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Short scientific paper

A problem-solving process using the Theory of Didactical Situations: 500 lockers problem

Abstract: The main focus of this study was to examine the mathematical thinking skills of the undergraduates in an adidactical situation. Didactical Situations Theory was adopted to explain and determine the complexity of students' mathematical thinking. The current case study was conducted with 16 volunteers, pre-service primary school teachers of mathematics and a task called "500 lockers" was used to challenge their reasoning process. The data obtained through observation and student works were analyzed deductively and according to the five stages of adidactical learning described by Brousseau (2002). One of the main results of the study is that the designed learning environment with the given problem context provoked participants to make conjectures and provided them with an opportunity to defend their own hypotheses. Consequently, the implementation of the problem resulted in invaluable reflections enhancing participants' mathematical thinking.

Keywords: *Didactical situations, adidactical learning setting, problem solving, 500 lockers problem*

Introduction

Mathematical thinking, which is an essential skill for effective mathematics education (National Council of Teachers of Mathematics [NCTM], 2000) is strictly related to problem solving (Schoenfeld, 1992). Therefore, teachers are advised to create an environment which will enhance mathematical thinking (Eisenhardt, Fisher, Schack, Tassell, & Thomas, 2011). This is possible when students encounter challenging problems (Harel & Sowder, 2005). According to contructivist approach, problem-solving is an important skill (Terhart, 2003; Tynjala, 1999; Yevdokimov & Passmore, 2008).

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Learners do not simply mirror and reflect what they read and the responsibility for learning falls upon a learner in constructivist environments (Glasersfeld, 1989). To imply the teacher's role, in his Theory of Didactical Situations [TDS] Brousseau (2002) states that "Doing mathematics does not consist only of receiving, learning and sending correct, relevant (appropriate) mathematical messages" (p.15). Regarding the development of mental structures, Cobb (1988) states that teachers have to facilitate a profound cognitive restructuring and conceptual reorganizations, rather than merely conveying to students information about mathematics. On the other hand, putting the students' own efforts to understand at the center of educational enterprise (Prawat, 1992), constructivism proposes that learners' knowledge is derived from a meaning-making search by engaging in a process of constructing individual interpretations (Brophy, 2002; Fosnot, 1996; Resnick, 1989). Hence, it can be claimed that TDS is very much a constructivist approach to the study of teaching situations (Artigue, 1994 as cited in Sriraman & Törner, 2008).

The TDS, developed by Guy Brousseau (2002), emerged in the second half of the 20th century and it has been a trend in mathematics education for the last two decades. According to this theory, knowledge is a property of a system constituted by a subject and a *milieu* in interaction. The core of the learning process lies in students' adaptation to this milieu. Students have to take responsibility without relying on teacher's feedback, which is what Brousseau defines as an *adidactical situation* (Ligozat & Schubauer-Leoni, 2010).

Brousseau (2002) specifies the responsibility of students in adidactical situations as follows: "The student learns by adapting herself to a milieu which generates contradictions, difficulties and disequilibria, rather as human society does." (p. 30). Therefore, the teacher's task is to arrange situations for students to discover knowledge and then depersonalize it. On the other hand, students' work consists of personal discovery followed by depersonalization (Winslow, 2005). Samaniego and Barrera (1999, p.3) identify three situations differentiated by Brousseau (2002) in the teaching process adapting from Bessot (1994, as cited in Samaniego & Barrera, 1999):

"*Non-didactical situation*: with respect to knowledge S, is that situation that is not explicitly organized to allow the learning of S. For instance, at the secondary level, all that has to do with operation with naturals may be considered as a non-didactical situation.

Didactical situation; with respect to knowledge S, is that situation designed explicitly to encourage S. We can consider as didactical all the tasks done in a classroom with which the teacher intents to teach S, and with which the student is forced to learn S.

A-didactical situation: with respect to knowledge S, is that situation that contains all the conditions that permit the student to establish a relationship with S, regardless of the teacher. The actions that the student does, and the answers and arguments that he/she produces depend on him/her relationship (no completely explicit) with S, i. e. with the "problem" that he/she must solve or wit the difficulty that she must overcome. In this case, a process of devolution of responsibility is in action."

