Analiza uticaja stanja oslone površine ležajeva velikog prečnika na pouzdanost u radu

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Uzroci pojave deplanacije površina za vezu ležajeva velikog prečnika potiču od spoljašnjih opterećenja i promenljivog su karaktera po intenzitetu i mestu dejstva. Formiranjem modela veza na osnovu kojih bih se izvršila teorijska analiza interakcije okretnih i neokretnih delova mašina transportne i građevinske mehanizacije, uz variranje geometrijskih veličina elemenata noseće konstrukcije, došlo bi se do parametara koji imaju dominantan uticaj na povećanje krutosti oslonih površina za vezu ležaja. U tom slučaju mora biti ispunjen uslov da deplanacija ne prelazi dozvoljene vrednosti. Na ovaj način bi se izdvojile tipične noseće konstrukcije koje povoljno utiču na funkcionalnost veze. Takođe uočili bi se i nedostaci tih konstrukcija koji treba da se eliminišu. Rezultat takvih istraživanja i analiza trebao bi omogućiti ispunjenje zahteva koji se postavljaju pred mašine građevinske i transportne mehanizacije, i to što bolje prilagodljivost mašine stazi po kojoj se ista kreće, kao i da se vitoperenja oslonih površina radijalno – aksijalnog ležaja zadrže u granicama koje omogućavaju normalno funkcionisanje veze.

Ključne reči: oslona površina, radijalno-aksijalni ležaj, vitoperenje, uticajni parametri

1. UVOD

Osnovna karakteristika veze okretnog i neokretnog dela posredstvom radijalno – aksijalnog ležaja, odnosno osnovni preduslov pravilnog funkcionisanja pa i montaže ovakvog tipa ležaja, je svakako zahtev za većom preciznošću i kvalitetom izrade oslone površine. Naravno potreba za uvek ravnom oslonom površinom na jednoj i pravilno funkcionisanje veze na drugoj strani čine apsolutno opravdanom preciznu izradu oslone površine.

Pre samog procesa montaže ležaja velikog prečnika preporučeno je merenje oslone površine odgovarajućim optičkim ili laserskim mernim uređajima (Sl. 1a). Ukoliko izmerene vrednosti prelaze dozvoljene vrednosti koje omogućavaju normalno funkcionisanje, preporučuje se i dodatna mašinska obrada, prenosnim obradnim mašinama (Sl. 1b). Naravno u nekim slučajevima dodatna mašinska obrada oslone površine može dovesti i do poteškoća, pa za izuzetne slučajeve gde se prethodno opisana procedura nemože primeniti, postoji mogućnost dalje konsultacije sa proizvođačem i otklanjanja neregularnosti oslone površine.

Imajući u vidu sve prethodno navedene činjenice, naravno uz druge zahteve za normalno funkcionisanje ležajeva (termički i fino obrađene radne površine po kojima se kotrljaju kuglice ili valjčići, tačna obrada delova za koji se vezuje ležaj, krutost noseće konstrukcije itd.), potrebno je analizirati uticaj stanja oslone površine, a sve u cilju ostvarenja konačnog cilja – povećanja sigurnosti i pouzdanosti u radu veze okretnog i neokretnog dela mašina građevinske i transportne mehanizacije.

Oslona površina ležaja ne sme imati vitoperenost veću od $\delta = r \cdot 10^{-4}$ [1], gde je r – poluprečnik ležaja. Ukoliko nije ispunjen ovaj uslov dolazi u pitanje normalno funkcionisanje veze okretnog i neokretnog dela, odnosno javlja se neravnomerna raspodela aksijalne sile na posredne elemente ležaja.

Sl.1. Merenje i obrada oslone površine ležajeva velikog prečnika [11]

Preporuke proizvođača ležajeva za dozvoljene vrednosti vitoperenja oslone površine (Sl. 2), koje zavisi od vrste radijalno – aksijalnog ležaja i poluprečnika ležaja, prikazane su u Tabeli 1.[1].

Slika 2. Vitoperenje oslone površine ležajeva velikog prečnika [11]

Tabela 1. Dopuštene vrednosti vitoperenja oslone površine ležajeva velikog prečnika

Poluprečnik r (mm)	Vitoperenje oslone površine δ (mm)		
	Dvoredni kuglični ležaj	Jednoredni kuglični ležaj	Valjčani ležaj
250	0.15	0.10	0.07
500	0.20	0.15	0.10
750	0.25	0.19	0.12
1000	0.30	0.22	0.15
1250	0.35	0.25	0.17
2000	0.40	0.30	0.20
3000	0.50	0.40	0.30
4000	0.60	0.50	0.40

Analizirajući ove preporuke kao bitne za normalno funkcionisanje veze okretnog i neokretnog dela, vitoperenje oslone površine u funkciji poluprečnika ležaja može se prikazati i grafičkim putem, tako da se na osnovu uporednih dijagrama dolazi do izvesnih zaključaka vezanih za uticaj stanja oslone površine na normalno funkcionisanje radijalno – aksijalnog ležaja. Uporedni dijagrami za zavisnost vitoperenja oslone površine od poluprečnika ležaja za različite vrste ležajeva i zavisnosti $\delta = r \cdot 10^{-4}$ prikazani su na Sl.3.

Sl. 3. Uporedni dijagrami zavisnosti vitoperenja od poluprečnika za: a) dvoredni kuglični ležaj; b) jednoredni kuglični ležaj; c) valjčani ležaj

Uporednom analizom dijagrama i funkcionalnih zavisnosti vitoperenja oslone površine i poluprečnika ležaja sa funkcijom $\delta = r \cdot 10^{-4}$, za dvoredni kuglični ležaj ($\delta = 1.13862r \cdot 10^{-4} + 16.2282 \cdot 10^{-2}$), jednoredni kuglični ležaj ($\delta = 1.0003r \cdot 10^{-4} + 10.4313 \cdot 10^{-2}$) i valjčani ležaj ($\delta = 0.8363r \cdot 10^{-4} + 5.5454 \cdot 10^{-2}$), nameću se i neki globalni zaključci:

- zavisnost $\delta = r \cdot 10^{-4}$ omogućava veći stepen sigurnosti normalnog funkcionisanja veze, odnosno predstavlja strožije zahteve koje oslona površina ležaja mora zadovoljiti u cilju normalnog funkcionisanja veze;
- zahtevi koji se postavljaju pred oslonom površinom zavise i od vrste ležaja, pa je samom konstrukcijom ležaja moguće obezbediti veće dozvoljeno pomeranje oslone površine;
- za prva dva tipa ležaja vidi se da je nagib prave isti, a različit za treći tip što samo odgovara činjenici da izuzimajući konstrukciju samog ležaja, možemo ostvariti dozvoljeno pomeranje oslone površine i ispunjenjem drugih uslova, odnosno ostvarivanjem potrebne krutosti same noseće konstrukcije ležaja. Drugim rečima valjčani ležaj zahteva najkruću noseću konstrukciju.

