

Analytical Approach to the Selection of Performer of the Project: Classification of Problems and an Illustration of an Example

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This paper provides an improvement in previously exposed to the general classification problem of selection of performers on the project behind a set of potential performers who are qualified for the relevant works. Thereby, one performer is not selected for the project as a whole, but more performers. The basic requirement is that for any stage is selected only one performer. First, there are three groups of problems from the standpoint of time and costs, as well as the parameters of activities: 1) unique parameters, 2) unique variants of all parameters of all potential performers of the same activities, and 3) various variants of parameters at least for one performer in relation to the other (one or more) performers of the same stage. In the framework of each group of problems are considered two subgroups: a) each activity is pre-assigned by the performer, and b) at least to one activity is made selection among several potential performers: 1. All performers have sufficient capacities, and 2. Some performers (at least one or all) have limited capacities. The aforementioned cases are described by mixed-integer linear mathematical models with the corresponding binary variables, considering more criteria. Group (i) is consisted by the criteria for the duration of the project and project costs, as the criteria of higher level of significance. Group (ii) is the criteria of lower level of significance for the values of the works, respectively the costs of the performers, which is justified to introduce appropriate priorities. Here is performed the minimization of the project criteria (according to the method of PERT/COST) and conditional maximization of performers' costs that does not distort the realized values for the project criteria (method of ϵ -limitation). Here is being specified a set of all or only characteristic Pareto-optimal solutions among which is selected the most appropriate solution for application. The mathematical problems are defined for the last two problems and are proposed algorithms to solve them. When there is limited capacities of performers, it is indispensable first to solve the linear problem without such limitations, and then for each solution to use a standard software for Project Management (PM) and to harmonize the required and available capacities of performers. The selection of performers is illustrated by sufficient capacities on a hypothetical project.

Keywords: project, selection of several performers, classification of problems, multi-criteria optimization.

1. INTRODUCTION

Many foreign and domestic authors decompose the problem of project management. Well known is the extreme complexity of construction projects (see, for example, [1, 2]). It is present that, to a lesser extent are exposed the aspects of selection of the performers, especially if the same activity has more potential performers. Typically is pointed the three ways of contracting works in construction of buildings (public bidding, collection of bids and direct negotiations) and it points out that an investor independently decides which way to apply. For example, in [3] are presented analytical approaches to selection of one among more potential performers for the project as a whole, or for individual stages of the project. There has been applied the theory of multi-criteria optimization (see, eg. [4, 5]).

General classification problem of selection of performers on the project is exposed to [6, 9, 10-13], indicating the choice of one performer for the project as a whole (which cannot introduce their sub-performer) and a selection of more performers by assigning appropriate project works. Some authors point out the other approach is more favorable which is explained in more details in continuation. Works

[7, 8] show the general mathematical models to the selected problems of performers' selection and their resolution on the hypothetical projects.

2. CRITERIA PROBLEMS TO SELECTION OF MORE PROJECT PERFORMERS

The problem of selection of the project performers can be considered in two sets of criteria groups. In doing so, it often favours appropriate potential performers.

The criteria of the project are to minimize the duration of the project and minimize project costs. In essence, if you are familiar with the time and costs of activities, project costs are known and is determined a plan with the minimum duration of the project. If at least one activity has a variety of time with appropriate costs, there is the problem of simultaneous minimization of the duration of the project costs of the project.

The criteria of potential performers usually are maximizing the value of their works, respectively maximizing carried out cost activities. In practice may be present and other requirements of performers. Eg., a particular performer expresses the desired minimum or maximum period of participation in the project (possibly given period), the earliest start to participate

in the project or given date, the latest participation in the completion of the project or given dates, etc.

Paragraph 1. The criteria of the project have a greater degree of significance in relation to the criteria of potential performers. After examining the criteria of performers it is not allowed to deteriorate the criteria of the project.

Paragraph 2. Potential performers may be equal to each other or to be assigned by the appropriate priorities. In the second case, the particular group of performers can be assigned by identical priorities. Also, it is possible to define several groups of performers including the following characteristics: (a) the different priorities of the group, and (b) all equal performers in the framework of the observed groups.

Paragraph 3. The main characteristic of the selection of more performers on the project is that for any observed activity or type of work, must be choose only one performer. If there are more alternatives for the time and costs of the observed activity, must be selected only one alternative.

3. ANALYTICAL APPROACH TO SELECTION OF MORE PERFORMERS OF THE PROJECT

Let be considered a project with m parts (phases) marked with $A_i, i \in I = \{1, \dots, m\}$. Let the potential performers on the project be marked with B_j and their number amounts to $n, j \in J = \{1, \dots, n\}$. The performers of the project can be equal to each other with determined priorities depending on the successful participation to other projects of the same owner or investor, as well as other criteria (technical-technological equipment, human resources, participation in other people's projects, etc.). Let them in general case have a strict lexicographic order according to ascending order of their indices ($B_1 \gg B_2 \gg \dots \gg B_n$), when B_1 has the highest and B_n the lowest

priority. In practice, it is often assumed that more performers to be of the same priorities.

Furthermore, some phases of A_i have n_i potential performers B_j qualified for relevant works, $1 \leq n_i \leq n, j \in J_i \subseteq J, i \in I$. Subgroup J_i contains indices of corresponding B_j . Therefore, each B_j may be potential for m_j phase $A_i, 1 \leq m_j \leq m, i \in I_j \subseteq I, j \in J$. Subgroup I_j contains indices of corresponding A_i . In limiting cases occur the following variants:

$n_i = 1$, is known B_j for observed A_i and a subgroup J_i has only one member, $j \in J_i \subset J, i \in I_j$,

$n_i = n$, svi B_j are potentials for the given A_i , so it applies to $j \in J_i = J, i \in I_j$,

$m_j = 1$, determined B_j can carry out only one A_i and a subgroup I_j with one member, $i \in I_j \subset I, j \in J_i$,

$m_j = m$, some B_j can carry out all A_i , that's why $i \in I_j = I, j \in J_i$.

