Dinamička analiza mehanizma kretanja stubne dizalice

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Ponudjen je novi pristup kompleksne dinamičke analize mehaničkih sistema uz upotrebu određenog skupa savremenih kompjuterskih sistema. Ovaj pristup obezbeđuje visok nivo dinamičke analize mehaničkih sistema, uključujući i prezentaciju dobijenih rezultata. To je posebno važno u najranijim fazama projektovanja mehaničkih sistema.

Ključne reči: mehanizam, modeliranje, dinamička analiza, dizalice

1. UVODNE NAPOMENE

U samom radu razmatra se metoda dinamičke analize mehaničkih sistema na primeru kranskog mehanizma kretanja u Mathcad-u. Ova metoda uključuje nekoliko faza. U prvoj fazi stvara se potreban dijagram rotirajućih tela (masa). U drugoj fazi se stvara matematički model - sistem diferencijalnih jednačina, opisujući funkcionisanje mehanizama za kretanje krana. Takav pristup obezbeđuje prezentaciju interakcije susednih tela mehanizma i značajno olakšava stvaranje odgovarajućeg matematičkog modela za dinamičku analizu mehanizma. U trećoj fazi definiše se maseno-inercione karakteristike osnovnih rotirajućih elemenata mehanizma i redukovane veličine (ulazni podaci). U četvrtoj fazi stvara se računarski model za izvršavanje računarskih eksperimenata u Mathcad sistemima. Sledi proces modeliranja.

2. DEFINISANJE DIJAGRAMA ROTIRAJUĆIH MASA

Prva faza. Mehanizam kretanja dizalice predstavlja se u formi međusobno povezanih rotacionih masa. Mehanizam se može predstaviti grafičkim putem pomoću dijagrama (slika 1).

Slika 1: Dijagram rotirajućih masa mehanizma z akretanje dizalice

Na slici, prikazane veličine predstavljaju, redom:

M_{06} – obrtni moment motora;

$I_{06}, I_1, I_2^*, I_{u0}^*$ - momenti inercije tela: rotora motora, puža, pužnog točka i izvršne jedinice;

c_1, c_2^*, c_3^* - koeficijent torzije krutosti tela: motor-puž, puž-pužni točak, pužni točak-izvršni uređaj;

μ_1, μ_2^*, μ_3^* - koeficijent rasipanja tela: motor-puž, puž-pužni točak, pužni točak-izvršni uređaj;

M_c^* - moment otpora kretanju;

i_1, i_2 - prenosni odnos pužnog reduktora i zupčanog para.

Pravougaoni ram pokreće (rotira ili pomera) telo mehaničkog sistema. Strele ulaze ili napuštaju pravougaonik, to su operativni momenti za tela rotacije i sile za pokretna tela. Strelice koje ulaze u pravougaonik, definišu se mlazom i ostavljaju aktivne momente za tela rotacije i odgovarajuće sile za pokretna tela.

Na svakom telu rotacije (kretanja) mlazni momenti (sile) mogu raditi: momenti (sile) inercije i momenti (sile) povezani sa disipativnim i elastičnim svojstvima tela.

Veličine povezane sa disipativnim i elastičnim svojstvima tela zavise od osobina susednih tela sa kojima povezano posmatrano telo.

3. MNEMONIČNO PRAVILO ZA FORMIRANJE MATEMATIČKOG MODELA

Za formiranje matematičkog modela - sistem diferencijalnih jednačina koji opisuju rad mehaničkog sistema, koji se sastoji od konzistentno povezanih tijela rotacije, ponuđeno je sljedeće mnemoničko pravilo:

- rotacija svakog i -tog tela sistema je opisana diferencijalnim jednačinama drugog reda;
- broj diferencijalnih jednačina drugog reda jednak je broju tela mehanizma;
- prvi član diferencijalnih jednačina za svako i -to telo jednako je $I_i \cdot \ddot{\varphi}_i$;
- broj članova u svakoj i -toj diferencijalnoj jednačini sistema jednaka je broju strelica koje ulaze ili izlaze iz odgovarajućeg tela plus prvi član;
- svaki član diferencijalne jednačine ima znak. Ukoliko strelica je usmerena ka telu sistem uzima znak plus u suprotnom minus;
- sledeći članovi diferencijalne jednačine jednaki su:
 - proizvodu koeficijenta krutosti susednog tela i razlike uglova okretanja susednih tela $c_j \cdot (\varphi_{j-1} - \varphi_j)$;
 - proizvodu koeficijenta rasipanja susednih tela i razlike ugona brzina susednih tela $\mu_j \cdot (\dot{\varphi}_{j-1} - \dot{\varphi}_j)$;

takođe je potrebno razmotriti prenosni odnos reduktora.

4. FORMIRANJE MATEMATIČKOG MODELA

Korišćenjem dijagrama rotirajućih masa mehanizma i mnemoničkog pravila moguće je napisati:

- Prva diferencijalna jednačina

$$I_{\partial\theta} \cdot \ddot{\varphi}_{\partial\theta} + c_1 \cdot (\varphi_{\partial\theta} - \varphi_1) + \mu_1 \cdot (\dot{\varphi}_{\partial\theta} - \dot{\varphi}_1) - M_{\partial\theta} = 0;$$

- Druga diferencijalna jednačina

$$I_1 \cdot \ddot{\varphi}_1 + c_2^* \cdot (\varphi_1 - \varphi_2 \cdot i_1) + \mu_2^* \cdot (\dot{\varphi}_1 - \dot{\varphi}_2 \cdot i_1) - c_1 \cdot (\varphi_{\partial\theta} - \varphi_1) - \mu_1 \cdot (\dot{\varphi}_{\partial\theta} - \dot{\varphi}_1) = 0.$$

