Slobodne oscilacije prostornog rama tipa X portalno-obrtne dizalice

Rade Vasiljević^{1*}, Milomir Gašić²

¹Visoka tehnička škola strukovnih studija, Beograd (Srbija)

²Fakultet za mašinstvo i građevinarstvo, Univerzitet u Kragujevcu, Kraljevo (Srbija)

U mašinskom inženjerstvu relativno je čest slučaj da izvedena rešenja konstrukcija nisu i optimalna. Dinamička analiza nosećih konstrukcija dizaličnih mašina je vrlo bitna za unapređenje tehničkih karakteristika dizalica. U dinamičkoj analizi frekventna analiza predstavlja prvu fazu u proceni dinamičke stabilnosti. U tom smislu, u ovom radu je razmatran problem slobodnih oscilacija noseće konstrukcije portalnih dizalica.

Ključne reči: Portalno-obrtne dizalice, Prostorni ram tipa X, MKE, Slobodne oscilacije, Frekvencije

1. UVOD

Problem oscilovanja konstrukcija je od velikog značaja u mašinskom inženjerstvu. On je posebno prisutan u dizaličnom inženjerstvu. Frekventna analiza idejnih rešenja dizaličnih konstrukcija je prvi i najbitniji element dinamičke analize za procenu njihove dinamičke stabilnosti. Postupak određivanja svojstvenih vrednosti kod složenih sistema (sistema sa velikim brojem stepeni slobode) predstavlja najskuplju fazu u dinamičkoj analizi, [1].

Ranija istraživanja određivanja sopstvenih frekvencija složenih nosećih struktura su bila bazirana na korišćenju približnih izraza i metoda [2]. Neki primeri približnih metoda prikazani su u [3]. Tačno određivanje sopstvenih frekvencija je bilo ograničeno na proste noseće strukture (prosta greda i konzola). Kod složenih elastičnih tela nalaženje rešenja frekventne jednačine je bilo teško jer je ona sadržala trigonometrijske i hiperbolične funkcije. Danas, matematički softveri omogućavaju rutinsko rešavanje frekventne jednačine oscilovanja složenih elastičnih tela. Tačno određivanje sopstvenih frekvencija je značajno sa aspekta optimizacije nosećih struktura.

Međutim, i dalje je kod suviše složenih nosećih konstrukcija korišćenje analitičkih metoda ograničeno. U tom slučaju za određivanje sopstvenih frekvencija nosećih struktura opredeljujemo se za neku od numeričkih metoda. Glavna prednost numeričkih metoda je da se veoma složene konstrukcije mogu posmatrati kao redukovani modeli, čija je analiza sa aspekta inženjerske tačnosti dovoljna u oceni ponašanja složenih struktura.

U ovom radu se razmatraju slobodne oscilacije prostorne noseće konstrukcije tipa X portalne dizalice, slika 1. Saglasno rečenom, frekventna analiza portala tipa X portalne dizalice je sprovedena numerički. Predstavljen konačno-elementni prilaz frekventnoj analizi, odnosno matematički model sa konzistentnim masama [4]. Teorijski rezultati numeričke analize su verifikovani primenom softverske simulacije.

Ovaj rad i rad [5] treba da postave podloge za ispitivanje dinamičke stabilnosti portalno-obrtne dizalice tipa X.

Slika 1: Portalna dizalica tipa X

2. KONAČNO-ELEMENTNI MODEL

Diskretni model noseće konstrukcije tipa X portalne dizalice prikazan je na slici 2. Prostorni ram tipa X je dosta sličan tipu H, za koji je dat detaljan opis u radu [6]. Ova dva tipa portala se razlikuju u horizontalnom ramu (ram H ili ram X).

Slika 2: Diskretizovani model portala tipa X

Slika 3: Konačno-elementni model portala tipa X

Prostorni ram (portal) je predstavljen preko 3D grednih konačnih elemenata sa dve čvorne tačke. Čvorovi 1...4 imaju četiri stepena slobode dok svi ostali čvorovi imaju šest stepeni slobode, slika 4.

Slika 4: Pomeranja čvorova

Vektor pomeranja noseće konstrukcije tipa X portalne dizalice glasi:

$$\mathbf{U}_{mn} = \{U_{y1} \ U_{\theta1} \ U_{\phi1} \ U_{\psi1} \ \dots \ U_{y4} \ U_{\theta4} \ U_{\phi4} \ U_{\psi4} \ U_{x5} \ U_{y5} \ U_{z5} \ U_{\theta5} \ U_{\phi5} \ U_{\psi5} \ \dots \ U_{x13} \ U_{y13} \ U_{z13} \ U_{\theta13} \ U_{\phi13} \ U_{\psi13}\}^T \quad (1)$$

$$\mathbf{t} = \begin{bmatrix} \lambda & \mu & \nu \\ \frac{-\lambda\mu \cos \gamma_l - \nu \sin \gamma_l}{\sqrt{\lambda^2 + \nu^2}} & \sqrt{\lambda^2 + \nu^2} \cos \gamma_l & \frac{-\nu\mu \cos \gamma_l - \lambda \sin \gamma_l}{\sqrt{\lambda^2 + \nu^2}} \\ \frac{\lambda\mu \sin \gamma_l - \nu \cos \gamma_l}{\sqrt{\lambda^2 + \nu^2}} & -\sqrt{\lambda^2 + \nu^2} \sin \gamma_l & \frac{\nu\mu \sin \gamma_l + \lambda \cos \gamma_l}{\sqrt{\lambda^2 + \nu^2}} \end{bmatrix} \quad (5)$$