3.1. Classification of selection problems of more performers

The aforementioned problems of selection can be classified into three groups 1) to 3) from the viewpoint of characteristic parameters t_i for duration and c_i for the costs of stages $A_i, i \in I, j \in J_i$ (Table 1). Each group is composed of two types of problems, a) and b), depending on whether A_i has one (known) or more potential performers of B_j . Every problem has two types, 1 and 2, from the viewpoint of capacity limitation or other limitations for the potential performers of B_j on the project (such as the terms of engagement, the overall value of the assigned works, etc.), $j \in J$.

Tabela 1. Opšta klasifikacija problema izbora izvođača projekta

Parameters of stage A_i (time and costs)	Potential features of B_j to perform A_i	Capacities for B_j	
		Sufficient	Limited
1) Unique parameters	a) every A_i known B_j	P.1.a.1	P.1.a.2
	b) at least one A_i with more B_j	P.1.b.1	P.1.b.2
2) Unique variants D_{ijv} of all B_j for the same A_i	a) every A_i known B_j	P.2.a.1	P.2.a.2
	b) at least one A_i with more B_j	P.2.b.1	P.2.b.2
3) Various variants D_{ijv} At least of one B_j for the same A_i	a) every A_i known B_j	P.3.a.1	P.3.a.2
	b) at least one A_i with more B_j	P.3.b.1	P.3.b.2
Warning 1. Trivial cases are excluded: every A_i known performer B_j and known variant D_{ijv} under 2), respectively B_j and D_{ijv} under 3)		Numeration of problems and mathematical models	

Group of problems 1) has unique parameters t_i and c_i on the same stage $A_i, i \in I$. Clearly, for type a) does not raise a problem to selection of performer B_j , since they're

assigned for each $A_i, i \in I, j \in J_i$. It is necessary to determine the minimum duration of the project T_p^{min} based on dependency of stages and their activities, and then

calculate the relevant total costs of the project $C_p(T_p^{min})$ and costs $C_j(T_p^{min})$ for performers B_j , $j \in J$. The presence at least of one A_i with more potential B_j in problems of the type b) enables optimization of values of the works, respectively maximization of costs $C_j(T_p^{min}, C_p(T_p^{min}))$ for preferred B_j on the project in the framework of the known values T_p^{min} and $C_p(T_p^{min})$, $j \in J$. Type 1 of the problem (P.1.a.1 and P.1.b.1) does not consider limitation for performers. For type 2 (P.1.a.2 and P.1.b.2) introduce restriction for some or for all potential performers B_j on the project, primarily, from the standpoint of their limited capacities, expressed in number of hours of engagement $w_{j0}(t)$ or with the required number of resource units $u_{j0}(t)$, in each unit of time of t on the project ($t=1,2,\dots$) or on the part of the project, $j \in J$.

Warning 2. All problems of type 2 (with limitation for performers) are characterized by the possibility to occur $T^* > T_p^{min}$, longer duration of project T^* in relation to its minimum duration T_p^{min} for a problem type 1 (no limitation for performers). Namely, type 1 expands to type 2, so the solving is conducted sequentially: *in the first stage* is solved the problem of type 1 and in the *second stage* it is going on to solve the problem of the type 2.

Warning 3. The presence of more variants for the stages (Figure 1.1 to 1.4) in groups of problems 2) and 3), in accordance with rules of the method PERT/COST (in literature of English language it often has the name ‘*Project Crashing Method*’ and ‘*Time-Cost Trade-Off Problem*’) enable to determine the final number of various values for duration and the costs of the project (Figure 1.5). In the framework of these values (higher level criteria) is performed cost optimization of the favoured performers (lower level criteria), based on the selection of performers and/or variant parameters.

When there are variants for time and costs of activities, the following terms are used:

Normal or maximum duration of the $(t_n)_i$ activities of A_i have normal or minimal costs $(c_n)_i$.

Forced (shortened) or a minimum duration of the $(t_u)_i$ activities have forced or maximum costs of $(c_u)_i$.

$$\Delta c_{iv} = c_{iv} - c_{i,v-1}, i \in I, v = 1, 2, \dots \quad (2)$$

$$c_{iv}^L(t) = -a_{iv}t + b_{iv}, i \in I, v = 1, 2, \dots \quad (3.1)$$

$$\Delta c_{iv}^{(k),L} = a_{iv} = \frac{c_{iv}^{(1)} - c_{i,v+1}^{(1)}}{t_{i,v+1}^{(1)} - t_{iv}^{(1)}}, i \in I, v \in 2, 3, \dots, k \in 2, 3, \dots \quad (3.2)$$

$$b_{iv}^k = \frac{c_{iv}^{(1)} t_{i,v+1}^{(1)} - c_{i,v+1}^{(1)} t_{iv}^{(1)}}{t_{i,v+1}^{(1)} - t_{iv}^{(1)}}, v \in 2, 3, \dots, k \in 2, 3, \dots \quad (3.3)$$

Applies: $(t_u)_i < (t_n)_i$ and $(c_u)_i > (c_n)_i$ (Figure 1.1, 1.2 and 1.3).

Normal times $(t_n)_i$ of all activities determine normal duration of the project $T_p(t_n) = M$ and normal total costs of the project $C_n = C_p(M)$

Forced times of $(t_u)_i$ of all activities determine the forced duration of the project $T_p(t_u) = m$ and forced total costs of the project $C_u = C_p(m)$. Applies: $m < M$ i $C_u > C_n$ (Figure 1.4).

Estimation and approximation of costs may be made in several ways and in accordance with the nature of each specific activity $A_i, i \in I$. Characteristic cases are shown in the following graphics.