Na sličan način su predstavljene diferencijalne jednačine i za druga rotirajuća tela. Parametri sa zvezdicom su rezultujući parametri. Konačan sistem diferencijalnih jednačina koji opisuju rad mehanizma kretanja kрана dati su u obliku:

$$I_{\partial\theta} \cdot \ddot{\varphi}_{\partial\theta} + c_1 \cdot (\varphi_{\partial\theta} - \varphi_1) + \mu_1 \cdot (\dot{\varphi}_{\partial\theta} - \dot{\varphi}_1) - M_{\partial\theta} = 0;$$

$$I_1 \cdot \ddot{\varphi}_1 + c_2^* \cdot (\varphi_1 - \varphi_2 \cdot i_1) + \mu_2^* \cdot (\dot{\varphi}_1 - \dot{\varphi}_2 \cdot i_1) - c_1 \cdot (\varphi_{\partial\theta} - \varphi_1) - \mu_1 \cdot (\dot{\varphi}_{\partial\theta} - \dot{\varphi}_1) = 0;$$

$$I_2^* \cdot \ddot{\varphi}_2 + c_3^* \cdot (\varphi_2 \cdot i_1 - \varphi_{uo} \cdot i_1 \cdot i_2) + \mu_3^* \cdot (\dot{\varphi}_2 \cdot i_1 - \dot{\varphi}_{uo} \cdot i_1 \cdot i_2) - c_2^* \cdot (\varphi_1 - \varphi_2 \cdot i_1) - \mu_2^* \cdot (\dot{\varphi}_1 - \dot{\varphi}_2 \cdot i_1) = 0;$$

$$I_{uo}^* \cdot \ddot{\varphi}_{uo} - c_3^* \cdot (\varphi_2 \cdot i_1 - \varphi_{uo} \cdot i_1 \cdot i_2) - \mu_3^* \cdot (\dot{\varphi}_2 \cdot i_1 - \dot{\varphi}_{uo} \cdot i_1 \cdot i_2) + \frac{M_c^*}{i_1 \cdot i_2} = 0.$$

Ovaj sistem diferencijalnih jednačina opisuje rad pogonskih elemenata sa dodatkom momenta rotora motora $M_{\partial\theta}$ i momenta otpora kretanja izvršnog (radnog) tela M_c .

5. FORMIRANJE KOMPJUTERSKOG MODELA

Za rešavanje sistema diferencijalnih jednačina na računaru neophodno ga je predstaviti u obliku sistema diferencijalnih jednačina prvog reda:

$$d\varphi_{\partial\theta} / dt = \dot{\varphi}_{\partial\theta};$$

$$d\varphi_1 / dt = \dot{\varphi}_1;$$

$$d\varphi_2 / dt = \dot{\varphi}_2;$$

$$d\varphi_{uo} / dt = \dot{\varphi}_{uo};$$

$$\frac{d\dot{\varphi}_{\partial\theta}}{dt} = \frac{1}{I_{\partial\theta}} \cdot (M_{\partial\theta} - c_1 \cdot (\varphi_{\partial\theta} - \varphi_1) - \mu_1 \cdot (\dot{\varphi}_{\partial\theta} - \dot{\varphi}_1));$$

$$\frac{d\dot{\varphi}_1}{dt} = \frac{1}{I_1} \cdot (c_2^* \cdot (\varphi_1 - \varphi_2 \cdot i_1) + \mu_2^* \cdot (\dot{\varphi}_1 - \dot{\varphi}_2 \cdot i_1) - c_1 \cdot (\varphi_{\partial\theta} - \varphi_1) - \mu_1 \cdot (\dot{\varphi}_{\partial\theta} - \dot{\varphi}_1));$$

$$\frac{d\dot{\varphi}_2}{dt} = \frac{1}{I_2^*} \cdot (c_3^* \cdot (\varphi_2 \cdot i_1 - \varphi_{uo} \cdot i_1 \cdot i_2) + \mu_3^* \cdot (\dot{\varphi}_2 \cdot i_1 - \dot{\varphi}_{uo} \cdot i_1 \cdot i_2) - c_2^* \cdot (\varphi_1 - \varphi_2 \cdot i_1) - \mu_2^* \cdot (\dot{\varphi}_1 - \dot{\varphi}_2 \cdot i_1));$$

$$\frac{d\dot{\varphi}_{uo}}{dt} = \frac{1}{I_{uo}^*} \cdot (c_3^* \cdot (\varphi_2 \cdot i_1 - \varphi_{uo} \cdot i_1 \cdot i_2) + \mu_3^* \cdot (\dot{\varphi}_2 \cdot i_1 - \dot{\varphi}_{uo} \cdot i_1 \cdot i_2) - \frac{M_c^*}{i_1 \cdot i_2}).$$

$\dot{\varphi}_{\partial\theta}$, $\dot{\varphi}_1$, $\dot{\varphi}_2$, $\dot{\varphi}_{uo}$ - shodno tome, ugaone brzine tela: motor, puž, pužni točak, izvršna jedinica. (Upravljački točak) 1/s;

Ovaj sistem u potpunosti opisuje rad elemenata pogona sa dodatkom momenta rotora motora i momenta otpora kretanja točka M_c .

Da bi se pojednostavila rešenje datog sistema, neophodno je predstaviti ga u obliku sistema diferencijalnih

jednačina koje se sastoje od četiri diferencijalne jednačine drugog reda i od osam diferencijalnih jednačina prvog reda.

6. DEFINISANJE ULAZNIH VELIČINA

Definisanje ulaznih veličina podrazumeva definisanje:

- Momenata inercije svih rotirajućih elemenata mehanizma za kretanje dizalice;
- Faktora krutosti veza susednih elemenata mehanizma;
- Faktora rasipanja veze susednih elemenata mehanizma.
- U krajnjim dinamičkim šemama sve stvarne parametre (momenti inercije, težine, krutosti, spoljna opterećenja ...) zameniti proisteklim.

Smanjenje pokretnih koncentriranih težina tela mehanizma kretanja dizalice svode se na osovinu motora na bazi jednakosti kinetičke energije.