Za dalju analizu ove problematike, u okviru ovog rada potrebno je formirati proračunski model i sagledati uticaj pojedinih parametara (stanja staze po kojoj se mašina kreće, noseće konstrukcije i dr.) na vitoperenje oslone površine, da bi se shodno tome smanjio uticaj prethodno navedenih parametara kao i ispunjenje zahteva koji se postavljaju pred mašine građevinske i transportne mehanizacije (što bolja prilagodljivost mašine stazi po kojoj se ista kreće, odnosno kontakt svih točkova sa stazom kao i da se vitoperenja oslonih površina radijalno – aksijalnog ležaja zadrže u granicama koje omogućavaju normalno funkcionisanje veze.

2. FORMIRANJE PRORAČUNSKOG MODELA

U osnovi sama noseća konstrukcija mašine predstavlja statički neodređeni sistem, a kao i što je već ranije ukazano [9], pomeranje oslonaca kod statičkih neodređenih sistema vrši njihovu deformaciju i stvara naprezanje u elementima strukture, tako da će struktura pri pomeranju oslonaca biti dopunski opterećena ili rasterećena. Pored metode sila, statički neodređeni sistemi se mogu rešavati i metodom pomeranja, koja je zasnovana na potpuno drugačijem konceptu u odnosu na metodu sila. U ovom slučaju osnovne nepoznate su pomeranja čvornih tačaka strukture, pri njoj deformaciji usled dejstva spoljnog opterećenja. Radi eliminisanja svih stepena slobode deformisanja u čvornim tačkama strukture (ugaona i linijska pomeranja) uvode se dopunske veze. Na taj način dobija se osnovni sistem, odnosno deljenje (diskretizacija) kontinualne strukture na jednostavnije karakteristične elemente, čije je ponašanje kao samostalnog elementa izloženog savijanju poznato.

U matricnoj formi sistem kanonskih jednačina, za osnovni sistem od nekoliko kinematskih nepoznatih, glasi:

$$[r] \cdot \{Z\} + \{R_F\} = \{0\} \quad (1)$$

gde su:

- r - matrica uticajnih koeficijenata krutosti (matrica reakcija usled jediničnih pomeranja);

$$[r] = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ r_{n1} & r_{n2} & \dots & r_{nn} \end{bmatrix}$$

- Z - vektor nepoznatih pomeranja;

$$\{Z\} = \begin{Bmatrix} Z_1 \\ Z_2 \\ \vdots \\ Z_n \end{Bmatrix}$$

- R_F - vektor reakcija usled zadatog opterećenja.

$$\{R_F\} = \begin{Bmatrix} R_{1F} \\ R_{2F} \\ \vdots \\ R_{nF} \end{Bmatrix}$$

Formiranje osnovnog sistema (Sl. 4.) se uvek vrši na isti način, a što je i prednost metode pomeranja u odnosu na metodu sila.

Sl. 4. Osnovni sistem noseće konstrukcije (H-tip)

Dijagrami momenata u osnovnom sistemu, od zadatog pomeranja oslonca Δ i od jediničnih pomeranja $\bar{Z}_i=1, (i=1,2,\dots,24)$ prikazani su na Sl. 5 i 6. Prema tim dijagramima, pomoću izrezivanja čvorova ili određenih delova rama primenom statičkih jednačina određuju se r_{ij} i $R_{j\Delta}$ ($i,j=1,2,\dots,24$). Kanonske jednačine metode pomeranja za slučaj, kada je zadato opterećenje pomeranje oslonaca, glase u opštem obliku:

$$\sum_{j=1}^{24} r_{ij} \cdot Z_j + R_{j\Delta} = 0 ; i = 1, \dots, 24 \quad (2)$$

gde su $R_{j\Delta}$ ($j=1,\dots,24$) reakcije na mestu pridodatih veza osnovnog sistema usled pomeranja oslonaca.

Sl. 5. Dijagrami momenata u osnovnom sistemu od jediničnih pomeranja $\bar{Z}_i=1, (i=1,2)$

Sl. 6. Dijagrami momenata u osnovnom sistemu od zadatog pomeranja Δ

Uticajni koeficijenti krutosti na glavnoj dijagonali (glavne reakcije) su uvek pozitivne veličine, dok su vandijagonalni članovi simetrični (r_{ij} i r_{ji}) i mogu biti kako pozitivni tako i negativni. Važno je napomenuti da se dijagrami momenata u osnovnom sistemu usled pomeranja dobijaju kada tablične ordinate momenata određene za jedinična pomeranja pomnožimo sa stvarnim vrednostima pomeranja.

Imajući u obzir broj reakcija uvedenih u čvorovima, postupak je prikazan na primeru čvora 1 (kome odgovaraju kinematičke nepoznate Z_1, Z_2 i Z_3). Isecanjem čvora 1 iz strukture (Sl. 7), reaktivnim momentima i silama treba promeniti smerove, dok reakcije imaju pretpostavljeni smer saglasno pretpostavljenim smerovima Z_1, Z_2 i Z_3 . Saglasno tome veličine reakcija date su u Tabeli 2.

Sl. 7. Veličine reakcija (r_{11}, r_{22} i r_{33}) određene isecanjem čvora 1

Posmatrajući izraze za veličine reakcija uvedenih na mestima pokretnih uklještenja (Tabela 2), a saglasno ranije napomenutoj osobenosti prostornih sistema, uočljivo je prisustvo krutosti na savijanje i uvijanje, tačnije odnosa

$$\alpha = \frac{GI_0}{EI}, [3].$$

Radi lakše generalizacije izraza potrebno je uvesti i vezu savojne i torzione krutosti sandučastog poprečnog preseka noseće konstrukcije. Zavisnost savojne i torzione krutosti, kao što je i ranije pomenuto, za izvedeno rešenje noseće konstrukcije moguće je prikazati u funkciji odnosa širine i visine istih [3,9]. Polazeći od izraza:

$$EI = \sqrt{3} \frac{h}{b} GI_0 \quad (3)$$

i odnosa savojne i torzione krutosti, dobijamo

$$\frac{EI}{GI_0} = \sqrt{3} \frac{h}{b} \Rightarrow \frac{1}{\alpha} = \sqrt{3} \frac{h}{b}$$

$$\alpha = \frac{\sqrt{3}b}{3h} = 0.57735 \frac{b}{h}$$

Ako se iskoriste usvojene zavisnosti kao i karakteristike izvedenog rešenja noseće konstrukcije moguće je dobiti veličine reakcija i nepoznata pomeranja u funkciji odnosa širine i visine nosača i veličine pomeranja oslonca Δ kao spoljašnjeg opterećenja (Tabela 2).