The didactical situation is made up of five phases which can be summarized briefly as; (*i*) *devolution* phase where the teacher transfers the responsibility to the students, (*ii*) the *action* phase where the students come up with new hypotheses on how to solve the problem, (*iii*) the *formulation* phase where the students articulate their hypothesis (*iv*) the *validation* phase where the hypotheses are tested for their validity, and finally (*v*) the *institutionalization* where the teacher offers possible solutions to the given problem and presents the problem in different contexts where the earlier solutions are the basis for understanding (Brousseau, 2002). Radford (2008) claims that the TDS works on the basis of these epistemic principles:

p1: knowledge is the result of the "optimal" solution to a certain situation or problem.

p2: learning is - in accordance to Piaget's genetic epistemology - a form of cognitive a d a p t a - tion.

p3: for every piece of mathematical knowledge there is a family of situations to give it an appropriate meaning.

p4: student autonomy is a necessary condition for the genuine learning of mathematics.

Apart from the theoretical base of TDS, some problems may occur within the milieu. In the first place, the teachers may have problems with implementing group work within the TDS. Davies (2009) listed some variables affecting group work such as motivation, tasks given, task complexity, recognition of effort, the size of the group and the effect of incentives and penalties. Although group work may result in unexpected failures, the teacher in TDS should try to organize the milieu to minimize the pitfalls of the group work. Furthermore, group tasks have to be evaluated to ensure that they are likely to result in effective group efforts (Davis, 1999). According to Michaelson, Fink and Knight (1997), group assignments should (i) require a high level of individual accountability of group members; (ii) require members to discuss issues and interact; (iii) ensure that members receive immediate, unambiguous, and meaningful feedback; (iv) provide explicit rewards for high levels of group performance to eliminate or minimize the difficulties that groups can face. When examined closely, it can be seen that TDS satisfies these conditions.

The TDS constitutes the framework for this research since the students endeavor to acquire knowledge on their own and, most importantly, since exploring how students learn within the process, rather than how teachers teach the subject, is the baseline for the present research. In this context, this study aims to examine the mathematical thinking skills of the students in an adidactical situation through an inquiry-based problem solving. Therefore, the study is important in terms of providing a basis on how to conduct a didactical situation within TDS, shifting the locker problem in a different context and examining the students' behaviors in an environment which requires of them to get involved in higher thinking processes. In addition, as Sriraman and English (2010) claim, various theories and philosophies that have informed and propelled the field forward should be tested in different contexts from time to time.

Method

Case study design (Yin, 2003) was used in the research in which the problem-solving process of the students was examined. The participants of the study were 16 (5 male and 11 female) voluntary undergraduate students of the Primary Mathematics Teaching Program at the state university in Turkey. An attractive problem situation was investigated to find out the mathematical thinking processes of the pre-service teachers. The problem situation known as the "locker problem" (Kimani, Olanoff, & Masingila, 2016), which the participants had never encountered before, was as follows:

"Assume that your school has 500 students and 500 lockers, one for each student. Both students and lockers are numbered from 1 to 500. When all the lockers are closed, the first student walks down the line and opens the doors of all 500 lockers. The second student closes the doors with even numbers. The third student changes the state of every third locker, i.e. if it is open, he/she closes it; if it is closed, he/she opens it. The fourth student does the same to every fourth locker, and the process is repeated with all 500 students. Each student changes ("change" means either closing an open door or opening a closed door) the state of those lockers numbered with multiple of their own id number. How many lockers will be open when all 500 students open or close the doors in the way described above?"

During the procedure, 2 researchers worked together with the four groups of students, each composed of 4 students. The groups were heterogeneous within themselves and homogeneous among themselves according to their academic success level. Furthermore, the activity was video-taped after the permission of the participants had been received. The ideas of different groups were put forward and an environment for discussion was formed to validate or falsify the expressed ideas. One of the researchers led the activity and discussions, while the other one was guiding the video camera and taking the observation notes. The researchers' notes, video camera recording, sketches of the groups on the delivered papers were used in the analysis. The data were analyzed by using deductive analysis, in which the data were analyzed according to an existing framework (Patton, 2002, p.453). The data analysis was conducted according to the TDS concepts, i.e., the stages of devolution, action, formulation, validation and institutionalization. Two researchers came together to compare the analysis results after they had analyzed the obtained data individually according to the themes created previously.

Results and Discussion

The findings obtained from the adidactical situation are presented, taking into consideration the five stages of the milieu.