Kinetička energija pri pokretanju mehanizma kretanja dizalice, sastavljenog od četiri rotirajuće i jedne težine kрана sa teretom, svedeno na vratilo motora, može se predstaviti u obliku

$$I_{np} \cdot \frac{\dot{\varphi}_{\partial\theta}^2}{2} = I_{\partial\theta} \cdot \frac{\dot{\varphi}_{\partial\theta}^2}{2} + I_1 \cdot \frac{\dot{\varphi}_1^2}{2 \cdot \eta_1} + I_2 \cdot \frac{\dot{\varphi}_2^2}{2 \cdot \eta_1 \cdot \eta_2} + I_{uo} \cdot \frac{\dot{\varphi}_{uo}^2}{2 \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}} + \frac{(m_{kp} + m_{cp})}{Z_M} \cdot \frac{v_{kp}^2}{2 \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}},$$

gde su:

I_{np} – glavni moment inercije vratila motora, kgm^2 ;

$I_{\partial\theta}$ – moment inercije rotora motora, kgm^2 ;

I_1 – moment inercije ugrađenog tela puža (spojnica i puž), kgm^2 ;

I_2 – moment inercije ugrađenog tela pužnog točka (pužni točak i vratilo zupčanika), kgm^2 ;

I_{uo} – moment inercije ugrađenog tela H_{cn} . Telo (zglob, vratilo i točak), kgm^2 ;

$\eta_1, \eta_2, \eta_{uo}$ – koeficijent korisnosti ugrađenih tela: puž, pužni točak, izvršna jedinica;

m_{kr} – masa dizalice, kg;

m_{cp} – masa tereta, kg;

V_{kp} – brzina kretanja dizlice, m/s;

Z_M – broj mehanizama kretanja dizalice.

Pošto smo podelili, levi i desni delovi poslednje jednačine sa $\dot{\varphi}_{\partial\theta}^2$, dobiće se izraz (kgm^2)

$$I_{np} = I_{\partial\theta} + \frac{I_1}{\eta_1} + \frac{I_2}{i_1^2 \cdot \eta_1 \cdot \eta_2} + \frac{I_{uo}}{i_1^2 \cdot i_2^2 \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}} + \frac{(m_{kp} + m_{cp})}{Z_M} \cdot \frac{(V_{kp} / \dot{\varphi}_{\partial\theta})^2}{\eta_1 \cdot \eta_2 \cdot \eta_{uo}}.$$

gde su: i_1, i_2 – odgovarajući prenosni odnosa pužnog reduktora i slobodnog zupčanika.

Takođe, moguće je predstaviti brzinu kretanja dizalice u obliku

$$V_{kp} = \varphi_{xk} \cdot R_{xk} = \varphi_{\partial\theta} \cdot R_{xk} / (i_1 \cdot i_2),$$

gde je: R_{xk} – poluprečnik točka dizalice, m.

Konačno, glavni moment inercije vratila motora, kgm^2 svih pokretnih masa mehanizma dizalice, dizalice i tereta svedenih na jedan mehanizam može se predstaviti u obliku

$$I_{np} = I_{os} + \frac{I_1}{\eta_1} + \frac{I_2}{i_1^2 \cdot \eta_1 \cdot \eta_2} + \frac{I_{uo}}{i_1^2 \cdot i_2^2 \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}} + \frac{(m_{kp} + m_p)}{Z_M} \cdot \frac{R_{xx}^2}{i_1^2 \cdot i_2^2 \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}}$$

7. PROAČUN MOMENATA INERCIJE OBRTNIH MASA I NJIHOVE

Momenti inercije rotirajućih tela mogu se definisati proračunom, koristeći poznate formule i crteže odgovarajućih detalja (čvorova), ili, stvaranjem odgovarajućeg modela detalja (čvorovi) u CAD-u. Na primer, moment inercije kočionog doboša može se definisati pomoću odgovarajućeg trodimenzionalnog modela konstruisanog u sistemu KOMPAS 3D (Slika. 2).

Slika 2: Rezultat određivanja momenta inercije kočionog doboša

Moment inercije rotora motora je određen iz tehničkih karakteristika motora.

$$I_{pom} = 0,065 \text{ kgm}^2.$$

Moment inercije ugrađenog tela puža I_l , kgm^2 (cilindrično puž) može se odrediti poravnanjem korišćenjem njegovih dimenzija (slika 3).

Slika 3: Dimenzije ugrađenog tela puža

$$I_l = \frac{\pi}{32} \cdot \rho \cdot \sum_{j=1}^{n_l} (I_{lj} \cdot d_{lj}^4) = \frac{3,14}{32} \cdot 7826 \cdot (0,088 \cdot 0,06^4 - 0,08 \cdot 0,035^4 - 0,055 \cdot 0,015^4 + 0,017 \cdot 0,042^4 + 0,028 \cdot 0,05^4 + 0,012 \cdot 0,055^4 + 0,013 \cdot 0,035^4 + 0,0465 \cdot 0,032^4 + 0,0735 \cdot 0,039^4 + 0,0405 \cdot 0,032^4 + 0,028 \cdot 0,03^4) = 1,274 \cdot 10^{-3},$$

gde su:

- ρ - gustina materijala tela, kg/m^3 ;
- n_l - broj koraka puža ;
- l_{lj} - dužina j-tog koraka puža, m;
- d_{lj} - prečnik j – tog koraka puža, m.

Slicno su definisani momenti inercije i njihove rezultujuće vrednosti sa poznatim formulama i za druga tela. Definisani su sa poznatim formulama i njihovih rezultirajućih vrijednosti faktori krutosti i rasipanja za svako telo, kao i parametri mehaničke karakteristike elektromotora.

Izračunavanje parametara mehaničkih karakteristika u sistemu Mathcad dati su u dodatku ovog rada.

8. ZAKLJUČAK

Predložena procedura za izračunavanje dinamičke analize rada mehaničkih pogona pomoću Mathcad softvera omogućava smanjenje vremena rada takvih proračuna kao i istraživanja koja se odnose na promenu parametara povezanih komponenti.