Matrica transformacije štapa glasi:

$$\mathbf{T} = \begin{bmatrix} \mathbf{t} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{t} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{t} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{t} \end{bmatrix} \quad (6)$$

2.1. Analiza grednog elementa u prostoru

Vektor generalisanih pomeranja i vektor generalisanih sila 3D grednog konačnog elementa glase:

$$\mathbf{q}^T = \{u_i \ v_i \ w_i \ \theta_i \ \varphi_i \ \psi_i \ u_k \ v_k \ w_k \ \theta_k \ \varphi_k \ \psi_k\} \quad (2)$$

$$\mathbf{R}^T = \{N_i \ T_{yi} \ T_{zi} \ M_{xi} \ M_{yi} \ M_{zi} \ N_k \ T_{yk} \ T_{zk} \ M_{xk} \ M_{yk} \ M_{zk}\} \quad (3)$$

Za obostrano uklješten štapa konstantnog poprečnog preseka vektor interpolacionih funkcija glasi:

$$\mathbf{N}^T = \begin{bmatrix} 1-\xi & 0 & 0 & 0 \\ 0 & 1-3\xi^2+2\xi^3 & 0 & 0 \\ 0 & 0 & 1-3\xi^2+2\xi^3 & 0 \\ 0 & 0 & 0 & 1-\xi \\ 0 & x-2l\xi^2+l\xi^3 & 0 & 0 \\ 0 & 0 & -x+2l\xi^2-l\xi^3 & 0 \\ \xi & 0 & 0 & 0 \\ 0 & 3\xi^2-2\xi^3 & 0 & 0 \\ 0 & 0 & 3\xi^2-2\xi^3 & 0 \\ 0 & 0 & 0 & \xi \\ 0 & l\xi^2-l\xi^3 & 0 & 0 \\ 0 & 0 & l\xi^2-l\xi^3 & 0 \end{bmatrix}, \quad (4)$$

gde je $\xi = x/l$.

Slika 5: 3D rotacije

Matrica transformacionog vektora $\mathbf{t}$ je jednaka, slika 5:

2.1.1. Matrica krutosti elementa

Odgovarajuća matrica krutosti linijskog elementa definisana je na osnovu interpolacionih funkcija (4):

$$\mathbf{k} = \int_V E \cdot \mathbf{N}^T \cdot \mathbf{N} \cdot dV \quad (7)$$

odnosno:

$$\mathbf{k} = \begin{bmatrix} \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{l^3} & 0 & 0 & 0 & \frac{6EI_z}{l^2} & 0 & -\frac{12EI_z}{l^3} & 0 & 0 & 0 & \frac{6EI_z}{l^2} \\ 0 & 0 & \frac{12EI_y}{l^3} & 0 & -\frac{6EI_y}{l^2} & 0 & 0 & 0 & -\frac{12EI_y}{l^3} & 0 & -\frac{6EI_y}{l^2} & 0 \\ 0 & 0 & 0 & \frac{GI_x}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{GI_x}{l} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & 0 \\ 0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l} & 0 & -\frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{2EI_z}{l} \\ -\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{l^3} & 0 & 0 & 0 & -\frac{6EI_z}{l^2} & 0 & \frac{12EI_z}{l^3} & 0 & 0 & 0 & -\frac{6EI_z}{l^2} \\ 0 & 0 & -\frac{12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} & 0 & 0 & 0 & \frac{12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} & 0 \\ 0 & 0 & 0 & -\frac{GI_x}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GI_x}{l} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & 0 \\ 0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{2EI_z}{l} & 0 & -\frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l} \end{bmatrix} \quad (8)$$

2.1.2. Matrica masa elementa

Odgovarajuća matrica masa linijskog elementa definisana je na osnovu interpolacionih funkcija (4):

$$\mathbf{m} = \int_V \rho \cdot \mathbf{N}^T \cdot \mathbf{N} \cdot dV \quad (9)$$

odnosno:

$$\mathbf{m} = \begin{bmatrix} 140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 156 & 0 & 0 & 0 & 22l & 0 & 54 & 0 & 0 & 0 & -13l \\ 0 & 0 & 156 & 0 & -22l & 0 & 0 & 0 & 54 & 0 & 13l & 0 \\ 0 & 0 & 0 & \frac{140I_x}{A} & 0 & 0 & 0 & 0 & 0 & \frac{70I_x}{A} & 0 & 0 \\ 0 & 0 & -22l & 0 & 4l^2 & 0 & 0 & 0 & -13l & 0 & -3l^2 & 0 \\ 0 & 22l & 0 & 0 & 0 & 4l^2 & 0 & 13l & 0 & 0 & 0 & -3l^2 \\ 70 & 0 & 0 & 0 & 0 & 0 & 140 & 0 & 0 & 0 & 0 & 0 \\ 0 & 54 & 0 & 0 & 0 & 13l & 0 & 156 & 0 & 0 & 0 & -22l \\ 0 & 0 & 54 & 0 & -13l & 0 & 0 & 0 & 156 & 0 & 22l & 0 \\ 0 & 0 & 0 & \frac{70I_x}{A} & 0 & 0 & 0 & 0 & 0 & \frac{140I_x}{A} & 0 & 0 \\ 0 & 0 & 13l & 0 & -3l^2 & 0 & 0 & 0 & 22l & 0 & 4l^2 & 0 \\ 0 & -13l & 0 & 0 & 0 & -3l^2 & 0 & -22l & 0 & 0 & 0 & 4l^2 \end{bmatrix} \frac{\rho Al}{420} \quad (10)$$

2.2. Analiza matrice krutosti i matrice masa sistema

2.2.1. Matrica krutosti sistema

Matrica krutosti elementa n u globalnom koordinatnom sistemu je jednaka:

$$\mathbf{K}_n = \mathbf{T}_n^T \mathbf{k}_n \mathbf{T}_n, n = 1 \dots 16 \quad (11)$$

Matrica krutosti noseće konstrukcije glasi:

$$[\mathbf{K}_{st}]_{78 \times 78} = \sum_{n=1}^{16} \mathbf{K}_n \quad (12)$$

Matrica krutosti elastičnih oslonaca glasi:

$$[\mathbf{K}_o]_{16 \times 16} = \sum_{n=0}^4 \mathbf{K}_n \quad (13)$$

$$\mathbf{K} = \begin{bmatrix} K_{1,1} + k_{1,1} & \dots & K_{1,16} + k_{1,16} & K_{1,17} & \dots & K_{1,78} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ K_{16,1} + k_{16,1} & \dots & K_{16,16} + k_{16,16} & K_{16,17} & \dots & K_{16,78} \\ K_{17,1} & \dots & K_{17,16} & K_{17,17} & \dots & K_{17,78} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ K_{78,1} & \dots & K_{78,16} & K_{78,17} & \dots & K_{78,78} \end{bmatrix} \quad (16)$$

2.2.2. Matrica masa sistema

Matrica masa elementa n u globalnom koordinatnom sistemu je jednaka:

$$\mathbf{M}_n = \mathbf{T}_n^T \mathbf{m}_n \mathbf{T}_n, n = 1 \dots 16 \quad (17)$$

Matrica masa sistema glasi:

$$\mathbf{M} = [\mathbf{M}_{st}]_{78 \times 78} = \sum_{n=1}^{16} \mathbf{M}_n \quad (18)$$

Matrica masa sistema u pravcu nepoznatih pomeranja glasi:

$$\mathbf{M}_{nn} = [\mathbf{M}_{st}]_{70 \times 70} = \sum_{n=1}^{16} \mathbf{M}_n \quad (19)$$

2.3. Analiza slobodnih oscilacija sistema

Jednačina slobodnih neprigušenih oscilacija sistema glasi:

$$\mathbf{M}_{nn} \ddot{\mathbf{U}}_{nn} + \mathbf{K}_{nn} \mathbf{U}_{nn} = \mathbf{0} \quad (20)$$

gde su:

- $\ddot{\mathbf{U}}_{nn}$ - vektor nepoznatih ubrzanja, i
- $\mathbf{U}_{nn}$ - vektor nepoznatih pomeranja.

Sopstvene frekvencije noseće konstrukcije se dobijaju rešavanjem algebarske jednačine:

$$\det(\mathbf{K}_{nn} - \omega^2 \mathbf{M}_{nn}) = 0 \quad (21)$$

Na osnovu kružnih sopstvenih frekvencija dobijaju se periodi oscilovanja preko izraza:

$$T_i = \frac{2\pi}{\omega_i} \quad (22)$$

Frekvencije noseće konstrukcije se dobijaju na osnovu izraza:

$$f_i = \frac{\omega_i}{2\pi} = \frac{1}{T_i} \quad (23)$$

3. BROJNI PRIMER

3.1. Polazni podaci

Brojni primer je urađen za portalno-obrtanu dizalicu tipa X, za koju su polazni podaci dati u tabelama 1, 2 i 3, [7-10].

