МЕТОДИКА НАСТАВЕ

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UDK: 37.091.3:51

BIBLID: 0353-7129, 25(2020)2, p.173-185

ERROR ANALYSIS – A WAY TO LEARN MATHEMATICS MORE EFFICIENTLY

Summary: Errors and overcoming failures are natural elements of the education process of all students. Our society tends not to look at them positively – teachers often try to avoid errors as well as they lead students to avoid them. They do not tend to use errors as a learning tool full of opportunities to better understanding. In this study we present the results of the observations of twelve lessons of mathematics, showing their common features with a special focus on errors. The core of the text also offers a detailed analysis of two lessons, where traditional teaching approach prevailed, and one lesson, mostly but not completely, conducted in a constructivist way. It includes beliefs and mindsets of those three teachers, showing their direct or indirect influence on their teaching. We also discuss their approach to math problems as well as the possible influence of their teaching on students. The paper stresses a discrepancy between beliefs of teachers related to errors and the reality in their practice.

Key words: mathematics, learning from errors, traditional teaching approach, constructivist teaching approach

INTRODUCTION

Teacher's beliefs have significant impact on students. Among other things, they strongly influence teacher's way of instruction, approach to math problems as well as their approach to errors (Thompson, 1992, Pajares, 1992).

In a traditional approach to teaching a teacher tends to present a student knowledge directly in good faith, it is the best and the fastest way to cognition (Hejný et al., 2004). Memorization is one of the main learning tools in this approach. Whitney

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(1987) claims that while students are frequently able to learn formulas and vocabulary through memorization, they are often unable to generalize their knowledge as a result of being provided limited examples by their teacher (Whitney, 1987). This may result in the effort of students to reproduce procedures without thinking about the task or trying to understand it. The attitude to error is unambiguous – since knowledge is presented in its final form, any error a student makes is a report of the error in the transfer of information from a teacher to a student. A student who makes an error indicates that he or she has misunderstood the information or has inaccurately memorized it. For that reason, there is a tendency to avoid errors – for fear that the knowledge will be fixed by students incorrectly.

Errors are perceived as something negative, shameful and self-threatening (Steuer et al., 2013). Therefore, it cannot be said that the error could be beneficial in teaching. That is why in math lessons errors are often pointed out immediately, the teacher gives a short explanation of what has been made wrongly or repeat the instruction once more, then the error is deleted and the lesson continues. According to Fischer and Malle (1985), we often do not see in schools situations, where a teacher starts to identify the cause of an error or discusses it with students. On the contrary, this attitude of the teacher is unusual and many students would interpret such a behavior of the teacher as an additional “drilling”.

Constructivist approach to teaching is characterized by an active creation of knowledge and parts of mathematics in the student’s mind. The individual actively constructs and reconstructs his own reality and his discoveries in order to make sense of his experience. The role of the teacher is to guide him in the right direction, to give him suitable problems and tasks, to organize and moderate a discussion among students (Kuřina, 2002). Kvasz (2016) proposed the term “genetic constructivism” to denote the Czech approach to constructivism. He aimed to distinguish it from radical constructivism, which it is often identified and criticized with. In the study, we use the term constructivism in the way, which Hejný (2014) denoted as “schema-oriented approach to teaching”.

Constructivist learning theories say that errors provide good opportunities for learning (Hejný, Kuřina, 2001, Zamora et al., 2018). Using errors as a learning tool is an instructional strategy and should be an important component of the teacher’s educational strategy. When a student overcomes obstacles by analyzing and explaining errors, there is progress in learning because the student not only strengthens the decision-making process, but he has to think much deeper about what he was doing and what he did not know. Success in mathematics comes from understanding of the error – instead of “how” the student knows “why” the problem was solved in that way. When the student understands “why” behind some mathematical concept, the understanding of mathematics is often better and deeper.

Teachers play an important role in the classroom when errors are used as opportunities to learn. In order to recognize and successfully deal with errors, students must understand how to recognize and successfully reflect on their errors. Teachers

must be armed with strategies for supporting students in recognizing and reconciling their errors (Lannin et al., 2006).

According to TALIS 2008, Slovak teachers believe that constructivist way of teaching prevails in lessons. More than 95% of teachers believe that students should think about the problem and look for the solution by themselves.