Tabela 2. Veličine reakcija u opštem obliku

$R_{i\Delta}$	Opšti oblik $R_{i\Delta}$	Vrednost x [$\Delta \cdot 10^3$]
$R_{1\Delta}$	$\Delta \left(\frac{8.4EI_1}{L_1^3} + \frac{8.4EI_2}{L_2^3} + \frac{6EI_1}{L_1^3} - \frac{3EI_1}{L_1^3} \right)$	7.83940
$R_{2\Delta}$	$\Delta \left(-\frac{4.2EI_1}{L_1^3} - \frac{3EI_1}{L_1^3} \right)$	-407.046937
$R_{3\Delta}$	$\Delta \left(\frac{4.2EI_1}{L_1^3} - \frac{3EI_1}{L_1^3} \right)$	77.312761
$R_{4\Delta}$	$\Delta \left(\frac{6EI_1}{L_1^3} + \frac{6EI_2}{L_2^3} + \frac{6EI_3}{L_3^3} - \frac{8.4EI_2}{L_1^3} - \frac{4.2EI_4}{L_2^3} - \frac{3.6EI_5}{L_3^3} \right)$	0.6451896
$R_{5\Delta}$	$\Delta \left(\frac{2.1EI_1}{L_1^3} - \frac{3EI_1}{L_1^3} \right)$	-47.622093
$R_{6\Delta}$	$\Delta \left(\frac{4.2EI_2}{L_2^3} - \frac{3EI_2}{L_2^3} + \frac{3EI_3}{L_3^3} - \frac{1.8EI_1}{L_1^3} \right)$	140.8088
$R_{7\Delta}$	$\Delta \left(\frac{3.6EI_1}{L_1^3} + \frac{3.6EI_2}{L_2^3} + \frac{6EI_3}{L_3^3} - \frac{4.2EI_4}{L_2^3} - \frac{6EI_5}{L_3^3} \right)$	2.8022979
$R_{8\Delta}$	$\Delta \left(\frac{1.2EI_4}{L_2^3} - \frac{1.8EI_1}{L_1^3} \right)$	-31.7480625
$R_{9\Delta}$	$\Delta \left(\frac{3EI_4}{L_2^3} + \frac{1.8EI_2}{L_2^3} - \frac{1.8EI_1}{L_1^3} \right)$	179.4652665
$R_{10\Delta}$	$\Delta \left(-\frac{3.6EI_2}{L_2^3} \right)$	-3.1991487
$R_{11\Delta}$	0	0
$R_{12\Delta}$	$\Delta \left(\frac{1.8EI_2}{L_2^3} \right)$	115.9691415
$R_{13\Delta}$	$\Delta \left(\frac{5.4EI_2}{L_2^3} + \frac{1.35EI_3}{L_3^3} - \frac{8.4EI_1}{L_1^3} - \frac{4.2EI_2}{L_2^3} \right)$	-5.34261103
$R_{14\Delta}$	$\Delta \left(-\frac{4.2EI_1}{L_1^3} - \frac{1.35EI_1}{L_1^3} \right)$	-336.8113875
$R_{15\Delta}$	$\Delta \left(\frac{2.7EI_1}{L_1^3} - \frac{2.1EI_1}{L_1^3} \right)$	38.6563805
$R_{16\Delta}$	$\Delta \left(\frac{4.2EI_2}{L_2^3} + \frac{4.2EI_3}{L_3^3} + \frac{4.2EI_4}{L_2^3} - \frac{8.4EI_1}{L_1^3} - \frac{4.2EI_2}{L_2^3} - \frac{6EI_5}{L_3^3} \right)$	-1.0663829
$R_{17\Delta}$	$\Delta \left(\frac{2.1EI_4}{L_2^3} - \frac{3EI_4}{L_2^3} \right)$	-47.62209
$R_{18\Delta}$	$\Delta \left(\frac{2.7EI_2}{L_2^3} - \frac{2.1EI_3}{L_3^3} + \frac{2.1EI_4}{L_2^3} - \frac{1.2EI_1}{L_1^3} \right)$	86.278474
$R_{19\Delta}$	$\Delta \left(\frac{2.4EI_1}{L_1^3} + \frac{2.4EI_2}{L_2^3} + \frac{2.4EI_3}{L_3^3} - \frac{3.6EI_4}{L_2^3} - \frac{4.2EI_5}{L_3^3} \right)$	0.14851192
$R_{20\Delta}$	$\Delta \left(\frac{2.1EI_4}{L_2^3} - \frac{1.8EI_1}{L_1^3} \right)$	15.87403125
$R_{21\Delta}$	$\Delta \left(\frac{2.1EI_2}{L_2^3} + \frac{1.2EI_2}{L_2^3} - \frac{1.2EI_3}{L_3^3} \right)$	124.9348548
$R_{22\Delta}$	$\Delta \left(-\frac{2.4EI_2}{L_2^3} \right)$	-2.1327658
$R_{23\Delta}$	0	0
$R_{24\Delta}$	$\Delta \left(\frac{1.2EI_2}{L_2^3} \right)$	77.312761

Naravno, treba imati u vidu da su momenti inercije poprečnog nosača I_2 i momenti inercije poprečnog i podužnog nosača na mestu oslanjanja ležaja I_3 i I_4 , jednaki. Na drugoj strani, koeficijenti b_1 i b_2 , koji figurišu u izrazima za veličine reakcija prikazane u Tabeli 2, predstavljaju odnose širine i visine poprečnih i podužnih nosača noseće konstrukcije ležaja. Radi daljeg sagledavanja uticaja stanja staze po kojoj se mašina kreće na vitoperenje oslone površine kao i verifikacije prethodno dobijenih zaključaka posmatrano je izvedeno rešenje nosećeg rama radijalno – aksijalnog ležaja sa geometrijskim parametrima (Tabela 3) (Sl.8.).

Sl. 8. Izvedeno rešenje nosećeg rama bagera

Tabela 3. Geometrijske veličine izvedenog rešenja nosećeg rama

Dimenzije	"H"	Dimenzije	"H"
L ₁ [mm]	725	h ₁ [mm]	300
L ₂ [mm]	800	b ₁ [mm]	350
l ₁ [mm]	1000	h ₂ [mm]	300
l ₂ [mm]	800	b ₂ [mm]	200
I ₁ [cm ⁴]	20270	h ₃ [mm]	300
I ₂ [cm ⁴]	16126	b ₃ [mm]	200
I ₃ [cm ⁴]	16126	h ₄ [mm]	300
I ₄ [cm ⁴]	16126	b ₄ [mm]	200

3. UTICAJ STANJA STAZE NA VITOPERENJE OSLONE POVRŠINE LEŽAJA

Radi sagledavanja uticaja stanja terena po kome se mašina kreće na vitoperenje oslone površine ležaja tj. pomeranje oslone površine u pravcu normalnom na oslonu površinu, pomeranja karakterističnih tačaka (Sl.9.) neophodna za takvu vrste analize, dobiće se rešavanjem sistema kanonskih jednačina korišćenjem programskog paketa MATLAB, a saglasno blok dijagramu (Sl. 10.).