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Dinamička analiza mehanizma za kretanje u Mathcad-u

Ulazni podaci:

- Moment inercije rotora motora, kgm^2
 $I_{en}=0.093$
- redukovani moment inercije puža, kgm^2
 $I_{1re}=0.00172$
- redukovani moment inercije pužnog točka, kgm^2
 $I_{2re}=0.00016$
- redukovani moment inercije izvršne jedinice, kgm^2
 $I_{3re}=0.202$
- koeficijent krutosti veze sa pužom, Nm/radian
 $c_1=7754$
- koeficijent krutosti veze sa pužnim točkom, Nm/radian
 $c_2=4215$
- koeficijent krutosti veze sa izvršnom jedinicom, Nm/radian
 $c_3=7122$
- rasipanje krutosti veze sa pužom, Nms/radian
 $\mu_1=0.731$
- rasipanje krutosti veze sa pužnim točkom, Nms/radian
 $\mu_2=0.336$
- rasipanje krutosti veze sa izvršnom jedinicom, Nms/radian
 $\mu_3=1.373$
- rezultujući moment otpora kretanja, kgm^2
 $M_r=103.161$
- prenosni odnos reduktora
 $i_1=39$
- prenosni odnos zupčanog para
 $i_2=2.52$
- kritično klizanje
 $S_c=0.551$
- max. (kritični) moment motora u statičkom modu, Nm
 $M_c=165.066$
- nominalni moment motora, Nm
 $M_n=52.469$
- ugaona brzina rotora motora, $1/\text{s}$
 $\omega_n=95.295$
- gravitaciono ubrzanje, m/s^2
 $g=9.81$

Potrebni parametri:

- ugao okretanja i ugaona brzina rotora motora sa kočionim dobošem, x_0, x_4
- ugao okretanja i ugaona brzina puža, x_1, x_5
- ugao okretanja i ugaona brzina pužnog točka, x_2, x_6
- ugao okretanja i ugaona brzina izvršne jedinice (pogonski točak), x_3, x_7

Algoritam proračuna:

1. Redukovane veličine

$$\begin{aligned} I_{en} &= I_{en} * g \\ I_{1re} &= I_{1re} * g \\ I_{2re} &= I_{2re} * g \\ I_{3re} &= I_{3re} * g \end{aligned}$$

2. Predstavljanje i rešavanje sistema diferencijalnih jednačina:

- vektor kolona početnih vrednosti potrebnih parametara

$$x := (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^T$$

- vektor kolona desnog dela diferencijalnih jednačina

$$D(t, x) := \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ x_7 \\ \frac{1}{I_{en}} \cdot \left[\frac{2 \cdot M_c}{1 - \frac{x_4}{\omega n}} - \mu_1 \cdot (x_4 - x_5) - c_1 \cdot (x_0 - x_1) \right] \\ \frac{\mu_1 \cdot (x_4 - x_5) + c_1 \cdot (x_0 - x_1) - \mu_2 \cdot (x_5 - x_6 \cdot i_1) - c_2 \cdot (x_1 - x_2 \cdot i_1)}{I_{1re}} \\ \frac{\mu_2 \cdot (x_5 - x_6 \cdot i_1) + c_2 \cdot (x_1 - x_2 \cdot i_1) - \mu_3 \cdot (x_6 \cdot i_1 - x_7 \cdot i_1 \cdot i_2) - c_3 \cdot (x_2 \cdot i_1 - x_3 \cdot i_1 \cdot i_2)}{I_{2re}} \\ \frac{1}{I_{3re}} \cdot \left[\mu_3 \cdot (x_6 \cdot i_1 - x_7 \cdot i_1 \cdot i_2) + c_3 \cdot (x_2 \cdot i_1 - x_3 \cdot i_1 \cdot i_2) - \frac{M_r}{i_1 \cdot i_2} \right] \end{bmatrix}$$

- parametri modeliranja:

Početno vreme modeliranja, s

$$t_0=0$$

Konačno vreme modeliranja, s

$$t_1=2$$

Broj koraka

$$N=100$$

- rešavanje diferencijalnih jednačina, formiranje funkcije

$$Rkadapt(x, t_0, t_1, N, D)$$

- metod Runge-Kutta sa podesivim korakom

$$Z := Rkadapt(x, t_0, t_1, N, D)$$

- rezultati potrebnih parametara:

$$\begin{aligned} t &:= Z^{(0)} \\ x_0 &:= Z^{(1)} \\ x_1 &:= Z^{(2)} \\ x_2 &:= Z^{(3)} \\ x_3 &:= Z^{(4)} \\ x_4 &:= Z^{(5)} \\ x_5 &:= Z^{(6)} \\ x_6 &:= Z^{(7)} \\ x_7 &:= Z^{(8)} \end{aligned}$$

3. Grafička interpretacija potrebnih parametara u funkciji od vremena t t:

- ugaono pomeranje tela: rotora motora (x_0) i pogonskog točka (x_3):

- ugaona brzina tela: rotora motora (x_4) i pogonskog točka (x_7):

4. Poređenje rezultata i simulacija:

- tabelarne vrednosti ugaone brzine rotora motora u funkciji od vremena, radian/s

$x_4^T =$	0	1	2	3	4	5	6	7
	0	0	2.939	5.929	8.967	12.058	15.198	18.39

- greška modeliranja ugaone brzine rotora motora - $\delta\omega_n$, %
 $(x_4)_N = 95.127$

$$\delta\omega_n := \left| \frac{\omega_n - (x_4)_N}{\omega_n} \right| \cdot 100$$

$$\delta\omega_n = 0.176$$

- tabelarne nominalne ugaone brzine pogonskog točka, radian/s

$$\omega_h := \frac{\omega_n}{i1 \cdot i2} = 0.97$$

- tabelarne vrednosti ugaone brzine obrtanja pogonskog točka u zavisnosti od vremena, radian/s

$x_7^T =$	0	1	2	3	4	5	6	7	8
	0	0	0.031	0.062	0.093	0.125	0.157	0.19	0.223

- greška modeliranja za ugaonu brzinu obrtanja pogonskog točka, %

$$(x_7)_N = 0.968$$

$$\delta\omega_h := \left| \frac{\omega_h - (x_7)_N}{\omega_h} \right| \cdot 100$$

$$\delta\omega_h = 0.176$$

5. Dinamički moment i dinamički faktor pri pokretanju motora:

- moment inercije težina mehanizma za kretanje dizalice na vratilu motora

$$I_{re} = I_{en} + I_{lre} + I_{2re} + I_{lre}$$

$$I_{re} = 2.912$$

- ugaono ubrzanje rotora motora

$$a_{en} := \frac{1}{I_{re}} \cdot \left[\frac{2 \cdot Mc}{1 - \frac{x_4}{\omega_n} + \frac{Sc}{1 - \frac{x_4}{\omega_n}}} - \mu l (x_4 - x_5) - c l (x_0 - x_1) \right]$$

- dinamički moment, Nm faktor vratila motora pri čijem kretanjem se rasipa mehanizam kretanja kрана

$$M_{dyn} := a_{en} \cdot I_{re}$$

$$k_{dyn} := \frac{M_{dyn} + Mr}{Mr}$$

- maksimalni dinamički moment, Nm i faktor vratila motora

$$M_{dyn\grave{a}\delta} := \max(M_{dyn})$$

$$M_{dyn\grave{a}\delta} = 157.556$$

$$k_{dynmax} := \max(k_{dyn})$$

$$k_{dynmax} = 2.527$$

- pprovera uslova: dinamički moment na vratilu motora u slučaju rasipanja mora biti manji od kritičnog momenta motora, Nm

$$M_{dyn\grave{a}\delta} = 157.556 < M_c = 165.066$$

Uslov je ispunjen.

