A theatre of mathematical history – a historical memoir

Summary: This paper summarises a particular aspect of using the history of mathematics to inspire and educate secondary level teachers. Whilst discussion about the uses of the history of mathematics in the classroom is mentioned with some of the most used approaches, the paper is itself a historical memoir. It reflects on the process in which, during a career in a mathematics classroom, and later, in author’s work with teachers, the author discovered a method that proves to be most efficient, simple, and at the same time inspiring for all involved – for pupils, teachers, and the author herself. The method consists of creating an imaginary theatre of mathematical objects and images. This imaginary theatre can be considered to be a place, real or imagined, that can be recreated in any circumstance where mathematics is learnt, taught, and discussed.

Keywords: National Curriculum in England, Prince’s Teaching Institute, ICME (International Congress of Mathematics Education), History and Pedagogy of Mathematics group (HPM), European Summer School (ESU).

Introduction

To begin with, we will concentrate on the uses of the history of mathematics in mathematics education in England, in order to offer some insight into the possible ways of how history can benefit mathematics education at a national and local level.

The history of mathematics has been around for some time – and an option has always existed of course, for every teacher of mathematics, whether they would make the use of it in their classroom. The official recommendation to link mathematics and its history can however be traced to the redesign of the National Curriculum in England in 2014 (National Curriculum in England, 2014). In this version of the National Curriculum in England, the connection...
between the two disciplines was for the first time explicitly established for all teachers in schools following the curriculum, i.e. state-sponsored schools. At the very start of this new curriculum, the *Purpose of study* is given very clearly and here mathematics is described as a ‘creative and highly inter-connected discipline that has been developed over centuries, providing solution to some of history’s most intriguing problems’ (National Curriculum in England, 2014: 2).

The National Curriculum in England itself was introduced into every state school in England in 1989 and since then it has undergone several iterations usually initiated by successive governments within a couple of years after they took office. The 2014 changes were done through the nationwide consultation including teachers, teacher educators, and mathematics education scholars from around the country. The author of this paper was involved with this project through her involvement with the Prince’s Teaching Institute (PTI), whose mathematics programme she led from 2009 to 2017. This Institute, founded in 2006 by HRH Prince Charles and Bernice McCabe, the latter the teacher and headmistress of a large secondary school in London, has a mission to initiate teacher networking and offer teachers opportunities to reinvigorate interest in their subjects through programmes of lectures and events. Through this programme, the UK’s leading artists, scientists, mathematicians and writers give lectures and talk to teachers usually in amazing surroundings of some of the most beautiful stately buildings in England. The reason this (the sumptuous settings and the involvement of artists, scientists and mathematicians) is mentioned here – otherwise perhaps unimportant detail – will become clear at the end of this paper.

In 2006 the subjects represented at the Institute were the English language, History, and Music, and from 2009 further subjects were included in the work of the Institute, mathematics among them. The Mathematics School’s Programme, which the author led for eight years, was aimed at the heads of mathematics teaching in state secondary schools around England and Wales. Additionally, for a couple of years, the author was involved in designing and working on yet another programme for teachers, but this time aimed at those who had just entered the profession, new teachers of mathematics. This second programme was called the *Subject Days Programme*.

Both of these programmes – the *Schools’ Programmes* and the *Subject Days*, had a specific role as mentioned earlier: to reinvigorate the interest in the subject in the teachers taking part in the programmes. The range of experience and abilities was very wide amongst the teachers represented by these two programmes, from complete novices to experienced professionals, and from teachers who retrained to teach mathematics from originally working in another subject area, to teachers who had obtained doctorates in mathematics.

Apart from the main aim of these programmes, the events were organised in such a way that they would give participants time out of the classroom and enable them to talk, work together, and attend lectures. The format of the programmes included one-day events, three-day residential schools, and the programmes which took place on Saturday over a period of a whole term. During their time with the PTI, the teachers were able to learn about new things, both *about* and *from* mathematics. This would take form in several ways: from discussions with their colleagues, through lectures and workshops given by leading professionals, and through a structured self-reflection of some kind.