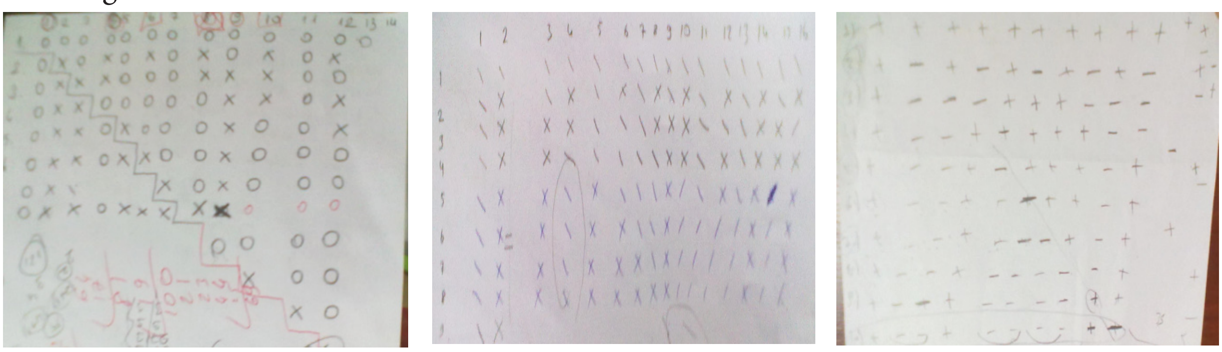
Devolution Stage: At the beginning of the activity, the researchers informed the students about the aim of the practice and important points of the process. The aforementioned problem was introduced to the students and the expectations from the groups were stated in order to have an effective problem-solving process. Hence the transfer of the task occurred and the researchers let the groups study on their own.

Action Stage: The students made an effort to solve the problem in groups after the problem was introduced. The most important indicator of this phase was that the students passionately discussed the possible solutions within the groups and put forth their strategies. The students mostly tried to find out a solution by trial and error, instead of suggesting a formal proof. Some of the strategies can be seen in Figure 1a, 1b, 1c.

Formulation Stage: The students presented formal hypotheses in this stage. The students who struggled for the solution through trial-and-error search also made mathematically reasonable and acceptable deductions in this stage. Three hypotheses that were thought to be worth discussing were suggested by the groups.

<u>*Hypothesis 1:*</u> *The doors of the lockers numbered with 1, 4, 9, 18, 35, 68, 133, 262 are open.*

S: It's a pattern having 2^n numbers between each consecutive numbers. For example, there are 2



a b c

Figure 1. The strategies to find out the open and closed lockers

numbers between 1 and 4 (i.e., 2 and 3), 4 numbers between 4 and 9 (i.e., 5, 6, 7 and 8), etc.

R: How did you come to that solution?

S: This is what we did: We first wrote down + for the doors numbered from 1 to 10. Then, we changed the even numbers with – sign. Then we changed the multiples of three, four, five and so on. After those markings, we noticed a pattern. There were 2 closed doors and 4 closed doors respectively and we thought that this pattern should go on like this.

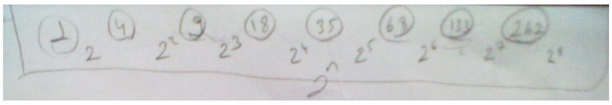


Figure 2. The strategy for Hypothesis 1

It can be seen that the students generalized the pattern they found for 10 lockers by trial and error for 500 lockers in the wrong way. Without any intervention regarding this hypothesis, another group was allowed to express their hypothesis.

<u>Hypothesis 2:</u> *The doors of the lockers numbered with prime numbers are always closed.*

The students first tried to find out whether there was something going on with prime numbers or not. So, it was the result of their curiosity with the primes. Although the students did not come up with a solution to the problem presented, they made a valid suggestion.