Figure 1.1. Known is the normal duration of $(t_n)_i$ and the forced duration of $(t_u)_i$ activities with the appropriate normal costs of $(t_n)_i$ and the forced costs of $(t_u)_i$. It is necessary to perform the linear approximation of costs, respectively to determine the linear costs of (1.1). Thereby is ensured equal average increase of costs (1.2) for the permissible time t_{iv} of the observed activities, $v=2,3$. Parameter $a_i = \Delta c_i^L$ is the coefficient of the direction and b_i (i.3) is the cut-out on the vertical axis.

Figure 1.2. It is a known a non-linear function of costs $c_i(t)$ on the interval $t \in [(t_u)_i, (t_n)_i]$. It is necessary to conduct a linear approximation of costs as previously.

Figure 1.3. Known are the costs for all allowable times of activities. There can be determined appropriate increase of costs (2).

Figure 1.4. Known are the costs for characteristic allowable times of activities. For the rest of times is required appropriate linear approximation of costs (3.1) with accompanying average increase of costs (3.2).

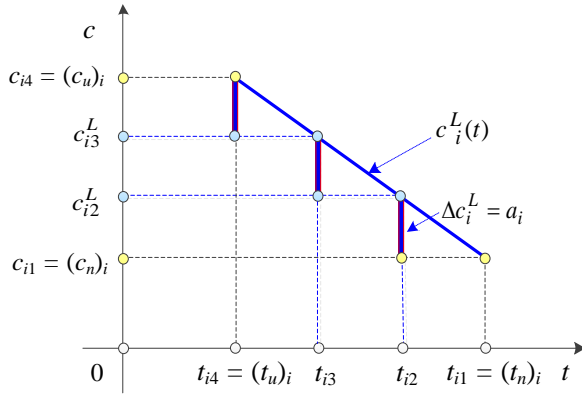
$$c_i^L(t) = -a_i t + b_i, i \in I \quad (1.1)$$

$$\Delta c_i^L = a_i = \frac{(c_u)_i - (c_n)_i}{(t_n)_i - (t_u)_i}, i \in I \quad (1.2)$$

$$b_i = \frac{(c_u)_i (t_n)_i - (c_n)_i (t_u)_i}{(t_n)_i - (t_u)_i}, i \in I \quad (1.3)$$

The method of PERT/COST enables to shorten only indispensable activities and determine optimal costs of the project for the appropriate times (Figure 1.5):

The minimum total project costs for each permissible duration of the project.



Slika 1.1

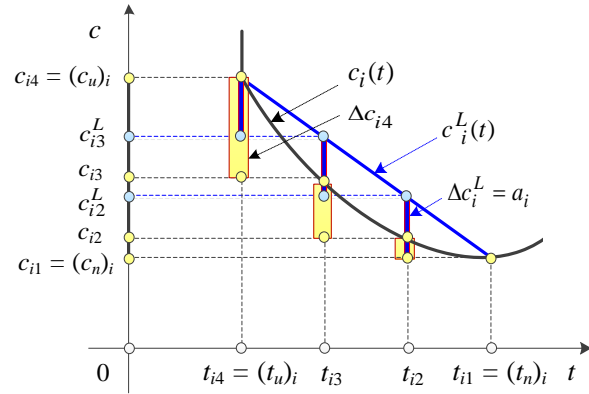


Figure 1.2

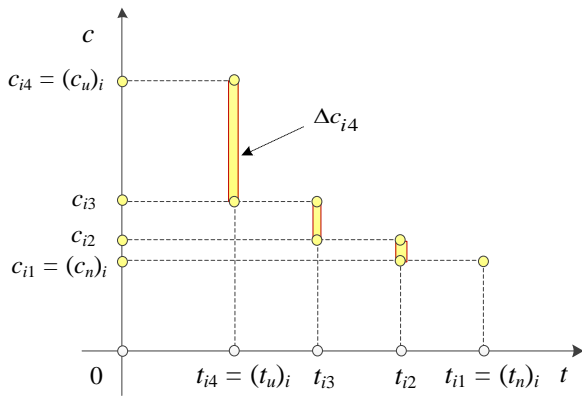


Figure 1.3

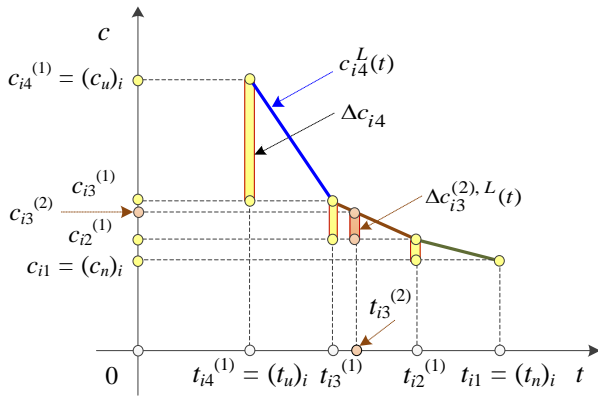


Figure 1.4

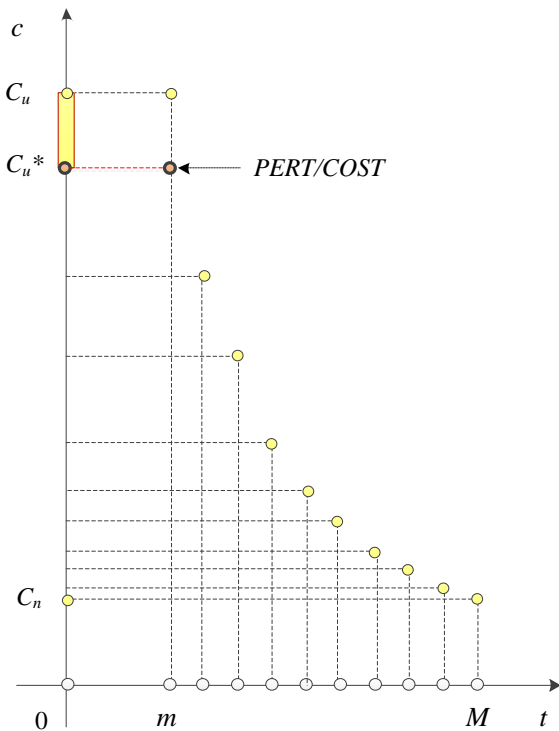


Figure 1.5

Lesser total project costs $C_u^* = C_p^*(m)$ for its forced duration, $C_u^* < C_u$, if it is not necessary that all activities to be reduced to the forced times.