Matrica krutosti celog sistema uključuje matricu noseće konstrukcije i matricu krutosti elastičnih oslonaca:

$$\mathbf{K} = [\mathbf{K}_{st}]_{78 \times 78} + [\mathbf{K}_o]_{16 \times 16} = \sum_{n=1}^{16} \mathbf{K}_n + \sum_{n=0}^4 \mathbf{k}_n \quad (14)$$

Matrica krutosti sistema u pravcu nepoznatih pomeranja glasi:

$$\mathbf{K}_{nn} = [\mathbf{K}_{st}]_{70 \times 70} + [\mathbf{K}_o]_{16 \times 16} \quad (15)$$

Da bi se izvršila jednačina (14), matrica krutosti elastičnih oslonaca $[\mathbf{K}_o]_{16 \times 16}$ se dodavanjem nula redova i nula kolona proširuje u matricu dimenzija 78×78 , osim za 16 stepeni slobode četiri čvora.

Tabela 1: Gabaritne dimenzije portala tipa X

Veličina	Vrednost
Dužina portala L [m]	6
Širina baze portala B [m]	6
Visina nogu portala H [m]	6
Prečnik prstena portala D [m]	4,243
Rastojanje između oslonaca strele H_0 [m]	3,675

Tabela 2: Dimenzije poprečnih preseka portala tipa X

n		1...4	5...8	9...12	13...16
H_n [mm]	i	530	524	1030	530
	j	1030	524	530	530
H_{nl} [mm]	i	500	500	1000	500
	j	1000	500	500	500
B_n [mm]	i	550	550	550	550
	j	550	550	550	550
B_{nl} [mm]	i	500	500	500	500
	j	500	500	500	500
δ_l [mm]	i	10	12	10	10
	j	10	12	10	10
δ_2 [mm]	i	15	12	15	15
	j	15	12	15	15

Tabela 3: Ostali podaci portala tipa X

Veličina	Vrednost
Modul elastičnosti E [N/mm ²]	2.1×10^{11}
Modul klizanja G [N/mm ²]	0.8×10^{11}
Gustina materijala ρ [kg/m ³]	7850
Krutost elastičnog oslonca k [N/m]	$1 \cdot 10^7$
Gravitaciona konstanta g [m/s ²]	9.81

3.2. Teorijski rezultati

Na osnovu teorijskih postavki u podnaslovu 2 i definisanih ulaznih podataka u programu Mathematica, [11] i [12], napisan je kod za izračunavanje sopstvenih frekvencija, frekvencija i perioda oscilovanja. Dobijene su njihove vrednosti za sve stepene slobode kretanja ($i=70$), ali su u tabeli 4 prikazane vrednosti za prvih 6 modova oscilovanja.

Tabela 4: Periodi i frekvencije oscilovanja

Br. moda	Period T [s]	Kružne frek. ω [rad/s]	Frekvencija f [Hz]
1	0.3677	17.09	2.72
2	0.3429	18.32	2.92
3	0.1902	33.03	5.26
4	0.1263	49.73	7.91
5	0.0732	85.86	13.66
6	0.0623	100.81	16.04

Sa aspekta optimizacije prva najniža (osnovna) frekvencija treba da bude u opsegu 0,5...3,5 [Hz] [9]. Kako prva frekvencija oscilovanja iznosi 2,72 [Hz], to znači da se nalazi u definisanom opsegu.

3.3. Verifikacija teorijskih rezultata softverskom simulacijom

U cilju verifikacije teorijskih rezultata, u softveru SAP2000 je formiran i rešen konačno-elementni model noseće konstrukcije dizalice, [13]. Konačno-elementni model noseće konstrukcije dizalice je prikazan na slici 6.

Slika 6: KE model u softveru SAP2000

Predstavljeni konačoelementni model je 3D model. Sprovođenjem modalne analize u softveru SAP2000, dobijene su vrednosti frekvencija i glavni oblici oscilovanja noseće konstrukcije razmatrane portalne dizalice.

U tabeli 5 dato je prvih šest frekvencija oscilovanja noseće konstrukcije razmatrane portalne dizalice.

Tabela 5: Periodi i frekvencije oscilovanja - SAP2000®

Br. moda	Period T [s]	Frkvencija f [Hz]
1	0.3667	2.73
2	0.3405	2.94
3	0.1901	5.23
4	0.1264	7.91
5	0.0727	13.75
6	0.0610	16.38

Prva dva moda oscilovanja noseće konstrukcije su prikazana na slikama 7 i 8.

Slika 7: 1 mod oscilovanja

Slika 8: 2 mod oscilovanja

Poređenjem vrednosti frekvencija dobijenih preko matematičkog modela sa konačno-elementnim pristupom i čistim numeričkim putem (FEM softver SAP2000), uočava se odlično poklapanje rezultata prve dve frekvencije sa relativnim odstupanjima $\Delta=(0,37; 0,68)$ [%]. I ostale frekvencije se dobro poklapaju. Tako npr. za sledeće 4 frekvencije relativno odstupanje iznosi do 2,08 [%], što je za prostorni model vrlo dobro.

4. ZAKLJUČAK

Problem oscilovanja u prostoru je prisutan u dizaličnom inženjerstvu. Posebno je izražen kod portalnih dizalica jer ove dizalice imaju proporcionalno malu osnovu u odnosu na visinu.