Sl. 9. Stvarna pomeranja karakterističnih tačaka za dati proračunski model

Sl.10. Blok dijagram programa korišćenog u MATLAB-u

Pri tome, saglasno stvarnoj deformaciji noseće odnosno oslone strukture ležajeva velikih prečnika, pomeranje karakteristične tačke sa brojem 7 (Sl. 9.) inače najmanje po svojoj vrednosti uzeto je za nulto pomeranje,

dok relativna pomeranja ostalih karakterističnih tačaka " u odnosu na nulto omogućavaju određivanje vitoperenja oslone površine ležaja. Rezultati vitoprenja oslone površine ležaja dobijeni su rešavanjem sistema kanonskih jednačina (proračunski model) i metodom konačnih elemenata (MKE) (Tabeli 4.).

Tabela 4. Vitoperenje oslone površine ležaja nosećeg rama u funkciji pomeranja oslonca

Tačka i	MKE- Z _i (mm)			Metoda pomeranja- Z _i (mm)		
	Δ=5 mm	Δ=10 cm	Δ=20 mm	Δ=5 mm	Δ=10 mm	Δ=20 mm
5	1.7540	3.5079	7.0147	1.91979	3.83958	7.679168
6	2.8392	5.6781	11.356	2.60506	5.21012	10.42024
7	0.9851	1.9701	3.9397	0.93544	1.87088	3.741768
8	1.5026	3.0052	6.0094	1.39622	2.7922	5.584905

Za Z ₁₉ =0	β -MKE (°)			β-Metoda pomeranja (°)		
	Δ=5 mm	Δ=10 mm	Δ=20 cm	Δ=5 mm	Δ=10 mm	Δ=20 mm
5	0.05506	0.110136	0.220229	0.070496	0.1409931	0.2819932
6	0.09389	0.187782	0.375576	0.0845541	0.1691079	0.3382125
8	0.03706	0.074133	0.1482310	0.0330009	0.0659846	0.1320047

Relativna pomeranja oslone površine u pravcu normalnom na oslonu površinu u ostalim karakterističnim tačkama, moguće je predstaviti i preko nagiba prave koja prolazi kroz nultu i odgovarajuću karakterističnu tačku tj. ugla zakretanja β (Sl. 11).

Sl. 11. Ugao zakretanja karakterističnih tačaka oslone površine ležaja

Analizirajući rezultate date u Tabelama 4, uočljivo je da su najveće vrednosti ugla zakretanja vezane za karakterističnu tačku 6, što je i u potpunoj saglasnosti sa stvarnim stanjem deformacija oslone površine.

Pomeranjem oslonca proračunskog modela za Δ, što je ranije ukazano, dolazi do deformacije odnosno naprezanja u elementima noseće strukture ležaja velikih prečnika. Oslona površina radijalno – aksijalnog ležaja (Sl. 11.) iz položaja 5-6-7-8 prelazi u položaj 5'-6'-7'-8' saglasno deformaciji strukture. Uzimanjem najmanjeg po vrednosti pomeranja Z₇ za nulto pomeranje, potrebno je potvrditi da pomeranje karakteristične tačke 6 u odnosu na nulto zajedno sa pomeranjem ostale dve tačke (5 i 8) ne pripada istoj ravni. Dakle, potrebno je napisati jednačinu ravni koja sadrži tri date nekolinearne tačke M₁ (x₁,y₁,z₁), M₂ (x₂,y₂,z₂), M₃ (x₃,y₃,z₃). Neka je tačka 6, odnosno M (x,y,z) proizvoljna tačka.

Ona će pripadati traženoj ravni ako i samo ako su vektori $\overrightarrow{M_1M_2}$, $\overrightarrow{M_1M_3}$ komplanarni, što se može zapisati u obliku

$$\left(\overrightarrow{M_1M_2} \times \overrightarrow{M_1M_3} \right) \cdot \overrightarrow{M_1M_3} = 0, \text{ ili}$$

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \quad (5)$$

Ako su koordinate tačaka $M_1(0,0,0)$, $M_2(800,0,-0.98435)$, $M_3(0,800,-0.46078)$, tražena jednačina ravni ima oblik:

$$\begin{vmatrix} x & y & z \\ 800 & 0 & -0.98435 \\ 0 & 800 & -0.46078 \end{vmatrix} = 0$$

$$787.48x + 368.624y + 640000z = 0 \quad (6)$$

Zamenom koordinata tačke $M_4(800,800,-1.66962)$ u jednačinu ravni vrši se provera prethodno navedne konstatacije.

$$787.48x + 368.624y + 640000z = 0$$

$$787.48 \cdot 800 + 368.624 \cdot 800 - 640000 \cdot 1.66962 = -143673.6 \neq 0 \quad (7)$$

Imajući u vidu stvarne vrednosti grešaka staza tako i grešaka nastalih prilikom izrade mašine, odnosno zadržavajući se na manjim vrednostima pomeranja oslonca, na uporednom dijagramu (Sl. 12.) je prikazana zavisnost ugla zakretanja β , karakteristične tačke 6, od pomeranja oslonca Δ dobijenog proračunskim modelom i modeliranjem iste strukture posredstvom računara. Razmatranjem uporednog dijagrama, uočljiv je porast procentualnog odstupanja ugla zakretanja dobijenog dvema metodama za iste vrednosti pomeranja oslonca.

Sl. 12. Uporedni dijagram zavisnosti ugla zakretanja β i pomeranja oslonca Δ

Odstupanje je prihvatljivo imajući u vidu ograničenja na kojima je analiza sprovedena na jednoj strani, dok na drugoj strani, činjenice dobijene kao rezultat ovakve uporedne analize, poklapaju se sa konstatacijom da se prilikom postupka projektovanja nosećih konstrukcija ležaja velikih prečnika uglavnom polazi od numeričkih metoda (kao što je metoda konačnih elemenata), jer noseće konstrukcije u osnovi predstavljaju višestruko statički neodređene sisteme.

Saglasno prethodnoj analizi, moguće je tako dobijene vrednosti vitoperenja za karakterističnu tačku 6 (najvećeg po vrednosti), uporediti sa vrednostima datim na uporednim dijagramima (Sl. 3).

Sl.13. Određivanje poluprečnika ležaja saglasno oslonom četvorougla izvedenog rešenja

Shodno tome, potrebno je pre toga odrediti poluprečnik ležaja koji odgovara oslonom četvorougla 5-6-7-8 izvedenog rešenja noseće konstrukcije (Sl. 13.). Najpovoljnija varijanta je uzimanje srednje vrednosti poluprečnika upisanog (r_u) i opisanog kruga (r_o) oslonog četvorougla izvedenog rešenja, čija je geometrija poznata. Za $l_2=800$ mm, dobijamo:

$$r_1 = \frac{r_u + r_o}{2} = \frac{\frac{l_2}{2} + \frac{l_2\sqrt{2}}{2}}{2} \approx 485 \text{ mm} \quad (8)$$

Upoređivanjem vrednosti za vitoperenje (Sl. 14.), dobijene proračunskim modelom i vrednosti koje su dobijene iz dijagrama zavisnosti vitoperenja oslone površine od poluprečnika za dva tipa ležaja, za vrednost poluprečnika koja odgovara oslonom četvorougla, uočava se veća vrednost vitoperenja nego što su preporuke proizvođača za date tipove ležaja.

