,1982), Day (1982, 1983) , Grandmont (1985), Goodwin (1990), Medio (1993), Lorenz (1993), Jablanović (2011, 2012, 2013, 2014), Puu, T. (2003), Zhang W.B. (2006) , etc.

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1. The chaotic economic growth model

Okun's (1962) paper regarding the unemployment – output relationship considers the measurement of potential output. Okun believed that the potential output should not be defined as the maximum output the economy could produce. Instead, he argued that the potential should be measured at full employment, which he characterized as the level of employment absent inflationary pressures.

We can write the Okun's Law in this form :

$$(Y_t - Y_p) = -\alpha \ (u_t - u_n) \tag{1}$$

where $\alpha > 0$. Namely, deviations of the real output from its natural level are inversely related with deviations of unemployment from its natural level. When unemployment increases above its natural level ($u_t > u_n$) real output tends to decrease below its natural level and vice versa.

Further, it is supposed:

$$u_n = \beta \ u_t \tag{2}$$

$$Y_p = \gamma \ Y_t \tag{3}$$

Where : u_t – unemployment rate ; u_n – natural rate of unemployment; Y_t – real output; Y_p – potential output; α – the coefficient which explains relation between deviations of real output from its natural level and deviations of unemployment from its natural level ; β – the coefficient which relates unemployment rate and natural rate of unemployment; γ – the coefficient which explains relation between real output and potential output.

Further, it is supposed that the growth rate of unemployment at time t should be proportional to 1 - u_t (the fraction of the labor force that is not used up by the unemployment at time t). Assuming that the unemployment is restricted by the labour force, the growth of the unemployment rate should change according to the following equation, after introducing a suitable parameter μ [Jablanović, 2011].

$$\frac{u_{t+1} - u_t}{u_t} = \mu \left(1 - u_t\right) \tag{4}$$

Where: u_t – unemployment rate; μ – the coefficient which explains relation between unemployment rate, growth rate and the fraction of the labor force that is not used up by the unemployment at time t

Solving (4) we obtain:

$$u_{t+1} = (1+\mu)\, u_t - \mu\, u_t^2 \tag{5}$$

By substitution one derives:

$$Y_{t+1} = (1+\mu)\, Y_t - \left[\frac{\mu(1-\gamma)}{\alpha(\beta-1)}\right] Y_t^2 \tag{6}$$

Further, it is assumed that the current value of the real output is restricted by its maximal value in its time series. This premise requires a modification of the growth law. Now, the real output growth rate depends on the actual value of the real output, Y, relative to its maximal size in its time series Y^m. We introduce y as $y = Y/Y^m$. Thus y ranges between 0 and 1. Again we index y by t, i.e., write y_t to refer to the size at time steps t = 0, 1, 2, 3... Now the real output growth rate is measured as

$$y_{t+1} = (1+\mu)\, y_t - \left[\frac{\mu(1-\gamma)}{\alpha(\beta-1)}\right] y_t^2 \tag{7}$$

This model given by equation (7) is called the logistic model. For most choices of α, β, γ, and μ there is no explicit solution for (7). Namely, knowing α, β, γ, μ, and measuring y_0 would not suffice to predict y_t for any point in time, as was previously possible. This is at the heart of the presence of chaos in deterministic feedback processes. Lorenz [1963] discovered this effect – the lack of predictability in deterministic systems. Sensitive dependence on initial conditions is one of the central ingredients of what is called deterministic chaos.

This kind of difference equation (7) can lead to very interesting dynamic behaviour, such as cycles that repeat themselves every two or more periods, and even chaos, in which there is no apparent regularity in the behavior of y_t. This difference equation (7) will possess a chaotic region. Two properties of the chaotic solution are important: firstly, given a starting point y_0 the solution is highly sensitive to variations of the parameters α, β, γ, and μ; secondly, given the parameters α, β, γ, and μ, the solution is highly sensitive to variations of the initial point y_0. In both cases the two solutions are rather close to each other for the first few periods, but later on they behave in a chaotic manner.