All these various programmes and events that the author worked on meant that she could talk to teachers about the sources of inspiration and a constant spring of ideas for teaching mathematics that the author, and the community of mathematics historians discovered: the history of the disci-
pline itself. What was learnt through these discussions was that teachers without exception wanted to introduce some historical elements into their classrooms, but that most of them thought they couldn't possibly have time to do so without huge investment of time and resources. The work of the author became therefore more focused on trying to solve this problem – to find a method which would allow teachers to introduce some historical elements in their teaching, within the incredibly busy schedules they already have.

The most common question asked in a mathematics classroom

Of course the author herself has been interested in how to use the history of mathematics since the completion of her own doctorate in the subject – but why would the teachers in these programmes become interested themselves? And how did the work with the teachers on these programmes reflect on the work they would have to do when they went back to their own mathematics classrooms?

Soon after entering the profession of mathematics educators, the author worked on a project which described in some detail the poor attitude towards mathematics raging from fear to complete disengagement in both middle and secondary school pupils (Lawrence, 2011a). Very soon a discovery that one of the reasons mathematics teachers are interested in motivating their students became apparent - it is much easier to teach mathematical skills to those who are interested in the subject itself, and not only in its utility. This was supported by the study which concluded that the question most commonly used in the mathematics classroom (Lawrence, 2012) can be with some certainly be said to be the following:

“When are we going to need this?”

Why is it that children are only worried about whether what they learn in a mathematics classroom will be useful to them? Could it be that kicking a ball during a sports lesson, or mucking about in their school's arts studio getting covered with paints and glue is more enjoyable than sitting in the mathematics classroom? In her own classroom the author experimented by keeping and using a crystal ball (Fig. 1) and whenever a pupil asked “when will we need this?” going into a theatrical game with putting her hands on the ball to see on what occasion that child will find the mathematics they were learning just then, useful in the future (Lawrence, 2012). But certainly, it is not a joke when a national survey, such as that done by Adrian Smith (2004) shows that the teenagers around the country (England) most commonly use phrases to describe their experience of mathematics as “boring” and “irrelevant”.

In the following section we will therefore give a description of how lessons learnt in the author's work with teenagers and with teachers in the Prince's Teaching Institute's programmes were used to formulate and formalise a method of using the history of mathematics in every day classrooms. The aim of this method is directly related to the reported negative perception many teenagers have of mathematics as a school subject, and our efforts to dispel the myth about irrelevance, and certainly make mathematics more interesting without trivialising its nature or content. These additional constraints - i.e., making sure that mathematics was not trivialised or simplified for the sake of making it more interesting or 'fun', were additionally considered and taken as a guide at all times in formulating our method.
The “uses” of the history of mathematics

Mathematics history in mathematics education has become a scholarly discipline in its own right (Lawrence & Đokić, 2014). The various international bodies therefore support such scholarly activity in different ways: the ICME (International Congress of Mathematics Education) has regular topic study groups related to the history of mathematics and mathematics education, and a global association of mathematics history education scholars is effectively organised through the History and Pedagogy of Mathematics group (an associate organisation of the International Mathematics Union). Both of these organisations have meetings every four years, as does the European Summer School in the History and Pedagogy of Mathematics group (HPM)3, when the possibilities to exchange new ideas and information about the recent outputs are utilised. Some national groups of mathematics history educationists also generate considerable scholarship in the field, such as the Canadian Society for the History of Mathematics and the British Society for the History of Mathematics, with two global journals dedicated to the history of mathematics: Historia Mathematica (published by Elsevier), and the British Journal for the History of Mathematics (published by Taylor & Francis)4.

But how does one connect the scholarship in the field with the experience of learners in the classroom? Well, this too has been a focus of research in the above mentioned groups of scholars in the past half a century. One of the prevailing theories of how history of mathematics can be used in the classroom, as a tool and as a goal itself (Jankvist, 2009), has informed various groups of researchers and teachers since it has been formulated. Within this framework there are different foci that researchers usually take:

- Looking at the theoretical and/or conceptual frameworks for integrating history in mathematics education.
- History and epistemology in students’ and teachers’ mathematical education: Classroom experiments and teaching materials.
- Original sources in the classroom and their educational effects.
- Mathematics and its relation to science, technology, and the arts: Historical issues interdisciplinary teaching and learning.
- Cultures and mathematics fruitfully interwoven.