Group of problems 2) imply certain or all stages A_i (or one at least A_i) with more variants D_{iv} for times t_{iv} and costs c_{iv} , $i \in I$, $v \in V_i = \{1, \dots, d_i\}$. Here d_i marks the number of variants D_{iv} for the observed A_i , $i \in I$. Parameters with D_{iv} are identical for the all potential performers of B_j on the observed A_i , $i \in I$, $j \in J_i$, $v \in V_i$. The problems of type a) with $n_i = 1$ require selection of variants D_{iv} for each A_i . The type b) of the problem

imposes selection of D_{iv} and B_j for A_i with $n_i > 1$, as well as the selection only for D_{iv} for the remaining A_i with $n_i = 1$ if such exist.

Group of problems 3) has at least one stage A_i with more variants D_{ijv} for the times t_{ijv} and the costs c_{ijv} , $i \in I$, $v \in V_{ij} = \{1, \dots, d_{ij}\}$. With d_{ij} is marked the number D_{ijv} of the potential performers of B_j for the

observed A_i , $i \in I$, $j \in J_i$. Now exists at least one A_i of various parameters with V_{ij} at least in a pair of potential B_j , $i \in I$, $j \in J_i$. Eg., B_{j_1} and B_{j_2} with $j_1, j_2 \in J_i$ can perform A_i , and applies: (i) $t_{ij_1v} \neq t_{ij_2v}$ and $c_{ij_1v} = c_{ij_2v}$, or (ii) $t_{ij_1v} = t_{ij_2v}$ and $c_{ij_1v} \neq c_{ij_2v}$, or (iii) $t_{ij_1v} \neq t_{ij_2v}$ and $c_{ij_1v} \neq c_{ij_2v}$. If the problems of the type a) have $n_i = 1$, is selected D_{ijv} of each A_i . With the problems of the type b) with $n_i > 1$, for appropriate A_i , is selected D_{ijv} if $d_{ij} = 1$, respectively B_j and D_{ijv} if $d_{ij} = 1$. If there exist A_i with $n_i = 1$ and $d_{ij} = 1$, selection is known with the given B_j and its D_{ijv} with one variant of parameter. The types of problems from the point of restriction of performers are valid by previously given interpretation.

3.2. Mathematical models and algorithms for solving of problems

With the all afore-mentioned problems we have defined the relevant general multi-criteria mathematical models of the type of mixed-integer linear programming (including specific binary variables) and set the algorithms of their solution. In continuation we consider the last two problems from the group 3 (3.b.1 and 3.b.2).

Paragraph 4. Modeling and solving of these problems are based on the use of of appropriate binary variables.

Binary variables of the first type h_{ijv} in models 3.b.1 and 3.b.2 have an intention to identify which performer and which his variant of time and costs is selected for the observed activity. The values are $h_{ijv} = 1$ (if for A_i is selected B_j with D_{ijv}), or $h_{ijv} = 0$ (on the contrary), $i \in I$, $j \in J_i$, $v \in V_{ij}$. Namely, $h_{ijv} = 0$ remains if for A_i is not selected B_j with D_{ijv} . There are two possibilities: (i) is selected B_j with a certain variant of parameter which is various from D_{ijv} , or (ii) is selected any other performer, but not the observed B_j .

Binary variables of the second type $h_{ijv}(t)$ in a model of 3.b.2 distinguish the activities that are performed in teh observed unit of time t . They have values $h_{ijv}(t) = 1$ (if is $h_{ijv} = 1$ and A_i is performed in the unit of time t), or $h_{ijv}(t) = 0$ (on the contrary), $i \in I$, $j \in J_i$, $v \in V_{ij}$, $t \in T$. There occurs $h_{ijv}(t) = 0$ in the following cases: (i) $h_{ijv} = 1$, but A_i is not performed in t , or (ii) A_i is performed in t , but is $h_{ijv} = 0$.

Paragraph 5. The essential characteristic of the problems, that for one A_i must be chosen only one B_j

and that can be used only one D_{ijv} , expresses a condition of a form $\sum_{j \in J_i} \sum_{v \in V_{ij}} h_{ijv} = 1$, $i \in I$ (the sum of all h_{ijv} is 1). With this rule is performed three tules for calculation of the following values:

duration of activitiea $t_i = \sum_{j \in J_i} \sum_{v \in V_{ij}} t_{ijv} \cdot h_{ijv}$,
 $i \in I$,

costs of activities $c_i = \sum_{j \in J_i} \sum_{v \in V_{ij}} c_{ijv} \cdot h_{ijv}$,
 $i \in I$,

required operational capacities of performers in the unit of time of the project (expressed in number of units of capacity) $\sum_{i \in I} \sum_{v \in V_{ij}} u_{ijv}(t) \cdot h_{ijv}(t)$, $j \in J$, $t \in T$.

Paragraph 6. Potential performers of B_j , who have n , are observed from the standpoint of their total costs C_j , $j \in J$. It must be kept in mind that performers are assigned appropriate priorities and is aptly to define their indices in accordance with the priorities, starting with favoured performers. Let be favoured performers with the indices in the group J^+ . To these performers are performed maximization of costs in the model of 3.b.1 and 3.b.2. There are two possibilities.

