Određivanje momenta inercije platforme merenjem oscilacija platforme

Stojko Biočanin

U radu je određen moment inercije obrtne platform borbenog sredstva na osnovu merenja na stvarnom objektu. Metoda je zasnovana na merenju slobodnih oscilacija platforme vezane za oprugu poznate krutosti.

Ključne reči: Obrtna platforme, moment inercije, oscilacije platforme.

1. UVOD

Cilj rada je određivanje momenta inercije platform borbenog sredstva. Platforma rotira oko vertikalne ose, a pokreće je elektromotor preko reduktora. Poznavanje momenta inercije platforme je neophodno za konstruisanje servosistema za pokretanje platforme i sintezu optimalnog regulatora. Određivanje momenta inercije platforme na drugi način je nepogodno zbog geometrijske složenosti platforme. U ovom radu određivanje momenta inercije obrtne platforme vrši se na osnovu merenja na samom objektu. Moment inercije je određen merenjima slobodnih oscilacija platforme vezane za oprugu poznate krutosti.

2. ODREĐIVANJE MOMENTA INERCIJE PLATFORME MERENJEM OSCILACIJA PLATFORME

Na sl.1 prikazana je instalacija za merenje slobodnih oscilacija platform borbenog sredstva. Prilikom merenja oscilacija motor se odvaja od reduktora. Na rastojanju l od ose rotacije za platform je vezana opruga poznate krutosti. Drugi kraj opruge je pričvršćen za krut oslonac (sl. 1). Za merenje ugla obrtanja platforme φ koristi se razlagač sa demodulatorom. Da bi se eliminisao šum demodulacije koristi se filter. Na ovaj način dobija se jednosmerni naponski signal U_r proporcionalan sinusu ugla obrtanja:

$U_r = k_r \sin \varphi$, gde je $k_r = 10,0285 \text{ V/rad}$.

Ovaj signal se preko A/D konvertora pretvara u digitalni oblik i memoriše u računar [2]. Nakon obrade primljenih podataka, sa računara se crtaju i štampaju dijagrami promene ugla φ u toku vremena t .

Slika.1 Alat za eksperimentalno određivanje momenta inercije platforme

Slika. 2. Instalacija za merenje oscilacija platforme

- platforma je izvedena iz ravnotežnog položaja ($\varphi = 0$, nenapregnuta opruga) za malu vrednost ugla i puštena da slobodno osciluje;
 - korišćenjem navedene opreme, prema sl. 1, snimana je promena ugla φ i dobijen je dijagram oscilovanja platforme, sl. 3.
- Isti postupak se ponavlja više puta.

3. MATEMATIČKI MODEL OSCILOVANJA PLATFORME

Kada se platforma borbenog sredstva, vezana oprugom za krut oslonac, izvede iz ravnotežnog položaja i pusti da slobodno osciluje [2], opruga stvara moment sile u opruzi u odnosu na z-osu (sl. 1 i sl.2):

$$M_c = c l^2 \sin \varphi \approx c l \varphi, \quad |\varphi| \leq 0,09 \text{ rad},$$

gde su:

- c – koeficijent krutosti opruge,
- l – rastojanje od ose rotacije do veze platforme sa oprugom,
- φ – ugao obrtanja platforme.

Prilikom kretanja platforme pokreće se i ležaj platforme što uzrokuje pojavu momenta trenja.

$$M_{v1} = v_1 \frac{d\varphi}{dt}$$

gde je v_1 koeficijent trenja.

Slika. 3 Dijagram prigušenih oscilacija

Iz uslova ravnoteže momenata za osu 0z:

$J_{pu} \frac{d^2 \varphi}{dt^2} + M_{v1} + M_c = 0$, sledi diferencijalna jednačina oscilovanja platforme

$$\frac{d^2 \varphi}{dt^2} + 2\delta \frac{d\varphi}{dt} + \omega^2 \varphi = 0 \quad (1)$$

$$\delta = \frac{v_1}{2J_p}, \omega^2 = \frac{cl^2}{J_p} \quad (2)$$

gde su:

- J_{pu} – ukupni moment inercije obrtnih delova sveden na osu rotacije platforme,
- M_{v1} – moment trenja,
- ω^2 – sopstvena učestanost,
- δ – koeficijent prigušenja.