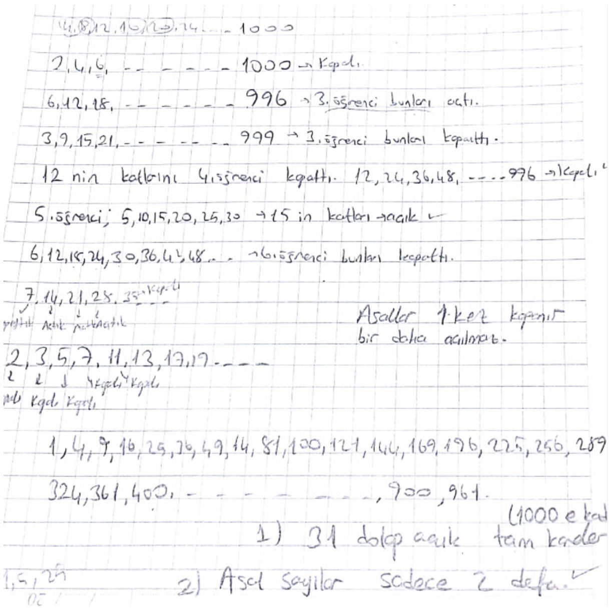


Figure 3. The strategy for Hypothesis 2 [The perfect-squares are written at the bottom. The primes are listed on the left side and labeled as "closed". Also, there is a note on the right side saying that "the prime numbers are closed once and never opened again."]

<u>Hypothesis 3:</u> *The doors of the lockers numbered with perfect squares (1, 4, 9, 16, …) are open.*

The students put the right solution forward with this hypothesis. Unlike the ones presenting the first hypothesis, these students worked with the first 30 or 40 numbers to generalize their reasoning. The researcher wrote down all the hypotheses suggested without mentioning the truth or falsity to let the students discuss among themselves in the next stage.

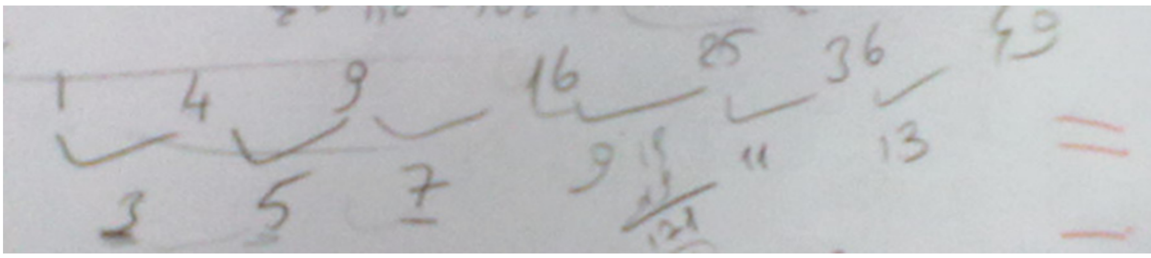


Figure 4. The strategy for Hypothesis 3

Validation Stage: The students started to discuss their arguments soon after they shared the hypotheses. In this context, the discussion was aimed at bringing out the results of their thinking processes. They were asked to provide justifications for what they thought about the truth of the statements suggested. Then the groups tried to convince the other groups about the truth of their arguments. In the meantime, the researcher addressed some questions about the deductions of the students.

S: We have to look at the number of the divisors. For example, 10 has four positive divisors (using the rule for the number of divisors): open, closed, open, closed (counting the state of the locker four times).

R: Why do you think that perfect-square-numbered lockers will be open?

S: Let's consider 16 this time. So, it has five divisors: open, closed, open, closed, open. Therefore, because of the odd number of divisors the locker will be open. If it were even, then it would be closed. Since the number of divisors for perfect squares is odd, the lockers numbered with perfect squares will be open.

Institutionalization Stage: The hypotheses which were stated and validated by the students themselves were expressed again explicitly.

S: We write down the factorization of the number such as $a^k.b^n.c^m$ *where a, b, c are primes and k,m,n are positive integers. Then the number has to have (k+1).(m+1).(n+1) divisors. The result of this multiplication is even, if there is at least one even multiplier. The result is odd only if all the multipliers are odd. So, the integers k, m, n have to be even to get the odd numbers when we add 1 to each of them. Consequently, since the powers of the multipliers are even, the numbers of the lockers should be perfect squares for them to be open.*

This explanatory statement of the students is necessary for them to get to the bottom of the solution and to understand why the hypothesis works. So the students are able to generalize the problem to 1000 lockers, or they can find out which lockers undergo two operations, i.e., opened just once and closed just once. These kinds of questions decontextualize the problem and make the students reason further.