However, the document of National Program for the Development of Education in Slovakia (2017) claims that traditional model of teaching is still commonly used in our education. This document appeals to the teachers to reconsider their teaching style. Many studies have proved its poor efficiency and unsuitability long ago and better learning outcomes are achieved when students actively construct their knowledge based on their own experience, activities as well as errors.

One of the reasons why teachers are turning more to traditional teaching methods is their lower difficulty and simpler application in teaching compared to other methods (OECD, 2016). Another reason may be as Lin and Cooney (2001) found out – they claim that they have never met with a teacher who would believe that he or she teaches math by memorization. Teachers say they should enable students to solve problems and develop their argumentative skills. However, evidence shows that the traditional teaching, respectively its form, is the dominant way of teaching in most schools.

It is important to stress the nature and the quality of instruction as it is fundamental to student learning (OECD, 2009). Scheerens and Bosker (1997) concluded that characteristics of instruction have a greater effect on student achievement than those of the school environment.

METHODOLOGY OF THE RESEARCH

All twelve teachers participating in the study work in the region of Banská Bystrica. All of them are secondary school teachers. Seriousness of the study is supported by the fact that a possible impact of even one teacher, who leads approximately 60 students in five-year turn, is overwhelming.

Research was conducted through: (i) independent observations of lessons (ii) semi-structured interviews with teachers and (iii) student questionnaires. The aim of the interviews and student questionnaires was the analysis of errors that occurred on the lessons from the teachers' and students' perspective. For that reason, these tools were used immediately after the observations.

The content analysis of the lessons gave us the opportunity to quantify the obtained data and statistically process them. However, we are not aiming at generalization of the results; the nature of the research is not adapted to this. Nevertheless, we consider statistical processing of data as important as it helps us to gain a deeper orientation in the amount of data obtained.

Regarding “pros and cons“ of research instruments our aim is to give a holistic view on errors caught during three specific lessons.

RESULTS OF THE RESEARCH

In the study we analyzed twelve math lessons. The lessons were selected randomly - we did not look for specific teaching approaches of teachers. It showed that on nine lessons (75%) traditional teaching method prevailed, three lessons (25%) were mostly but not completely conducted in the constructivist way. Based on the observations, we captured 90 situations, in which errors occurred.

Lessons that were instructed in the same teaching style showed the same common features with direct or indirect focus on errors:

A. Lessons in which the traditional teaching style prevailed:

- teachers authoritatively and directly managed the whole instruction;
- teachers required students to memorize definitions and rules for solving tasks by heart;
- teachers built the solutions of the tasks on a multi-step process that required each student to master;
- the focus of the teachers when solving a task was on the “correct steps” of the procedure that was presented as the only possible way of solving the task;
- teachers did not give students time to think about the task – they expected them to start solving the task immediately;
- when there was an error, teachers did not tend to give students time to think about it, but corrected it immediately;
- teachers did not search for causes of errors, they did not ask students why they solved tasks the way they did;
- teachers did not tend to use errors as a learning tool – only in 8,8% error cases teachers clarified the correct answer by suitable questions or gave students time to discuss the solution;
- 87,5% of errors were captured and pointed out by the teacher;
- in 73,8% of error cases, it was the teacher who corrected the error by saying the correct answer.

B. Lessons with dominating constructivist style:

- teachers and students were more like partners in the learning process;
- providing the tasks was not followed by the instruction which procedure should be used;
- teachers tended to support creativity and interest in mathematics;

- when solving a task, teachers gave students time to think about it as well as to come up with the individual solution of the task;
- teachers asked students not only what was the solution of the task but also on the way of solving the task – why students solved the task the way they did;
- when there was an error teachers created space for a discussion among students;
- in 50% of cases it was the teacher who reacted to the error but did not correct it immediately;
- in 40% of cases it was the teacher, who announced the correct answer;
- the error was used as a learning tool in 90% of cases – by suitable questions teachers guided students to correct answers or provided space for discussion.

ANALYSIS OF MATH CLASSES

In the detailed analysis of math lessons we selected two lessons with traditional approach to teaching and one lesson with dominating constructivist style. This selection was based on the ratio of these two approaches detected from the observations. Analysis includes beliefs of teachers as well as facts and commentaries from the observations supported by illustrations of specific situations from the lessons.