Sl. 14. Dijagrami zavisnosti vitoperenja od poluprečnika ležaja

Drugim rečima, uočljiva je razlika krutosti koju treba obezbediti da bi vitoperenje imalo vrednosti koje omogućavaju normalno funkcionisanje veze okretnog i neokretnog dela, i da ta razlika koju je potrebno obezbediti osim potrebnog odnosa geometrijskih veličina zavisi i od same konstrukcije ležaja odnosno tipa ležaja velikog prečnika.

Razlika krutosti $\Delta C = k_r \cdot C_p$, potrebna za dovođenje vitoperenja oslone površine u granice obuhvaćene preporukama proizvođača ležaja, za konstantan poluprečnik istog za dva tipa ležaja, a u funkciji pomeranja oslonca mašine Δ prikazana je na Sl. 15. Gde je C_p - krutost saglasno preporukama proizvođača,

Sl. 15. Promena koefijenta razlike krutosti k_r za dva tipa ležaja od pomeranja oslonca

4. ZAKLJUČAK

Oslanjajući se na rezultate prethodnih analiza uticaja geometrijskih veličina noseće konstrukcije na vezu okretnog i neokretnog dela građevinskih i transportnih mašina, kao i na teorijske zavisnosti dobijene u ovom radu, vidi se da se pogodnim izborom geometrijskih veličina može uticati na povećanje ili smanjenje krutosti noseće strukture, a samim time i oslone površine radijalno – aksijalnog ležaja.

Naravno, nemoguće je uvek u potpunosti pogodnim izborom odnosa geometrijskih veličina, obezbediti dovoljnu krutost, kao bitan preduslov normalnog funkcionisanja veze okretnog i neokretnog dela i boljeg naleganja ili praćenja staze po kojoj se mašina kreće.

Sastavni elemenat, čija ugradnja omogućuje da se osim izbora odnosa geometrijskih veličina bitno utiče na efekat povećanja krutosti oslone površine radijalno – aksijalnog ležaja, kod većine izvedenih rešenja mašina građevinske i transportne mehanizacije je cilindrični nosač.

Cilindrični nosač koji se ugrađuje kao posredni element između samog radijalno – aksijalnog ležaja i noseće konstrukcije, utiče na ostvarenje potrebne krutosti, odnosno doprinosi i smanjenju vitoperenja oslone površine. Uradnjom posrednog elementa ostvario bi se efekat povećanja krutosti oslone površine ležaja, odnosno smanjenje uticaja stanja staze po kojoj se mašina kreće na vitoperenje oslone površine ležaja. Imajući u vidu preduslove pouzdanog i sigurnog rada veze okretnog i neokretnog dela, kao i pogodan izbor odnosa geometrijskih veličina nosećeg rama, uočljivo je da dovoljnu krutost oslone površine možemo ostvariti ugradnjom posrednog elementa kao i izborom odgovarajućeg tipa ležaja.

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Analysis of the Influence of Large Diameter Bearing Support Structure Condition on Reliability in Operation

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The causes of support structure deformation of the large diameter bearing arise from external loads and they are variable by the magnitude and point where load acts. By forming a model of bonds, on the basis of which would be performed a theoretical analysis of the interaction of revolving and non-revolving parts in construction and transport mechanization machines, with the variation of geometric values of supporting structure elements, could be obtained some parameters that have a dominant influence on the increasing of necessary support surface stiffness for bearing bond. Therefore, the deformation of support surface must be smaller than allowed value. In this way, will be separated a typical carrying structures which favorably effect on functionality of bonds. Also, we noticed the shortcomings of these structures that should be eliminated. The result of such research and analysis should enable the fulfillment of the demands which placed before of construction and transport mechanization machines, i.e. the better adaptability of the excavator running machine with the leaning surface, as well as the radial-axial bearing support surface deformation must be kept under the limits that allows the proper functioning of bonds.

Key words: support surface, radial-axial bearing, deformation, influence parameters

1. INTRODUCTION

The basic characteristic of revolving and non-revolving parts bond with radial-axial bearing, i.e. the basic prerequisite for proper functioning and the mounting of this type of bearing, is certainly a requirement for greater precision and quality of support surface. Of course, the need for always flat a support surface on one side and the proper functioning of the bond on the other side make it an absolutely justified a precise machining of the support surface.

Before mounting of large diameter bearing, there is recommendation for measuring of support surface with appropriate optical or laser measuring devices (Fig. 1a). If the measured values exceed the values that are allowed a proper functioning, there is recommendation for additional machining, with portable machines (Fig.1b). Of course, in some cases, additional machining of support surface can lead to difficulties, and for exceptional cases where the previously described procedure could not be applied, there is the possibility for further consulting with the manufacturer and for removing the irregularity of the support surface.

Bearing in mind all the previously mentioned facts, of course with other requirements for the normal functioning of bearings (thermal and finely machining of surfaces for balls and rollers rolling, precise machining of the parts for which the bearing is attached, the stiffness of the supporting structure, etc.), it is necessary to analyse the influence of the condition of support surface, all in order to achieve the final goal - to increasing the safety and reliability of revolving and non-revolving bonds in construction and transport mechanization machines.

The deformation of bearing support surface must be smaller than $\delta = r \cdot 10^{-4}$ [1], where r is the bearing radius.

If this condition is not met, the proper functioning of the revolving and non-revolving parts bond is disabled, i.e. there is occurred a no uniform distribution of the axial force to the indirect elements of the bearing.

Fig.1. Measuring and machining of large diameter bearing support surface [11]

The recommendations of the bearing manufactures for support surface deformation (Fig.2), which depends from the type of radial - axial bearing and the bearing radius, are shown in Table 1. [11].

Fig. 2. Radial-axial bearing support surface deformation [11]

Table 1. Allowed values of radial-axial bearing support surface deformation

Radius r (mm)	Support structure deformation δ (mm)		
	Two-way ball bearing	One-way ball bearing	Roller bearing
250	0.15	0.10	0.07
500	0.20	0.15	0.10
750	0.25	0.19	0.12
1000	0.30	0.22	0.15
1250	0.35	0.25	0.17
2000	0.40	0.30	0.20
3000	0.50	0.40	0.30
4000	0.60	0.50	0.40

Analyzing these recommendations as essential for the proper functioning of revolving and non-revolving parts bond, the support surface deformation in the function of the bearing radius can be shown graphically, so that on the basis of that comparative diagrams a certain conclusions could be obtained about a influencing of the support surface condition on proper functioning of radial-axial bearing.