Conclusion and Discussion

Reflections from an adidactical learning situation organized to determine students' mathematical thinking processes have been presented in this research. Adidactical learning environment is encouraging for the students as they learn without being aware of the fact that they learn (Brousseau, 2002). The research posed a challenging problem to the students to let them try to analyze the problem. As Kaplan and Moskowitz (2000) and Torrence and Wagon (2007) stated, this locker problem is a rich benchmark problem appearing in both secondary and university curricula. The students endeavored to hypothesize the solution and to verify or falsify these hypotheses. Furthermore, students interacted with the milieu to reach the conclusion in an additional trial and error approach. On the other hand, group discussions gave the students an opportunity to defend their hypotheses and argue for their statements on the basis of mathematical reasoning, as well as to present their mathematical arguments. Seshaiyer, Suh and Freeman (2012) also concluded that this problem was accessible to all students and the use of models, together with acting-out strategies, seemed to engage and motivate students. In this research, however, students were made to think abstractly and create their own hypotheses. Lester and Mau (1993) claim that problem-solving mathematics instruction enables pre-service teachers to understand and appreciate the value of the classroom climate that allows students to take charge of their own learning. They also used the locker problem to evaluate the classroom climate in which the responsibility lies mostly on students and concluded that students were motivated and excited to come up with their own products.

Calder (2010) and Schoenfeld (1992) state that problem-solving strategies are fundamental aspects of mathematical thinking which emerge through engagement in mathematical practices. The students make significant gains in mathematical understanding when teachers carefully choose tasks that require of students to engage in mathematical thinking and problem-solving. Additionally, teachers should encourage students' thinking processes by asking questions and encourage reflection and sense making (Papadopoulos, 2017; Rigelman, 2007). Accordingly, students should justify their reasoning or refute the hypotheses suggested for an effective mathematical thinking process (Harel & Sowder, 2005). Seshaiyer, Suh and Freeman (2012) state that the locker problem is a great mathematical puzzle that not only furnishes multiple entry points to access a variety of mathematical content, but also encourages the skills such as multiple problem-solving strategies, multiple representations, critical thinking, justification and proof, which reinforce the Process Standards specified by NCTM (2000). Lester and Mau (1993) believe that this type of problem-solving can result in the development of social norms in the classroom that are useful for promoting independent problem-solving behavior in students by emphasizing the teachers' role as a guide who asks probing questions, rather than leading questions. As a conclusion, it can be asserted that the students accomplished the five stages of adidactical learning situation willingly and unwittingly. Parallel with the findings of this research, Çelik, Güler, Özüm-Bülbül and Özmen (2015) concluded that an adidactical learning setting reveals the mathematical thinking process of students. The participants also expressed their opinions about their experience in the milieu, stating that they enjoyed the process more than the product and adding that this experience had broadened their horizons and made them think about their future practices in the classroom. Importantly, the pre-service teachers claimed that experiencing a constructivist, problem-solving process helped them understand the importance of developing exploratory thinking skills in their own students. Furthermore, this form of problem-solving contexts prepared in advance can help teachers to create hypothetical learning trajectories as a way to deal with questions such as "what could this student learn next and how could they learn it?" (Empson, 2011, pp. 573-574).

Setting and reinforcing the norms for group behavior was one of the difficulties encountered in the resesearch. As TDS has a differentiated view on the in-classroom work, the students adapted to this new situation with difficulty. At first, they tried to solve the problem individually, rather than through exchanging ideas. Arslan, Taşkın and Kirman Bilgin (2015) also conclude that individual work yields better results than group work in adidactical learning situations. However, when coordinated appropriately, teamwork allows for standardization of knowledge among peers; it fosters discussion on different solutions and strategies; it develops in students the ability to communicate on mathematical ideas; and it also encourages the development of arguments that validate the statements made in the process (Samaniego & Barrera, 1999). The second difficulty was that the students tried to solve the problem directly, skipping the task of "conjecturing the hypotheses". This may be the result of the examination-focused educational system which seeks the final results and does not focus on the process. Thirdly, generalizing the problem setting, justifying their hypotheses and convincing their peers was difficult for the students. Arslan, Baran and Okumuş (2011) admit that students may encounter some difficulties in some stages of an adidactical game. Skemp (1986) concludes that the process of mathematical generalization is a sophisticated and powerful activity. Students have to abstract from a specific situation to formulate generalizations (Krutetskii, 1976). Hence, the milieu should include a motivating problem, letting students get involved in the problem-solving process and reflect on their thoughts. Sriraman (2004) drew attention to the fact that the problem selection is quite important if a teacher wants to establish an environment allowing students to have problem-solving experiences that enable them to generalize.