Lesson 1

Theme: Direct and inverse proportionality

Class: 7th grade

Aim of the lesson: Practicing word problems with the use of direct and inverse proportionality

Teacher's approach: Traditional approach to teaching prevails

Key characteristics of the lesson. During the whole lesson students worked on word problems from workbook. While one student was solving the task on the board, the rest of the class was solving the task in their exercise books; students in front of the table took turns. The lesson was directed by the teacher – she managed every step in a word problem that was solved. When there was more complex word problem, she divided it for students into smaller parts.

Teacher's belief. The teacher believes that she combines constructivist and traditional teaching styles. She, in her words, builds a solution of a word problem on a multi-step process that students are supposed to remember. She emphasizes its memorization as well as the memorization of the specific steps.

Observation of the lesson. The observation of the lesson showed that constructivist elements were not used at all. Students were not wondering how to find the

solution or what the task required to be solved – they merely focused on the application of individual steps of the procedure. If a student was not able to react immediately, the teacher “helped” him or her by telling him or her the next step. The teacher did not let students think about the task, but she divided the whole task into smaller parts in which students just needed to perform simple mathematical operations. Students did not see the task as a whole, but as a summary of simple mathematical operations. Thus, even when they made errors, they were, in their view, errors in calculations – “I did not write the equation correctly”, “I wrote the arrows incorrectly”, “I wrote numbers reversely”, “I swapped the proportion in the calculation”.

Teacher's belief. The teacher is convinced that the error on the blackboard should be corrected immediately so that students do not write it down incorrectly in their exercise books.

Observation of the lesson. The observation showed that her belief is so strong that she did not leave students space to reflect on errors, but corrected them immediately by a commentary. However, the results of the student questionnaires showed that, despite the above mentioned, 36% of students felt that the teacher was discussing errors during the lesson. It seemed that students considered the statement of the teacher whether the answer is correct or not for the “discussion over error”. The observation also showed that the teacher did not search for the cause of the error herself, which was reflected in the fact that she did not ask students to explain their solutions, there was no question “why?” or “explain ...”. However, none of the students indicated in the questionnaire that the teacher corrected the error without explanation – which was her most common reaction to the error. 43% of students stated that the teacher explained why the error had occurred.

Teacher's belief. The teacher considers inattention as the most common cause of errors that students make. She also thinks that today's children have great difficulty in understanding word problems and do not think about them.

Observation of the lesson. It was apparent from the observation that the teacher was strongly focused on the procedure and its individual steps. She missed the fact that a large part of the students did not understand the proportionality. This was the most obvious due to an incorrect use of arrows – almost in every task – but this error was always corrected by the teacher without explanation. This failure to notice the cause of the error was also from the side of students – only 14% of them said that the cause of the error lied in the fact that they did not understand the subject matter. The observation also showed that after the teacher assigned the task to students, she expected them to start solving it immediately. She did not leave them time and space to process the task. If they did not start to solve the task immediately, the teacher would enter the solution and began to dictate to students what to do and how to do it. Consequently, students not only had no space to think about the task, but they did not have to do that either – as the teacher told them every following step in the procedure, and so they only needed to perform necessary mathematical operations. They did not have to look at the task as a whole and think about what the task required from them and how they could solve

it. We think that the students had problems with understanding the tasks. On one hand, the teacher mentions problems of students with understanding tasks, on the other hand, she does not create space for students to process them.

Lesson 2

Theme: Direct and inverse proportionality

Class: 7th grade

Aim of the lesson: Practicing word problems with the use of direct and inverse proportionality

Teacher's approach: Traditional approach to teaching prevails

Key characteristics of the lesson. The teacher assigned tasks to students on the work list and students solved the tasks one by one in front of the table. Meanwhile, some students were trying to solve the tasks by themselves, some of them were waiting for the student in front of the blackboard to solve it. The whole lesson was strongly directed by the teacher herself. The teacher required students to follow exact steps in the process of solving the tasks.

Teacher's belief. The teacher states that she uses only traditional teaching style. In her words, math is about procedures that students need to learn.

Observation of the lesson. The whole lesson was visibly guided in the traditional teaching style. The teacher managed the whole lesson very authoritatively and directly. There was no time to process the word problems – the teacher expected students to start solving the tasks immediately after she assigned them. If a student did something differently than what he or she was supposed to do or wanted to go his or her own way in the solution, the teacher immediately stopped the student and told him or her how to do it.