$n^+ = n$, favouring all potential performers B_j ,

$j \in J^+ = J$,

$n^+ < n$, not favouring all potential performers B_j ,

so that for the favoured performers is valid $j \in J^+ = \{1, \dots, n^+\}$ i $J^+ \subset J = \{1, \dots, n^+, \dots, n\}$. It can be said that the non-favouring performers with the indices $j \in J \setminus J^+ = \{n^+ + 1, \dots, n\}$ have the lowest priority.

Paragraph 7. Potential performers can have limited operational capacities from the resource standpoint of the type of 'labor' or 'work' (man power and machinery), which considers the model 3.b.2. Let has n^- performers B_j with the limited capacities and the indices of such performers make a group J^- . Applies $j \in J^-$ with the characteristics from continuation.

Limited capacities may have all potential performers (occurs $n^- = n$), or just any potential performers (present is $n^- < n$),

Limited capacities may have favoured and non-favoured performers,

Limited capacities must be considered independantly whether such feature have the favoured or the non-favoured performers.

3.2.1. General mathematical model of problems 3.b.2

Is performed an optimization of $2 + n^+$ criteria: project duration minimization (1), project costs duration (2), and costs maximization of favoured performers (3.j).

Unknown or variables are: binary variables h_{ijv} i duration of project T_p that simultaneously expresses $h_{ijv}(t)$, commencement and ending for A_i (x_i, y_i), criterion (1).
 commencement and ending for A_λ (x_λ, y_λ), and

$$\min T_p(h) = T_p \quad (1)$$

$$\min C_p(h) = \sum_{i \in I} \sum_{j \in J_i} \sum_{v \in V_{ij}} c_{ijv} \cdot h_{ijv} \quad (2)$$

$$\max C_j(h) = \sum_{i \in I} \sum_{v \in V_{ij}} c_{ijv} \cdot h_{ijv}, j \in J^+ \quad (3.j)$$

p.o.

$$x_i \geq 0, i \in I \wedge R_i^- = \emptyset \quad (4)$$

$$x_i \geq y_\lambda, i \in I \wedge R_i^- \neq \emptyset \quad (5)$$

$$y_\lambda = x_\lambda + \sum_{j \in \Lambda_j} \sum_{v \in V_{\lambda j}} t_{\lambda jv} h_{\lambda jv}, \lambda \in R_i^- \neq \emptyset, i \in I \quad (6)$$

$$y_i = x_i + \sum_{j \in I_j} \sum_{v \in V_{ij}} t_{ijv} \cdot h_{ijv}, i \in I \quad (7)$$

$$y_i \leq T_p, i \in I \wedge R_i^+ = \emptyset \quad (8)$$

$$\sum_{j \in J_i} \sum_{v \in V_{ij}} h_{ijv} = 1, i \in I \quad (9)$$

$$\sum_{i \in I} \sum_{v \in V_{ij}} u_{ijv}(t) \cdot h_{ijv}(t) \leq u_{j0}(t), j \in J^-, t \in T \quad (10)$$

$$h_{ijv} = \begin{cases} 1, & A_i \rightarrow (B_j \rightarrow D_{ijv}) \quad i \in I, j \in J_i, v \in V_{ij} \\ 0, & \text{otherwise} \end{cases} \quad (11)$$

$$h_{ijv}(t) = \begin{cases} 1, & h_{ijv} = 1 \wedge A_i \text{ at } t \quad i \in I, j \in J_i, v \in V_{ij}, t \in T \\ 0, & \text{otherwise} \end{cases} \quad (12)$$

$$x_i, y_i \geq 0, i \in I, \quad (13.1)$$

$$x_\lambda, y_\lambda \geq 0, \lambda \in R_i^- \neq \emptyset, i \in I \quad (13.2)$$

$$T_p \geq 0 \quad (14)$$

where: (t_{ijv} , c_{ijv}) duration and costs of activities of A_i if is chosen the performer B_j with the variant D_{ijv} , $i \in I$, $j \in J_i$, $v \in V_{ij}$, x_i beginning A_i , $i \in I$, y_i ending A_i , $i \in I$, R_i^- sum of indices λ for activities of 'predecessors', ie. Activities of A_λ from which depends A_i , $\lambda \in R_i^- \neq \emptyset$, $i \in I$, (x_λ, y_λ) beginning and ending of A_λ , $\lambda \in R_i^- \neq \emptyset$, $i \in I$, $t_{\lambda jv}$ duration of A_λ if is selected the performer B_j including the variant $D_{\lambda jv}$, $\lambda \in R_i^- \neq \emptyset$, $i \in I$, $j \in J_\lambda$, $v \in V_{\lambda j}$, R_i^+ sum of activity index of 'successors' of the observed A_i , ie. Activities that depend from A_i , $i \in I$, T_p duration of the project, t unit of time on the project, T sum of units in time, $t \in T = \{1, \dots, T_p\}$, $u_{ijv}(t)$ engagement of operational capacities of the performers B_j in the unit of time t on A_i with the variant D_{ijv} for t_{ijv} and c_{ijv} , $i \in I$, $j \in \{J_i \cap J^-\}$, $v \in V_{ij}$, $t \in T$,

$u_{j0}(t)$ available operational capacities of the performer B_j in the unit of time t , $j \in J^-, t \in T$.

