Prilog geometrijskoj identifikaciji nosača aksijalnih ležajeva velikih prečnika

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Kod većeg broja mašina građevinske i transportne mehanizacije sa okretnom platformom, veza sa nosećom konstrukcijom se ostvaruje radijalno-aksijalnim ležajevima velikih prečnika. Primena tih ležajeva zahteva i određene uslove koje treba da ispune nosači tih ležajeva, koji se, pre svega, odnose na krutost elemenata nosača. U radu su definisani neki od uticajnih parametara nosača tih ležajeva koji imaju dominantne uticaje na sprečavanje vitoperenja (deplanaciju) oslone površine ležajeva. Definisane su vrednosti odnosa visine i širine poprečnog preseka sandučastog tipa (koeficijent k), odnos momenata inercije pri savijanju i uvijanju poprečne i uzdužne grede nosača (koeficijent ε), pri konstantnim vrednostima ugiba nosača f i krutosti podužne grede B_2 . Takođe, definisana je teorijska zavisnost odnosa vrednosti dopunskih sila u osloncu u funkciji promene rastojanja između podužnih greda nosača.

Ključne reči: Radijalno-aksijalni ležaj, geometrija nosača, ugib oslonca, dopunska sila u osloncu.

1. UVOD

Nosači radijalno-aksijalnih ležajeva velikih prečnika, zastupljeni su po pravilu, kod mašina građevinske i transportne mehanizacije sa okretnom platformom. Predstavljaju ravanske noseće strukture koje moraju da ispunjavaju određene uslove kako bi ti ležaji pravilno i dugotrajno funkcionisali. S obzirom da su poprečni preseki radijalno-aksijalnih ležajeva relativno mali (mala im je krutost) to njihovi nosači moraju posedovati krutost koja će sprečiti pojavu deplanacije površine za vezu ležajeva. Vodeći svetski proizvođači radijalno-aksijalnih ležajeva propisuju uslove ugradnje tih ležajeva, pre svega uslove koji se odnose na veličinu ugiba nekog od poprečnog ili uzdužnog segmenta nosača.

Karakteristike izvođenja veza posredstvom radijalno-aksijalnih ležajeva zavise od konstruktivnog izvođenja nosača tih ležajeva. Ta izvođenja se ostvaruju u dve grupe:

- 1) Izvođenje "H" tipa (Sl.1)
- 2) Izvođenje "X" tipa (Sl.2)

Slika 1. Nosač ležaja "H" tipa

Slika 2. Nosač ležaja "X" tipa

S obzirom da dominantna opterećenja na nosače deluju u vertikalnoj ravni, to su noseće konstrukcije nosača izložene naprezanju na savijanje i uvijanje, između ostalog i što krajevi ramova ne leže u jednoj ravni. Jasno je, da sa takvim naprezanjima, poprečni preseki elemenata nosača ležajeva su, po pravilu, sandučastog tipa (Sl.3.) sa geometrijskim karakteristikama, koje su definisane sa sledećim geometrijskim veličinama:

b - širina preseka,

h - visina preseka,

δ - debljina limova.

Slika 3. Geometrijski parametri preseka sandučastog tipa

2. FORMIRANJE MODELA NOSEĆEG RAMA

Analiza uticaja odnosa geometrijskih veličina elemenata nosača radijalno-aksijalnih ležajeva velikih prečnika zavisi, pre svega, od mogućnosti variranja velikog broja parametara. Kako nije moguće teorijskim proračunskim metodama obuhvatiti sve te parametre, to se analiza sprovodi sa izvesnim stepenom redukcije tih parametara. Ovim prilazom (postupkom) se daje mogućnost uspostavljanja i definisanja teorijskih relacija između odgovarajućih geometrijskih veličina elemenata noseće strukture nosača, pri čemu treba zadržati opštost razmatranog problema.

U opštom slučaju (Sl.4) noseću strukturu nosača aksijalnih ležajeva velikih prečnika formiraju podužne i poprečne grede, preko kojih se, dalje, opterećenja prenose na podlogu.

U slučaju kada su svi oslonci u ravni oslanjanja (Sl.4), vertikalni pritisci se mogu odrediti iz uslova ravnoteže. Kako kod realnih konstrukcija, nije moguće ostvariti oslanjanje u sva četiri oslonca, to na veličinu pritisaka u osloncima znatan uticaj ima i odnos geometrijskih veličina elemenata noseće strukture rama.