Teacher's belief. In the interview the teacher says that her most frequent reaction to the error is: "I'd just tell him to rewrite it". She says she simply corrects the student's error and goes on in the lesson. She does not discuss the error with students.

Observation of the lesson. When a student made an error, the teacher would not only correct it immediately, but she would react quite angrily. During the lesson the teacher raised her voice many times when someone did or said something incorrectly. At the end of the lesson, there was an unpleasant situation for a student who had not been present in the class for a week and solved the word problems in a different way – we do not know whether her way of solving the task was correct or not as the teacher was not interested in it. She did not ask the student how she had proceeded. Here is the situation:

When solving the word problem, the student gets to the step where she is supposed to edit the equation, that was constructed with the help of the teacher. However, she does not know how to proceed.

Teacher: We repeated it a moment ago! The product of inner numbers equals to the product of outer numbers – that tells us about multiplication. Anna, have you solved at least one task of the four that we solved during the lesson?

Student: I did it in a different way.

Teacher: Well, here we go! So, you did it differently! I say that you have to do it in this way, you weren't present for a week and you do it your own way?! Then don't be surprised that you can't do it! Does it seem normal to you to do it otherwise?! I have been explaining it for the whole lesson, repeating the same sentence five times! Anna, I guess you are smart enough to repeat one thing!

Student: But I did it likewise.

The student feels awkward, she is exposed in front of the class, with her head down, the teacher's voice is raised.

Teacher: No! I think you were doing something else! Keep going! If you are not present, you are supposed to do it like the rest of us!

The whole situation was very unpleasant for the student. However, this was the way how the teacher communicated with students during the class. 15% of students said that the teacher had reacted angrily to the error and 8% of students said that the teacher had said something unpleasant. We do not understand why students responded about angry reactions of the teacher the way they did. Possible explanation for that could be that they were afraid of the teacher to read the results of the questionnaire, even though we had assured them about the anonymity of the questionnaire, or they were simply used to her teaching style. 85% of students in the class agreed that when an error occurred, the teacher would point it out. The teacher, in the interview, denies any discussion about errors or explanation of the causes of errors. Although, 62% of students in the questionnaire said that errors had been discussed with the teacher and 46% of them stated that the teacher had explained why the error had been made.

Teacher's belief. The teacher, in her words, does not think about the causes of errors, nor does she search for them during the lesson. “I would tell a student „Don't you know how to do that?“ or „Don't you see that it is different on the board? Do it again!“

Observation of the lesson. The observation confirmed the teacher's words – she did not search for the cause of the error, nor did she ask students how and why they proceeded the way they did. Essentially, by requiring to solve the tasks in a certain way, these questions wouldn't even have a place in the lesson. Students had visible gaps in the subject matter, which the teacher did not see. She drew her attention to the procedure that students should remember. She disregarded the causes of students' errors – the interview with the teacher showed that the reason for this was that students had not learned the “definition” – cross-multiplication rule, at home. The observation showed that a large part of students did not fully understand the proportionality. For example, they did not know when to use direct or inverse proportionality – it was noticeable when they were supposed to “determine the arrows”. The teacher, without

further explanation, said which type of the proportionality would be used and students continued to the next step of the procedure. None of the students in front of the board knew how to construct the equation and how to calculate the unknown x from the equation. They repeated the rule in each task, but it did not help students to know how to do these steps in the procedure. Students were solving tasks automatically, trying to think more about what steps to take and what to do in those steps, rather than stopping and thinking about tasks and what they require. Despite the visible gaps in the subject matter, the most common causes of errors according to student questionnaire were: “I made some errors in counting” (54%) and “I misread the word problem” (23%). Other causes, such as “I did not have enough knowledge”, “I did not know how to solve a task” or “I did not understand the subject matter” were not mentioned by any of the students.

Teacher's belief. From the teacher's point of view, the biggest problem that students had during the lesson was with the memorization of the cross-multiplication rule by heart. She says that even if they often know the definitions by heart, they do not understand them and do not know how they relate to solutions of tasks.