Rešenje diferencijalne jednačine (1) za početne uslove $\varphi(0) = \varphi_0$ i $\frac{d\varphi(0)}{dt} = 0$, uz pretpostavku da je $\omega > \delta$, što odgovara obavljenom eksperimentu, je:

$$\varphi(t) = \varphi_0 e^{-\delta t} \left(\cos \omega_p t + \frac{\delta}{\omega_p} \sin \omega_p t \right)$$

(3)

gde je: $\omega_p = (\omega^2 - \delta^2)^{1/2}$.

Iz (3) se vidi da je period oscilovanja:

$$T_p = \frac{2\pi}{\omega_p} = \frac{2\pi}{(\omega^2 - \delta^2)^{1/2}}$$

(4)

i logaritamski dekrement (logaritam odnosa dve uzastopne amplitude koje se razlikuju za $T_p/2$, sl. 2):

$$\ln \left| \frac{\varphi[iT_p/2]}{\varphi[(i+1)T_p/2]} \right| = \left| \ln \frac{\varphi_i}{\varphi_{i+1}} \right| = \frac{\delta T_p}{2}, \quad i = 0, 1, 2 \dots$$

odnosno:

$$\delta = \frac{2}{T_p} \ln \left| \frac{\varphi_i}{\varphi_{i+1}} \right|$$

(5)

Iz (2) i (4) sledi da je moment inercije platforme:

$$J_{pu} = \frac{cl^2 T_p^2}{4\pi^2 + \delta^2 T_p^2}$$

(6)

4. IZRAČUNAVANJE MOMENTA INERCIJE PLATFORME

Korišćenjem dijagrama oscilovanja platforme, moment inercije platforme J_p izračunava se na sledeći način [2]:

- sa dijagrama oscilacija (sl. 4) očitava se period oscilovanja T_p , kao najkraći interval vremena između dva nulta položaja platforme ($\varphi = 0$ rad) u kojima je ugaona brzina ($d\varphi/dt$) istog znaka;
- sa dijagrama oscilacija (sl. 4) očitavaju se dve amplitude (φ_i, φ_{i+1}) pomeene za pola periode ($T_p/2$);
- korišćenjem jednačina (5) i (6) izračunava se moment inercije platforme J_{pu} .

Slika.4 Dijagram oscilovanja platforme

Poznati su: $c=12000 \text{ N/m}$, $l=1,478 \text{ m}$. Izmerene su vrednosti T_p , φ_i , φ_{i+1} sa dijagrama oscilovanja (sl.3).

Izračunate vrednosti momenta inercije platforme J_p upisane su u tabeli 1.

Tabela 1

Red. br. merenja	T_p	φ	φ_{i+1}	$\delta = \frac{2}{T_p} l$	$J_p = \frac{cl^2 T_p^2}{4\pi + \delta^2 T_p^2} \text{ kgm}^2$
1	1,476	0,48	0,33	0,507	1426
2	1,458	0,54	0,39	0,446	1397
3	1,458	0,46	0,30	0,586	1386
4	1,422	0,36	0,21	0,758	1305
5	1,392	0,33	0,19	0,793	1248
6	1,422	0,33	0,18	0,852	1295
7	1,452	0,41	0,26	0,627	1371

Srednja vrednost momenta inercije: $\bar{J}_p = 1346 \text{ kgm}^2$

Maksimalna apsolutna greška:
 $\Delta I_p = |1426 - 1346| = 80 \text{ kgm}^2$

Maksimalna relativna greška: $\frac{80}{1346} \cdot 100 = 5\%$

Na ovaj način dobijen je ukupni moment inercije obrtnih delova za osu O_z , a on je jednak zbiru momenata inercije platforme (J_p) i momenta inercije obrtnih delova reduktora svedenog na osu O_z (J_{pr}).

$$J_{pu} = J_p + J_{pr}$$

Moment inercije reduktora [3], čija je kinematska šema data na sl. 5, sveden na osu O_z određen je na sledeći način:

- pretpostavljeno je da su zupčanici diskovi čiji je prečnik jednak prečniku podeonog kruga a debljina jednaka širini zupčanika. U tom slučaju važi da je

$$J_{ij} = \frac{\pi Q b_j}{32} D_j^4, \quad j = 1, 2, \dots, 7,$$

gde su:

- Q – gustina čelika,
- b_j – širina zupčanika,
- D_j – prečnik podeonog kruga zupčanika j .

