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Original Scientific Paper
UDC 101 Кант И.
101.1:51
Received: August 17, 2024
Revised: January 16, 2025
Accepted: March 11, 2025

FOR LACK OF OBJECTS: DENYING THE CONSTRUCTIVE CHARACTER OF ARITHMETIC

Abstract

Kant's arithmetical judgements do not meet his standard for mathematical construction. In his view, mathematics is synthetic a priori because it constructs mathematical objects a priori. The method of constructing is not one that philosophy can mimic but the resulting intuitive objects are an indication of how ampliative judgments can have a secure ground and do serve as guide to philosophy. While it can be separately argued that arithmetic is synthetic a priori, construction requires the exhibition of an object in intuition and mathematical construction in particular produces a continuous magnitude. In geometry, geometrical concepts have non-conceptual content via elementary constructions that correspond to a priori intuitive activities. A priori intuition conditions the successive nature of considering arithmetical judgements but it does not provide arithmetic with non-conceptual content. Absent arithmetical intuitive objects, we cannot characterize arithmetic as constructive, something Kant views as endemic to the mathematical method. To deny the in concreto constructive method of arithmetic is to deny its status as an organon of mathematics. While J. Michael Young's tracing of the arithmetical process closely aligns with the framework of construction it falls short of mathematical construction in subtle but significant ways. Arithmetic remains synthetic a priori but achieves syntheticity because arithmetical concepts cannot be exhaustively and exclusively contained within one another.

Keywords: Kant · Mathematics · Synthetic A Priori · Intuition · Arithmetic

More than two hundred years later, Kant's work continues to influence various fields of philosophy. Though brief, his remarks on mathematics have had an enduring impact. One of the most recognizable assertions of Kant's theoretical philosophy is the claim that all mathematical judgments are synthetic *a priori*. That is, for Kant, mathematical judgments offer content that is more than logical analysis can provide while remaining apodictically certain independent of experience. This claim has persistently served as a point of debate in understanding Kant's transcendental idealism and in the ensuing development of the foundations of mathematics.[1] In Kant scholarship, these discussions often overlook the distinction between syntheticity and construction. On my view, Kant's characterization of arithmetic does not meet his own standard of mathematical construction because it does not construct mathematical objects of its own domain. In "Arithmetic and the Categories," Charles Parsons holds that "[i]n one way or another, Kant must regard some objects of arithmetic and algebra as at a conceptual remove from the intuitions that found statements about them". (Parsons 1983, 48) This conceptual remove is far from inconsequential. In what follows I show that while this conceptual mediation does not undermine the syntheticity of arithmetic, it requires denying the constructive character of arithmetic.[2]

While some of Kant's reflections on mathematics motivated his critical philosophy, Kant does not offer an explicit philosophy of mathematics. Much of what he does say about mathematics is in the service of a broader philosophical point about cognition. When Kant first argues that mathematical judgments are all synthetic a priori in the Introduction to the first Critique, he is offering evidence that such judgments exist and that we have warrant to ask about their possibility in developing a system of metaphysical cognition. In contrast, when Kant argues for the constructive character of mathematics in the Discipline of Pure Reason, he is distinguishing it from the discursive method of philosophy. In so doing, Kant holds that philosophy cannot avail itself of the constructive method. These are two distinct points of emphasis that draw on two distinct—though related—features of mathematics.

Mathematical judgments are synthetic *a priori* because they turn to a priori intuition to construct. Yet Kant describes arithmetic's turn to intuition in a manner distinct from that of geometry. For Kant, geometry synthesizes concepts with the a priori intuition of space to construct spaces while arithmetic turns to intuition to represent the successive (*hintereinander*) nature of arithmetical operations. J. Michael Young argues that arithmetical activity meets the standard for construction because, for Kant, fingers or strokes on a page are examples of an exhibition in intuition that corresponds to the concept. My view is that while such intuitions relate to the concept, they do not constitute an arithmetical object. Much of the secondary literature justifiably focuses on the syntheticity of pure mathematics, but the unintended consequence is that syntheticity and construction are often

1 Several important figures (Frege, Hilbert, Einstein, etc.) in the development of mathematics since Kant reference him as influential in the development of their thought, even if they are challenging or rejecting claims he made.

2 Although Parsons makes this remark about arithmetic and algebra, I limit my discussion to concerns about arithmetic. In the final analysis, the distinction at work between algebra and geometry is different from that which distinguishes geometry and arithmetic. Algebra for Kant concerns a method rather than a separate mathematical domain.

implicitly considered biconditional. Kant's own remarks about arithmetic appear to betray other claims he makes about construction and pure mathematics. Kant is certainly not intending a separation between geometry and arithmetic regarding how they meet the criteria of mathematical activity. My point is that his account of the respective activities leaves room for a divide. If we distill out some version of a philosophy of mathematics in Kant, we must recognize Kant's accounts of geometry and arithmetic as differing regarding construction.

In the first section, I detail Kant's account of mathematical construction. In the second section, I outline the differences in Kant's description of geometrical and arithmetical activity. In the third section I show how such differences leave arithmetic without the requisite objects and the necessarily constructive character. Finally, I consider how a non-constructive arithmetic remains synthetic and fits into a coherent system of pure mathematics more broadly.

1. Mathematical Construction

For Kant, mathematical judgments go beyond the concept of their subjects with great security because in the turn to intuition, mathematics always treats its objects *in concreto* and concepts alone can only treat objects *in abstracto*. When we treat objects *in concreto*, we can always keep our objects of investigation immediately before us and treat them as particulars. *In abstracto* treatment of the objects of investigation is mediated and indirect as it treats them as universals. When we treat an object *in concreto* we deal with *this object*. When we treat an object *in abstracto* we must relate this object to a concept that holds for all objects with a subset of its features. In *abstracto* treatment of objects is discursive and as such is mediated.