Slika 4. Noseća struktura nosača ležaja rama

Naime u jednom od oslonaca noseće strukture će se pojaviti zazor u odnosu na ravan π , zbog neravnina staza kao i netačnosti izrade noseće strukture nosača ležajeva. Zbog postojanja zazora f_c , (Sl.4) stvarne vrednosti pritisaka u osloncima noseće strukture nosača ležajeva, se određuju iz izraza:

$$R = R_s \pm C \cdot f_c \quad (1)$$

gde su:

R_s - pritisci u osloncima kada svi oslonci leže u π ravni,
 C - ekvivalentna krutost noseće konstrukcije i oslone podloge.

Znak (-) u izrazu (1) se odnosi na dijagonalne oslonce sa zazorom, a znak (+) na druga dva oslonca.

Jasno je da će u jednom od oslonaca noseće strukture rama doći do povećanja sile pritiska što može uticati na pojavu deplancije oslone površine nosača ležaja.

3. DOPUNSKA SILA U OSLONCU D

Ako se, na primer, oslonac C spusti za f_c (Sl.4) osnovni proračunski model u tom slučaju dobija vid kao na Sl.5. Umesto oslonca D uvedena je dopunska sila X_1 , a umesto odbačenih veza u rasečenim poprečnim gredama uvedene su nepoznate: sile X_2 i moment X_3 . Sistem kanonskih jednačina, za proračunski model na Sl.5 glasi:

$$A \vec{X} = \vec{f} \quad (2)$$

gde su:

A - kvadratna matrica uticajnih koeficijenata δ_{ik} ,

$\vec{X}$ - vektor kolona nepoznatih sila i momenata,

$\vec{f}$ - vektor kolona pomeranja u vertikalnom pravcu.

Slika 5. Proračunski model nosača ležaja

Za rešavanje sistema kanonskih jednačina (2), potrebno je odrediti članove kvadratne matrice, odnosno uticajne koeficijente δ_{ik} .

Primenom nekih od metoda (Maksvel-Morov integral ili Vereščaginov postupak) mogu se odrediti uticajni koeficijenti. S obzirom da izrazi za uticajne koeficijente sadrže savojnu (EI) i torziju (GI_t) krutost, to korišćenjem odnosa $EI = \sqrt{3}kGI_t$ kao i veza (3) i (4) dodatno pojednostavljuju izraze za uticajne koeficijente kao i dalju analizu.

$$I_1 = \varepsilon I_2, \quad I = 2I_2 \quad (3)$$

$$I_{11} = \varepsilon I_{12} = \varepsilon \frac{3I_2}{2k} \quad (4)$$

U izrazima (3) i (4) su:

ε - koeficijent srazmere $I_1(I_{11})$ i $I_2(I_{12})$.

k - koeficijent odnosa visine i širine nosača sandučastog tipa (Sl.3)

Vrednosti uticajnih koeficijenata sa vezama (3 i 4) su date u Tabeli 1.

Tabela 1

$\delta_{11} = \frac{4l^3}{3EI_2} + \frac{2l_2^3}{3EI_2} \left[3 \left(\frac{l_1}{l_2} \right)^2 + 3 \frac{l_1}{l_2} + 1 \right] + \frac{4l_2L_2^2(1+\nu) \cdot k}{3EI_2} + \frac{4L_1l_2^2(1+\nu) \cdot k}{3EI_2}$
$\delta_{22} = \frac{L_1^3}{6\varepsilon EI_2} + \frac{2l_2L_1^2(1+\nu) \cdot k}{3EI_2} + \frac{2l_2^3}{3EI_2};$
$\delta_{33} = \frac{2l_2}{EI_2} + \frac{8L_1(1+\nu) \cdot k}{3\varepsilon EI_2}$
$\delta_{12} = \frac{l_2^2}{3EI_2}(3l_1 + l_2) - \frac{2l_2L_1L_2(1+\nu) \cdot k}{3EI_2};$
$\delta_{13} = -\frac{l_2}{EI_2}(2l_1 + l_2) - \frac{4l_2L_1(1+\nu) \cdot k}{3\varepsilon EI_2}$
$\delta_{23} = -\frac{l_2^2}{EI_2}$