Slika. 5 Šema reduktora

Moment inercije reduktora svedenog na osu rotacije O_z je:

$$J_{rp} = \frac{J_{r1}}{i_{12}^2 i_{34}^2 i_{56}^2 i_{78}^2} + \frac{J_{r2}}{i_{34}^2 i_{56}^2 i_{78}^2} + \frac{J_{r3}}{i_{34}^2 i_{56}^2 i_{78}^2} + \frac{J_{r4}}{i_{34}^2 i_{56}^2} + \frac{J_{r5}}{i_{34}^2 i_{56}^2} + \frac{J_{r6}}{i_{78}^2} + \frac{J_{r7}}{i_{78}^2}$$

gde je $i_{j,j+1}$ – prenosni odnos od zupčanika j do $j+1$.

Računski je na osnovu podataka iz tabele 2, $Q=78000 \text{ kg/m}^3$, dobijeno $J_p=0,0005315 \text{ kgm}^3$ (tabela 2), što je za sedam redova veličine manje u odnosu na očekivane vrednosti J_{pu} , pa je praktično $J_p=J_{pu}$.

Tabela 2

Zupčanik broj, j	Broj zubaca, z_j	Podeoni prečnik, $D_j(\text{m})$	Širina zupčanika, $b_j (\text{m})$	Moment inercije, $J_{rj} (\text{kgm}^2)$	Prenosni odnos, $i_{j,j+1} = \frac{z_{j+1}}{z_j}$
1	26	0,065	0,036	0,00050	2,23077
2	58	0,145	0,026	0,00896	
3	18	0,054	0,020	0,00033	3,55556
4	64	0,192	0,020	0,02075	
5	18	0,054	0,024	0,00026	3,55556
6	64	0,192	0,024	0,02504	
7	18	0,090	0,060	0,00303	7,5
8	135	0,675	0,060	nepokretan	

ZAKLJUČAK

Metodom merenja oscilacija platforme borbenog sredstva određen je moment inercije platforme. Ova metoda je pogodna za skromnije opremljene laboratorije. Zbog malog statističkog rastruranja rezultata dovoljan je relativno mali broj ponavljanja ispitivanja. S druge strane, priprema eksperimenta dugo traje i povlači određene materijalne izdatke. Na kraju se može zaključiti da je navedena metoda za određivanje momenta inercije univerzalna i da se može efikasno koristiti za određivanje momenta inercije rotirajućih objekata.

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Defining The Moment Of Inertia Of The Platform By Measuring Platform Oscillations

Stojko Biocanin

This paper defines the moment of inertia of the rotating platform of the warfare artillery based on the measuring on real objects. This method is based on the measuring of free oscillations of the platform attached to the spring of known stiffness.

Keywords: Rotating platform, moment of inertia, platform oscillation

0. INTRODUCTION

The aim of this paper is defining the moment of inertia of the platform of the warfare artillery. The platform rotates around vertical axis and is moved by electro engine via reducers. Knowing the moment of inertia is essential for constructing the power system for moving the platform and the synthesis of optimal regulators. Defining the moment of inertia in some other way is unsuitable due to the geometrical complexity of the platform. In this paper, defining the moment of inertia of the rotating platform is done by measuring on the object itself. The moment of inertia is defined by measuring free oscillations of the platform attached to the spring of known stiffness.

1. DEFINING THE MOMENT OF INERTIA BY MEASURING THE OSCILLATIONS OF THE PLATFORM

In the picture 1. there is an installation for measuring of the free oscillations of the warfare artillery platform. While measuring the oscillations the engine is being separated from the reducer. At the distance l from the rotating axis, the spring is attached to the platform of the known stiffness. The other end of the spring is attached to the unmovable foundation (picture 1.). For measuring the angle of the rotating of the platform φ the decomposer with demodulator is used. The filter is used for eliminating the noise of demodulating. The one direction voltage signal U_r is formed, which is proportional to the sinus of the rotating angle:

$$U_r = k_r \sin \varphi, \text{ where } k_r = 10,0285 \text{ V/rad.}$$

This signal is introduced into the computer where it is converted to the digital form via A/D converter and is memorized [2]. After the processing of the received data, the computer designs and prints the diagrams of the angle changing over a period of time.