Restrictions have the following meanings: (4) determines the beginning of the starting activities of A_i which does not depend on other activities (A_i has $R_i^- = \emptyset$), (5) defines that the beginning of A_i with dependencies of (has $R_i^- \neq \emptyset$) cannot be earlier from the ending of its each predecessors A_λ , $\lambda \in R_i^-$, (6) calculates ending of each A_λ (beginning plus duration) for application in (5), (7) forms ending of each A_i (beginning plus duration), (8) conditions that the project duration of T_p , respectively the ending of the project cannot be earlier than the end of each completed A_i without followers (without further activities, there is $R_i^+ = \emptyset$), (9) provides that any of A_i can perform only one B_j using only one D_{ijv} , (10) conditions that utilization of capacities of each performers of B_j in any unit of time does not exceed its available capacity, (11) defines binary

variables of the first type, (12) defines binary variables of the second type, (13.1) and (13.2) are natural conditions on non-negativity of the real variables for initial activities of A_i and A_λ , respectively, and (14) is a condition for the real variable that express duration of the project. The aforementioned variables receive integer values, when are integer numbers of variants t_{ijv} during the activity times.

Paragraph 8.1. The model does not contain a definition of the latest completion of the project and non-linear calculations of the latest term for endings and beginnings of activity. Calculation of the above parameters is not necessary in terms of a general model of problems. The same is determined with subsequent application of software for PM.

Paragraph 8.2. Unknown for beginnings and endings of activities are not always the earliest times from the usual analysis on the project. Namely:

The linear conditions (5) replace non-linear definitions (15) for earlier beginning of each A_i which depend from more A_λ (the earliest beginning x_i^E for A_i is equal to the earliest ending of y_λ^E for A_λ which is ending last).

The linear conditions (8) replace non-linear conditions (16) for ending of the project (duration, respectively the earlier and the latest ending of the project T_p is equal to the earliest ending y_i^E for A_i from the set of activities without follower which ending the last).

$$x_i^E = \max_{\lambda \in R_i^-} y_\lambda^E, \quad i \in I \wedge R_i^- \neq \emptyset \quad (15)$$

$$T_p = \max_i y_i^E, \quad i \in I \wedge R_i^+ = \emptyset \quad (16)$$

Paragraph 9. The general mathematical model of problem 3.b.1 arises by omitting restrictions (10) that consider available capacities of performers, and conditions (12) for the corresponding binary variables of the second type.

3.2.2. Resolving of models

Algorithm 1. Multi-criteria model 3.b.1 is solved with the simplest combined application method of ε -limitation and lexicographical methods, using software for mixed-integer linear programming. There are determined appropriate Pareto-optimal solutions or Pareto-solutions.

Step 1. There are determined minimal project costs for the given duration of the project in accordance with the method of PERT/COST. Permissible duration times are considered starting from the normal duration of the project of $T_p = M$ and is ended with the forced duration of $T_p = m$. There is applied the method of ε -limitation and is determined extreme Pareto-solutions from the standpoint of criteria of the first level significance.

Step 2. With each solution from the step 1 is performed conditional maximization of costs of the favoured performers in accordance with their priorities. There is applied the lexicographical method for costs of the mentioned performers (as a criterion of low level significance), with conditions that do not disrupt the

earned values for duration and the project costs (as a criterion of higher level significance).

Step 3. Chosen is the solution for the application, as the most acceptable solution under the given operational conditions. Prepared is an appropriate project plan using a standard software for PM. On the basis of the requested start date of the project and adopted calendar is determined the appropriate elements of the project: beginning and ending dates of activities, dates of completion of the project, the costs for the corresponding periods and the like.

Paragraph 10. Solution of the model 3.b.1 in the mathematical sense makes the set of all Pareto-solutions. However, in practice it is appropriate to specify only the selected Pareto solutions. It can be solutions primarily with the characteristic values for the duration and costs of the project. Then, in the context of these values are found solutions with the most acceptable costs for certain favored performers, who have more priorities over others.

Algorithm 2. Multicriteria model of 3.B.2 is not easily solved by using linear programming. Therefore, it is relatively complicated to use restrictions (10) for the available capacities of performers. Due to this, easier is an interactive application of software for the PM that supports leveling of resources.

Stage 1. Determined is the selected solutions of the model 3.b.1 using the algorithm 1.

Stage2. With every decision from stage 1 is defined the project plan using the software for PM. Then are added data for the required and available capacities of relevant performers in accordance with the relations (10). Followed is the leveling of resources by reducing the required capacity of performers in their available limits. It further observes the resulting minimum duration of the project.

If is not extended duration of the project from stage 1, is obtained Pareto-solution that stands out in a matching set of E_{Par} .

If the extended duration of the project from stage 1, the obtained solution is not Pareto-optimal.

Stage 3. From set E_{Par} is selected the most acceptable solution for application.

4. ILLUSTRATIVE EXAMPLE

Let be considered the problem of 3.2.1 for the project with $m = 9$ activities $A_i, i \in I = \{1, \dots, 9\}$, which have dependencies of the type 'Finish to Start' (Figure 2). There are $n = 4$ potential performers of $B_j, j \in J = \{1, 2, 3, 4\}$. There are determined the required for activities and the performers (Table 2). Duration of activities has been given in weeks and the costs are shown in monetary units (m.u.).

Normal duration activity times $(t_n)_i$ determine normal project duration of $T_p = M = 20$ weeks, when the normal activity costs $(c_n)_i$ give forced total project costs $C_p(M) = 130$ n.j. Forced times of all activities $(t_u)_i$ and corresponding forced costs $(c_u)_i$ determine forced project duration $T_p = m = 14$ weeks with the forced total project

costs $C_p(m)=174$ n.j. However, will be proven that the forced project duration can be achieved with the lesser total costs including an amount of $C_p^*(m)=166$ m.u.