Analizirajući izvedena rešenja noseće konstrukcije nosača ležajeva za veličine L , l_0 i L_1 (Sl.4), mogu se usvojiti sledeće vrednosti:

$$L = 5,0m, \quad l_0 = 5,5m, \quad L_1 = L_2 = 3,5m$$

iz kojih, dalje slede ostale veličine:

$$l_1 = \frac{l_0 - l_2}{2} = \frac{5,5 - 3,5}{2} = 1,0m$$

$$L_2 = \frac{L - L_1}{2} = \frac{5,0 - 3,5}{2} = 0,75m$$

$$l = \sqrt{l_1^2 + L_2^2} = \sqrt{1^2 + 0,75^2} = 1,25m$$

Uticajni koeficijenti δ_{ik} ($i, k = 1, 2, 3$), definisani u Tabeli 1, zavise od koeficijenata k , ε i savojne krutosti $B_2 = EI_2$, odnosno biće određeni izrazima:

$$\left. \begin{aligned} \delta_{11} &= \frac{61,86}{B_2} + k \left[\frac{6,06}{B_2} + \frac{74,32}{\varepsilon B_2} \right] \\ \delta_{22} &= \frac{28,6}{B_2} + \frac{37,15k}{B_2} + \frac{7,14}{\varepsilon B_2} \\ \delta_{33} &= \frac{7}{B_2} + \frac{12,13k}{\varepsilon B_2} \\ \delta_{12} &= \frac{40,83}{B_2} - \frac{10,6k}{\varepsilon B_2} \\ \delta_{13} &= -\frac{19,25}{B_2} - \frac{21,2k}{\varepsilon B_2} \\ \delta_{23} &= -\frac{12,25}{B_2} \end{aligned} \right\} \quad (5)$$

Kako je $f_{1C} = -f_i$ i $f_{2C} = f_{3C} = 0$, to se iz formiranih kanonskih jednačina vrednosti koeficijenata $\varepsilon = 0,5; 1,0; 1,5; 2,0; 3,0$ i $k = 1,0; 1,5; 2,0; 3,0$ dobijaju se novoformirane jednačine iz kojih se određuju nepoznate X_1 .

Prvo se određuju uticajni koeficijenti δ_{ik} ($i, k = 1, 2, 3$) iz zavisnosti (5) za vrednosti koeficijenata $\varepsilon = 0,5$ i $k = 1,0$:

$$\left. \begin{aligned} \delta_{11} &= \frac{216,56}{B_2}; \delta_{22} = \frac{80,00}{B_2}; \\ \delta_{33} &= \frac{31,26}{B_2}; \delta_{12} = \frac{30,23}{B_2}; \\ \delta_{13} &= -\frac{61,65}{B_2}; \delta_{23} = -\frac{12,25}{B_2}; \end{aligned} \right\} \quad (6)$$

Unošenjem vrednosti za δ_{ik} (6) sistem kanonskih jednačina (2) poprima sledeći oblik:

$$\left. \begin{aligned} 216,56X_1' + 30,23X_2' - 61,65X_3' &= f \cdot B_2 \\ 30,23X_1' + 80,00X_2' - 12,25X_3' &= 0 \\ -61,65X_1' - 12,25X_2' + 31,26X_3' &= 0 \end{aligned} \right\} \quad (7)$$

Rešavanjem sistema jednačina (7) po nepoznatoj X_1' dobija se njena vrednost:

$$X_1' = 0,01158f \cdot B_2 \quad (8)$$

Na osnovu sprovedene procedure za $\varepsilon = 0,5$ i $k = 1,0$, određuju se vrednosti za nepoznatu X_1 i pri $\varepsilon = 1,0; 1,5; 2,0; 3,0$, čije su vrednosti prikazane u Tabeli 2.

Tabela 2

ε	$X_1'(L=5)$	k
0,5	$11,58 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$8,15 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$6,2 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$4,04 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
1,0	$17,81 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$13,57 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$10,6 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$7,51 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
1,5	$23,13 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$16,74 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$13,26 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$9,47 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
2,0	$27,30 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$20,01 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$15,97 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$11,86 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
3,0	$33,56 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$22,3 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$18,08 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$13,8 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0

Sprovođenjem proračunske metodologije sa drugim vrednostima za L , l_0 i L_1 , dolazi se do novih kanonskih jednačina. Za iste vrednosti koeficijenata ε i k iz kanonskih jednačina se mogu odrediti nepoznate sa novim vrednostima: $L = 10\text{ m}$; $l_0 = 8,0\text{ m}$; $L_1 = L_2 = 4,8\text{ m}$; $l_1 = 1,6\text{ m}$; $L_2 = 2,6\text{ m}$; $l = 3,05\text{ m}$.