> Philosophy confines itself solely to general concepts, [mathematics] cannot do anything with the mere concepts but hurries immediately to intuition, in which it considers the concept *in concreto*, although not empirically, but rather solely as one which it has exhibited *a priori*, i.e., constructed, and in which that which follows from the general conditions of the construction must also hold generally of the object of the constructed concept. (B743f)[3]

Kant's concern is that when we consider objects in a mediated or indirect manner, we could let the concepts progress unchecked without something to discipline our use of those concepts. When the object remains before us, that risk is avoided.

3 As is customary, I will refer to the *Critique of Pure Reason* with edition (A or B) and page number. Other references to Kant will refer to Kant's *Gessammelte Schriften* (Berlin: Georg Reimer, later Walter de Gruyter & Co., 1900-) by volume and page number. Unless otherwise noted, all translations are from the *Cambridge Edition of the Works of Immanuel Kant*.

Kant speaks of both constructing a concept and constructing an object. This seeming terminological distinction does not amount to a difference in geometry since for Kant "[w]e cannot think of a line without drawing it in thought" (B154). To think or cognize the concept of a geometrical object is to think through the procedure for producing the object in thought. Thus, to construct a circle is both to produce the composite pieces of its concept and to think through the production of a circle in intuition. Any geometrical construction is a procedure that is a composition (*Zusammenfassung*) of the elementary constructions of drawing a line, extending a line, and circumscribing a circle. For Kant, the geometrical object is not the image left behind, but the activity of producing it. Images are of value in geometry only insofar as they serve as correlates of a procedure. This constructive procedure is a figurative synthesis of the productive imagination that is not guided by the features of images but by the *a priori* that we have ourselves put into the construction—both conceptual and intuitive.

For Kant, the relevant features of a mathematical construct are owed to and always exhibited in intuition. The a priori intuitions of space and time structure our receptivity to experience and are given prior to any conceptual syntheses. So that mathematics is conditioned by something more than logical analysis. This relationship to pure intuition assures the objective validity of pure mathematics. Mathematical judgments are evidence that we have a priori access to real possibility. Real possibility tells us something more than mere logical possibility. In order to move beyond mere logical possibility, we must turn to intuition.

> That in such a concept no contradiction must be contained is, to be sure, a necessary logical condition; but it is far from sufficient for the objective reality of the concept, i.e., for the possibility of such an object as is thought through the concept. Thus in the concept of a figure that is enclosed between two straight lines there is no contradiction, for the concepts of two straight lines and their intersection contain no negation of a figure; rather the impossibility rests not on the concept in itself, but on its construction in space, i.e., on the conditions of space and its determinations; but these in turn have their objective reality, i.e, they pertain to possible things, because they contain in themselves *a priori* the form of experience in general. (B268)[4]

4 This idea that the a priori intuition of space rules out the possibility of a figure enclosed by two lines has received plenty of attention in the secondary literature and is viewed as evidence of Kant's commitment to Euclid. Though I think a Kantian view of mathematics can remain coherent while holding that space rules out the real possibility of certain logically possible concepts I do not think the two sided figure can be considered an intuitively impossible though logically possible object. As Heis (2020) points out, Kant had some reservations about the status of the parallel postulate (or parallel axiom as Heis calls it) his deference to Euclid notwithstanding.

Intuition makes possible such concepts and thus conditions mathematical judgments. Intuition conditions not only our activity of forming the concept of the object but also the content of the concept as our geometrical concepts can only be of spatial referents.

The a priori intuition of space is the ground of geometrical concepts. Space is necessary to generate geometrical concepts like line, circle, plane, etc. Kant recognizes the possibility of manipulating such concepts without the turn to intuition, but such manipulation would be a matter of mere play because it would not be about anything and thus would not amount to cognition. $(x\text{-}a)^2 + (y\text{-}b)^2 = r^2$ offers a formula for a circle, but for Kant this is inseparable from the intuitive sweeping out of a circle in space. The significance or meaning of this formula is necessarily tied to construction in intuition. As Heis notes, Kant "believed that possessing a concept, having its definition, and being able to construct instances of it were all coeval abilities. On Kant's view, one cannot possess a mathematical concept without knowing its definition, and one cannot know the definition without knowing how to give oneself objects that fall under the concept" (Heis 2014, 608). There must be objects about which such judgments are true. Such a priori objects are made possible by the a priori intuition of space. They are constructed because they are the result of an a priori synthesis that exhibits a construction a priori in intuition. When we consider such objects, we have an experience, but it is only the a priori that we have put into the experience that counts as the mathematical, not the images left behind,

> A new light broke upon the first person who demonstrated the isosceles triangle. For he found that what he had to do was not to trace what he saw in this figure, or even trace its mere concept, and read off, as it were, from the properties of the figure; but rather that he had to produce the latter from what he himself thought into the object and presented according to a priori concepts, and that in order to know something securely a priori he had to ascribe to the thing nothing except what followed necessarily from what he himself had put into it in accordance with its concept. (Bxii)

This is a necessary feature of mathematics. Kant thinks that recognizing this is what set math on the right path.

Moving beyond the concept of the subject is necessary but not sufficient to denote a judgment as mathematical. That is, a judgment can be synthetic a priori but that alone would not denote it as mathematical. Kant is trying to establish a critical philosophy comprised of synthetic a priori judgments. The necessary or distinguishing feature is that mathematics constructs objects in intuition.

> Philosophical cognition is rational cognition from concepts, mathematical cognition that from the construction of concepts.

> But to construct a concept means to exhibit a priori the intuition corresponding to it. For the construction of a concept, therefore, a non-empirical intuition is required, which consequently, as intuition, is an individual object, but that must nevertheless, as the construction of a concept (of a general representation), express in the representation universal validity for all possible intuitions that belong under the same concept. (B741)

Philosophical cognition can turn to intuition and be synthetic a priori but it would not be able to construct. For construction, the exhibition of a corresponding object in intuition is necessary. We must understand *a priori* construction as a distinguishing feature of pure mathematical judgments within the class of synthetic a priori judgments. It is not merely that pure mathematics turns to intuition as all synthetic judgments must do, but that pure mathematics turns to intuition in a particular way that conditions mathematical constructions.