1. The logistic equation

The logistic map is often cited as an example of how complex, chaotic behaviour can arise from very simple non-linear dynamical equations. The map was popularized in a seminal 1976 paper by the biologist Robert May. The logistic model was originally introduced as a demographic model by Pierre François Verhulst.

It is possible to show that iteration process for the logistic equation

$$z_{t+1} = \pi z_t (1 - z_t), \quad \pi \in [0,4], \quad z_t \in [0,1] \tag{8}$$

is equivalent to the iteration of growth model (4) when we use the identification

$$z_t = \frac{\mu(1-\gamma)}{\alpha(1+\mu)(\beta-1)} y_t \quad \text{and} \quad \pi = 1+\mu \tag{9}$$

Using (7) and (9) we obtain

$$z_{t+1} = \frac{\mu(1-\gamma)}{\alpha(1+\mu)(\beta-1)} y_{t+1} = \frac{\mu(1-\gamma)}{\alpha(1+\mu)(\beta-1)}$$

$$\left\{(1+\mu) y_t - \left[\frac{\mu(1-\gamma)}{\alpha(\beta-1)}\right] y_t^2\right\} =$$

$$= \left[\frac{\mu(1-\gamma)}{\alpha(\beta-1)}\right] y_t - \left[\frac{\mu^2(1-\gamma)^2}{\alpha^2(\beta-1)^2(1+\mu)}\right] y_t^2$$

Using (8) and (9) we obtain

$$z_{t+1} = \pi z_t (1 - z_t) = (1+\mu)$$

$$\left[\frac{\mu(1-\gamma)}{\alpha(1+\mu)(\beta-1)}\right] y_t \left\{1 - \left[\frac{\mu(1-\gamma)}{\alpha(1+\mu)(\beta-1)}\right] y_t\right\} =$$

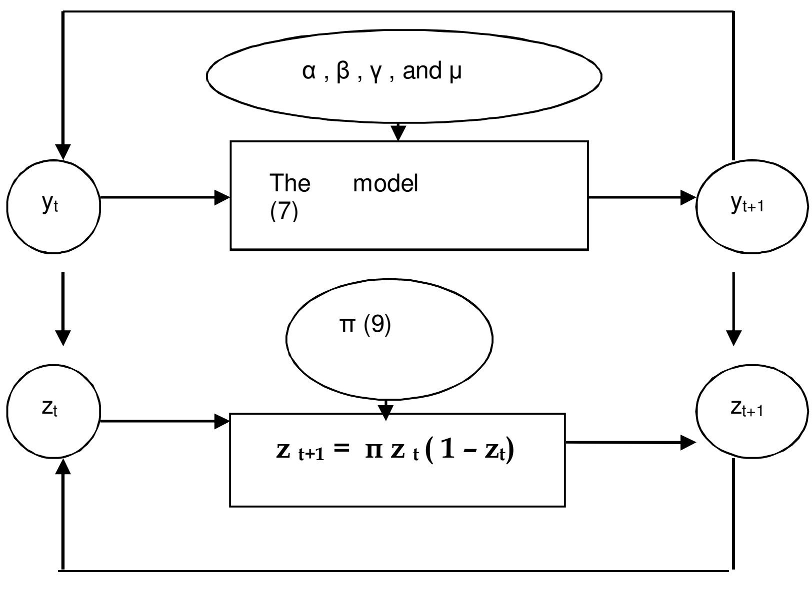
$$= \left[\frac{\mu(1-\gamma)}{\alpha(\beta-1)}\right] y_t - \left[\frac{\mu^2(1-\gamma)^2}{\alpha^2(\beta-1)^2(1+\mu)}\right] y_t^2$$

Thus we have that iterating (7) is really the same as iterating (8) using (9) . It is important because the dynamic properties of the logistic equation (8) have been widely analyzed [Li and Yorke, 1975, May 1976].