Novе vrednosti za uticajne koeficijente δ_{ik} , saglasno Tabeli 1, će zavisiti takođe od koeficijenata k , ε i savojne krutosti $B_2 = EI_2$, odnosno biće određeni izrazima:

$$\left. \begin{aligned} \delta_{11} &= \frac{190,94}{B_2} + k \left[\frac{57,54}{B_2} + \frac{196,11}{\varepsilon B_2} \right] \\ \delta_{22} &= \frac{73,72}{B_2} + \frac{98,05 \cdot k}{B_2} + \frac{18,43}{\varepsilon B_2}; \\ \delta_{33} &= \frac{9,6}{B_2} + \frac{17,02 \cdot k}{\varepsilon B_2}; \\ \delta_{12} &= \frac{110,59}{B_2} - \frac{53,11 \cdot k}{\varepsilon B_2}; \\ \delta_{13} &= -\frac{38,4}{B_2} - \frac{21,2 \cdot k}{\varepsilon B_2}; \\ \delta_{23} &= -\frac{23,04}{B_2} \end{aligned} \right\} \quad (9)$$

Prikaz vrednosti za dopunsku silu u osloncu D dat je u Tabeli 3.

Tabela 3

ε	$X_1'(L=10)$	k
0,5	$2,97 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$2,09 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$1,62 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$1,09 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
1,0	$4,48 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$3,42 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$2,78 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$2,03 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
1,5	$5,76 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$4,3 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$3,45 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$2,53 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
2,0	$6,49 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$4,90 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$3,97 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$2,93 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
3,0	$8,21 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$5,72 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$4,41 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$3,43 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0

4. KOMPARATIVNI PREGLED ODNOSA SILA U OSLONCU D PRI $L = 5,0\text{ m}$ I $L = 10\text{ m}$

Prikaz zavisnosti $X_1 = X_1(\varepsilon, k)$, pri $f = \text{const}$ i $B_2 = \text{const}$ dat je na grafiku, Sl.6.

Slika 6. Grafički prikaz zavisnosti $X_1 = X_1(\varepsilon, k)$ pri $L = 5,0$ i 10 m

Ako se sa X_1' označe vrednosti sila u osloncu D, koje odgovaraju rastojanju između podužnih glavnih nosača sandučastog tipa $L = 10\text{ m}$, sa X_1 vrednosti sila u osloncu D koje odgovaraju rastojanju $L = 5,5\text{ m}$, mogu se formirati odnosi X_1/X_1' , čije su vrednosti prikazane u Tabelama 4,a i 4,b.

Tabela 4,a

ε	$k = 1,0$	$\frac{X_1}{X_1'}$	$k = 1,5$	$\frac{X_1}{X_1'}$
0,5	$\frac{11,58}{2,97}$	3,9	$\frac{8,15}{2,09}$	3,9
1,0	$\frac{17,81}{4,48}$	4,0	$\frac{13,57}{3,48}$	3,9
1,5	$\frac{23,13}{5,76}$	4,0	$\frac{16,74}{4,30}$	3,9
2,0	$\frac{27,3}{6,49}$	4,2	$\frac{20,01}{4,90}$	4,0
3,0	$\frac{33,56}{8,21}$	4,0	$\frac{22,3}{5,57}$	4,0

Tabela 4,b

ε	$k = 2,0$	$\frac{X_1}{X'_1}$	$k = 3,0$	$\frac{X_1}{X'_1}$
0,5	$\frac{6,2}{1,62}$	3,8	$\frac{4,04}{1,09}$	3,7
1,0	$\frac{10,6}{2,78}$	3,8	$\frac{7,51}{2,03}$	3,7
1,5	$\frac{13,26}{3,45}$	3,8	$\frac{9,47}{2,53}$	3,7
2,0	$\frac{15,97}{3,97}$	4,0	$\frac{11,86}{2,93}$	4,0
3,0	$\frac{18,08}{4,41}$	4,1	$\frac{13,8}{3,43}$	4,0