2. Arithmetic and Geometry

While arithmetical and geometrical activity may not be separable to Kant, he certainly distinguishes between them and their respective activities. Kant has good philosophical reasons to find unity in the syntheticity of arithmetic and geomerty. Daniel Sutherland has shown that Kant's view of magnitude draws on the Greek theory of proportions where the relevant concern is the consideration of homogenous continuous magnitudes, irrespective of numbers.

> Numbers counted as magnitudes, and in particular, discrete magnitudes. Because numbers cannot stand in many of the ratios that continuous magnitudes can, however, numbers appear to be just a special, limited case of magnitudes more generally. As a result, the Euclidean tradition focused on the continuous magnitudes of geometry rather than on arithmetic and developed mathematics as far as possible without appeal to numbers. The Greek mathematical tradition passed down through Euclid was shaped by this priority of geometry over arithmetic and influenced thinkers into the eighteenth century, including Kant. (Sutherland 2006, 537)

This accounts for the unity Kant sees in arithmetical and geometrical activity. The fundamental concern with proportion also suggests how an emphasis on geometry would be consistent with such a unity since the discrete magnitudes of arithmetic would be presumed by the continuous magnitudes of geometry. In this sense, the construction of both discrete and continuous magnitudes has its ground in a notion of composition.

When considering geometry, Kant has in mind something like the proof procedures of Euclid's Elements. In arithmetic, Kant has in mind counting and the basic operations of addition, multiplication, and exponentiation (and their inverses). There is a prima facie case for geometry's dependence on the content in question, "[t]he synthetic and intuitive character of geometry gets a considerable plausibility from the fact that geometry can naturally be viewed as a theory about actual space and figures constructed in it" (Parsons 1983, 128). What Kant considers to be the elementary operations of geometry, which are postulated in Euclid's Elements, each correspond to a phenomenological activity in the a priori intuition of space. In arithmetic, it is less clear how intuition—time or space—provides necessary content to arithmetical judgments. As Emily Carson notes, "whereas the link between geometry and intuition is direct (geometry is about the form of outer intuition), that between arithmetic and intuition is not so direct (arithmetic is not the science of time, the form of inner intuition). It is clear from this, in any case, that any role for intuition in grounding arithmetic is going to look rather different" (Carson 2020, 234). Put another way, space does not ground arithmetic in the same manner as it grounds geometry and time cannot ground arithmetic in the manner in which space grounds geometry. Sutherland (2014) for example thinks that the proper analogy is that geometry is the science of space as mechanics is the science of time. In describing geometry and arithmetic, Kant gives further indication of the different relationship with intuition.

The clearest distinction Kant draws between arithmetic and geometry, which gets the most attention in secondary literature, concerns their use of axioms. Kant held that "concerning magnitude (*quantitas*), i.e., the answer to the question, 'How big is something?', although various of these propositions are synthetic and immediately certain (*indemonstrabilia*), there are nevertheless no axioms in the proper sense" (B204). Such immediately certain synthetic a priori principles qualify as postulates if they are practically necessary. Kant grants the indemonstrable yet immediately certain principles of geometry because its constructions are always considered *in concreto*. This *in concreto* treatment is what allows for a synthesis that is not mediated, "mathematics, on the contrary, is capable of axioms, e.g., that three points always lie in a plane, because by means of the construction of concepts in the intuition of the object it can connect the predicates of the latter a priori and immediately" (B760). There are, however, synthetic self-evident propositions in arithmetic, but they are not general. Instead, they take the form of singular numerical formulas like the oft cited 7+5=12.

Contemporary perspectives tend to include axioms among the essential principles of arithmetic, but Kant has in mind a slightly different notion of axiom. Axioms are immediately certain propositions, and their certainty has its ground in intuition.

> Discursive principles are therefore something entirely different from intuitive ones, i.e., axioms. The former always require a deduction, with which the latter can entirely dispense, and, since the latter are on the same account self-evident, which the philosophical principles, for all their certainty, can never pretend to be, any synthetic proposition of pure and transcendental reason is infinitely less obvious than the proposition that Two times two is four. (B760f)

These immediately certain propositions serve as the given intuitive rules of construction. While there are rules for arithmetical judgments, Kant seems to grant that no arithmetical rules are conditioned by the nature or structure of intuition. Whatever its indispensability for arithmetic, a priori intuition does not contribute to the content of arithmetical judgments. In this passage Kant uses an arithmetical example as an example of an axiom although he explicitly rules out arithmetical axioms.

For Kant, geometry has axioms and arithmetic does not because the latter lacks *quanta* as objects. In a letter to Schulz concerning arithmetic, he writes that arithmetic "has no axioms, because it actually does not have a quantum, i.e., an object of intuition as magnitude, for its object, but merely quantity, i.e., a concept of a thing in general by determination of magnitude" (Br 10:555f). In this letter Kant reiterates that the thought of the addends cannot at the same time include the thought of the sum. He grants that while particular numbers when applied to intuition have spatiotemporal referents, the mere consideration of the relationships of the concepts of magnitudes does not construct any objects. Thus, the consideration of arithmetical magnitudes requires a mediating representation for the application to objects. It is precisely this separation from objects that is problematic for characterizing arithmetic as constructive. There is something that intuition provides to geometry that it does not provide to arithmetic.