3 History and Pedagogy of Mathematics Group has a website and publishes a Newsletter three times a year. This is accessible from two sources: https://grouphpwm.wordpress.com/ and http://www.clab.edc.uoc.gr/hpm/NewsLetters.htm (both accessed 2nd January 2020).
• Topics in the history of mathematics education.

and more recently,

• Looking at a particular focus depending on the meeting, for example a History of Mathematics in China and Eastern Asia.5

The focus of this paper is to define more narrowly a type of approach of how to make mathematics teaching more meaningful and engaging for students so that we do not end up with the results as those mentioned in the previous section (Smith, 2004). Rooted in practice, first with the secondary-age children, and then with their teachers, we sought to answer this challenge and in turn posed the following question: How can teachers structure their mathematics curriculum so the learning of mathematics becomes more meaningful for students and keeps teachers inspired to learn at the same time? The question is not original, and there is a disparity of whether learning mathematics in historical context, which can become more meaningful for one group of people (for example teachers), may not have the same or similar effect on another group (for example students). This in itself is an important research project that has a considerable existing bibliography (Panasuk & Horton, 2013) and is mentioned here as it is important to consider this aspect of using the history to engage and inspire learners.

As we now get into discussing history as part of mathematics as a school subject, let us digress a little and compare mathematics with art, and mathematics education with art education. Mathematics is often compared with art, and art education, in our experience, is usually something secondary school students rarely complain about. Why should this be? Again, from experience, it seems that everyone is able to enjoy, to a certain degree, working in an art classroom as there is freedom and expectation that students, in order to learn about art, will be given opportunities to explore and ‘muck-about’.

Let us draw another parallel. In educating an artist, history has an important role. One cannot become a competent ‘artist’ without knowing some of the history of art. This may, some argue, pose a problem: and very briefly, that problem is that the artist-teacher would have adopted some stance in relation to the history of art having understood the complexities (including political attitudes) of certain artists (Day, 1986). Problem of this nature may be resolved if the teacher-artist adopts a role of an artist when teaching some aspects of art and becomes a teacher (and not an artist) when teaching some others – the question here is of the ability of the teacher-artist to distance themselves from what they ‘like’ about art. The goal of art education however remains the same: to teach creativity and self-expression, using all the tools one may have at one’s disposal, creating an aesthetically engaging piece (Day, 1986).

But the difference between an art teacher (in the context which we describe in this paper and bibliography therein) and a mathematics teacher is both qualitatively and substantively different. Whilst an art teacher is usually an artist at home, a mathematics teacher does not have to be a mathematician. In fact, the author reports elsewhere (Lawrence & Ransom, 2011) that almost a half of mathematics teachers she has come across during her work as a teacher educationalist in England do not have a degree in mathematics or a related discipline. This does not mean teachers without the first degree in mathematics are not good teachers of mathematics – but it does mean that they are not necessarily aware of the complexities of the development of mathematics through history. This state of affairs can, apart from many other problems that can materialise in the classroom, lead to mathematics teachers feeling that they would have to know all of the history of mathematics before they can start introducing some historical elements into their mathematics teach-

ing repertoire. This fallacy can be avoided, the author argues (Lawrence, 2013) if one adopts a method which identifies the teaching of mathematics to the phases of the development of mathematics. This method is not new – Heppel for example used it towards the end of the 19th century (Heppel, 1893) and Barwell at the beginning of the 20th century (Barwell, 1913).6

Fauvel and Van Maanen (1997), Katz et al. (2000) and Furinghetti (2013) discuss these and other approaches at length, offering the insights into the examples on how to structure teaching within historically rich context, and how the teaching of mathematics with its history enhances engagement and motivation.