Picture 1. Tool for experimental defining the moment of inertia platform

Picture 2. Installation for measuring the oscillation of the platform

For defining the moment of inertia the experiment is performed in the following way:

- The platform is brought into disbalance position (when the spring is not in motion) at a little angle and let to be oscillating freely.
- Using the mentioned equipment (picture 1.) the angle change was recorded and the diagram of the oscillating of the platform was formed (picture 3.)

The same procedure is being repeated many times.

2. MATHEMATICAL MODEL OF THE OSCILLATION OF THE PLATFORM

When the platform of the warfare artillery is brought into a disbalance position and let to oscillate freely [2], the

spring creates the moment of force in the spring in relation to the z axis.

$$M_c = c l^2 \sin \varphi \cong c l \varphi, \quad |\varphi| \leq 0,09 \text{ rad},$$

where:

- c – spring stiffness coefficient,
- l – the distance between the rotation axis and the point of connection of the spring and the platform,
- φ – the angle of the rotating of the platform.
- During the motion of the platform, the hub bearing is moving which is causing the occurrence of the moment of friction.

$$M_{v1} = v_1 \frac{d\varphi}{dt}$$

where: v_1 - coefficient of friction.

Picture 3. The diagram of undertoned oscillations

From the conditions of balance for the axis Oz:

$J_{pu} \frac{d^2\varphi}{dt^2} + M_{v1} + M_c = 0$, the differential equation of the platform oscillation is following

$$\frac{d^2\varphi}{dt^2} + 2\delta \frac{d\varphi}{dt} + \omega^2 \varphi = 0 \quad (1)$$

$$\delta = \frac{v_1}{2J_p}, \quad \omega^2 = \frac{cl^2}{J_p} \quad (2)$$

where:

- J_{pu} – total moment of inertia of the rotating parts related to the axis of rotation of the platform,
- M_{v1} – moment of friction,
- ω^2 – the frequency of its own,
- δ – coefficient of undertoning

The solution of the differential equation (1) for the starting conditions $\varphi(0) = \varphi_0$ i $\frac{d\varphi(0)}{dt} = 0$, with the assumption that $\omega > \delta$, which corresponds to the performed experiment is:

$$\varphi(t) = \varphi_0 e^{-\delta t} \left(\cos \omega_p t + \frac{\delta}{\omega_p} \sin \omega_p t \right) \quad (3)$$

where: $\omega_p = (\omega^2 - \delta^2)^{1/2}$.

From (3) we can see that the period of oscillating:

$$T_p = \frac{2\pi}{\omega_p} = \frac{2\pi}{(\omega^2 - \delta^2)^{1/2}} \quad (4)$$

and logarithm decrement:

$$\ln \left| \frac{\varphi[iT_p/2]}{\varphi[(i+1)T_p/2]} \right| = \left| \ln \frac{\varphi_i}{\varphi_{i+1}} \right| = \frac{\delta T_p}{2}, \quad i = 0, 1, 2 \dots$$

$$\delta = \frac{2}{T_p} \ln \left| \frac{\varphi_i}{\varphi_{i+1}} \right| \quad (5)$$

From (2) and (4) it follows that the moment of the inertia of the platform:

$$J_{pu} = \frac{cl^2 T_p^2}{4\pi^2 + \delta^2 T_p^2} \quad (6)$$

3. CALCULATING THE MOMENT OF INERTIA OF THE PLATFORM

Using the diagram of the oscillating of the platform, moment of the inertia of the J_p is calculated in the following way [2]:

- From the diagram of oscillations (Picture 4.) we can see the period of oscillating T_p , as the shortest time interval between two platform positions ($\varphi = 0$ rad) in which the angle speed ($d\varphi/dt$) is of the same sign,

From the diagram of oscillations (Picture 4.) we can see

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two amplitudes (ϕ_i, ϕ_{i+1}) which are shifted for half a period $T_p/2$,

- Using the equations (5) and (6) the moment of inertia of the platform J_{pu} is being calculated.

Slika 5 – Dijagram oscilovanja platforme

Picture 4. The diagram of the oscillating of the platform

The known parameters are: $c=12000 \text{ N/m}$,
 $l=1,478 \text{ m}$. The measured values are T_p , φ_i , φ_{i+1} from
 the diagram of oscillating (Picture 3.).