5. ZAKLJUČAK

Analizirajući rezultate, koji su prikazani u Tabelama 2 i 3, kao i grafički prikaz $X_1 = X_1(\varepsilon, k)$ (Sl.6), dolazi se do sledećih zaključaka:

- Sa rastom koeficijenta k dolazi do smanjenja vrednosti za silu X_1 . Za $k = 2,0 \div 3,0$ vrednosti dopunske sile X_1 u odnosu D opada skoro dva puta u odnosu na $k = 1,0$
- Sa rastom koeficijenta ε (preko 1,0), vrednost sile X_1 se povećava čak za 70% u odnosu na $\varepsilon = 0,5$. S obzirom na unifikaciju geometrije nosača sandučastog tipa treba usvojiti $\varepsilon = 1$
- Analiza odnosa X_1/X'_1 kada je $L = 5m$ i $L = 10m$ (Tabela 4,a i 4,b), pri istim vrednostima koeficijenata ε i k , pokazuje da se taj odnos kreće preko 4, odnosno menja se kvadratom odnosa rastojanja $[L_{10}/L_5]$

$$\frac{X_1}{X'_1} = \left(\frac{L_{10}}{L_{5,0}} \right)^2 = \left(\frac{10}{5} \right)^2 = 4$$

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Contribution to Geometrical Identification of Carriers in Radial Axial Bearings with Big Diameters

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A large number of building and transporting machines with rotating platform have a carrying structure with radial axial bearings of big diameters. Application of such bearings demands certain conditions which have to be fulfilled by the bearing carriers which are, primarily, related to the rigidity of carrier members. The paper defines some influential parameters of bearing carriers which have domineering effects on preventing deflection of the supporting area of bearings. The values of relation between height and width of box-like cross section (coefficient k) are defined, as well as relation between moments of inertia during bending and twisting of transverse and longitudinal beam of the carrier (coefficient ε) in case of constant values of carrier deflection f and rigidity of longitudinal beam B_2 . Also, theoretical dependence between the values of additional forces at support is defined in terms of change of distance between longitudinal beams of the carrier.

Keywords: Radial-axial bearing, carrier geometry, support deflection, additional force at support.

1. INTRODUCTION

The carriers of radial axial bearings with big diameters are widely used in building and transporting machines with rotating platform. They represent carrying plane structures which have to fulfill certain conditions so that the bearings have proper and long operation. Since the cross sections of radial axial bearings are relatively small (their rigidity is small), their carriers have to possess rigidity which is able to prevent deflection of the area connected to the bearings. Leading manufacturers of radial axial bearings worldwide set conditions of installing these bearings, primarily the conditions related to the size of deflection of a transverse or longitudinal member of the carrier.

Characteristics of connections by means of radial axial bearings depend on structural design of bearing carriers. These structures are designed in two types:

- 1) "H" type (Fig. 1), and
- 2) "X" type (Fig. 2).

Figure 1. Bearing carrier type "H"

Figure 2. Bearing carrier type "X"

Domineering loads on carriers act in vertical plane so the carrying structures of bearings are exposed to bending stress and torsion because the ends of the frames are not in the same plane. It is obvious that due to such stresses, the cross sections of the carrier members are box-like (Fig. 3) and their geometrical characteristics are defined by the following geometrical quantities:

- b - cross section width,
- h - cross section height,
- δ - thickness of sheet metal.

Figure 3. Geometrical parameters of the box-like cross section

2. FORMING THE MODEL OF CARRYING FRAME

The analysis of relationship between geometrical values of carrier members of radial axial bearings with big diameters depends, above all, on the possibility to vary a large number of parameters. Since theoretical calculating methods cannot include all the parameters, the analysis is done by reducing these parameters to a certain degree. This approach makes it possible to establish and define theoretical relations between appropriate geometrical values of carrying members but generality of the problem has to be maintained.

In a general case (Fig. 4) the carrying structure of axial bearings with big diameters is formed by transverse and longitudinal beams through which the loads are further transferred to the ground.

In case where all supports are in the support plane π (Fig. 4), vertical pressures can be determined from the balance conditions. As support in all four points cannot be achieved in real structures, the relation between geometrical quantities of carrying frame structure has considerable influence on the pressure value in supports.