Warning 4.1. Observed are the cases that the potential performers of the same stages have similar variants in time and costs. Also, any performer has a favorable variation in relation to one or more other performers on the same stage. Examples:

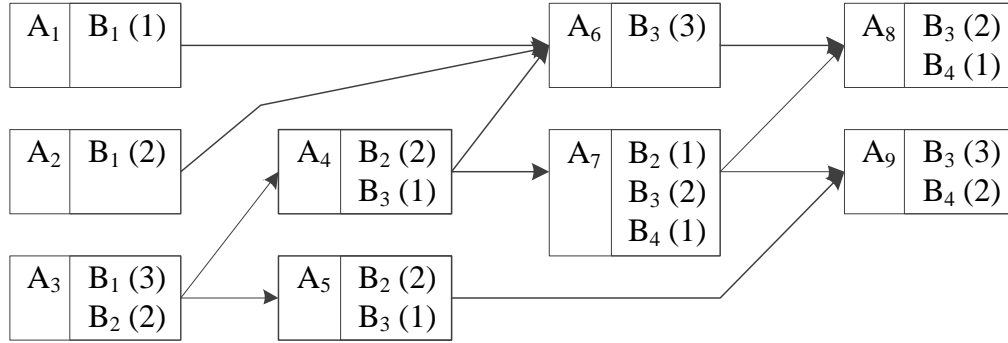


Figure 2. Project network with A_i, B_j and $d_{ij}, i \in I, j \in J_i$

Warning 4.2. Can be omitted the consideration of adverse variations of parameters. This reduces the number of binary variables of the first type, or dimensions of mathematical models. However, this does not affect the resolution of the model and adverse variants will not be selected.

A_3 has two similar variants for B_1 i $B_2 : D_{311} = D_{321}$ i $D_{312} = D_{322}$

A_4 has better variant D_{422} for B_2 in relation to D_{432} for B_3 , due to $t_{422} = t_{432}, c_{422} < c_{432}$

By shortening the time of the project for the 1 week (Table 3, Figure 3) is determined the Pareto-solution with the matching minimal project costs $C_p^*(T_p)$. In accordance with the adopted lexicographic order of criteria significance arise the following solutions:

Tabela 2. Polazni podaci projekta (različite D_{ij} nekih potencijalnih B_j iste faze A_i)

Phases	Predecessors	Contractors	Variants D_{ijv}		Durations			Costs			Normal		Crashed	
			Number	Indexes	t_{ij1}	t_{ij2}	t_{ij3}	c_{ij1}	c_{ij2}	c_{ij3}	Dur.	Cost	Dur.	Cost
A_i	K_i	B_j	d_{ij}	V_{ij}							$(t_n)_i$	$(c_n)_i$	$(t_u)_i$	$(c_u)_i$
A_1	\emptyset	B_1	1	{1}	4	-	-	14	-	-	4	14	4	14
A_2	\emptyset	B_1	2	{1,2}	3	2	-	13	16	-	3	13	2	16
A_3	\emptyset	B_1	3	{1,2,3}	5	4	3	15	20	25	5	15	3	25
		B_2	2	{1,2}	5	4	-	15	20	-				
A_4	{3}	B_2	2	{1,2}	5	4	-	15	17	-	5	15	4	17
		B_3	1	{2}	-	4	-	-	18	-				
A_5	{3}	B_2	2	{1,2}	4	3	-	14	15	-	4	14	3	15
		B_3	1	{1}	5	-	-	14	-	-				
A_6	{1,2,4}	B_3	3	{1,2,3}	6	5	4	16	22	30	6	16	4	30
A_7	{4}	B_2	1	{1}	4	-	-	14	-	-	4	14	3	18
		B_3	2	{1,2}	4	3	-	15	18	-				
		B_4	1	{2}	-	3	-	-	20	-				
A_8	{6,7}	B_3	2	{1,2}	4	3	-	14	16	-	4	14	3	16
		B_4	1	{1}	4	-	-	17	-	-				
A_9	{5,7}	B_3	2	{1,2,3}	5	4	3	15	19	23	5	15	3	23
		B_4	3	{1,2}	5	4	-	16	18	-				
											M	$C_p(M)$	m	$C_p(m)$
											= 20	= 130	= 14	= 174

Table 3. All Pareto-solutions for T_p and C_p with the best consequences $C_j^*(T_p, C_p), j \in J$

Solutions			1	2.1	2.2	3	4	5	6	7	Warnings	
Optimization	T_p		20	19		18	17	16	15	14	$T_p^* = 14$	
	C_p		130	132		134	139	144	153	166	$C_p^* = 130$	
Determination of the best consequences	C_1		42	42		42	47	52	52	52	$C_1^* = 52$	
	C_2		43	45	43	45	45	45	45	45	$C_2^* = 45$	
	C_3		45	45	47	47	47	47	56	69	$C_3^* = 69$	
	C_4		0	0		0	0	0	0	0	$C_4^* = 0$	
Stages, performers, binary variants of the first and the selected time	A_1	B_1	h_{111}	4	4	4	4	4	4	4	4	$t_u = t_n = 4$
	A_2	B_1	h_{211}	3	3	3	3	3	3	3	3	NOT $t_u = 2$
	A_3	B_1	h_{311}	5	5	5	5 ↓	-	-	-	-	YES $t_u = 3$
			h_{312}	-	-	-	-	4 ↓				
			h_{313}	-	-	-	-	-	3	3	3	
	A_4	B_2	h_{421}	5 ↓	-	5 ↓	-	-	-	-	-	YES $t_u = 4$
			h_{422}	-	4 ↑	-	4	4	4	4	4	
	A_5	B_2	h_{521}	4	4	4	4	4	4	4	4	NOT $t_u = 3$
	A_6	B_3	h_{631}	6	6	6	6	6	6 ↓	-	-	YES $t_u = 4$
			h_{632}	-	-	-	-	-	-	5 ↓		
			h_{633}	-	-	-	-	-	-	-	4	
	A_7	B_2	h_{721}	4	4	4	4	4	4	4	4	NOT $t_u = 3$
A_8	B_3	h_{831}	4	4 ↓	-	-	-	-	-	-	YES $t_u = 3$	
		h_{832}	-	-	3	3	3	3	3	3		
A_9	B_3	h_{931}	5	5	5	5	5	5 ↓	-	-	YES $t_u = 3$	
		h_{932}	-	-	-	-	-	-	4 ↓	-		
		h_{933}	-	-	-	-	-	-	-	3		

Solutions 1, 2.1, 3 to 7 correspond to the priorities $(T_p, C_p) \gg C_1 \gg C_2 \gg C_3 \gg C_4$.