Kant's concern for axioms depends on intuitive matter or content. Space and time are the forms of intuition, but space provides the content of geometrical investigation. Kant's famed footnote at B160 might create more disagreements than it settles but perhaps its clearest claim is that in geometry space must be treated as an object, "Space, represented as object (as is really required in geometry), contains more than the mere form of intuition, namely the comprehension of the manifold given in accordance with the form of sensibility in an intuitive representation, so that the form of intuition merely gives the manifold, but the formal intuition gives unity of the representation..." (B160n). Contemporary perspectives of arithmetic that draw on an axiomatic system like Peano's might argue that Kant was wrong about arithmetic lacking axioms though he might be right in his claim that arithmetic does not turn to intuitive objects (Friedman 1992). In the same letter to Schulz, Kant grants that though arithmetic may rely on time, time has no bearing on the properties of numbers:

> Time, as you correctly notice, has no influence on the properties of numbers (considered as pure determinations of magnitude), as it may have on the property of any alteration (considered as alteration of a quantum) that is itself possible only relative to a specific state of inner sense and its form (time): the science of number, notwithstanding succession, which every construction of magnitude requires, is a pure intellectual synthesis that we represent to ourselves in thoughts. (Br, 10:557)

In this sense, the intuitive element in arithmetic concerns succession. Arithmetical activity requires succession. That is, the synthesis is intellectual save for succession. Succession is not part of the content of the synthesis, but the time required to execute the synthesis. Put another way, it is not a synthesis of time, but a synthesis made in time.[5] Kant concedes that the properties of numbers are not intuitive. This separation between intuition and numbers is the conceptual remove that is not present in geometrical construction and cannot be in any mathematical construction. Kant here refers to arithmetic as constructive but is in my view only defending its intuitive character.

3. Constructing Mathematical Objects

In geometrical construction, conceptual syntheses limit the a priori intuition of space to generate particular spaces. The geometrical space in which we perform geometrical constructions is also a construct, the product of an a priori synthesis that is distinct from metaphysical space. Kant makes this distinction explicitly in On *Kästner's Treatises*,

> Metaphysics must show how one can have the presentation of space, geometry however teaches how one can describe a space, viz., exhibit one in the representation a priori (not by drawing). In the former, space is considered in the way it is given, before all determination of it in conformity with a certain concept of object. In the latter, one [i.e. a space] is constructed (*gemacht*). (AA20:419)

In order to treat space in geometry, we must construct it. The a priori intuition of space grounds the kind of geometrical spaces we can construct. That constructed space is distinct from the given original representation of space. Counting and adding as Kant describes them, however, are not limitations on time the way squares and rectangles are limitations on space, "Kant certainly

5 This distinction is used as part of the motivation to argue that Kant is working with what today we would call an ordinal theory of number as opposed to a cardinal theory. Sutherland (2020) has a thorough discussion of what is at stake in that distinction. It might be that with an ordinal theory of number the case for denying construction might be more straightforward but on my view arithmetic's lack of objects would remain if we view Kant a s having a cardinal theory.

did not regard arithmetic as a special theory of, say, time, in the sense in which he regarded geometry as a special theory of space" (Parsons 1983, 133). Of course, all intuitive content is always temporal, but numbers are not uniquely conditioned by intuition nor do they result from intuitive activity. Neither numbers nor sums are discrete times. Counting and adding require time for the sake of successive synthesis but not in the sense that the resulting constructs are made a synthetic unity with an a priori intuition—of time or space. But while this presupposes temporal intuition in mathematics for the sake of successive synthesis, it also underscores the incompleteness of its contribution. The one-after-another-ness of mathematical synthesis does not provide content. Rather, because mathematical synthesis deals with intuitive content, it must do so in a one-after-another manner,

> Since the mere intuition in all appearances is either space or time, every appearance as intuition is an extensive magnitude, as it can only be cognized through successive synthesis (from part to part) in apprehension. All appearances are accordingly already intuited as aggregates (multitudes of antecedently given parts), which is not the case with every kind of magnitude, but rather only with those that are represented and apprehended by us as extensive. (B203f)

Regarding extensive magnitudes, the representation of the parts is required and thus presupposed for the representation of the whole. This part-whole relationship is necessary for any intuitive representation and is one reason Parsons challenges the immediacy condition, questioning whether infinite features are given immediately or require a going over.[6] In geometrical construction, there is not much of a problem in suggesting that the parts must be constructed by delimiting the whole. Time admits parts, but its parts are not part of the content of any mathematical construction, rather the parts of time condition our notion of composition. A sum is not a composition of temporal content.

This lack of intuitive content is problematic since the emphasis on construction in pure mathematics is supposed to produce objects in its turn to intuition, rather than a merely intellectual synthesis:

> Although all these principles, and the representation of the object with which [mathematics] occupies itself, are generated in the mind completely *a priori*, they would still not signify anything at all if we could not always exhibit their significance in appearances (empirical objects).... Mathematics fulfills this requirement by means of the construction of the figure, which is an appearance present to the senses

6 In "Infinity and Kant's Conception of the 'Possibility of Experience'", (1983) Parsons makes the case that the infinite or indefinite features of intuition cannot be singularly intuited. These infinite features are supposed to be part of why concepts are deficient in accounting for them but if the features require a going over to intuit them, then there is a tension between them and the realm of "possible experience."

> (even though brought about *a priori*). In the same science, the concept of magnitude seeks its standing and sense in number, but seeks this in turn in the fingers, in the beads of an abacus, or in strokes and points that are placed before the eyes. (B299)

This affirms the necessity of our concepts' applicability to objects of possible experience and the need to keep the objects before us. In the geometrical example, we have produced the appearance with the relevant features. Interestingly, Kant here suggests that in constructing a figure we *fulfill* (*erfüllt*) the requirement, while fingers and beads are where arithmetic seeks (*sucht*) to meet this requirement. I doubt that we can read Kant as having such wordsmithing in mind, but we can recognize a clear distinction. On the one hand, we can construct a priori a spatial geometric figure that has the spatial features we are considering. On the other hand, we can turn to spatial referents to consider a spatial exhibition of a nonspatial property or relationship. Since we often apply numerical values to our geometrical constructs, such numerical activity would be related or associated and might seem amenable to geometrical representations. That is, as I often assign numerical values to geometrical constructs, I can associate geometrical constructs to numerical considerations. That would, however, be distinct from those arithmetical activities requiring such construction. For example, the geometer may know 3, 4, 5 as a Pythagorean triple, but the arithmetician does not need to construct a triangle to consider the series and its relationships. When we plot the unit circle on a graph, we use the numerical values of cosine and sine for the x and y axes, but we do not have to construct a 30/60/90 triangle in order to consider the value of ½.