Dynamic Analysis of Tower Crane Movement Mechanism

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It is offered the new approach to complex dynamic analysis of mechanical systems with use of a certain set of modern computer systems. This approach provides high level of dynamic analysis of mechanical systems, including and presentation of received results. It is especially important at the earliest design stages of mechanical systems.

Keywords: mechanism, modeling, dynamic analysis, crane

1. INTRODUCTION

Considers method of the dynamic analysis of mechanical systems on an example of modeling movement mechanism crane in system Mathcad. This method includes several stages. On the first stage creates the marked state graph of rotating boldies. On the second stage creates mathematical model - differential equations system, describing functioning movement mechanism of crane. Such approach provides presentation of interaction of adjacent bodies of the mechanism and significantly facilitates creation of corresponding mathematical model for the dynamic analysis of the mechanism. On the third stage defines mass-inertial characteristics of the basic rotating elements of the mechanism and reduced value (input data). On the fourth stage creates the computer model for execution of the computer experiments in Mathcad systems. Next carries out modeling process.

2. CREATION MARKED STATE GRAPH

The first stage. The movement mechanism of the crane represents in the form of consistently connected bodies of rotation. It can be represented in a graphic kind by means of the marked graph (fig. 1).

Figure 1: The marked state graph of rotating bodies movement mechanism of the crane

where:

M_{06} - the rotational moment of the engine;
 $I_{06}, I_1, I_2^*, I_{10}^*$ - moment of inertia of bodies: an engine rotor; a worm-shaft; a worm-wheel and an executive unit;
 c_1, c_2^*, c_3^* - coefficients of torsion rigidity bodies: the engine-worm; a worm-worm a wheel; a worm wheel-executive office;

μ_1, μ_2^*, μ_3^* - coefficients of dissipation bodies: the engine-worm shaft; a worm shaft-worm-wheel; a worm-wheel-executive unit;

M_c^* - moment of resistance to movement;

i_1, i_2 - gear-ratio of a worm reducer and open gear.

The rectangle of the count is driving (rotating or moving) a body of mechanical system. The arrows entering or leaving a rectangle, it is the operating moments for bodies of rotation and force for moving bodies. The arrows entering into a rectangle, define themselves jet, and the leaving active moments for bodies of rotation and corresponding forces for moving bodies.

On each body of rotation (moving) the jet moments (forces) can operate: the moments (forces) of inertia and the moments (forces) connected with диссипативными and elastic properties of a body.

The sizes connected with dissipative and elastic properties of a body, depend on properties of adjacent bodies with which the considered body cooperates.

3. MNEMONIC RULE FOR CREATION OF MATHEMATICAL MODEL

For creation of mathematical model - systems of the differential equations describing work of mechanical system, consisting of consistently connected bodies of rotation, the following mnemonic rule is offered:

- rotation of each i -th body of system is described by the differential equation of the second order;
- number of the differential equations of the second order to equally number of bodies of rotation of the mechanism;
- first member of the differential equation for each i -th body of rotation is equal $I_i \cdot \ddot{\varphi}_i$;
- number of members in each i -th differential equation of system to equally number of the arrows entering or leaving the considered body of rotation, plus the first member;
- each member of the differential equation has the sign. If the arrow is directed to an investigated body of system the member undertakes with a sign plus if from a body with a sign a minus;
- the subsequent members of the differential equation are equal:

to multiply coefficient of rigidity adjacent body on a difference of turning angles of adjacent bodies $c_j \cdot (\varphi_{j-1} - \varphi_j)$;

to multiply coefficient dissipation adjacent body on a difference of angular speeds of adjacent bodies $\mu_j \cdot (\dot{\varphi}_{j-1} - \dot{\varphi}_j)$;

it is necessary to consider gear-ratios also (see below).

4. CREATION OF MATHEMATICAL MODEL

Using the marked state graph of rotating bodies of the mechanism and a mnemonic rule it is possible to write:

- first differential equation

$$I_{\partial\partial} \cdot \ddot{\varphi}_{\partial\partial} + c_1 \cdot (\varphi_{\partial\partial} - \varphi_1) + \mu_1 \cdot (\dot{\varphi}_{\partial\partial} - \dot{\varphi}_1) - M_{\partial\partial} = 0;$$

- second differential equation

$$I_1 \cdot \ddot{\varphi}_1 + c_2^* \cdot (\varphi_1 - \varphi_2 \cdot i_1) + \mu_2^* \cdot (\dot{\varphi}_1 - \dot{\varphi}_2 \cdot i_1) - c_1 \cdot (\varphi_{\partial\partial} - \varphi_1) - \mu_1 \cdot (\dot{\varphi}_{\partial\partial} - \dot{\varphi}_1) = 0.$$