U radu je razmatrana frekventna analiza prostorne noseće konstrukcije tipa X portalne dizalice. Primenjen je prilaz zamene raspodeljenih masa konzistentnim (ekvivalentnim) masama. Saglasno usvojenom prilazu, formiran je konačno-elementni model razmatrane noseće konstrukcije. Za formirani konačno-elementni model postavljena je matematička formulacija slobodnih oscilacija. U programskom paketu Mathematica® napisan je kod za automatizovano izračunavanje frekvencija i perioda oscilovanja.

Validacija teorijskih rezultata frekventne analize sprovedena je softverskom simulacijom (SAP2000®).

Formirani konačno-elementni model u ovom radu je podloga za dalje istraživanje uticajnih parametara na dinamičko ponašanje portalnih dizalica visokih performansi. Takođe, rad ostavlja prostor za optimizaciju ovakvih vrsta konstrukcija, na osnovu ograničenih vrednosti svojstvenih frekvencija.

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Free Vibrations of the Spatial Frame Type X of Portal-rotating Cranes

Rade Vasiljevic^{1*}, Milomir Gasic²

¹ High Technical School, Belgrade (Serbia)

² Faculty of Mechanical and Civil Engineering, University of Kragujevac, Kraljevo (Serbia)

In Mechanical engineering it is a relatively common case that existing designs and constructions are not optimal. Dynamic analysis of support structures of crane machinery is essential for the improvement of technical characteristics of portal crane. In a dynamic analysis, frequency analysis is the first stage in the evaluation of dynamic stability. In this sense, in this paper discusses the problem of free oscillations of the carrying structure of portal cranes.

Keywords: Portal-rotating cranes, Spatial Frame Type X, FEM, Free vibrations, Frequencies

1. INTRODUCTION

Problem concerning structures oscillating is of great importance in Mechanical Engineering. It is especially present in the crane engineering. Frequency analysis of conceptual designs of crane design is the first and most essential element of dynamic analysis for estimation of their dynamic stability. The process of determining the eigenvalues in complex systems (systems with a large number of degrees of freedom) is the most expensive phase in the dynamic analysis, [1].

Earlier studies determining the natural frequencies of complex carrying structures have been based on the use of approximate expressions and methods [2]. Some examples of the approximate methods are shown in [3]. Accurate determination of natural frequencies was limited to simple carrying structure (simple beams and brackets). In complex elastic body for a solution frequency equation was difficult because it contained trigonometric and hyperbolic functions. Today, mathematical software enables routine solving of complex oscillation frequency equations of elastic bodies. Accurate determination of natural frequencies is important in optimizing the carrying structure.

However, using analytical methods at complex carrying structures is limited. In this case, for the determination of natural frequencies by the carrying structure we opt for one of the numerical methods. The main advantage of numerical methods is that they very complex structures can be viewed as a reduced model, whose analysis the aspect of engineering accuracy is sufficient to evaluate the behavior of complex structures.

In this paper discusses the free oscillations of spatial carrying structure type X portal cranes, Figure 1. Pursuant to the aforesaid, the frequency analysis discussed portal crane is conducted numerically. The is represented by finite element approach to frequency analysis, ie the mathematical model consistent with the masses [4]. The theoretical results of numerical analyzes are verified using the simulation software.

This paper and the paper [5] should lay foundations for stability testing of dynamic portal-rotating crane type X.

Figure 1: Portal crane type X

2. FINITE ELEMENT MODEL

The discrete model of the carrying structure type X of portal-rotating cranes is shown at Figure 2. Spatial frame type X is very similar to the type H for which is described in detail in [6]. These two types portal differ in the frame horizontal (frame H or frame X).

Figure 2: Discrete model of portal type X

Figure 3: Finite element model of portal type X

Spatial frame (portal) is presented via 3D beam finite elements with two nodal points. Nodes 1...4 have four degrees of freedom, while all other nodes have six degrees of freedom, Figure 4.

Figure 4: Nodal displacements

Vector of displacements carrying structure type X of portal-rotary crane read:

$$\mathbf{U}_{mn} = \{U_{y1} \ U_{\theta1} \ U_{\phi1} \ U_{\psi1} \ \dots \ U_{y4} \ U_{\theta4} \ U_{\phi4} \ U_{\psi4} \ U_{x5} \ U_{y5} \ U_{z5} \ U_{\theta5} \ U_{\phi5} \ U_{\psi5} \ \dots \ U_{x13} \ U_{y13} \ U_{z13} \ U_{\theta13} \ U_{\phi13} \ U_{\psi13}\}^T \quad (1)$$

$$\mathbf{t} = \begin{bmatrix} \lambda & \mu & \nu \\ \frac{-\lambda\mu \cos \gamma_l - \nu \sin \gamma_l}{\sqrt{\lambda^2 + \nu^2}} & \sqrt{\lambda^2 + \nu^2} \cos \gamma_l & \frac{-\nu\mu \cos \gamma_l - \lambda \sin \gamma_l}{\sqrt{\lambda^2 + \nu^2}} \\ \frac{\lambda\mu \sin \gamma_l - \nu \cos \gamma_l}{\sqrt{\lambda^2 + \nu^2}} & -\sqrt{\lambda^2 + \nu^2} \sin \gamma_l & \frac{\nu\mu \sin \gamma_l + \lambda \cos \gamma_l}{\sqrt{\lambda^2 + \nu^2}} \end{bmatrix} \quad (5)$$