All solutions 1 to 7 are determined with priorities $(T_p, C_p) \gg C_1 \gg (C_2, C_3) \gg C_4$.

Table shows the values of criteria and the selected performers of B_j with the times t_{ijv} for stages $i A_i$ in accordance with the selected variants of D_{ijv} , that have binary variables $h_{ijv} = 1$. Warnings 'YES t_u ' from the last column indicate which stages are shortened at forced times in the last solution 7 or any other previous solution. Solution for the forced project duration of $T_p^* = m = 14$ weeks have the characteristics from continuation, from the standpoint of optimal times and costs, $t_i^* \leq c_i^*$.

A_1 could not be shortened, it has $t_1^* = t_u = t_n = 4$ i $c_1^* = c_u = c_n = 4$

A_2, A_5 i A_7 are partially shortened, they have $t_i^* > (t_u)_i$ i $c_i^* = c_i(t_i^*) < (c_u)_i$

Other stages were shortened completely, they have

$$t_i^* = (t_u)_i \text{ i } c_i^* = (c_u)_i$$

The mentioned shortening of stages have enabled achievement of lower total project costs $C_p^*(m) = 166$ m.u. compared to the costs $C_p(m) = 174$ m.u. when all stages have forced times $(t_u)_i$ and forced costs $(c_u)_i$. There has been achieved saving amounting $C_p(m) - C_p^*(m) = 174 - 166 = 8$ m.u., respectively 4,60%.

On the basis of the analysis of the the achieved solutions the following conclusions are performed:

It's not necessary to engage the performer B_4 , since all solutions have $C_4 = 0$.

When the criteria are equal C_2 and C_3 , only for $T_p = 19$ with $C_p^* = 132$ there are two alternatives of Pareto-solution: solution 2.1 with $C_2 = C_3 = 45$, and solution 2.2 with $C_2 = 43 < C_3 = 47$.

Ideal values of criteria for the project are the minimum duration $T_p^* = 14$ with solution 7, and minimum costs $C_p^* = 130$ with solutions 1.

Conditioned ideal values for the costs of the performers $C_j^*(T_p, C_p^*(T_p))$ amounts $C_1^* = 52$ (solutions 5, 6, 7), $C_2^* = 45$ (solutions 3 to 7), and $C_3^* = 59$ (solution 7).

The selection of the final solution is carried out on dependency from acceptance of time and project costs. If adopted $T_p = 19$ sa $C_p = 132$, is selected solution 4.1 if is favoured the performer B_1 in relation to B_2 , respectively solution 4.2 if B_2 has higher significance than B_1 .

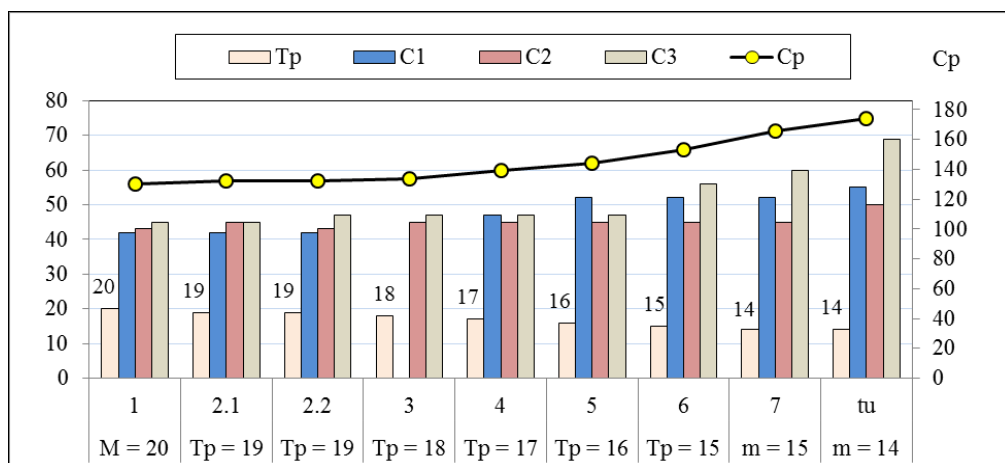
5. CONCLUSION

This paper shows classification problem of choice of the project performer from the set of potential performers who are qualified for the relevant works. There was carried out the selection of more performers for specific works, according to their specialties, and not the choice of one performer for the project as a whole. The basic requirement is that for any stage selects only one

performer. There were formed three groups of problems, each group with two subgroups and each subgroup with two types of data for the working capacity of potential performers.

From the standpoint of the project was carried out minimization of the duration and costs (according to the method of PERT/COST), as a criterion of a higher level of significance. Then was performed conditional maximization of criteria of a lower level of significance - the costs of the performers, so as not to worsen the realized values for the criteria of the project (method ϵ -limitation). Potential performers are defined by the appropriate number of priorities. Thus is determined the set of all or only characteristic of Pareto-optimal solutions among which selects the best solution for the application.

There was defined a general mathematical model for the last problem 3.a.2 which included the problem 3.a.1. There were proposed algorithms for solving both models. There has been displayed resolving 3.a.1 on a hypothetical project with 9 stages and 4 potential performers.



Slika 3. Vrednosti kriterijuma za generisana Pareto-rešenja

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