This study has implications for both practitioners and researchers. Teachers can organize activities that will enable their students to get involved in higher-level thinking processes through scientific research process in which they have to make their own conjectures. Additionally, the students should experience taking responsibility of their own learning and value the importance of the process of an exciting mathematical activity, rather than just solving the problem to get the correct answer. On the other hand, researchers should take into consideration the difficulties and take the necessary precautions. The research has also contributed to the problem-solving literature, decontextualizing the locker problem on the basis of the TDS.

References

- Arslan, S., Baran, D., & Okumuş, S. (2011). Brousseau's Theory of Didactical Situations in mathematics and an application of adidactical situations. *Necatibey Faculty of Education Electronic Journal of Science and Mathematics Education, 5*(1), 204-224.
- Arslan, S., Taşkın, D., & Kirman Bilgin, A. (2015). Effect of individual and group works on students' success in adidactical situations. *Turkish Journal of Computer and Mathematics Education, 6*(1), 47-67.
- Brophy, J. (Ed.). (2002). *Social constructivist teaching: Affordances and constraints.* Oxford, UK: Elsevier Science.
- Brousseau, G. (2002). *Theory of didactical situations in mathematics*. London: Kluwer Academic Publisher.
- Calder, N. (2010). Using stratch: An integrated problem-solving approach to mathematical thinking. *Australian Primary Mathematics Classroom*, *15*(4), 9-14.
- Cobb, P. (1988). The tension between theories of learning and instruction in mathematics education. *Educational Psychologist, 23*(2), 87-103.
- Çelik, D., Güler, M., Özüm-Bülbül, B., & Özmen, Z. M. (2015). Reflections from a learning setting designed to investigate mathematical thinking. *International Journal of Educational Studies in Mathematics, 2*(1), 11-23.
- Davies, W. M. (2009). Groupwork as a form of assessment: Common problems and recommended solutions. *Higher Education, 58*, 563-584.
- Davis, B. G. (1999). Cooperative learning: Students working in small groups. *Speaking of Teaching, 10*(2), 1-4.
- Eisenhardt, S., Fisher, M., Schack, E., Tassell, J., & Thomas, J. (2011). Noticing numeracy Now (N^3): A collaborative research project to develop pre-service teachers' abilities to professionally notice children's mathematical thinking. In S. Reeder, (Ed.). *Proceedings of the 38th Annual Meeting of the Research Council on Mathematics Learning 2011* (1-8). Cincinnati, OH.
- Empson, S. B. (2011). On the idea of learning trajectories: Promises and pitfalls. *The Mathematics Enthusiast, 8*(3), 571-596.
- Fosnot, C.T. (1996). Constructivism: A psychological theory of learning. In C.T. Fosnot (Ed.), *Constructivism: Theory, Perspectives, and Practice.* New York: Teachers College Press.
- Glasersfeld, E. (1989). Cognition, construction of knowledge, and teaching. *Synthese, 80*(1), 121-140.