The transformation matrix element read:

$$\mathbf{T} = \begin{bmatrix} \mathbf{t} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{t} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{t} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{t} \end{bmatrix} \quad (6)$$

2.1. Analysis of beam element in space

Vector of generalized displacements and vector of generalized forces of 3D of beam element read:

$$\mathbf{q}^T = \{u_i \ v_i \ w_i \ \theta_i \ \phi_i \ \psi_i \ u_k \ v_k \ w_k \ \theta_k \ \phi_k \ \psi_k\} \quad (2)$$

$$\mathbf{R}^T = \{N_i \ T_{yi} \ T_{zi} \ M_{xi} \ M_{yi} \ M_{zi} \ N_k \ T_{yk} \ T_{zk} \ M_{xk} \ M_{yk} \ M_{zk}\} \quad (3)$$

For both sides wedged rod of constant cross-section vector interpolation function read:

$$\mathbf{N}^T = \begin{bmatrix} 1-\xi & 0 & 0 & 0 \\ 0 & 1-3\xi^2+2\xi^3 & 0 & 0 \\ 0 & 0 & 1-3\xi^2+2\xi^3 & 0 \\ 0 & 0 & 0 & 1-\xi \\ 0 & x-2l\xi^2+l\xi^3 & 0 & 0 \\ 0 & 0 & -x+2l\xi^2-l\xi^3 & 0 \\ \xi & 0 & 0 & 0 \\ 0 & 3\xi^2-2\xi^3 & 0 & 0 \\ 0 & 0 & 3\xi^2-2\xi^3 & 0 \\ 0 & 0 & 0 & \xi \\ 0 & l\xi^2-l\xi^3 & 0 & 0 \\ 0 & 0 & l\xi^2-l\xi^3 & 0 \end{bmatrix}, \quad (4)$$

where $\xi = x/l$.

Figure 5: 3D rotation

The transformation matrix of vector $\mathbf{t}$ is equal to, Figure 5:

2.1.1. Stiffness matrix of the element

The corresponding stiffness matrix of the line element is defined based on interpolation functions (4):

$$\mathbf{k} = \int_V E \cdot \mathbf{N}^T \cdot \mathbf{N} \cdot dV \quad (7)$$

or:

$$\mathbf{k} = \begin{bmatrix} \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{12EI_z}{l^3} & 0 & 0 & 0 & \frac{6EI_z}{l^2} & 0 & -\frac{12EI_z}{l^3} & 0 & 0 & 0 & \frac{6EI_z}{l^2} \\ 0 & 0 & \frac{12EI_y}{l^3} & 0 & -\frac{6EI_y}{l^2} & 0 & 0 & 0 & -\frac{12EI_y}{l^3} & 0 & -\frac{6EI_y}{l^2} & 0 \\ 0 & 0 & 0 & \frac{GI_x}{l} & 0 & 0 & 0 & 0 & 0 & -\frac{GI_x}{l} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & 0 \\ 0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l} & 0 & -\frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{2EI_z}{l} \\ -\frac{EA}{l} & 0 & 0 & 0 & 0 & 0 & \frac{EA}{l} & 0 & 0 & 0 & 0 & 0 \\ 0 & -\frac{12EI_z}{l^3} & 0 & 0 & 0 & -\frac{6EI_z}{l^2} & 0 & \frac{12EI_z}{l^3} & 0 & 0 & 0 & -\frac{6EI_z}{l^2} \\ 0 & 0 & -\frac{12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} & 0 & 0 & 0 & \frac{12EI_y}{l^3} & 0 & \frac{6EI_y}{l^2} & 0 \\ 0 & 0 & 0 & -\frac{GI_x}{l} & 0 & 0 & 0 & 0 & 0 & \frac{GI_x}{l} & 0 & 0 \\ 0 & 0 & -\frac{6EI_y}{l^2} & 0 & \frac{2EI_y}{l} & 0 & 0 & 0 & \frac{6EI_y}{l^2} & 0 & \frac{4EI_y}{l} & 0 \\ 0 & \frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{2EI_z}{l} & 0 & -\frac{6EI_z}{l^2} & 0 & 0 & 0 & \frac{4EI_z}{l} \end{bmatrix} \quad (8)$$