Comparative diagrams for different types of bearings are shown in Figure 3.

Fig. 3. Deformation - bearing radius comparative diagrams: a) two-way ball bearing; b) one-way ball bearing; c) roller bearing

With comparative analysis of the diagrams and functional dependencies of the support surface deformation and bearing radius with the function $\delta = r \cdot 10^{-4}$ for two-way ball bearing ($\delta = 1.13862r \cdot 10^{-4} + 16.2282 \cdot 10^{-2}$), single-row ball bearing ($\delta = 1.0003r \cdot 10^{-4} + 10.4313 \cdot 10^{-2}$) and roller bearing ($\delta = 0.8363r \cdot 10^{-4} + 5.5454 \cdot 10^{-2}$), some global conclusions are obtained:

- The function $\delta = r \cdot 10^{-4}$ allows a greater degree of reliability of the proper bond functioning, i.e. there is more stringent requirements that the support surface must satisfy for the proper bond functioning;
- Requirements that are placed before the surface depending of the type of bearing, so with the construction of the bearing itself it is possible to provide greater allowed displacement of the support surface;
- For the first two types of bearings, it can be seen that the slope of lines is the same, and different for the third type, which corresponds only to the fact that, except for the construction of the bearing itself, we can achieve the permissible displacement of support surface with the fulfillment of other conditions, i.e. by achieving the required stiffness of the bearing carrying structure itself. In other words, the roller bearing requires the greatest value of stiffness of supporting structure.

For further analysis of this problem, in this paper, it is necessary to forming a calculation model and analysing the influence of certain parameters (the condition of the path by which the balls moves, the supporting structures, etc.) to the support surface deformation, in order to reduce the influence of the previously mentioned parameters, as well as the fulfillment of requirements imposed before construction and transport mechanization machines (the better adaptability of the running machine with the leaning surface, i.e. the full contact of all the wheels with the track, and in that case the radial - axial bearing deformations could not exceeds the values that enable a proper bond functioning.

2. FORMING A CALCULATION MODELS

Basically, the excavator carrying structure is a statically indeterminate system, and as previously indicated [9], the displacement one of the frame supports in that systems makes their deformations and creates stresses in the structural elements, so that structure during the displacement of supports is additionally loaded or unloaded.

In addition to the force method of analysis, statically indeterminate systems can also be solved by the displacement method, which is based on a completely different concept in relation to the force method. In this case, basic displacements of the joints caused by external load, are unknown.

In order to eliminate all joints deformation degrees of freedom (rotation and translation), additional bonds are introduced. In this way, the basic system is formed, i.e. the dividing (discretization) of the continuous structure into simpler characteristic elements is obtained, which behavior as the independent element exposed to bending is known.

In the matrix form, the system of canonical equations, for the basic system with several kinematic unknowns, is:

$$[r] \cdot \{Z\} + \{R_F\} = \{0\} \quad (1)$$

where are:

- r - matrix of influential coefficients of stiffness (matrix of reactions due to unit displacements);

$$[r] = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{1n} \\ r_{21} & r_{22} & \dots & r_{2n} \\ \cdot & \cdot & \dots & \cdot \\ \cdot & \cdot & \dots & \cdot \\ r_{n1} & r_{n2} & \dots & r_{nn} \end{bmatrix}$$

- Z - vector of unknown displacements;

$$\{Z\} = \begin{Bmatrix} Z_1 \\ Z_2 \\ \cdot \\ \cdot \\ Z_n \end{Bmatrix}$$

- R_F - vector of reaction causes by given loads.

$$\{R_F\} = \begin{Bmatrix} R_{1F} \\ R_{2F} \\ \cdot \\ \cdot \\ R_{nF} \end{Bmatrix}$$

Formation of the basic system (Fig. 4) is always done in the same way, which is the advantage of the displacement method in relation to the force method.

Fig. 4. The basic system of excavator carrying frame (H-type)

The moment diagrams in the basic system from given displacement of the support Δ and unit displacements $\bar{Z}_i=1$ ($i=1,2,\dots,24$), are shown on Fig. 5 and 6). According to these diagrams, with isolating of joints or certain frame parts and solving a equilibrium equations, are determined r_{ij} i $R_{j\Delta}$ ($i,j=1,2,\dots,24$).

The canonical equations of the displacement method in the case when the given load is the displacement one of the supports, in the general form are:

$$\sum_{j=1}^{24} r_{ij} \cdot Z_j + R_{j\Delta} = 0 \quad ; \quad i = 1, \dots, 24 \quad (2)$$

where are $R_{j\Delta}$ ($j=1,\dots,24$) reactions at the place of the attachment bonds in the base system due to the displacement of the supports.

Fig. 5. The moment diagrams of the basic system from $\bar{Z}_i=1$, ($i=1,2$)

Fig. 6. The moment diagrams in the basic system from displacement Δ

The influential stiffness coefficients on the main diagonal (the main reactions) are always positive, while the van diagonal members are symmetric (r_{ij} and r_{ji}) and can be both positive and negative. It is important to note that the moment diagrams due to movement are obtained when the ordinate of the moment specified for the unit displacements is multiplied by the actual displacement values.

Considering the number of reactions introduced in joints, the procedure is shown for example of joint 1 (to which corresponds the kinematic unknown Z_1 , Z_2 and Z_3). Isolating a joint 1 from the structure (Fig.7), reactive moments and forces should change the directions, while the r_{ij} have the assumed direction in accordance with the assumed directions Z_1 , Z_2 and Z_3 . Accordingly, the values of the reaction are given in Table 2.

Fig. 7. The values of reaction (r_{11}, r_{22} i r_{33})

Observing the expressions for the values of reactions introduced at the places of fixed ends (Table 2), and according to the previously mentioned feature of indeterminate systems, the presence of bending and torsional stiffness is noticeable, i.e. their relationship

$$\alpha = \frac{GI_0}{EI}, [3].$$

In order to facilitate the generalization of the expression, it is necessary to introduce the relation between the bending and torsional stiffness of the box-shaped cross-section of the excavator carrying structure. The relation between the bending and torsional stiffness, for the realized excavator carrying structure can be shown in the function of the ratio of width and height of the same [3,9]. Starting from the expression:

$$EI = \sqrt{3} \frac{h}{b} GI_0 \quad (3)$$

we get:

$$\frac{EI}{GI_0} = \sqrt{3} \frac{h}{b} \Rightarrow \frac{1}{\alpha} = \sqrt{3} \frac{h}{b}$$

$$\alpha = \frac{\sqrt{3}b}{3h} = 0.57735 \frac{b}{h}$$

If obtained relationship as well as the realized solution characteristics of the excavator carrying structure are used, it is possible to obtain the reactions and unknown displacements in terms of the carrier width and height ratio and the support displacement Δ causes by external load (Table 2).