Mathematical reasoning makes objectively valid claims because it deals with objects *in concreto*. It accesses the universal in the particular by keeping objects before us. When we abstract away features of intuitive objects to deal with them discursively, we are not guided by the object and instead are guided by our concepts. By abstracting we select features that we deem relevant but in so doing we reduce the object to a subset of its features. When we consider an object *in concreto*, we must reason about this object, not a category of objects. For an empirical object, we must abstract to consider a category of objects. In the Doctrine of Method, Kant argues the *in concreto* method is exclusively for pure mathematics,

> Give a philosopher the concept of a triangle, and let him try to find his way how the sum of its angles might be related to a right angle. He has nothing but the concept of a figure enclosed by three straight lines, and in it the concept of equally many angles. Now he may reflect on this concept as long as he wants, yet he will never produce anything new. He can analyze and make distinct the concept of a straight line, or of an angle, or of the number three, but he will not come upon any other properties that do not already lie in these concepts. (B744)

Later in the Doctrine of Method Kant distinguishes between the technical and the architectonic unity of sciences. The former is the contingent historical unity of how questions in different sciences emerge. The latter is an internal a priori unity that "makes a system out of a mere aggregate" of cognition (B860). In this sense, there must be an internal end or question that allows for the systematicity of mathematics. A key element of that internal end is a constructive procedure. A full discussion of Kant's account of the architectonic is beyond the scope of this paper, but my contention is that in drawing so sharp a contrast between philosophical and mathematical cognition Kant has denoted constructing magnitudes a priori an element of the architectonic unity of mathematics.

Geometrical construction draws on activities that are conditioned by and exhibited in the *a priori* intuition of space. Michael Friedman identifies these elementary processes as drawing a line, extending a line, and circumscribing a circle. Geometrical activity is a matter of iterating these elementary constructions in myriad ways. These elementary geometrical constructions stem from our phenomenological a priori activities of translocating and rotating our perspective within the formal intuition of space, an intuition that has already been made a unity. These elementary geometrical activities correspond to *a priori* phenomenological activity that has been united with a concept to produce an *a priori* synthesis. For Kant, the possibility of such constructions is uniquely conditioned by the *a priori* intuition of space. As he puts it:

> That, however, the possibility of a straight line and a circle cannot be demonstrated **mediately** through inferences, but only immediately through the construction of these concepts (which is not at all empirical), is due to the fact that among all constructions (exhibition which are determined according to a rule in intuition *a priori*) some must be **the first**, such as the **drawing** (*Ziehen*) or the describing (in thought) of a straight line and the **rotation** thereof around a fixed point, where neither the latter can be derived from the former, nor these from any other construction of the concept of magnitude. (20: 411) (Kant's emphasis)

Geometrical activities are inseparable from the magnitudes provided by intuition. To construct mathematically is to remain in immediate relation to the intuition of space. The elementary geometrical procedures are inherently spatial, but arithmetic lacks an analogous connection to intuition. What seems like the basic or elementary process of arithmetic is not a mathematical construction.

In some of his earlier work, Friedman argues that we ought to contextualize Kant's appeal to intuition. Kant must turn to intuition because the logic at his disposal cannot ground infinity and derivative properties like continuity. These infinite features are present in geometry and arithmetic but Friedman notes a difference between the two,

> the successor function is not a specific function at all for Kant; rather, it expresses the general form of succession or iteration common to all functional operations whatsoever. So it is not necessary to postulate any specific initial functions in arithmetic: whatever initial functions there may be, the existence and well-definedness of the successor function is guaranteed by the mere form of iteration in general (that is time). Thus, in explaining why geometry has axioms while arithmetic does not at A164/B205-206, Kant refers to the need in geometry for general "functions of the productive imagination" such as our ability to construct a triangle.... The point, presumably, is that no such specific functional operations need be postulated in arithmetic. (Friedman 1992, 89)

Friedman recognizes the difference according to Kant between the two fields and holds that an underlying difference concerns whether we require intuition apart from succession. Geometry does, arithmetic does not. The elementary constructions of geometry are the specific operational functions postulated in geometry. In arithmetic, succession is a general form necessary to connect a sum to its addends, i.e., simply counting to 12 is not the arithmetical activity but rather the equating of twelve to the sum of 7 and 5. For Kant, the turn to intuition is required to verify that the conjunction of 7 and 5 is 12. We cannot stipulate or analytically derive that 12 is the conjunction of 7 and 5. For Kant, the synthetic process of arithmetic must be exhibited by turning to strokes on a page or the fingers of our hand. My contention is that this process involves intuition but is not a mathematical construction because what is exhibited is not arithmetical. Any twelve numerically distinct things in intuition will suffice for this exhibition and there is nothing about the strokes or fingers that constitutes the 'twelveness' of the sum. The twelve is without content absent the strokes or fingers. In geometry, not just any intuition will suffice; it requires this construction, the construction that is spelled out in the definition of our geometrical concept. There may be red ink or white chalk that is part of our experience, but the ink and chalk are not the geometrical construction. The construction has content because of the drawing, extending, or circumscribing. The process is *in concreto* because it keeps the relevant object before us. In cognizing 7+5=12, one must turn to intuition, but the content of the cognition is not an intuitive object. There is no synthesizing of the a priori intuition of time. To cognize duration, I must draw something extended in space but to cognize a line one must construct a line.

The successive strokes Kant has in mind do not constitute an intuitive construction or a limitation of time or space. They do not constitute a synthesis of intuition and a limiting concept. Kant wants mathematics to keep from the mere play of concept manipulation by turning to content *in concreto*. We turn to intuition to consider strokes on a page or successive moments in time, but in the construction of a circle, the turn to intuition is required in order to even have the concept of a circle. The concept is a result of a limitation on space that occurs via a synthesis with the *a priori* intuition of space and that is required for conceiving of circle. With strokes on a page, however, the content is not required to arrive at the definition in the first place. The content is not *in concreto* because it turns to the universal properties of numbers and considers their exhibition in other objects rather than being exhibited in the numbers themselves. That is, the exhibition is of other objects with number or magnitude as a property. We turn to intuition to have cognition, but the feature displayed is categorial. As such, fingers or strokes on a page equally suffice and neither is standing in for something else. Arithmetical and geometrical activity cannot be understood simply as two species of mathematical construction. There is a unique route to mathematical construction, by constructing magnitudes *in concreto*.