The calculated values of the moment of inertia of
 the platform are written in the table 1.

Table 1: Calculated values

Ordinal numb. of measure	T_p	φ_i	φ_{i+1}	$\delta = \frac{2}{T_p} \ln\left(\frac{\varphi_i}{\varphi_{i+1}}\right)$	$J_p = \frac{cl^2 T_p^2}{4\pi + \delta^2 T_p^2} \text{ kgm}^2$
1	1.476	0.48	0.33	0.507	1426
2	1.458	0.54	0.39	0.446	1397
3	1.458	0.46	0.30	0.586	1386
4	1.422	0.36	0.21	0.758	1305
5	1.392	0.33	0.19	0.793	1248
6	1.422	0.33	0.18	0.852	1295
7	1.452	0.41	0.26	0.627	1371

The average value of the moment of inertia: $\bar{J}_p = 1346 \text{ kgm}^2$

Maximum absolute error: $\Delta I_p = |1426 - 1346| = 80 \text{ kgm}^2$.

Maximum relative error: $\frac{80}{1346} \cdot 100 = 5\%$. In this way

we calculate the total moment of inertia of rotating parts for the axis Oz, and it is equal to the total amount of the moments of inertia of the platform (J_p) and the inertia of the rotating parts of the reducers in relation to the axis Oz (J_{pr}). $J_{pu} = J_p + J_{pr}$

The moment of inertia of the reducer [3], whose kinematic scheme is given in the Picture 5. in relation to the axis Oz is formed in the following way:

- Supposing the gears are discs whose radius is equal to the radius of the radius of the dividing head, and the width is equal to the width of a gear. In that case it is

$$J_{rj} = \frac{\pi Q b j}{32} D_j^4, \quad j = 1, 2, \dots, 7,$$

gde su:

- Q – the iron density,
- b_j – the width of a gear,
- D_j – the radius of the dividing head j .

$$J_{rp} = \frac{J_{r1}}{i_{12}^2 i_{34}^2 i_{56}^2 i_{78}^2} + \frac{J_{r2}}{i_{34}^2 i_{56}^2 i_{78}^2} + \frac{J_{r3}}{i_{34}^2 i_{56}^2 i_{78}^2} + \frac{J_{r4}}{i_{34}^2 i_{56}^2} + \frac{J_{r5}}{i_{34}^2 i_{56}^2} + \frac{J_{r6}}{i_{78}^2} + \frac{J_{r7}}{i_{78}^2}$$

where: $i_{j,j+1}$ – gear ratio from the gear zupčanika j to the $j+1$.

By calculating the parameters from the table 2., $Q=78000 \text{ kg/m}^3$, it is calculated $J_{rp}=0,0005315 \text{ kgm}^3$ (Table 2.), which is by seven units less compared to expected values J_{pu} , so it is practically $J_p=J_{pu}$

Picture 5. The scheme of a reducerThe moment of inertia of a reducer in relation to the axis Oz is:

Table 2: *Parameters*

Gear numb. j	Numb. of gear heads, z_j	Dividing head radius, $D_j(\text{m})$	Width of a gear, $b_j (\text{m})$	Moment of inertia, $J_{rj} (\text{kgm}^2)$	Gear ratio, $i_{j,j+1} = \frac{z_{j+1}}{z_j}$
1	26	0.065	0.036	0.00050	2.23077
2	58	0.145	0.026	0.00896	
3	18	0.054	0.020	0.00033	3.55556
4	64	0.192	0.020	0.02075	
5	18	0.054	0.024	0.00026	3.55556
6	64	0.192	0.024	0.02504	
7	18	0.090	0.060	0.00303	7.5
8	135	0.675	0.060	immobile	

CONCLUSION

Using the method of measuring the oscillations of the platform of the warfare artillery, the moment of inertia of the platform is defined. This method is suitable for modestly equipped laboratories. Due to the small amount of result deviations relatively small amount of test repetitions is sufficient. On the other hand, the preparation for this experiment is time-consuming and initiates certain material expenses. Finally it can be concluded that the mentioned method for defining the moment of inertia is universal and can be efficiently used for defining the moment of inertia of the rotating objects.

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