- Harel, G. & Sowder, L. (2005). Advanced mathematical-thinking at any age: It's nature and its development. *Mathematical Thinking and Learning*, 7 (1), 27-50.
- Lester, F. K., & Mau, S. T. (1993). Teaching mathematics cia problem solving: A course for prospective elementary teachers. *For the Learning of Mathematics, 13*(2), 8-11.
- Ligozat, F. & Schubauer-Leoni, F. (2010). The joint action theory in didactics: Why do we need it in the case of teaching and learning mathematics? In V. Durand Guerrier, S. Maury& F. Arzarello, *CERME 6 Proceedings* (pp. 1615-1624). Lyon: INRP.
- Kaplan, J., & Moskowitz, M. (2000). *Mathematics Problem Solving.* New York: Triumph Learning.
- Kimani, P. M., Olanoff, D., & Masingila, J. O. (2016). The locker problem: An open and shut case. *Mathematics Teaching in the Middle School, 22*(3), 144-151.
- Krutetskii, V. A. (1976). *The psychology of mathematical abilities in school children* (J. Teller, Trans., & J. Kilpatrick & I. Wirszup, Eds.). Chicago: University of Chicago Press.
- Michaelsen, L., Fink, D., & Knight, A. (1997). Designing effective group activities: Lessons for classroom teaching and faculty development. In D. DeZure (Ed.), *To improve the academy.* Stillwater, OK: POD Network.
- National Council of Teachers of Mathematics (2000). *Let's Count in Curriculum and Evaluation Standards for School Mathematics.* Reston, VA: NCTM.
- Papadopoulos, I. (2017). Opening inquiry mathematics to parents: Can they be engaged as teachers' partners in mathematical work?. *Journal of Pedagogical Research, 1*(1), 1-20.
- Patton, M. Q. (2002). *Qualitative research and evaluation methods.* Thousand Oaks, CA: Sage.
- Prawat, R. S. (1992) Teachers beliefs about teaching and learning: A constructivist perspective. *American Journal of Education, 100,* 354-395.
- Radford, L. (2008). Theories in mathematics education. A brief inquiry into their conceptual differences. Working Paper. Prepared for the ICMI Survey Team 7. The notion and role of theory in mathematics education research. Retrieved from http://www.luisradford.ca/pub/44_radfordicmist7.pdf
- Resnick, L. B. (1989). Introduction. In L. B. Resnick (Ed.), *Knowing, learning, and instruction: Essays in honor of Robert Glaser* (pp. 1-24). Hillsdale, NJ: Erlbaum.
- Rigelman, N. R. (2007). Fostering mathematical thinking and problem solving: The teacher's role. *Teaching Children Mathematics,* 13(6), 308-314.
- Samaniego, A. H. F., & Barrera, S. V. (1999). *Brousseau in action: Didactical situation for learning how to graph functions.* Paper presented at the 4th Asian Technology Conference in Mathematics.
- Schoenfeld, A. H. (1992). Learning to think mathematically: Problem solving, metacognition and sense making in mathematics. In D. A. Grouws (Ed.), *Handbook of research on mathematics teaching* (pp. 334-370). New York: MacMillan Publishing.
- Seshaiyer, P., Suh, J. M., & Freeman, P. (2012). Unlocking the locker problem. Teaching children mathematics, 18(5), 322-326.
- Skemp, R. (1986). *The psychology of learning mathematics.* London: Penguin Books.
- Sriraman, B. (2004). Reflective abstraction, uniframes and the formulation of generalizations. *Journal of Mathematical Behavior, 23,* 205-222.

- Sriraman, B., & English, L. (2010). Surveying theories and philosophies of mathematics education. In *Advances in Mathematics Education: Theories of Mathematics Education: Seeking New Frontiers (pp.* 7-32). Berlin Heiderberg: Springer.
- Sriraman, B., & Törner, G. (2008). Political union/mathematical education disunion. In L. D. English (Ed.), *Handbook of International Research in Mathematics Education* (2nd ed., pp. 656-690). London: Routledge, Taylor & Francis.
- Terhart, E., 2003. Constructivism and teaching: A new paradigm in general didactics? *Journal Curriculum Studies, 35*(1), 25-44.
- Torrence, B., & Wagon, S. (2007). The locker problem. *Crux Mathematicorum, 33*(4), 232-236.
- Tynjala, P. (1999). Towards expert knowledge? A comparison between a constructivist and a traditional learning environment in university. *International Journal of Educational Research,* 31, 357-442.
- Winslow, C. (2005). Introduction: A Graduate Course on Four French Frameworks for Reseacrh on Didactics of Mathematics. In C. Winslow (Ed.) *The Didactics of Mathematics: The French Way* (pp. 7-20). Copenhagen: Center For Naturfagenes Didaktik.
- Yevdokimov, O., & Passmore, T. (2008). Problem solving activities in a constructivist framework: Exploring how students approach difficult problems. In M. Goos, R. Brown, & K. Makar (Eds.), *Proceedings of the 31st Annual Conference of the Mathematics Education research Group of Australasia* (pp.629-636). Sydney: MERGA.
- Yin, R. K. (2003). *Applications of case study research.* Newbury Park, CA: Sage.

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ПРОЦЕС РЕШАВАЊА ПРОБЛЕМА ПОМОЋУ ТЕОРИЈЕ ДИДАКТИЧКИХ СИТУАЦИЈА: ПРОБЛЕМ 500 ОРМАРИЋА

Проширени резиме

Фокусирајући се на активно учешће појединаца, Брусоова (Brousseau) Теорија дидактичких ситуација [ТДС] (2002) наводи да се „савлађивање математике не састоји само од примања, учења и слања исправних, релевантних (одговарајућих) математичких порука” (стр. 15). Дидактичка ситуација састоји се од пет фаза које се укратко могу описати на следећи начин: (1) фаза *деволуције* у којој наставник преноси одговорност на ученике, (2) фаза *деловања* у којој ученици износе нове хипотезе о томе како решити одређени математички проблем, (3) фаза *формулације* у којој ученици артикулишу своју хипотезу (4) фаза *валидације* у којој се тестира ваљаност хипотеза и на крају (5) *институционализација*, у којој наставник нуди могућа решења за дати проблем и представља проблем у различитим контекстима, док су ранија решења основа за разумевање (Brousseau, 2002).