In geometry, elementary construction produces an object. But the pure synthesis of arithmetic does not produce arithmetical objects. Numerals are not arithmetical objects. The drawn numeral may constitute a spatiotemporal object, but it is an image that exhibits no arithmetical properties and merely serves as a convention to denote a number. Even numbers, however, are not themselves arithmetical objects. Numbers are indicative of a structure applicable to objects but are not constructs of objects. Numbers more closely resemble schemata than objects. We can have an intuitive representation of six objects but 6 is not an object of intuition for us in the way that a square or triangle is. Numbers serve as indications of the successive and conjunctive properties of objects but not as objects. The magnitudes produced by the processes of arithmetic can be applied to *a priori* geometrical constructions as well as to empirical objects, but they are not themselves objects of intuition for us. Arithmetical magnitudes have their application to sensibility, in that the intuitions of space and time provide the quanta to be enumerated but not their ground.

If we deny that arithmetic meets Kant's standard for mathematical construction, Kant can still use 7+5=12 as an example of synthetic a priori judgments. To ask how pure mathematics is possible would serve Kant's aim as long as we clarify that 7+5=12 does not exhibit anything uniquely mathematical. That is, 7+5=12 exhibits conditions of syntheticity even if it does not serve as an example of mathematical construction. To the extent that the discussion in the Doctrine of Method displays a philosophy of mathematics, constructing magnitudes is indispensable for mathematics.

> The essential difference between these two kinds [philosophical and mathematical] of rational cognition therefore consists in this

> form, and does not rest on the difference in their matter, or objects. Those who thought to distinguish philosophy from mathematics by saying of the former that it has merely quality while the latter has quantity as its object have taken the effect for the cause. The form of mathematical cognition is the cause of its pertaining solely to quanta. For only the concept of magnitudes can be constructed, i.e., exhibited *a priori* in intuition, while qualities cannot be exhibited in anything but empirical intuition. Hence a rational cognition of them can be possible only through concepts. Thus no one can ever derive an intuition corresponding to the concept of reality from anywhere except experience, and can never partake of it *a priori* from oneself and prior to empirical consciousness. (B742f)

It is not simply that mathematics deals with quanta, as arithmetic surely does, but rather that it constructs magnitudes. Magnitudes must be continuous and intuitive but there is no intuitive magnitude constructed in turning to fingers on a hand or strokes on a page. While these two appeals to intuition are unproblematic for speaking of syntheticity, they do not both meet the standard of mathematical construction.

Though arithmetic may deal with quanta, it receives those objects from elsewhere, rather than construct them. So, while arithmetic may have something to say about magnitudes, they are not arithmetical objects. As Friedman points out

> On this conception, therefore, algebra and arithmetic are not conceived as we would understand them today: that is, as bodies of general truths concerning specific domains of objects—the domains of natural, integral, rational, or real numbers for example. The only mathematical science that concerns a special domain of objects is geometry, and neither arithmetic nor algebra has a special domain of its own. Instead, algebra and arithmetic are conceived as techniques of calculation for solving particular problems for finding the magnitudes of any objects there happen to be—where the latter are not given by the sciences of arithmetic and algebra themselves. (Friedman 1992 113)

These objects of their own domain are precisely the marker of a constructive approach that treats objects *in concreto*. That is, if we are treating objects generated by another domain, we must abstract away some of their relevant features, without the object or its definition at hand as guide.

Many thinkers have worked to understand Kant's claims of the synthetic or constructive character of mathematics. J. Michael Young specifically considers the constructive character of arithmetic arguing that arithmetical activity fits Kant's recipe for construction,

> When Kant says that to construct a concept is to exhibit *a priori* the intuition that corresponds to it, what he means, therefore, is that it is

> to provide ourselves with the intuition of a particular object, one which somehow 'displays' or 'exhibits' the concept in question. His view, accordingly, is that mathematical judgments have to be based upon such intuitions. (Young 1982, 18)

Young is right that this is the way to understand construction in Kant, but the *particular* object must be generated by a rule or concept to be constructed. The object cannot be an empirical or received object. Put another way, the object must be the result of the productive imagination, as opposed to the reproductive imagination.

Mathematical construction exhibits concepts that are conditioned by the extensive magnitudes of intuition. As noted earlier, the exhibition itself must be non-empirical and thus have only that which belongs to the construction. To do so the exhibition must be immediately related to the concept it represents. It is to exhibit *the* intuition that corresponds to the concept, not an arbitrary intuition that can be related to it. Mathematics always has at hand the procedure for producing the object in a proper mathematical definition. Otherwise, the exhibited intuition is mediated and our consideration of it is *in abstracto*. Only mathematics is capable of this since its very concepts are generated from elementary intuitive activities. Young cites Kant's example at how such a relation is possible, "[i]n this empirical intuition [that results from a priori construction] we consider only the act whereby we construct the concept, and abstract from the many determinations (for instance, the magnitude of the sides and of the angles), which are quite indifferent, as not altering the concept 'triangle' (B742). In my view, Kant can only turn to a geometrical example to show this. He cannot use an arithmetical example here because there is no arithmetical analog with an exhaustive definition that can be sure to have all the relevant features and nothing else. In the case of my fingers as aid, I have not produced any of the particulars via my concept.

To remove the particulars that are not immediately related to the concept is to remove all the particulars, a point Young seems to concede,

> The numeral string '79' does not exemplify or instantiate the corresponding arithmetical concept, while geometrical diagrams do exemplify (at least approximately) the concepts they serve to construct. In Kant's terminology, a numeral string does not provide an ostensive construction of the corresponding arithmetical concept. The point remains, nonetheless, that it does exhibit or display what is thought in that concept. Since the mode of exhibition is symbolic, the numeral string can be said to provide a symbolic construction of the corresponding arithmetical concept. The column of numeral strings that we use in calculation can likewise be said to provide a symbolic construction of the concept of the sum that we seek to determine. The notion of symbolic construction is one that Kant himself suggests albeit with different examples in mind. (Young 1982, 23)

The sum is not an intuitive construction and so it cannot be exhibited immediately in intuition. It requires a mediate exhibition in intuition, but this is different from what Kant deems mathematical. Young tries to accommodate this difficulty by separating ostensive and symbolic construction, but this is problematic as it obfuscates Kant's intended meaning with regard to the status of symbolic construction.