But in the era when students are heavily oriented towards the uses of technology that allow them new pathways of exploring the mathematical experiences, a new approach is perhaps needed. This approach, it is suggested, is about giving students the insights into the real mathematicians’ experiences, in order that they can develop an understanding of the importance mathematics has for those who

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6 See also Furinghetti (2012).
Figure 3. Barwell (1913: 97), similar idea to Heppel (Figure 2), but given as an inverted image of the similar structure.
contribute to its development. This of course depends on how much we can conjure such examples in a mathematics classroom, and there are numerous examples where groups of people, or nations, or indeed movements which are mainly anonymous (such as for example the abacus schools in mediaeval Europe) contributed to mathematics. A task to conjure such a culture of mathematical practice may be more difficult, in order to make it accessible to younger audiences, in comparison to giving details related to historical personalities (Robson & Stedall, 2008). Of course this ‘conjuring’ is itself a modern interpretation of the use of ‘role models’ (Lawrence, 2007; Wiest & Johnson, 2005) but with an added layer – accessibility of original sources which are now mostly freely available to all with an access to the Internet.

We are now getting ‘into’ the mechanics of methods employed in the reported practice of teaching mathematics through, and alongside, its history.

Mathematics as a way of life and mathematical history as an art of memory

The history of mathematics can teach many things, and as teachers we have to make a choice between the topics we want to introduce to our students. After years of searching for most effective methods and topics to incorporate into my mathematics curriculum, we realised that one such method was represented very well in all historical examples the author both used herself. This method (and subsequently the topics that illustrate it) that can be perhaps best articulated as the subtitle of this section. To say that there was an ‘option’ and a conscious decision that this method would be used as opposed to any other methods described for example in Katz (2000), would be an over-confident, and probably exaggerated claim. On the contrary, the method came through an accumulated experience, as years went by, and the author’s research was further enlightened by her work and engagement with mathematical bodies and societies, and constant further learning about the lives and activities of such a wide range of mathematicians including those working in industry, government, education, and finance. As the present work with mathematicians and within their organisations developed (Lawrence, 2016), parallels were (or at least some correspondences) drawn with what we learnt about their predecessors, the mathematicians from the past. A realisation eventually crystalized that to mathematicians, mathematics is as a way of life, and they are mainly people who inhabit a mathematical landscape as a type of universe within which they live. This description, whilst it is, of course, more poetical and philosophical rather than physical, we put forward as being more accurate than the alternatives (for example that mathematics is a profession). For mathematicians, just as for artists, their discipline offers a world which they can happily inhabit, and many do so well beyond their professional lives. In this mathematical universe, mathematicians learn from others and are linked to them regardless of the distances both geographical or in time. There they form alliances, friendships and have arguments. In most instances, the new mathematics that they discover and articulate illuminates other mathematical inventions.

So the question then arose about finding a way to make this mathematical world accessible to pupils and teachers and involve them, perhaps at the same time. How can this mathematical world be opened up and which methods could we employ to show to teachers that they themselves can do the similar in their own classrooms? Such method would help bring about not only greater engagement of students in mathematics classrooms, but would in fact directly link to the purpose of study of mathematics, as given in the introduction to this paper.

The answer to the first of these two questions is given in other places (Lawrence, 2011, 2015, 2019). At the end of the author’s secondary teaching work, a Langton Centre for Young Mathematicians
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was created by the author and her pupils. This little institute was set within an ordinary mathematical classroom and had research, publishing, and exchange programmes. This was the culmination of the author’s work in a school classroom, which brought both the recognition within the scholarly community and, more importantly, some life-long engagement with mathematics in many of my students (Lawrence, 2012).

How to translate that into a work with teachers so that they can also benefit from the experience of what works in the mathematical classroom (Lawrence, 2009) and where history is part of learning? The first aspect of that work is to engage teachers in learning some new mathematical procedures within rich mathematical contexts (Lawrence, 2011; Foster, 2013). Once teachers realise that what they have taught one way may be shown in many different ways, seen from many historical examples, they can re-evaluate their position towards both their pedagogy and their discipline.