The differential equations and for other bodies of rotation are similarly represented. Parameters with asterisks are the resulted parameters. Final system of the differential equations describing work of the mechanism of movement of the crane will register so:

$$I_{\partial\partial} \cdot \ddot{\varphi}_{\partial\partial} + c_1 \cdot (\varphi_{\partial\partial} - \varphi_1) + \mu_1 \cdot (\dot{\varphi}_{\partial\partial} - \dot{\varphi}_1) - M_{\partial\partial} = 0;$$

$$I_1 \cdot \ddot{\varphi}_1 + c_2^* \cdot (\varphi_1 - \varphi_2 \cdot i_1) + \mu_2^* \cdot (\dot{\varphi}_1 - \dot{\varphi}_2 \cdot i_1) -$$

$$- c_1 \cdot (\varphi_{\partial\partial} - \varphi_1) - \mu_1 \cdot (\dot{\varphi}_{\partial\partial} - \dot{\varphi}_1) = 0;$$

$$I_2^* \cdot \ddot{\varphi}_2 + c_3^* \cdot (\varphi_2 \cdot i_1 - \varphi_{uo} \cdot i_1 \cdot i_2) + \mu_3^* \cdot (\dot{\varphi}_2 \cdot i_1 - \dot{\varphi}_{uo} \cdot i_1 \cdot i_2) -$$

$$- c_2^* \cdot (\varphi_1 - \varphi_2 \cdot i_1) - \mu_2^* \cdot (\dot{\varphi}_1 - \dot{\varphi}_2 \cdot i_1) = 0;$$

$$I_{uo}^* \cdot \ddot{\varphi}_{uo} - c_3^* \cdot (\varphi_2 \cdot i_1 - \varphi_{uo} \cdot i_1 \cdot i_2) - \mu_3^* \cdot (\dot{\varphi}_2 \cdot i_1 - \dot{\varphi}_{uo} \cdot i_1 \cdot i_2) + \frac{M_c^*}{i_1 \cdot i_2} = 0.$$

This system of the differential equations describes work of elements of a drive at the appendix to a rotor of the engine of moment $M_{\partial\partial}$, and to executive (working) body of the moment resistance of movement M_c .

5. CREATION OF COMPUTER MODEL

For the decision of system of the differential equations on the computer it is necessary to present it in the form of system of the differential equations of the first order:

$$d\varphi_{\partial\partial} / dt = \dot{\varphi}_{\partial\partial};$$

$$d\varphi_1 / dt = \dot{\varphi}_1;$$

$$d\varphi_2 / dt = \dot{\varphi}_2;$$

$$d\varphi_{uo} / dt = \dot{\varphi}_{uo};$$

$$\frac{d\dot{\varphi}_{\partial\partial}}{dt} = \frac{1}{I_{\partial\partial}} \cdot (M_{\partial\partial} - c_1 \cdot (\varphi_{\partial\partial} - \varphi_1) - \mu_1 \cdot (\dot{\varphi}_{\partial\partial} - \dot{\varphi}_1));$$

$$\frac{d\dot{\varphi}_1}{dt} = \frac{-1}{I_1} \cdot (c_2^* \cdot (\varphi_1 - \varphi_2 \cdot i_1) + \mu_2^* \cdot (\dot{\varphi}_1 - \dot{\varphi}_2 \cdot i_1) - c_1 \cdot (\varphi_{\partial\partial} - \varphi_1) - \mu_1 \cdot (\dot{\varphi}_{\partial\partial} - \dot{\varphi}_1));$$

$$\frac{d\dot{\varphi}_2}{dt} = \frac{-1}{I_2^*} \cdot (c_3^* \cdot (\varphi_2 \cdot i_1 - \varphi_{uo} \cdot i_1 \cdot i_2) + \mu_3^* \cdot (\dot{\varphi}_2 \cdot i_1 - \dot{\varphi}_{uo} \cdot i_1 \cdot i_2) - c_2^* \cdot (\varphi_1 - \varphi_2 \cdot i_1) - \mu_2^* \cdot (\dot{\varphi}_1 - \dot{\varphi}_2 \cdot i_1));$$

$$\frac{d\dot{\varphi}_{uo}}{dt} = \frac{-1}{I_{uo}^*} \cdot (c_3^* \cdot (\varphi_2 \cdot i_1 - \varphi_{uo} \cdot i_1 \cdot i_2) + \mu_3^* \cdot (\dot{\varphi}_2 \cdot i_1 - \dot{\varphi}_{uo} \cdot i_1 \cdot i_2) - \frac{M_c^*}{i_1 \cdot i_2}).$$

$\dot{\varphi}_{\partial\partial}$, $\dot{\varphi}_1$, $\dot{\varphi}_2$, $\dot{\varphi}_{uo}$ - accordingly angular speeds of bodies: engine, the worm-shaft, the worm-wheel, executive unit. (A running wheel), 1/s;

This system completely describes work of elements of a drive at the appendix to a rotor of the engine of the moment, and to a running wheel of the moment of resistance to movement M_c .

For simplification of the decision of the given system it is expedient to present it in the form of the system of the differential equations consisting not from four differential equations of the second order, and from eight differential equations of the first order.

6. DETERMINATION INPUT DATA

Calculation of initial data includes calculation:

- the moments of inertia of all rotating elements of the mechanism of movement of the crane;
- factors rigidity of connections of adjacent elements of the mechanism;
- factors dissipation connections of adjacent elements of the mechanism.
- In settlement dynamic schemes all real parameters of cars (the moments of inertia, weights, rigidity, external loadings, ...) are replaced with the resulted.

Reduction of the moving concentrated weights of bodies of the mechanism of movement of the crane we will make to a shaft of the engine on the basis of equality kinetic energy.

Kinetic energy at start-up of the mechanism of movement of the crane consisting of four rotating and one forward weights (the crane with cargo), resulted at an engine shaft, can be presented in such kind

$$I_{np} \cdot \frac{\dot{\varphi}_{\partial\partial}^2}{2} = I_{\partial\partial} \cdot \frac{\dot{\varphi}_{\partial\partial}^2}{2} + I_1 \cdot \frac{\dot{\varphi}_1^2}{2 \cdot \eta_1} + I_2 \cdot \frac{\dot{\varphi}_2^2}{2 \cdot \eta_1 \cdot \eta_2} + I_{uo} \cdot \frac{\dot{\varphi}_{uo}^2}{2 \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}} + \frac{(m_{kp} + m_{zp})}{Z_M} \cdot \frac{v_{kp}^2}{2 \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}},$$

where:

I_{np} - the led moment of inertia to an engine shaft, kgm^2 the mechanism of movement of the crane and the crane with the cargo, falling to one mechanism;

$I_{\partial\partial}$ - moment of inertia of a rotor of the engine with полумуфтой, kgm^2 ;

I_1 - moment of inertia of an incorporated body the worm-shaft (half-coupling and a worm-shaft), kgm^2 ;

I_2 - moment of inertia of an incorporated body the worm-wheel (a worm-wheel and a shaft a gear wheel), kgm^2 ;

I_{uo} - moment of inertia of incorporated body II_{cn} . Body (a cogwheel, a shaft and a running wheel), kgm^2 ;

$\eta_1, \eta_2, \eta_{uo}$ - accordingly coefficient of efficiency incorporated bodies: worm-shaft, worm-wheel, executive unit;

m_{kr} - weight of the moved crane, kg;

m_{zp} - weight of moved cargo, kg;

V_{kp} - speed of movement of the crane, m/c;

Z_M - number of mechanisms of movement of the crane.