It is obtained that :

- For parameter values $0 < \pi < 1$ all solutions will converge to $z = 0$;
- (II) For $1 < \pi < 3{,}57$ there exist fixed points the number of which depends on π;
- (III) For $1 < \pi < 2$ all solutions monotonically increase to $z = (\pi - 1) / \pi$;
- (IV) For $2 < \pi < 3$ fluctuations will converge to $z = (\pi - 1) / \pi$;
- (V) For $3 < \pi < 4$ all solutions will continuously fluctuate;
- (VI) For $3{,}57 < \pi < 4$ the solution become "chaotic" which means that there exist totally aperiodic solution or periodic solutions with a very large, complicated period. This means that the path of z_t fluctuates in an apparently random fashion over time, not settling down into any regular pattern whatsoever.

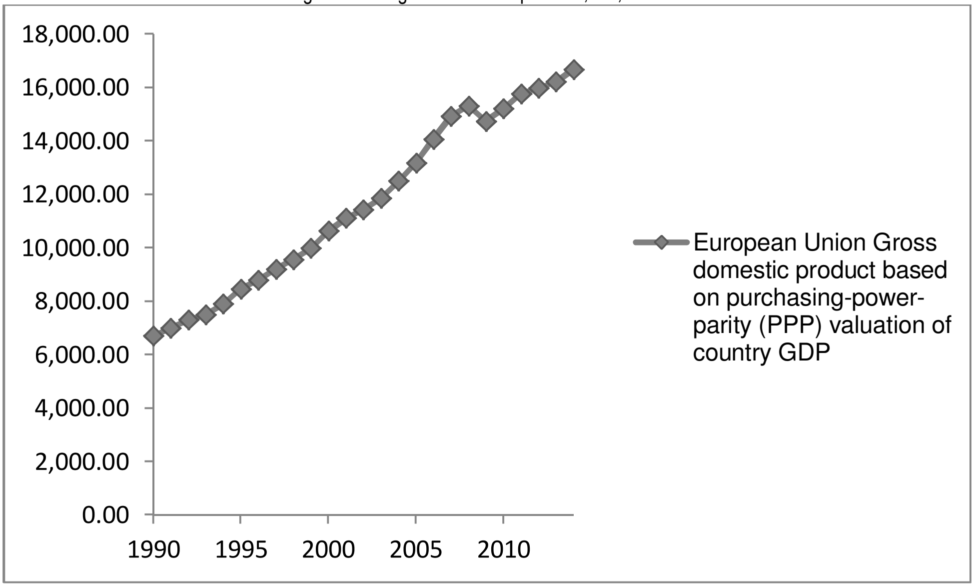
Figure 1. Two quadratic iterations running in phase are tightly coupled by the transformations indicated



2. Empirical evidence

The main aim of this paper is to analyze the real GDP growth stability in EU in the period 1990–2014, by using the presented non-linear, logistic economic growth model (10) :

Figure 2. The gross domestic product , EU, 1990-2014.



Source: (www.imf.org)

$$y_{t+1} = \pi y_t - \vartheta y_t^2 \qquad (10)$$

where: y – the real gross domestic product, $\pi = (1+ \mu)$, $\vartheta = \mu (1-\gamma)/\alpha (\beta-1)$.

Firstly, data on the real gross domestic product based on purchasing – power parity (PPP) are transformed [www.imf.org] from 0 to 1, according to our supposition that actual value of GDP, Y, is restricted by its highest value in the time-series, Y^m. Further, we obtain time-series of $y = Y / Y^m$.

Table 1. The estimated model (10): EU, 1990–2014.
(R= 0.99579 Variance explained: 99.15%)

	π	ϑ
Estimate	1.09839	0.080780
Std.Err.	0.02863	0.03168
t(21)	42.86169	2.549888
p-level	0.00000	0.018648

Source: www.imf.org

Conclusion

This paper suggests conclusion for the use of the chaotic economic growth model in predicting the fluctuations of the real gross domestic product. The model (7) has to rely on specified parameters α, β, γ, and μ, and initial value of the real gross domestic product, y_0. However, even slight deviations from the values of parameters: α, β, γ, and μ and initial value of the real gross domestic product, y_0 show the difficulty of predicting a long-term the real gross domestic product.