2.1.2. Mass matrix of the element

The corresponding mass matrix of the line element is defined based on interpolation functions (4):

$$\mathbf{m} = \int_V \rho \cdot \mathbf{N}^T \cdot \mathbf{N} \cdot dV \quad (9)$$

or:

$$\mathbf{m} = \begin{bmatrix} 140 & 0 & 0 & 0 & 0 & 0 & 70 & 0 & 0 & 0 & 0 & 0 \\ 0 & 156 & 0 & 0 & 0 & 22l & 0 & 54 & 0 & 0 & 0 & -13l \\ 0 & 0 & 156 & 0 & -22l & 0 & 0 & 0 & 54 & 0 & 13l & 0 \\ 0 & 0 & 0 & \frac{140I_x}{A} & 0 & 0 & 0 & 0 & 0 & \frac{70I_x}{A} & 0 & 0 \\ 0 & 0 & -22l & 0 & 4l^2 & 0 & 0 & 0 & -13l & 0 & -3l^2 & 0 \\ 0 & 22l & 0 & 0 & 0 & 4l^2 & 0 & 13l & 0 & 0 & 0 & -3l^2 \\ 70 & 0 & 0 & 0 & 0 & 0 & 140 & 0 & 0 & 0 & 0 & 0 \\ 0 & 54 & 0 & 0 & 0 & 13l & 0 & 156 & 0 & 0 & 0 & -22l \\ 0 & 0 & 54 & 0 & -13l & 0 & 0 & 0 & 156 & 0 & 22l & 0 \\ 0 & 0 & 0 & \frac{70I_x}{A} & 0 & 0 & 0 & 0 & 0 & \frac{140I_x}{A} & 0 & 0 \\ 0 & 0 & 13l & 0 & -3l^2 & 0 & 0 & 0 & 22l & 0 & 4l^2 & 0 \\ 0 & -13l & 0 & 0 & 0 & -3l^2 & 0 & -22l & 0 & 0 & 0 & 4l^2 \end{bmatrix} \frac{\rho Al}{420} \quad (10)$$

2.2. Analysis of stiffness matrix and mass matrix of the system

2.2.1. Stiffness matrix of the system

Stiffness matrix of element n in the global coordinate system is equal to:

$$\mathbf{K}_n = \mathbf{T}_n^T \mathbf{k}_n \mathbf{T}_n, n = 1 \dots 16 \quad (11)$$

Stiffness matrix of the carrying structure:

$$[\mathbf{K}_{st}]_{78 \times 78} = \sum_1^{16} \mathbf{K}_n \quad (12)$$

Stiffness matrix of the elastic supports:

$$[\mathbf{K}_o]_{16 \times 16} = \sum_0^4 \mathbf{K}_n \quad (13)$$

$$\mathbf{K} = \begin{bmatrix} K_{1,1} + k_{1,1} & \dots & K_{1,16} + k_{1,16} & K_{1,17} & \dots & K_{1,78} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ K_{16,1} + k_{16,1} & \dots & K_{16,16} + k_{16,16} & K_{16,17} & \dots & K_{16,78} \\ K_{17,1} & \dots & K_{17,16} & K_{17,17} & \dots & K_{17,78} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ K_{78,1} & \dots & K_{78,16} & K_{78,17} & \dots & K_{78,78} \end{bmatrix} \quad (16)$$

2.2.2. Mass matrix of the system

Mass matrix of element n in the global coordinate system is equal to:

$$\mathbf{M}_n = \mathbf{T}_n^T \mathbf{m}_n \mathbf{T}_n, n = 1 \dots 16 \quad (17)$$

Mass matrix of the system read:

$$\mathbf{M} = [\mathbf{M}_{st}]_{78 \times 78} = \sum_1^{16} \mathbf{M}_n \quad (18)$$

Mass matrix of the system in the direction of unknown displacements read:

$$\mathbf{M}_{uu} = [\mathbf{M}_{st}]_{70 \times 70} = \sum_1^{16} \mathbf{M}_n \quad (19)$$

2.3. Analysis of free vibrations of the system

The equation of the free undamped oscillation of the system read:

$$\mathbf{M}_{uu} \ddot{\mathbf{U}}_{uu} + \mathbf{K}_{uu} \mathbf{U}_{uu} = \mathbf{0} \quad (20)$$

where:

- $\ddot{\mathbf{U}}_{uu}$ - vector of unknown accelerations, and
- $\mathbf{U}_{uu}$ - vector of unknown displacements.

Eigenfrequencies of the carrying structure are obtained by solving algebraic equation:

$$\det(\mathbf{K}_{uu} - \omega^2 \mathbf{M}_{uu}) = 0 \quad (21)$$

Based on the circular natural frequencies obtained by the oscillation periods via expression:

$$T_i = \frac{2\pi}{\omega_i} \quad (22)$$

Frequencies of the carrying structure are obtained based on expression:

$$f_i = \frac{\omega_i}{2\pi} = \frac{1}{T_i} \quad (23)$$

3. NUMERICAL EXAMPLE

3.1. Starting data

Numerical example is made of portal-rotating crane type X, for which the 3.1. starting data given in the tables 1, 2 i 3, [7-10].