Figure 4. Carrying structure of the frame bearing carrier

Namely, in one of the supports of carrying structure there is a clearance in terms of the plane π , due to rough paths and inaccurate fabrication of carrying structure of bearing carriers. Due to the clearance f_c (Fig. 4) the real values of pressures in supports of bearing carrier structure are defined by the following expression:

$$R = R_s \pm C \cdot f_c \quad (1)$$

where:

R_s - pressures in supports when all supports lie in the plane π ,

C - equivalent rigidity of carrying structure and supporting ground.

The sign (-) in (1) refers to diagonal supports with clearance while the sign (+) refers to other two supports.

It is obvious that pressure force is increased in one of the supports of frame structure, which may cause deflection of supporting area of bearing carrier.

3. ADDITIONAL FORCE IN SUPPORT D

If, for instance, support C is lowered for f_c (Fig. 4), the basic calculation model has the form as shown in Figure 5. Instead of support D the additional force X_1 is introduced; instead of split connections in split transverse beams the following unknown values are introduced: force X_2 and moment X_3 . The system of canonical equations for calculation model presented in Figure 5 is:

$$A \vec{X} = \vec{f} \quad (2)$$

where:

A - square matrix of influential coefficients δ_{ik} ,

$\vec{X}$ - column vector of unknown forces and moments,

$\vec{f}$ - column vector of displacement in vertical direction.

Figure 5. Calculation model of the bearing carrier

To solve the system of canonical equations (2) it is necessary to define the members of square matrix, i.e. the influential coefficients δ_{ik} .

Some of the methods (Maxwell-Mors integrals [2] or Vereshchagin method) can help to determine influential coefficients. The expressions for influential coefficients contain bending rigidity (EI) and torsional rigidity (GI_t), so the relation $EI = \sqrt{3}kGI_t$ as well as the relation (2.3) and (2.4) simplify the expressions for influential coefficients and further analysis, too.

$$I_1 = \varepsilon I_2, \quad I = 2I_2 \quad (3)$$

$$I_{t1} = \varepsilon I_{t2} = \varepsilon \frac{3I_2}{2k} \quad (4)$$

The expressions (3) and (4) contain the following coefficients:

ε - coefficient of proportion between $I_1(I_{t1})$ and $I_2(I_{t2})$.

k - coefficient of relation between height and width of box-like carrier (Fig.3).

The quantities of influential coefficients δ_{ik} with these relations (3 and 4) are given in Table 1.

Table 1

$\delta_{11} = \frac{4l^3}{3EI_2} + \frac{2l_2^3}{3EI_2} \left[3 \left(\frac{l_1}{l_2} \right)^2 + 3 \frac{l_1}{l_2} + 1 \right] +$ $+ \frac{4l_2 L_2^2 (1+\nu) \cdot k}{3EI_2} + \frac{4L_1 l_2^2 (1+\nu) \cdot k}{3EI_2}$
$\delta_{22} = \frac{L_1^3}{6\varepsilon EI_2} + \frac{2l_2 L_1^2 (1+\nu) \cdot k}{3EI_2} + \frac{2l_2^3}{3EI_2};$
$\delta_{33} = \frac{2l_2}{EI_2} + \frac{8L_1 (1+\nu) \cdot k}{3\varepsilon EI_2}$
$\delta_{12} = \frac{l_2^2}{3EI_2} (3l_1 + l_2) - \frac{2l_2 L_1 L_2 (1+\nu) \cdot k}{3EI_2};$
$\delta_{13} = -\frac{l_2}{EI_2} (2l_1 + l_2) - \frac{4l_2 L_1 (1+\nu) \cdot k}{3\varepsilon EI_2}$
$\delta_{23} = -\frac{l_2^2}{EI_2}$

By analyzing the existing designs of carrying structure of bearing carriers for quantities L , l_0 and L_1 (Fig.4) the following quantities can be adopted for the sake of further analysis:

$$L = 5,0 \text{ m}, l_0 = 5,5 \text{ m}, L_1 = l_2 = 3,5 \text{ m}$$

from which the following values are derived:

$$l_1 = \frac{l_0 - l_2}{2} = \frac{5,5 - 3,5}{2} = 1,0 \text{ m}$$

$$L_2 = \frac{L - L_1}{2} = \frac{5,0 - 3,5}{2} = 0,75 \text{ m}$$

$$l = \sqrt{l_1^2 + L_2^2} = \sqrt{1^2 + 0,75^2} = 1,25 \text{ m}$$

Influential coefficients δ_{ik} ($i, k = 1, 2, 3$), defined in Table 1, depend on coefficients k , ε and bending rigidity $B_2 = EI_2$, i.e. they are defined by the following expressions:

$$\left. \begin{aligned} \delta_{11} &= \frac{61,86}{B_2} + k \left[\frac{6,06}{B_2} + \frac{74,32}{\varepsilon B_2} \right] \\ \delta_{22} &= \frac{28,6}{B_2} + \frac{37,15k}{B_2} + \frac{7,14}{\varepsilon B_2} \\ \delta_{33} &= \frac{7}{B_2} + \frac{12,13k}{\varepsilon B_2} \\ \delta_{12} &= \frac{40,83}{B_2} - \frac{10,6k}{\varepsilon B_2} \\ \delta_{13} &= -\frac{19,25}{B_2} - \frac{21,2k}{\varepsilon B_2} \\ \delta_{23} &= -\frac{12,25}{B_2} \end{aligned} \right\} \quad (5)$$

Since $f_{1C} = -f_i$ i $f_{2C} = f_{3C} = 0$, for coefficient values $\varepsilon = 0,5; 1,0; 1,5; 2,0; 3,0$ and $k = 1,0; 2,0; 3,0$, we get newly-formed equations which define the unknowns X_1 .

Influential coefficients δ_{ik} ($i, k = 1, 2, 3$) are first determined from relation (5) for coefficient values $\varepsilon = 0,5$ and $k = 1,0$:

$$\left. \begin{aligned} \delta_{11} &= \frac{216,56}{B_2}; \delta_{22} = \frac{80,00}{B_2}; \\ \delta_{33} &= \frac{31,26}{B_2}; \delta_{12} = \frac{30,23}{B_2}; \\ \delta_{13} &= -\frac{61,65}{B_2}; \delta_{23} = -\frac{12,25}{B_2}; \end{aligned} \right\} \quad (6)$$

After inserting the quantities for δ_{ik} (6), the system of canonical equations (2) has the following form:

$$\left. \begin{aligned} 216,56X_1' + 30,23X_2' - 61,65X_3' &= f \cdot B_2 \\ 30,23X_1' + 80,00X_2' - 12,25X_3' &= 0 \\ -61,65X_1' - 12,25X_2' + 31,26X_3' &= 0 \end{aligned} \right\} \quad (7)$$

The unknown X_1' has the following value after solving the system of equations (7):

$$X_1' = 0,01158 f \cdot B_2 \quad (8)$$

The procedure conducted for $\varepsilon = 0,5$ and $k = 1,0$, is applied to the values of unknown X_1 for $\varepsilon = 1,0; 1,5; 2,0; 3,0$ and $k = 1,0; 2,0; 3,0$ as shown in Table 2.

Table 2

ε	$X_1' (L = 5)$	k
0,5	$11,58 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$8,15 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$6,2 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$4,04 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
1,0	$17,81 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$13,57 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$10,6 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$7,51 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
1,5	$23,13 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$16,74 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$13,26 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$9,47 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
2,0	$27,30 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$20,01 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$15,97 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$11,86 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
3,0	$33,56 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$22,3 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$18,08 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$13,8 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0

New canonical equations are obtained by the calculating methodology for other values of L , l_0 and L_1 . New values can be identified from canonical equations for the same quantities of ε and k :

$$L = 10\text{ m}; l_0 = 8,0\text{ m}; L_1 = l_2 = 4,8\text{ m}; l_1 = 1,6\text{ m}; \\ L_2 = 2,6\text{ m}; l = 3,05\text{ m}.$$

New values for influential coefficients δ_{ik} , in compliance with Table 1, depend on coefficients k and ε and bending rigidity $B_2 = EI_2$, i.e. they are defined by the following expressions:

$$\left. \begin{aligned} \delta_{11} &= \frac{190,94}{B_2} + k \left[\frac{57,54}{B_2} + \frac{196,11}{\varepsilon B_2} \right] \\ \delta_{22} &= \frac{73,72}{B_2} + \frac{98,05 \cdot k}{B_2} + \frac{18,43}{\varepsilon B_2}; \\ \delta_{33} &= \frac{9,6}{B_2} + \frac{17,02 \cdot k}{\varepsilon B_2}; \\ \delta_{12} &= \frac{110,59}{B_2} - \frac{53,11 \cdot k}{\varepsilon B_2}; \\ \delta_{13} &= -\frac{38,4}{B_2} - \frac{21,2 \cdot k}{\varepsilon B_2}; \\ \delta_{23} &= -\frac{23,04}{B_2} \end{aligned} \right\} \quad (9)$$

The quantities of additional forces in support D are presented in Table 3.