Table 2. The reaction values in general form

R_{ij}	General form of R_{ij}	Value x $[\Delta \cdot 10^3]$
R_{11}	$\Delta \left(\frac{8.4EI_1}{L_1^3} + \frac{8.4EI_2}{L_1^3} - \frac{6EI_1}{L_1^2} - \frac{3EI_2}{L_1^2} \right)$	7.83940
R_{21}	$\Delta \left(-\frac{4.2EI_1}{L_1^2} - \frac{3EI_2}{L_1^2} \right)$	-407.046937
R_{31}	$\Delta \left(\frac{4.2EI_2}{L_1^2} - \frac{3EI_1}{L_1^2} \right)$	77.312761
R_{41}	$\Delta \left(\frac{6EI_1}{L_1^3} + \frac{6EI_2}{L_1^3} + \frac{6EI_3}{L_1^3} - \frac{8.4EI_1}{L_1^2} - \frac{4.2EI_2}{L_1^2} - \frac{3.6EI_3}{L_1^2} \right)$	0.6451896
R_{51}	$\Delta \left(\frac{2.1EI_1}{L_1^2} - \frac{3EI_2}{L_1^2} \right)$	-47.622093
R_{61}	$\Delta \left(\frac{4.2EI_2}{L_1^2} - \frac{3EI_1}{L_1^2} - \frac{1.8EI_3}{L_1^2} \right)$	140.8088
R_{71}	$\Delta \left(\frac{3.6EI_1}{L_1^3} + \frac{3.6EI_2}{L_1^3} + \frac{6EI_3}{L_1^3} - \frac{4.2EI_1}{L_1^2} - \frac{6EI_2}{L_1^2} \right)$	2.8022979
R_{81}	$\Delta \left(\frac{1.2EI_1}{L_1^2} - \frac{1.8EI_2}{L_1^2} \right)$	-31.7480625
R_{91}	$\Delta \left(\frac{3EI_1}{L_1^2} + \frac{1.8EI_2}{L_1^2} - \frac{1.8EI_3}{L_1^2} \right)$	179.4652665
R_{101}	$\Delta \left(-\frac{3.6EI_2}{L_1^3} \right)$	-3.1991487
R_{111}	0	0
R_{121}	$\Delta \left(\frac{1.8EI_2}{L_1^2} \right)$	115.9691415
R_{131}	$\Delta \left(\frac{5.4EI_1}{L_1^3} + \frac{1.35EI_2}{L_1^3} - \frac{8.4EI_1}{L_1^2} - \frac{4.2EI_2}{L_1^2} \right)$	-5.34261103
R_{141}	$\Delta \left(-\frac{4.2EI_1}{L_1^2} - \frac{1.35EI_2}{L_1^2} \right)$	-336.8113875
R_{151}	$\Delta \left(\frac{2.7EI_1}{L_1^2} - \frac{2.1EI_2}{L_1^2} \right)$	38.6563805
R_{161}	$\Delta \left(\frac{4.2EI_2}{L_1^3} + \frac{4.2EI_1}{L_1^3} + \frac{4.2EI_3}{L_1^3} - \frac{8.4EI_2}{L_1^2} - \frac{4.2EI_1}{L_1^2} - \frac{6EI_3}{L_1^2} \right)$	-1.0663829
R_{171}	$\Delta \left(\frac{2.1EI_1}{L_1^2} - \frac{3EI_2}{L_1^2} \right)$	-47.62209
R_{181}	$\Delta \left(\frac{2.7EI_2}{L_1^3} - \frac{2.1EI_1}{L_1^3} + \frac{2.1EI_3}{L_1^3} - \frac{1.2EI_1}{L_1^2} \right)$	86.278474
R_{191}	$\Delta \left(\frac{2.4EI_1}{L_1^3} + \frac{2.4EI_2}{L_1^3} + \frac{2.4EI_3}{L_1^3} - \frac{3.6EI_1}{L_1^2} - \frac{4.2EI_2}{L_1^2} \right)$	0.14851192
R_{201}	$\Delta \left(\frac{2.1EI_1}{L_1^2} - \frac{1.8EI_2}{L_1^2} \right)$	15.87403125
R_{211}	$\Delta \left(\frac{2.1EI_1}{L_1^2} + \frac{1.2EI_2}{L_1^2} - \frac{1.2EI_3}{L_1^2} \right)$	124.9348548
R_{221}	$\Delta \left(-\frac{2.4EI_2}{L_1^3} \right)$	-2.1327658
R_{231}	0	0
R_{241}	$\Delta \left(\frac{1.2EI_2}{L_1^2} \right)$	77.312761

Of course, it should be borne in mind that the moments of the inertia of the transverse carrier I_2 and the moments of the inertia of the transverse and longitudinal carrier of bearing support I_3 and I_4 are the same. On the other hand, the coefficients b_1 and b_2 , which are figured in the reaction terms are shown in Table 2, represent the ratio of the width and height of the transverse and longitudinal support structure carrier. In order to further considering of the influence of the condition of leaning surface on support surface deformation, as well as the verification of the previously obtained conclusions, the realized solution of the carrying frame of the radial axial bearing with geometric parameters (Table 3) is observed (Fig. 8).

Table 3. Geometric parameters of realized solution of excavators carrying frame [9]

Dimension	"H"	Dimension	"H"
L ₁ [mm]	725	h ₁ [mm]	300
L ₂ [mm]	800	b ₁ [mm]	350
l ₁ [mm]	1000	h ₂ [mm]	300
l ₂ [mm]	800	b ₂ [mm]	200
I ₁ [cm ⁴]	20270	h ₃ [mm]	300
I ₂ [cm ⁴]	16126	b ₃ [mm]	200
I ₃ [cm ⁴]	16126	h ₄ [mm]	300
I ₄ [cm ⁴]	16126	b ₄ [mm]	200

3. THE INFLUENCE OF ROUGHNESS OF LEANING SURFACE ON BEARING STRUCTURE SUPPORT SURFACE

In order to examine the influence of roughness of terrain by which the machine moves to the bearing support surface deformation, i.e. displacement of support surface in the direction normal to the surface, displacement of the characteristic points (Fig. 9) necessary for this type of analysis will be obtained by solving the canonical equations using the MATLAB program according to the block diagram (Fig. 10).

Fig. 9. The displacement of characteristic point in calculation model

Fig. 10. Block diagram of MATLAB program

In accordance with the actual deformation of the large diameter bearings carrying or support structure, the displacement of the characteristic point with the number 7 (Fig.9) was taken for zero movement, while the relative displacements of the other characteristic points in relation to the zero could be used for determination of support surface deformation.

The results of this deformation can be obtained by solving the canonical equations system (calculation model) and by Finite Element Method (FME) (Table 4.).