Observation of the lesson. The teacher gave students the cross-multiplication rule. She required each student to memorize this rule by heart and apply it. She asked for the exact wording of the rule in each task. However, students had problems with the rule. If they repeated it, they would not know how to apply it correctly. They did not see how it was related to the task. Here is the situation from the last word problem that students solved during the lesson:

Teacher: And now, here you have the equality of two ratios. At the beginning we repeated when two ratios are equal. Just tell me the definition.

Student: When the inner equals ...

Teacher: But what inner ...?

Student: ... outer ...

Teacher: But what about inner and outer? What equals what? In order to two ratios to equal ... what must be equal to what?

The teacher turns to the class, students stay quiet and do not know how to respond.

Student_2: ... hm, outer and inner ...

Teacher: But what? ... the product of what ...? The product of outer numbers must be equal to the product of inner numbers. Thus, I do not want you to say that outer must be equal to the inner, but the product of outer numbers must be equal to the product of inner numbers.

The teacher did not provide any explanation of that rule or give students the opportunity to solve the task in a different way. She raised her voice and repeated the rule every time herself. It could be seen that the rule was very important to the teacher.

Lesson 3

Theme: Variables, expressions and equations

Class: 8th grade

Aim of the lesson: Practicing using variables and expressions. Practicing word problems with the use of linear equations

Teacher's approach: Constructivist approach to teaching prevails

Key characteristics of the lesson. In the first part of the lesson, students worked on the tasks from the work list on their own. Their solutions were discussed in pairs or small groups, subsequently, they presented their solutions on the blackboard. In the second part of the lesson, students worked on the word problems using linear equations, again, on their own. Afterwards, the teacher checked their solutions by the discussion.

Teacher's belief. The teacher, in her words, is trying to use the constructivist way of teaching.

Observation of the lesson. From the observation it was apparent that the teacher was trying to provide students with enough time to think about the tasks. Firstly, after assigning the task, she gave students time to solve the task independently on their own and to discuss their solutions in pairs or small groups. After that, students showed their solutions on the blackboard and discussed it. The teacher asked students on their way of solution, not just on the outcome. If someone had a different procedure, he or she would go and show it on the board. The teacher made a space for students to express their opinions, solutions and arguments freely. Also, she did not give instructions to students, which procedures they were supposed to use. As students presented their solutions on the blackboard, the teacher no longer created space for the discussion among students. She entered the discussion with students herself – in this part of the lesson, the discussion took place only at the teacher-student level.

Teacher's belief. The teacher in the interview says, that if there is an error, she tries not to react on it. “*Some time ago, students relied on me – I was the one, who told them, whether the solution is correct or not. Now, I have learnt not to react on the error, I have learnt to act as if I do not know whether there is an error or not*”. She says, she usually sends a student who makes an error to another student to discuss their solutions together. Sometimes, she discusses their errors by herself.

Observation of the lesson. When there was an error on the board, the teacher reacted similarly in all cases – firstly, she asked a student why he or she had solved the task the way that was presented. She did not respond by rejecting it. However, she did not ask other students on their opinion about the solution – whether they agreed. She kept the initiative on her side and entered the discussion with the student herself – by questions she prompted the student to correct answer. Students were not afraid to express their opinions or to enter the discussion with the teacher. If a student presented a correct solution, the teacher would not just agree with it, but she would ask the class whether they agree with the solution or not. By this reaction, the teacher put right and wrong solutions of students into equality – she made students aware of the fact,

that she knew, they made an effort to solve it, whether it was correct or not. 67% of students said that the teacher had discussed the error with them. However, the teacher created a space for discussion within each task. Some students may have not realized this fact, apparently. 52% of students said that the teacher explained, why the error had occurred.

Teacher's belief. In the interview, the teacher admits that she is not always able to identify the cause of the error during the lesson. However, she says that the error is essential because students find it logical.

Observation of the class. This belief was also evident in the class. After each task, she would ask students if the explanation of the solution was clear. If a student said no, she would go and search for the problem of that student – she would not leave the no-answer unnoticed. By questions and discussions she led the student to the correct answer. The most common cause of errors was, according to 57% of students, miscalculation. However, based on the observation, students did not have many possibilities to make errors in the calculation. Of course, it could have happened, but we concluded that students did not realize the nature of their errors. Another cause of the error, according to 43% of students, was that they did not know, how to solve the task.