Table 3

ε	$X'_1(L=10)$	k
0,5	$2,97 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$2,09 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$1,62 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$1,09 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
1,0	$4,48 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$3,42 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$2,78 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$2,03 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
1,5	$5,76 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$4,3 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$3,45 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$2,53 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
2,0	$6,49 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$4,90 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$3,97 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$2,93 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0
3,0	$8,21 \cdot f \cdot B_2 \cdot 10^{-3}$	1
	$5,72 \cdot f \cdot B_2 \cdot 10^{-3}$	1,5
	$4,41 \cdot f \cdot B_2 \cdot 10^{-3}$	2,0
	$3,43 \cdot f \cdot B_2 \cdot 10^{-3}$	3,0

4. COMPARATIVE ANALYSIS OF FORCES IN SUPPORT D WHEN $L = 5,0\text{ m}$ AND $L = 10\text{ m}$

Dependence between $X_1 = X_1(\varepsilon, k)$ when $f = \text{const}$ and $B_2 = \text{const}$ is shown in Figure 6.

Figure 6. Graphical presentation of dependence $X_1 = X_1(\varepsilon, k)$ when $L = 5,0$ i 10 m

If X'_1 and X_1 refer to values of forces in support D which are related to the distance between main box-like longitudinal carriers and $L = 5\text{ m}$, respectively, then we can form the relations X_1/X'_1 whose values are shown in Tables 4,a and 4,b.

Table 4,a

ε	$k = 1,0$	$\frac{X_1}{X'_1}$	$k = 1,5$	$\frac{X_1}{X'_1}$
0,5	$\frac{11,58}{2,97}$	3,9	$\frac{8,15}{2,09}$	3,9
1,0	$\frac{17,81}{4,48}$	4,0	$\frac{13,57}{3,48}$	3,9
1,5	$\frac{23,13}{5,76}$	4,0	$\frac{16,74}{4,30}$	3,9
2,0	$\frac{27,3}{6,49}$	4,2	$\frac{20,01}{4,90}$	4,0
3,0	$\frac{33,56}{8,21}$	4,0	$\frac{22,3}{5,57}$	4,0

Table 4,b

ε	$k = 2,0$	$\frac{X_1}{X'_1}$	$k = 3,0$	$\frac{X_1}{X'_1}$
0,5	$\frac{6,2}{1,62}$	3,8	$\frac{4,04}{1,09}$	3,7
1,0	$\frac{10,6}{2,78}$	3,8	$\frac{7,51}{2,03}$	3,7
1,5	$\frac{13,26}{3,45}$	3,8	$\frac{9,47}{2,53}$	3,7
2,0	$\frac{15,97}{3,97}$	4,0	$\frac{11,86}{2,93}$	4,0
3,0	$\frac{18,08}{4,41}$	4,1	$\frac{13,8}{3,43}$	4,0

5. CONCLUSION

Analysis of results presented in Tables 2 and 3, as well as graphical presentation $X_1 = X_1(\varepsilon, k)$ (Fig.6) leads to the following conclusions:

- The bigger the coefficient k , the less the values of force X_1 . For $k = 2,0 \div 3,0$ quantities of additional force X_1 in support D are nearly twice smaller in comparison to $k = 1,0$
- If the coefficient ε is increased (over 1,0), the quantity of force X_1 increased for even 70% in comparison to $\varepsilon = 0,5$. Having in mind the unification of geometry of the box-like carrier, it has to be adopted that $\varepsilon = 1$
- Analysis of relation X_1/X'_1 when $L = 5m$ and $L = 10m$ (Tables 4,a and 4,b) for the same coefficients ε and k , shows that the relation is about 4, i.e. it changes with the square of relation between distances $[L_{10}/L_5]$

$$\frac{X_1}{X'_1} = \left(\frac{L_{10}}{L_{5,0}} \right)^2 = \left(\frac{10}{5} \right)^2 = 4$$

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