A key hypothesis of this work is based on the idea that the coefficient $\pi = (1+\mu)$ plays a crucial role in explaining local growth stability of the real gross domestic product, where, μ – the coefficient which explains relation between unemployment rate growth rate and the fraction of the labor force that is not used up by the unemployment at time t.

The estimated value of the coefficient π is 1.09839. This result confirms stable but monotonically increasing values of the real gross domestic product in EU in the period 1990–2014. Decreasing unemployment rate is important ingredient of the real gross domestic growth rate stability in the EU.

References

Benhabib, J., & Day, R. H., "Rational choice and erratic behaviour", *Review of Economic Studies*, 48, 1981, 459–471.

Benhabib, J., & Day, R. H., "Characterization of erratic dynamics in the overlapping generation model", *Journal of Economic Dynamics and Control*, 4, 1982, 37–55.

Day, R. H., "Irregular growth cycles", *American Economic Review*, 72, 1982, 406–414.

Day, R. H., "The emergence of chaos from classical economic growth", *Quarterly Journal of Economics*, 98, 1983, 200–213.

Goodwin, R. M., *Chaotic economic dynamics*, Clarendon Press, Oxford, 1990.

Grandmont, J. M., "On endogenous competitive business cycles", *Econometrica*, 53, 1985, 994–1045.

Jablanovic, V., "A Chaotic Unemployment Rate Growth Model", *Hyperion International Journal of Econophysics & New Economy*, 4(1), 2011, 45–51.

Jablanovic, V., *Budget Deficit and Chaotic Economic Growth Models*, Aracne editrice S.r.l., Roma, 2012.

Jablanovic, V., "The Chaotic Economic Growth Model: China", *Advances in Management and Applied Economics*, 3(4), 2013, 89–94.

Jablanovic, V., "The Chaotic Inflation Growth Model: The Euro Area", *Advances in Management and Applied Economics*, 4(2), 2014, 91–98.

Li, T., & Yorke, J., “Period three implies chaos“, *American Mathematical Monthly*, 8, 1975, 985–992.

Lorenz, E. N., “Deterministic nonperiodic flow“, *Journal of Atmospheric Sciences*, 20, 1963, 30–141.

Lorenz, H. W., *Nonlinear dynamical economics and chaotic motion* (2nd ed.), Springer-Verlag, Heidelberg, 1993.

May, R. M., “Mathematical models with very complicated dynamics“, *Nature*, 261, 1976, 459–467.

Medio, A., *Chaotic dynamics: Theory and applications to economics*, Cambridge University Press, Cambridge, 1993.

Okun, A., *Potential GNP, its measurement and significance*, Cowles Foundation, Yale University, 1962; http://cowles.econ.yale.edu/P/cp/p01b/p0190.pdf

Puu, T., *Attractors, Bifurcations, and Chaos – Nonlinear Phenomena in Economics*, Springer Verlag, Berlin, Heidelberg, 2003.

Zhang W. B., *Discrete Dynamical Systems, Bifurcations and Chaos in Economics*, Elsevier B. V., Amsterdam, 2012.

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Summary

Chaos embodies three important principles: (i) extreme sensitivity to initial conditions ; (ii) cause and effect are not proportional; and (iii) nonlinearity. The basic aim of this paper is to provide a relatively simple chaotic economic growth model that is capable of generating stable equilibria, cycles, or chaos depending on parameter values. Okun’s Law is included in this paper. A key hypothesis of this work is based on the idea that the coefficient $\pi = (1+\mu)$ plays a crucial role in explaining local growth stability of the real gross domestic product, where, μ – the coefficient which explains relation between unemployment rate growth rate and the fraction of the labor force that is not used up by the unemployment at time t. The estimated chaotic economic growth model confirms stable but monotonically increasing values of the real gross domestic product in EU in the period 1990–2014.