The stiffness matrix of the whole system includes the matrix of the carrying structure and the stiffness matrix of elastic supports:

$$\mathbf{K} = [\mathbf{K}_{st}]_{78 \times 78} + [\mathbf{K}_o]_{16 \times 16} = \sum_1^{16} \mathbf{K}_n + \sum_0^4 \mathbf{k}_n \quad (14)$$

The stiffness matrix of the system in the direction of unknown displacements read:

$$\mathbf{K}_{uu} = [\mathbf{K}_{st}]_{70 \times 70} + [\mathbf{K}_o]_{16 \times 16} \quad (15)$$

In order to execute Equation (14), the stiffness matrix of elastic supports $[\mathbf{K}_o]_{16 \times 16}$ is, by adding zero rows and zero columns, extended to the matrix with dimensions 78×78 , except for 16 degrees of freedom of four nodes.

Table 1: Overall dimensions for the portal type X

Parameter	Amount
Length of the portal base L [m]	6
Width of the portal base B [m]	6
Height of the leg H [m]	6
Diameter of the circular ring D [m]	4,243
Distance between the supports and the boom H_0 [m]	3,675

Table 2: Dimensions of the cross-sections of the portal type X

n		1...4	5...8	9...12	13...16
H_n [mm]	i	530	524	1030	530
	j	1030	524	530	530
H_{nl} [mm]	i	500	500	1000	500
	j	1000	500	500	500
B_n [mm]	i	550	550	550	550
	j	550	550	550	550
B_{nl} [mm]	i	500	500	500	500
	j	500	500	500	500
δ_1 [mm]	i	10	12	10	10
	j	10	12	10	10
δ_2 [mm]	i	15	12	15	15
	j	15	12	15	15

Table 3: Other data for the portal type X

Parameter	Amount
Modulus of elasticity E [N/mm ²]	2.1×10^{11}
Modulus of slipping G [N/mm ²]	0.8×10^{11}
Mass density of material ρ [kg/m ³]	7850
Stiffness of elastic supports k [N/m]	$1 \cdot 10^7$
Gravitational constant g [m/s ²]	9.81

3.2. Theoretical results

Based on theoretical assumptions in the subtitle 2 and defined starting data in Mathematica, [11] and [12], was written **cod** for calculating the natural frequencies, the frequency and the oscillation period. Obtained are their value for all degrees of freedom ($i = 70$), but are in Table 4 shows the values for the first 6 modes of oscillation.

Table 4: Periods and frequencies

Mode No	Period T [s]	Circular freq. ω [rad/s]	Frequency f [Hz]
1	0.3677	17.09	2.72
2	0.3429	18.32	2.92
3	0.1902	33.03	5.26
4	0.1263	49.73	7.91
5	0.0732	85.86	13.66
6	0.0623	100.81	16.04

Along with aspect of the optimization of the first lowest (basic) frequency should be in the range 0,5...3,5 [Hz] [9]. As the first oscillation frequency is 2,72 [Hz], that means it is located in the defined range.

3.3. Verification of the theoretical results of simulation software

In order to verify the theoretical results, the software SAP2000 is formed and solved finite element model of the carrying structure of cranes, [13]. The FE model of the structure of the crane is shown in Figure 6.

Figure 6: FE model in software SAP2000

Presented finite element model is a 3D model. By implementation the modal analysis in SAP2000, obtained the values frequencies and main mode shapes of carrying structure considered portal crane.

Table 5 shows the values of the first six frequency oscillating support structures portal crane.

Table 5: Periods and frequencies - SAP2000®

Mode No	Period T [s]	Frequency f [Hz]
1	0.3667	2.73
2	0.3405	2.94
3	0.1901	5.23
4	0.1264	7.91
5	0.0727	13.75
6	0.0610	16.38

The first two modes oscillation of carrying structures are shown in Figure 7 and 8.

Figure 7: 1st mode shape Figure 8: 2nd mode shape

By comparing the values of frequencies obtained through the mathematical model with the finite element approach and in a purely numerical way (FEA software SAP2000), excellent coinciding between the results of the first two frequencies and the relative deviations $\Delta = (0,37; 0,68)$ [%] is noticed. Other frequencies coincide well, too. Thus, for example, for the next 4 frequencies the relative deviation is up to 2,08 [%], which is very good for the spatial model.

4. CONCLUSION

The problem of oscillation in the space is present in lifting engineering. It is particularly pronounced in portal-crane because these cranes have small base in proportion to their height.

In this paper discussed the frequency analysis of spatial carrying structure type X portal cranes. Has been applied approach to replace distributed mass consistently (equivalent) mass. In adopted accordance with the approach was formed in finite-element model considered carrying structure. To formed by the final-element model set is a mathematical formulation of free oscillations. The software package Mathematica® written code for automated calculation of frequency and the oscillation period.

Validation of the theoretical results of frequency analysis was conducted simulation software (SAP2000®).

Formed finite-element model in this paper is the basis for further research of influential parameters on the dynamic behavior of portal crane, high performance. The paper also leaves room for optimization of these types of constructions.

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