Another aspect of this effort in the work with teachers was the examining of mathematical scenes from history so that they could re-examine their own role within this rich and long history of abstract thought that we call mathematics. The most successful of such projects, as reported by the students, was the one in which teachers began looking into a painting of a mathematician and recreated their own portrait along with the tools they themselves would most like to be able to use in their classrooms. The first case in this project was the portrait of Luca Pacioli, painted around 1500 (Lawrence, 2019). Through this project we realised that the setting that the Prince’s Teaching Institute uses for our various programmes for teachers itself can be used. The beautiful settings of some of the historical buildings in which the events took place could be imagined to be settings in which we place our mathematical objects and concepts. Perhaps this needs a clarification – any setting is as good as another, but the historical buildings reminded of the classical tradition and one of its more beautiful (and useful) aspects: the mnemonic system of Symonides of Ceos (Yates, 1966). In this system of memory, we can place objects around the ‘theatre of memory’, a real or imagined space set as a theatrical scene, and as we later move around, we are able to recreate in our mind the concepts that relate to such object and discuss it with our friends. This simple method can be used for teacher events as much as it can be used in the classroom. Yates’ work (1966), and the sources she mentioned there, suddenly found application in the most unlikely of places – a mathematics classroom.

One does not have to have a physical classroom, as such spaces can be created in the virtual world as well as in the real one (Lawrence, 2004); in fact, the Internet makes such spaces much easier to create and use. To shine a little bit of light into the scenes on mathematicians from the past and the work they do has been a perpetual interest of the author, and within the framework of the ‘art of memory’ this ancient method is recommended as one that can be replicated easily in any circumstances and by teachers in any and all mathematics classrooms. The ‘theatre of mathematical history’ has opened up the scenes in which mathematics and mathematicians play an eternal game of discovery that enchants as it engages all that see it. It is an open theatre in which audience can always get involved.
References

ГДЕ МОЖЕМО ДА НАУЧИМО МАТЕМАТИКУ – ИСТОРИЈСКИ МЕМОАР

У овом раду је је прихватао употребу историје математике у математичком образовању у Енглеској, како би се понудио увид у могуће начине на које историју математике може да допринесе математичком образовању на националном и локалном нивоу. Поред тога, овај рад је прихватао употребу историје математике у настави као средство за понашање наставника и студенте у њеним активностима. Истражујемо однос између историје, филозофије математике и математичког образовања. Испитују се одређени примери употребе историје математике, као и корист коју наставници и студенти математике могу имати од проучавања ове области. Поред тога, предлажу се начине на које наставници математике могу да инкорпорирају историју математике у наставу.

У раду представљамо неколико међународних тела која подржавају истраживање начина на које историја математике може да се употреби у образовању. Овај рад је поред многих других научних студија, признат за важан у овом смеру. Уводним делом рада је представљена историја настава у Мидлесексу и разгледана могућности употребе историје математике у образовању. У цртежу, на пример, је већина учења геометрије из историје, како би се скоришћена разликовања и представа о математикама у различитим периодима у историји математике.

Основну разлог за употребу историје математике у образовању, као и европску моделу историје математике, позвали су наставници и студенти математике, као што су представљена у овом раду. Они су показали како историја математике може да допринесе настави човека и онога који су штители математике. У цртежу је приказана њихова ставка и уздао и посебне питање која су постављена у контексту историје математике. Овај рад је признат за важан у овом смеру и је признат за могућност употребе историје математике у образовању.
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Our investigations over two decades are summarized in our work, as well as our involvement in (contemporary) mathematical organizations and societies. Based on these experiences, as well as the continuous learning about the lives and work of many mathematicians, including those working in industry, government agencies, education, and the financial sector, we propose a method by which all these experiences can be metaphorically transferred into the classroom. We are, in fact, proposing a new approach to integrating the history of mathematics into the teaching of mathematics that is based on the ancient technique of memory, revived in the age of the Renaissance. This approach involves creating an imaginary theatre that includes mathematical objects and images. How to present the mathematicians of the past and their work is a topic that has occupied us for years, and our innovative approach, set within the framework of “memory techniques”, is useful because it can easily be replicated in any situation and can be used by teachers in any form of mathematics teaching. This “theater of mathematical history” is a stage where mathematics and mathematicians play an eternal game of discovery that inspires and engages everyone who sees and experiences it, including mathematics teachers and their students.

Key words: National Curriculum in England, Princess Training Institute, MKMO (International Congress on Mathematical Education), Group for the History and Pedagogy of Mathematics (IHPM), European Summer School (ELSH).