As Lisa Shabel (1998) points out, Kant's language of symbolic construction borrows from a particular question in early modern mathematical practice. Early modern mathematics did not consider algebra to be its own subfield or domain of mathematics. Rather it was considered a way to reason about the domains of arithmetic and geometry. Mathematicians of the day had to deal with the question of the application of algebra to geometry. Thus, early modern mathematicians outlined a method for constructing equations. Shabel notes that it is algebra that Kant describes as constructing with symbols while holding that geometry and arithmetic construct without symbols. By symbolic construction Kant understands an ostensive construction with symbols, it is not a different kind of construction in mathematics with different requirements. Rather in ostensive construction we construct with determinate magnitudes and in symbolic constructions with symbols or indeterminate magnitudes. Kant views arithmetic and geometry to employ ostensive construction and symbolic construction can be employed to consider the indeterminate magnitudes of arithmetic and geometry. It is algebra that turns to symbolic constructions,[7] "the algebraist ignores the qualitative aspect of the object, considering it only in accordance with the pure concept, or category, of magnitude (*quantitatem*). Thus, according to Kant, the algebraist can construct a mathematical object qua quantity only, without regard for shape or figure" (Shabel 1998, 610). For Kant algebra is closer to a method for engaging in geometry and arithmetic than a subfield of mathematics. The algebraic method considers the unknown magnitudes of arithmetic and geometry, not magnitudes of a different kind or different domain. Since Kant does not think of symbolic construction as mediated, we cannot account for the mediation between intuition and numeral strings through an appeal to symbolic construction. Secondly, if a symbolic construction in Kant were as Young described here then it would betray Kant's comments about *in concreto* treatment of objects since it would seem to openly call for mediation between concepts and intuitive objects.

Most importantly, Young seems to grant that arithmetical construction is not identical to geometrical construction with the status of concepts as the most significant difference between them,

> Kant's thesis regarding arithmetical judgments rests on his view of arithmetical concepts and of their relation to the universal procedures

7 There is some terminological confusion. If we think of algebra as a method then there is a category problem in listing algebra alongside geometry and arithmetic.

> of schemata by which we apply them to intuited objects. He holds that we have a pure concept of quantity. He means, I take it, both that we have a pure or intellectual concept of quantity in general (the concept of a collection of things) and also that we have concepts of determinate quantities (concepts of collections of this or that many things). He also holds, however, that the pure concepts are in themselves rather meager things.... We may have the concept of a collection of seven plus five things, but to know that this is a collection of twelve things we must both command and exercise the ability to identify a collection of seven plus five things and to enumerate its members. (Young 1982, 37f)

This is really an argument for the syntheticity of 7+5=12. Even a discursive process that could derive some of these concepts could not cognize anything without turning to intuition, but this a priori turn to intuition is not a mathematical construction. Although intuition does identify for us which concepts are more than merely logically possible in mathematics, the concepts must have an intuitive origin to be considered mathematical construction. But here Young has eliminated that possibility in arithmetic since the pure concept of quantity precedes application to objects. And though we employ intuitive objects in our arithmetical deliberation, the sum is not itself the intuitive object. Finally, in this framework the intuitive object conditions the verification of the sum rather than the arithmetical judgment.

4. Situating A Non-Constructive Arithmetic in the Kantian View of Mathematics

It is important that we distinguish between mere syntheticity and construction. All construction is synthetic but not all synthetic judgments are constructive. Kant often invokes mathematics as evidence that synthetic a priori judgments are possible. 7+5=12 can still be invoked to make the point that the content of synthetic a priori judgments is what we have put into them and that such judgments require a turn to intuition. Secondary literature often considers whether Kant was right in characterizing pure mathematics as synthetic a priori. Affirming the constructive character of pure mathematics is often a step in affirming its syntheticity. My point here is that denying the constructive character of arithmetic is not to deny the a priori syntheticity of pure mathematics. Rather it is to suggest that arithmetic does not adhere to the mathematical method that Kant outlines and thus cannot be an organon of mathematics, that is, the arithmetic that Kant has described does not produce or expand mathematical knowledge but serves to determine boundaries and guard against errors (B823). This distinction would not undermine many of the reasons that motivate thinking of arithmetic as synthetic. Nor would it necessarily compromise any of the philosophical steps Kant justifies by considering the syntheticity of pure geometry or arithmetic as a given.

Denying the constructive character of arithmetic does not require that we view it as analytic. We have seen in Young why 7+5=12 requires an a priori turn to intuition. R. Lanier Anderson (2004)[8] shows another way to understand arithmetical syntheticity arguing that a priori logical concept hierarchies that express containment cannot account for operator concepts like addition without overlap. Per Anderson, there is only one way that a subject can contain its predicate and that is via concept hierarchy in which the genera concept is a constitutive part of the species concept. Genera concepts can be exclusively and exhaustively divided into species concepts. The range of species and subspecies would constitute the extension of the genera concept and the hierarchy of genera concepts of a species would exhaust the concepts contained within it. Arithmetical operations cannot be so situated because the division would not be exclusive or exhaustive.