Having divided, the left and right parts of last expression on $\dot{\varphi}_{\partial\partial}^2$, we will receive expression, kgm^2 :

$$I_{np} = I_{o6} + \frac{I_1}{\eta_1} + \frac{I_2}{i_1^2 \cdot \eta_1 \cdot \eta_2} + \frac{I_{uo}}{i_1^2 \cdot i_2^2 \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}} + \frac{(m_{kp} + m_{ep}) \cdot (V_{kp} / \dot{\varphi}_{o6})^2}{Z_M \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}}$$

where: i_1, i_2 - accordingly gear-ratios of a worm reducer and an open tooth gearing.

But it is possible to present speed of movement of the crane in such kind

$$V_{kp} = \varphi_{xk} \cdot R_{xk} = \varphi_{o6} \cdot R_{xk} / (i_1 \cdot i_2),$$

where: R_{xk} - radius of a running wheel of the crane, m.

Definitively led moment of inertia to an engine shaft, kgm^2 all moving bodies of one mechanism of movement of the crane and the crane with the cargo, falling to one mechanism, буде to look so

$$I_{np} = I_{o6} + \frac{I_1}{\eta_1} + \frac{I_2}{i_1^2 \cdot \eta_1 \cdot \eta_2} + \frac{I_{uo}}{i_1^2 \cdot i_2^2 \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}} + \frac{(m_{kp} + m_{ep}) \cdot R_{xk}^2}{Z_M \cdot i_1^2 \cdot i_2^2 \cdot \eta_1 \cdot \eta_2 \cdot \eta_{uo}}$$

7. CALCULATION OF THE MOMENTS OF INERTIA OF ROTATING BODIES AND THEIR RESULTED VALUES

The moments of inertia of rotating bodies can be defined calculation, using known formulas and drawings of corresponding details (knots), or, having created corresponding models of details (knots) in CAD in which they automatically pay off. For example, the moment of inertia of a brake pulley can be defined, using the corresponding three-dimensional model constructed in system the KOMPAS 3D (Fig. 2.)

Figure 2: Result of definition of the moment of inertia of model the Brake pulley

The moment of inertia of a rotor of the engine is defined from engine technical characteristics.

$$I_{rom} = 0,065 \text{ kgm}^2.$$

The moment of inertia I_l , kgm^2 an incorporated body the worm (the cylindrical-shaft and the worm-shaft) it is possible to define and a settlement method with use of its sizes fig. 3

Figure 3: The sizes of an incorporated body the worm-shaft (the cylindrical shaft and the worm-shaft)

$$I_l = \frac{\pi}{32} \cdot \rho \cdot \sum_{j=1}^{n_l} (I_{lj} \cdot d_{lj}^4) = \frac{3,14}{32} \cdot 7826 \cdot (0,088 \cdot 0,06^4 + 0,08 \cdot 0,035^4 + 0,015^4 + 0,017 \cdot 0,042^4 + 0,028 \cdot 0,05^4 + 0,012 \cdot 0,055^4 + 0,013 \cdot 0,035^4 + 0,0465 \cdot 0,032^4 + 0,0735 \cdot 0,039^4 + 0,0405 \cdot 0,032^4 + 0,028 \cdot 0,03^4) = 1,274 \cdot 10^{-3},$$

where:

ρ - material density of which the body is made, kg/m^3 ;

n_l - Number of steps on an incorporated body the worm-shaft;

l_{lj} - Length j - oh steps of an incorporated body the worm-shaft, m;

d_{lj} - Diameter j - oh steps of an incorporated body the worm-shaft, m.

The moments of inertia and their resulted values under known formulas and for other bodies are similarly defined. Further factors of rigidity and диссипации (демпфирования) for each body under known formulas and their resulted values, and also parameters of the mechanical characteristic of the electric motor are defined. Calculation of parameters of the mechanical characteristic in system Mathcad (See below)

8. CONCLUSION

The proposed procedure of calculation of the dynamic analysis of work of mechanical drives using Mathcad software enables to decrease by an order of magnitude the time of labour inputs of such calculation, as well as pursue investigations related to changing parameters of attachment components.

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The dynamic analysis of the crane movement mechanism in system Mathcad

Initial data:

- moment inertia of an join body an engine rotor, kgm^2
 $I_{en}=0.093$
- reduced moment of inertia of an join body the Worm, kgm^2
 $I_{1re}=0.00172$
- reduced moment of inertia of an join body the: Worm heel, kgm^2
 $I_{2re}=0.00016$
- reduced moment of inertia of an join body the Execution unit, kgm^2
 $I_{3re}=0.202$
- rigidity coefficient of connections of an join body the Worm, Nm/radian
 $c_1=7754$
- rigidity coefficient of connections of an join body the Worm heel, Nm/radian
 $c_2=4215$
- rigidity coef. of connections of an join body the Execution unit, Nm/radian
 $c_3=7122$
- rigidity dissipation of connections of an join body the Worm, Nms/radian
 $\mu_1=0.731$
- rigidity dissipation of connections of an join body the Worm heel, Nms/radian
 $\mu_2=0.336$
- rigidity dissipation of con. of an join body the Execution unit, Nms/radian
 $\mu_3=1.373$
- resulted moment of resistance of a crane movement mechanism, kgm^2
 $M_r=103.161$
- gear-ratio of the reductor
 $i_1=39$
- gear-ratio of the open gear
 $i_2=2.52$
- size of critical sliding
 $S_c=0.551$
- maximum (critical) moment of the engine in a static mode, Nm

$$M_c=165.066$$

- nominal moment of the engine, Nm

$$M_n=52.469$$

- angular velocity of rotation of the engine rotor, $1/\text{s}$

$$\omega_n=95.295$$

- acceleration of gravity, m/s^2

$$g=9.81$$

Required parameters:

- angle of turn and angular velocity of the engine rotor with brake pulley, x_0, x_4
- angle of turn and angular velocity of the Worm, x_1, x_5
- angle of turn and angular velocity of the Worm wheel, x_2, x_6
- angle of turn and angular velocity of the Execution unit (the running wheel), x_3, x_7

Algorithm of calculation:

1. Reduction of dimensions

$$I_{en}=I_{en} \cdot g$$

$$I_{1re}=I_{1re} \cdot g$$

$$I_{2re}=I_{2re} \cdot g$$

$$I_{3re}=I_{3re} \cdot g$$

2. Representation and the decision of system of the differential equations:

- a vector-column of initial values required parameters

$$x := (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0)^T$$

- a vector-column of the right part of system of the differential equations

$$D(t, x) := \begin{bmatrix} x_4 \\ x_5 \\ x_6 \\ x_7 \\ \frac{1}{I_{en}} \cdot \left[\frac{2 \cdot M_c}{1 - \frac{x_4}{\omega n} + \frac{S_c}{S_c}} - \mu_1 \cdot (x_4 - x_5) - c_1 \cdot (x_0 - x_1) \right] \\ \frac{\mu_1 \cdot (x_4 - x_5) + c_1 \cdot (x_0 - x_1) - \mu_2 \cdot (x_5 - x_6 \cdot i_1) - c_2 \cdot (x_1 - x_2 \cdot i_1)}{I_{1re}} \\ \frac{\mu_2 \cdot (x_5 - x_6 \cdot i_1) + c_2 \cdot (x_1 - x_2 \cdot i_1) - \mu_3 \cdot (x_6 \cdot i_1 - x_7 \cdot i_1 \cdot i_2) - c_3 \cdot (x_2 \cdot i_1 - x_3 \cdot i_1 \cdot i_2)}{I_{2re}} \\ \frac{1}{I_{3re}} \cdot \left[\mu_3 \cdot (x_6 \cdot i_1 - x_7 \cdot i_1 \cdot i_2) + c_3 \cdot (x_2 \cdot i_1 - x_3 \cdot i_1 \cdot i_2) - \frac{M_r}{i_1 \cdot i_2} \right] \end{bmatrix}$$

- modelling parameters:

Initial time of modelling, s

$$t_0=0$$

Final time of modelling, s

$$t_1=2$$

Number of steps of integration

$$N=100$$

- decision of system of the differential equations by means of built in function

$$Rkadapt(x, t_0, t_1, N, D)$$

- method Runge-Kutta with adaptive step

$$Z := Rkadapt(x, t_0, t_1, N, D)$$

- results of definition of required parameters:

$$\begin{aligned} t &:= Z^{(0)} \\ x_0 &:= Z^{(1)} \\ x_1 &:= Z^{(2)} \\ x_2 &:= Z^{(3)} \\ x_3 &:= Z^{(4)} \\ x_4 &:= Z^{(5)} \\ x_5 &:= Z^{(6)} \\ x_6 &:= Z^{(7)} \\ x_7 &:= Z^{(8)} \end{aligned}$$

3. Graphic representation of required parameters depending on time - t :

- angular movings of bodies: an engine rotor (x_0) and the running wheel (x_3):

- angular speeds of bodies: an engine rotor (x_4) and the running wheel (x_7):

4. Comparison of results of calculation and simulation:

- settlement tabular values of angular speed of rotation of a rotor of the engine depending on time by results of modelling, radian/s

x_4^T	0	1	2	3	4	5	6	7
	0	2.939	5.929	8.967	12.058	15.198	18.39	...

- an error of modelling of angular speed of rotation of a rotor of the engine - $\delta\omega_n$, %

$$(x_4)_N = 95.127$$

$$\delta\omega_n := \left| \frac{\omega_n - (x_4)_N}{\omega_n} \right| \cdot 100$$

$$\delta\omega_n = 0.176$$

- settlement nominal angular speed of rotation of a running wheel, radian/s

$$\omega_h := \frac{\omega_n}{iI \cdot i2} = 0.97$$

- settlement tabular values of angular speed of rotation of a running wheel depending on time by results of modelling, radian/s

x_7^T	0	1	2	3	4	5	6	7	8
	0	0.031	0.062	0.093	0.125	0.157	0.19	0.223	...

- an error of modelling of angular speed of rotation of a running wheel, %

$$(x_7)_N = 0.968$$

$$\delta\omega_h := \left| \frac{\omega_h - (x_7)_N}{\omega_h} \right| \cdot 100$$

$$\delta\omega_h = 0.176$$

5. The dynamic moment and factor of dynamism in the course of engine start-up:

- moment of inertia of weights of the crane movement mechanism, led to an engine shaft

$$I_{re} = I_{en} + I_{lre} + I_{2re} + I_{lre}$$

$$I_{re} = 2.912$$

- angular acceleration of the engine rotor

$$a_{en} := \frac{1}{I_{re}} \cdot \left[\frac{2 \cdot Mc}{1 - \frac{x_4}{\omega n} + \frac{Sc}{1 - \frac{x_4}{\omega n}}} - \mu l (x_4 - x_5) - c l \cdot (x_0 - x_1) \right]$$

- dynamic moment, Nm and dynamism factor on an engine shaft under which action there is a dispersal of the crane movement mechanism

$$M_{dyn} := a_{en} \cdot I_{re}$$

$$k_{dyn} := \frac{M_{dyn} + Mr}{Mr}$$

- maximum dynamic moment, Nm and dynamism factor on an engine shaft

$$M_{dyn\grave{a}\grave{o}} := \max(M_{dyn})$$

$$M_{dyn\grave{a}\grave{o}} = 157.556$$

$$k_{dynmax} := \max(k_{dyn})$$

$$k_{dynmax} = 2.527$$

- condition check: the dynamic moment on a shaft of the engine in the course of dispersal should be less than the critical moment of the engine, Nm

$$M_{dyn\grave{a}\grave{o}} = 157.556 < Mc = 165.066$$

The condition is carried out.