Table 4. The bearing support surface deformation in relation from support displacement

Point i	FME			Displacement method- Z _i (mm)		
	Δ=5 mm	Δ=10 cm	Δ=20 mm	Δ=5 mm	Δ=10 mm	Δ=20 mm
5	1.7540	3.5079	7.0147	1.91979	3.83958	7.679168
6	2.8392	5.6781	11.356	2.60506	5.21012	10.42024
7	0.9851	1.9701	3.9397	0.93544	1.87088	3.741768
8	1.5026	3.0052	6.0094	1.39622	2.7922	5.584905

Z ₁₉ =0	β - FME (°)			β - Displacement method (°)		
	Δ=5 mm	Δ=10 mm	Δ=20 cm	Δ=5 mm	Δ=10 mm	Δ=20 mm
5	0.05506	0.110136	0.220229	0.070496	0.1409931	0.2819932
6	0.09389	0.187782	0.375576	0.0845541	0.1691079	0.3382125
8	0.03706	0.074133	0.1482310	0.0330009	0.0659846	0.1320047

The relative displacements of the support surface can also be represented by the slope of the straight line which passes through the zero and the corresponding characteristic point, i.e. rotation angle β (Fig. 11).

Fig. 11. Rotation of characteristic points of bearing support surface

Analyzing the results given in Tables 4, it is notable that the highest values of the angle of rotation are related to the characteristic point 6, which is in complete agreement with the actual state of the surface deformation. By moving the support of the calculation model for Δ, as previously indicated, there is a deformation or stress in the elements of large diameter bearing carrying structure. The radial-axial bearing carrying surface (Fig. 11) from position 5-6-7-8 goes to the position 5'-6'-7'-8' in accordance with the deformation of the structure. By taking the smallest Z₇ value for zero movement, it is necessary to confirm that the displacement of the characteristic point 6 in relation to the zero along with the displacement of the other two points (5 and 8) does not belong to the same plane. Therefore, it is necessary to write a plane equation that contains three given nonlinear points M₁ (x₁, y₁, z₁), M₂ (x₂, y₂, z₂), M₃ (x₃, y₃, z₃).

Let point 6 or M (x, y, z) is an arbitrary point. It will belong to the required plane if and only if the vectors are complement, which can be written in the form

$$\begin{aligned} & (\overrightarrow{M_1M_2} \times \overrightarrow{M_1M_3}) \cdot \overrightarrow{M_1M_6} = 0, \text{ or} \\ & \begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0 \end{aligned} \quad (5)$$

If the coordinates of points are $M_1 (0,0,0)$, $M_2 (800,0,-0.98435)$, $M_3 (0,800,-0.46078)$, the required equation of plane is:

$$\begin{vmatrix} x & y & z \\ 800 & 0 & -0.98435 \\ 0 & 800 & -0.46078 \end{vmatrix} = 0 \quad (6)$$

$$787.48x + 368.624y + 640000z = 0$$

By changing the coordinate of point $M_4 (800,800, -1.66962)$ in the equation, the above-mentioned statement can be checked.

$$\begin{aligned} 787.48x + 368.624y + 640000z &= 0 \\ 787.48 \cdot 800 + 368.624 \cdot 800 - 640000 \cdot 1.66962 &= -143673.6 \neq 0 \end{aligned} \quad (7)$$

Bearing in mind the real values of the terrain roughness and the errors that occurred during the machine construction, i.e. retaining on the smaller displacement values, the comparative diagram (Fig.12) shows the dependence between the rotation angle β of the characteristic point 6 and the displacement of the support Δ obtained by the calculation model and modeling the same structure by FME. By considering a comparative diagram, an increase in the percentage deviation obtained by two methods for the same displacement values is noticeable.

Fig. 12. Comparative diagram of relation between rotation angle β and support displacement Δ

The deviation is acceptable in view of the limitations on which the analysis was carried out on one side, while on the other hand, the facts obtained as a result of this comparative analysis coincide with the conclusion that the process of designing of large diameter bearings carrying structures, it mainly starts from numerical methods (FME), because the supporting structures are basically multi-statically indeterminate systems.

In accordance with the previous analysis, it is possible comparing an obtained value of deformation for the characteristic point 6 (the highest by value) with the values given on the comparative diagrams (Fig. 3).

Fig. 13. The determining of bearing radius

Consequently, it is necessary to determine the bearing radius that corresponds to the supporting quadrilateral 5-6-7-8 of realized excavator carrying frame (Fig. 13). The most favorable variant is the taking of the mean value of the radius of inscribed (r_u) and described circle (r_o) of the supporting quadruple whose geometry is known. For $l_2 = 800$ mm, we get:

$$r_1 = \frac{r_u + r_o}{2} = \frac{\frac{l_2}{2} + \frac{l_2\sqrt{2}}{2}}{2} \approx 485 \text{ mm} \quad (8)$$

By comparing the deformation values (Fig.14), obtained by calculation model and the values derived from the diagrams for two types of bearings, for the radius value corresponding to the supporting quadruple, there is noticeable a higher value of deformation than the manufacturer recommendations for that bearing types. In other words, it is noticeable following facts: there is stiffness difference necessary for ensuring the proper functioning of the revolving and non-revolving parts bond, and that required difference depends from required ratio of geometric values of carrying structure and type of large diameter bearing.

Fig. 14. Relation diagrams of deformation and bearing radius

The difference in stiffness $\Delta C = k_r \cdot C_p$, necessary for bringing the support surface under the limits covered by the bearings manufacturer's recommendations, for a same constant radius for two bearing types, and in the function of the machine support displacement Δ is shown in the Fig. 15. Where C_p is stiffness according to the manufacturer's recommendations.

Fig. 15. Comparative diagram of stiffness difference quotient dependence k_r for two bearing types

4. CONCLUSION

Drawing on the results of previous analyzes as well as the theoretical dependence obtained in this paper, it can be seen that by favoring the selection of geometric values can be achieved an increase or decrease of bearing structure stiffness itself, and consequently the support surface of the radial - axial bearing.

Naturally, it is impossible always to be completely suitable for choosing the ratio of geometric parameters, to provide sufficient stiffness, as an essential prerequisite for

the normal functioning of the bond and better track supervision.

An integral element, whose installation allows to significantly influence the effect of increasing the stiffness of support surface of the radial - axial bearing, in most of the implemented solutions of construction and transport mechanization machines is a cylindrical carrier.

A cylindrical carrier, which is installed as an intermediate element between the radial axial bearing and the supporting structure itself, affects the achievement of the required stiffness, that is, contributes to and reduces the deformation of support surface. By creating an intermediate element, the effect of increasing the stiffness of the bearing support surface will be realized, i.e. the reduction in the influence of the condition of the track by which the machine moves. Bearing in mind the preconditions of reliable and safe functioning of bonds of revolving and no revolving parts, as well as a suitable choice of the ratio of the geometric dimensions of the carrying frame, it is noticeable that sufficient stiffness of the support surface can be achieved by the installation of the intermediate element as well as by the choice of the appropriate bearing type.

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