CONCLUSION

There is a discrepancy between beliefs of teachers related to errors and the reality in their practice. On one hand, 92% of interviewed teachers agree that errors are very important element of a student's learning, and 83% of them express the conviction that it is important to look for the causes of errors in order to know what they should focus on while teaching. On the other hand, they do not pay attention to them as well as they do not tend to detect them. A deeper analysis of errors does not occur very often – they are simply corrected by the teachers. The logical impacts of this practice on students are:

- students often see math as a subject full of formulas, definitions and rules. Based on that, their relationship to mathematics is often negative as they feel they cannot understand it, they just have to memorize it;
- students do not think when solving a task – they just look for the algorithms or procedures to apply;
- students do not have to process the task or to look for a suitable way of solving it as teachers are often the ones who, through step-by-step procedure, lead them to the correct solution;
- as students do not have time to think about the task on their own, they tend to rely on the teacher or other students to suggest specific steps of problem-solving;

- as teachers do not lead students to search for the causes of their errors, students are not aware of the nature of their errors as well as they do not search for it;
- by “only” correcting errors – saying the right answers – students do not learn from them, errors are not beneficial for students;
- as a result of a negative reaction of a teacher to students’ errors, students tend to avoid errors and when they make an error, they are afraid to admit it.

However, if teachers use more constructivist elements on their lessons as well as use errors as a learning tool, this would have the following logical consequences on students:

- students like mathematics as they see the purpose of it;
- as students do not know what algorithm or procedure should be used, they are forced to think about the task and search for the best way to solve it;
- students work on their creativity as well as critical thinking as they look for the best way to find the solution;
- as students discuss their solutions, they have to clarify and argue over it, their understanding of mathematics becomes better and deeper;
- students are not afraid of errors, they see them as the next step in gaining knowledge;
- errors function as motivation to learn – revealing their cause brings a joy to students.

Despite the fact that we observed only twelve lessons of mathematics, we captured 90 error situations – it requires a lot of skills to manage that and to make the most of it. Based on that, we recommend adequate attention to errors on the side of teachers as well as in the preparation of future teachers of mathematics.

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Анализ ошибок - способ более эффективного изучения математики

Абстракт: Ошибки и преодоление отказов являются естественными элементами в процессе образования всех учеников. Наше общество обычно не глядит на это положительно – учителя часто стремятся избегать ошибок и ведут учеников, чтобы они тоже избегали их. Обычно не используют ошибки как инструмент обучения, который полон возможностей для лучшего понимания математики. В этой статье представляются результаты наблюдения двенадцати уроков математики, их общие черты вместе с направлением на ошибки. Ядро текста предлагает подробный анализ двух уроков математики, в которых преобладает традиционный подход к обучению, а также один урок, в котором в большей мере, но не в большинстве, преобладает конструктивистский подход. Мы стремились подвести убеждения этих трёх учителей и показать их прямое или косвенное влияние на учёбу. Анализируем также их подход к упражнениям по математике, как и возможное влияние их учёбы на учеников.

Ключевые слова: математика, образование с ошибками, традиционный подход к обучению, конструктивистский подход к обучению

Анализа грешака – учење математике на ефикаснији начин

Резиме: Грешке и превазилажење неуспеха природни су елементи процеса образовања свих ученика. Наше друштво на њих не гледа позитивно – наставници често покушавају да избегну грешке, као и да наведу ученике да их избегну. Они немају тенденцију да користе грешке као алат за учење пун могућности за боље разумевање. У овој студији представљамо резултате посматрања дванаест лекција математике, показујући њихове заједничке карактеристике – са посебним фокусом на грешке. Језгро текста такође нуди детаљну анализу две лекције, где је преовладавао традиционални наставни приступ, и једне лекције, углавном али не у потпуности, изведене на конструктивистички начин. Истраживање испитује веровања и начин размишљања три наставника, показујући њихов директан или индиректан утицај на њихов начин инструкције. Такође разговарамо о њиховом приступу математичким проблемима, као и о могућем утицају њиховог начина инструкције на ученике. У раду се наглашава несклад између веровања наставника у вези са грешкама и стварности у њиховој пракси.

Кључне речи: математика, учење на грешкама, традиционални наставни приступ, конструктивистички наставни приступ