> Thus, Kant's invitations to consider what is "thought in" the concepts of numbers and sums should be understood as demands to consider carefully how the concepts could be related to one another as genera and species. On this reading, Kant's denial that the concept <12> is contained in the sum concept <7+5> amounts to the claim that a hierarchy conforming to the rules of logical division cannot express the truth that 7+5=12 in terms of containment relations between <12> and <7+5>. As it turns out, the division rules do block the construction of an appropriate hierarchy, and so arithmetic must be synthetic. (Anderson 2004, 517-518)

We cannot hold that 7+5 is an exclusive constituent concept of 12 because it would have to be a constituent concept of infinitely many numbers. Arithmetical judgments thus would be outside the limits of the logical analysis available to Kant. In this sense, arithmetic must turn to intuition. Anderson's explanation supports Kant's claim about the need to turn to intuition when considering 7+5=12. Neither side of the equation can be exclusively and exhaustively contained within the concept of the other. Thus the judgment that equates the sum of 7+5 with 12 requires something extralogical.

Recognizing the distinction between syntheticity and construction does not leave all other defenses of Kant's views on mathematics intact. In contextualizing Kant, Friedman (1992) argues that much of Kant's appeal to intuition is motivated by concern for infinity owed to the limits of monadic logic. In later work, Friedman grants that the elementary constructions of geometry, while still drawing on infinite features that are unavailable to eighteenth-century logic, have analogs to phenomenological activities that

8 Many have made a similar claim. Some discuss this in terms of the genus/species relationship that is possible with subject and predicate of analytic judgments. Others still do this with reference to concept intension and extension. To my mind the overall concern is roughly the same, that logical or merely analytic relationships can be neatly mapped onto taxonomies whereas mathematical judgments cannot.

are grounded in a priori intuition. If the distinction between construction and syntheticity highlights the conceptual remove between arithmetic and intuitive objects, it reinforces the concern about the limits of eighteenth-century logic. If the reliance on intuition in arithmetic is a matter of successive iteration, criticisms that concern infinite iterability will be of greater consequence. Another way to understand the distinction between syntheticity and construction is to acknowledge that the limits of logic might require synthesis in intuition but not necessarily construction in intuition, a special class of synthesis. Other criticisms of Kant's account of mathematics that argue in some way or another that 7+5=12 is not the kind of synthesis Kant wants it to be will remain problematic for Kant. On the other hand, distinguishing between construction and syntheticity also shields Kant from some criticisms. Arguments that take aim at the significance of 7+5=12 will have little impact, if any, on the question of the constructability of geometrical concepts. Frege, for example, agrees with Kant that the truths of geometry are synthetic a priori but holds that logic can generate arithmetic without the appeal to intuition. One could defend Kant by suggesting as McFarlane (2002) does that Frege is simply working with a different notion of logic, but one could also see the argument about the syntheticity of arithmetic as inconsequential for the constructive character of geometry.

Pure mathematics is often cited as evidence of synthetic a priori judgments but its significance for the critical project is not limited to this contribution. For Kant, we have access to enough synthetic a priori judgments to form an entire system of such cognition but only pure mathematics can construct. We can grant that basic arithmetical operations, while necessary for a robust mathematics, do not amount to mathematical operations. Thus, arithmetic is a necessary instrument for some—though certainly not all—mathematical activities, but not sufficient to produce mathematical knowledge on its own. So, arithmetic can, like logic, be a part of regulating or constraining what mathematical judgments can do but it is not an organon of mathematics, one that produces new content or knowledge. For Kant, logic alone does not produce mathematical objects, but it is indispensable for the proof procedure of mathematics. In the same way, arithmetic falling short of mathematical construction means it can be regarded as somehow protomathematical in that it anticipates magnitudes and provides a language for relating those magnitudes although it cannot construct them. In the philosophy of mathematical practice, the basic arithmetical procedures Kant describes would be considered reckoning arithmetic,[9] a kind of universal human cognition that admits of certainty without yet importing more robust mathematical principles or assumptions. Arithmetic provides a language and system for making sense of the relationships and ratios of the magnitudes that are constructed in mathematics, but it does not produce the mathematical objects.

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9 For a clear account of this sense of reckoning arithmetic see José Ferrirós' *Mathematical Knowledge and the Interplay of Practices*, Princeton; 2016.

For Kant, dealing with magnitudes is not what denotes mathematical activity. Philosophy can deal with magnitudes also. Rather what denotes a mathematical method is the construction of magnitudes. Intuition conditions mathematical construction by providing intuitive content but that content alone does not make mathematics. That intuitive content must be made a synthetic unity to form concepts and judgments,

> In fact it is not images of objects but schemata that ground our pure sensible concepts. No image of a triangle would ever be adequate to the concept of it. For it would not attain the generality of the concept, which makes this valid for all triangles, right or acute, etc., but would always be limited to one part of this sphere. The schema of the triangle can never exist anywhere except in thought, and signifies a rule of the synthesis of the imagination with regard to pure shapes in space. (B180)

Mathematics must reach a certain level of generality to attain knowledge and knowledge is always expressed in concepts. The *in concreto* method allows mathematics to attain universal conceptual knowledge while still being tethered to an *a priori* intuitive representation,

> Philosophical cognition thus considers the particular only in the universal, but mathematical cognition considers the universal in the particular, indeed even in the individual, yet nonetheless a priori and by means of reason, so that just as this individual is determined under certain general conditions of construction, the object of the concept, to which this individual corresponds only as its schema, must likewise be thought as universally determined. (B742)

The emphasis on the intuitive content of pure mathematics should not mislead us into thinking that that is all that is required for mathematical construction. As such it is not problematic to include arithmetical judgments as part of something mathematical so long as the content from construction is still owed to a priori intuition.

Arithmetic does not have objects of its own domain and has only a mediated turn to spatial intuition. This results in an *in abstracto* method that is synthetic but cannot be constructive. Arithmetic's ground in composition is distinct from geometry's. Proportion may be a unifying ground, but it is no guarantee of the production of objects or the mathematical method. Most importantly, proportion can be considered with distinct methods and in order to be mathematical, an *in concreto* method that treats of objects is necessary. Of course, arithmetic is still necessary for a full account of mathematics more broadly. A complete historical description of mathematics as well as thorough mathematical instruction would require some arithmetical prolegomena. In a strict sense, arithmetic does not meet Kant's standard of mathematical activity, but proportions and the special case of discrete magnitudes are still necessary for robust mathematical activity.

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