ТДС представља оквир за ово истраживање зато што ученици покушавају да самостално стекну знање и, што је најважније, зато што је анализирање начина на који ученици уче у овом процесу, а не како наставници предају предмет, полазна основа у нашем истраживању. У томх контексту, циљ овог рада је да се испитају вештине математичког мишљења ученика у адидактичној ситуацији кроз решавање проблема заснованог на промишљању. Наше истраживање је важно зато што пружа основу за спровођење дидактичке ситуације у оквиру ТДС-а пребацивањем проблема ормарића у другачији контекст и испитивањем понашања ученика у окружењу које захтева од њих да се укључе у процесе виших нивоа размишљања.

У овој студији случаја учествовало је 16 студената на добровољној основи. Циљ решавања проблема са ормарићима био је да се утврде математички процеси размишљања будућих учитеља. Проблем се састојао од отварања и затварања врата свих ормарића, односно, конкретније, први студент отвара све ормариће, други затвара врата ормарића са парним бројевима, трећи мења стање сваког трећег ормарића.

„Колико ће ормарића бити отворено када свих 500 ученика отвори или затвори ормариће на горе описани начин?”

У дедуктивној анализи, у којој су подаци анализирани према постојећем оквиру, коришћене су белешке истраживача, видео снимци, скице група на предатим папирима

(Patton, 2002, стр. 443). Анализа података је спроведена у складу са поставкама ТДС-а, тј. фазама деволуције, деловања, формулације, валидације и институционализације.

Фаза деволуције: Наведени проблем представљен је студентима и речено је шта се од група очекује како би се постигао ефективан процес решавања проблема.

Фаза деловања: Најважнији индикатор у овој фази био је да су учесници страствено расправљали о могућим решењима унутар група и износили су своје стратегије.

Фаза формулисања: Учесници који су покушавали да реше задатак помоћу принципа погрешке и исправљања такође су у овој фази доносили математички разумне и прихватљиве закључке. Групе су предложиле три хипотезе.

Хипотеза 1: Ормарићи означени бројевима 1, 4, 9, 18, 35, 68, 133, 262 су отворени.

Хипотеза 2: Ормарићи обележени простим бројевима увек су затворени.

Хипотеза 3: Ормарићи обележени потпуним квадратима (1, 4, 9, 16, ...) су отворени.

Фаза валидације: Учесници су почели да расправљају о својим аргументима убрзо након што су изнели своје хипотезе. Од њих се тражило да образложе зашто мисле да су њихова решења исправна. Затим су групе покушале да убеде једне друге да су њихови аргументи исправни.

Фаза институционализације: Изнете и образложене хипотезе су потом поново експлицитно наведене. На тај начин студенти могу да генерализују ѕадати проблем и до 1000 ормарића, или могу да открију на које ормариће се примењују две операције и тако дектонтекстуализују проблем.

Учесници су настојали да хипотетизују решење и да потврде или оповргну изнете хипотезе. Штавише, учесници су били у интеракцији са задатом проблематиком, а да би дошли до закључка, користили су и принцип учења кроз грешке и исправљање погрешног размишљања. С друге стране, групне дискусије пружиле су учесницима прилику да бране своје хипотезе и доказују ставове на основу математичког расуђивања, као и да представе сопствене математичке аргументе. Сешајер, Сух и Фриман (Seshaiyer, Suh and Freeman, 2012) су такође закључили да је овај проблем погодан за све студенте и да коришћење модела, уз стратегије уживљавања у проблем, привлачи и мотивише студенте. Међутим, у овом истраживању учесници су морали да размишљају апстрактно и да створе сопствене хипотезе. Можемо да закључимо да су учесници у истраживању добровољно и ненамерно остварили пет ступњева адидактичког учења. Учесници су такође изразили мишљење о свом искуству у датом окружењу, изјавивши да су уживали у процесу решавања проблема више него у исходу, а навели су и да је ово искуство проширило њихове видике и натерало их да размишљају о свом будућем раду у учионици.

Кључне речи: *дидактичке ситуације, адидактичке ситуације, решавање проблема, проблем 500 ормарића.*