Uporedna modalna analiza portala tipa "H" i "X" portalnih dizalica

Rade Vasiljević^{1*}

¹Kosančićeva 1A/4, 31109 Užice (Serbia)

U ovom radu je opisana uporedna modalna analiza sa FEM pristupom dva portala tipa "H" i "X" portalnih dizalica. Ova dva tipa portala imaju iste glavne geometrijske mere. Razlikuju se samo na srednjem nivou u horizontalnoj ravni. Prvi portal na srednjem nivou ima ram tipa "H", dok drugi portal na srednjem nivou ima ram tipa "X". U radu su upoređeni odgovarajući modalni parametri dva tipa portala, koji su dobijeni tokom modalne analize. Data je prva ocena koji od dva razmatrana tipa noseće konstrukcije ima povoljniju dinamičku stabilnost protiv prevrtanja.

Keywords: Portal, Tip "H", Tip "X", FEM, Modalna analiza

1. UVOD

U konstrukcionom inženjerstvu relativno je čest slučaj da izvedena rešenja konstrukcija nisu optimalna. Problem oscilovanja nosećih konstrukcija je od velikog značaja u mašinskom inženjerstvu. On je posebno prisutan kod dizaličnih mašina.

Dinamička analiza nosećih konstrukcija je vrlo značajna za unapređenje tehničkih karakteristika dizalica. Modalna analiza idejnih rešenja dizaličnih mašina predstavlja prvu fazu u proceni njihove dinamičke stabilnosti. Postupak određivanja svojstvenih vrednosti kod složenih sistema (sistema sa velikim brojem stepeni slobode) predstavlja najskuplju fazu u dinamičkoj analizi. Približni izrazi za određivanje prirodnih frekvencija portalnog rama mogu se dobiti pomoću Rejljeve metode (Ref. [1]). Neki primeri približnih metoda prikazani su u [2]. Takođe, problem modalne analize razmatran je u referencama [3,4,5,6]. FEM softver se široko primenjuje za modalnu analizu različitih konstrukcija [7,8,9]. FE tehnike za izračunavanje dinamičkog odgovora struktura su date u [10].

Saglasno aktuelnosti pomenute problematike, u ovom radu je data uporedna modalna analiza dva portala tipa "H" i "X" portalnih dizalica. Modalna analiza tipskih portala je sprovedena numerički primenom metoda sa konzistentnim masama. Formirani i rešeni su konačno-elementni modeli portala. Teorijski rezultati numeričke analize metodom konzistentnih masa verifikovani su primenom softverske simulacije u programskom paketu SAP2000®.

2. OPIS PORTALA

Kao podloga za modalnu analizu u ovom podnaslovu dat je kratak opis dva tipa portala ("H" i "X") portalnih dizalica. Više detalja o ovim portalima dato je u [11].

Glavne mehaničke karakteristike portala jednake su:

- Modul elastičnosti, $E=2.1 \times 10^{11} \text{ N/m}^2$;
- Modul klizanja, $G=0.8 \times 10^{11} \text{ N/m}^2$; i
- Gustina materijala, $\rho=7850 \text{ kg/m}^3$.

2.1. Opis portala - tip "H"

Diskretni model portala tipa "H" portalne dizalice prikazan je na slici 1. Portal je kruti prostorni ram. Ovaj portal ima četiri noge visine $H=6 \text{ m}$, koje su na srednjem nivou u horizontalnoj ravni povezane ramom tipa "H". Osnova portala ima dimenzije $L \times B=6 \times 6 \text{ m}$. Glavni strukturni elementi portala su noge, kosi stubovi, ram tipa "H" i kružni prsten prečnika $D=3 \text{ m}$. Noge su identične i stoje na istom nivou. Kosi stubovi su identični i vezuju se za noge, ram tipa "H" i kružni prsten. Rastojanje između rama tipa "H" i kružnog prstena jednako je $H_s=3.675 \text{ m}$. Kružni prsten ima funkciju gornjeg oslonca obrtne strele. Ram tipa "H" ima funkciju donjeg oslonca obrtne strele.

Slika 1: Diskretni model portala - tip "H"

2.2. Opis portala - tip "X"

Diskretni model portala tipa "X" portalne dizalice prikazan je na slici 2. Ovaj portal ima četiri noge visine $H=6 \text{ m}$, koje su na srednjem nivou u horizontalnoj ravni povezane ramom tipa "X". Ostali detalji opisa portala tipa "X" su isti kao kod portala tipa "H".

Slika 2: Diskretni model portala - tip "X"

3. MODALNA ANALIZA PORTALA

U ovom radu, za modalnu analizu portala (opisanih u prethodnom podnaslovu) koristi se metod konzistentnih masa. Analiza 3D grednog konačnog elementa korišćenjem metoda konzistentnih masa data je u radu [12]. Takođe, u radu [12] je detaljno data matematička formulacija modalne analize prostornih grednih nosača. Jednačina slobodnih neprigušenih oscilacija sistema glasi [12,13,14]:

$$\mathbf{M}_{uu} \ddot{\mathbf{U}}_{uu} + \mathbf{K}_{uu} \mathbf{U}_{uu} = \mathbf{0} \quad (1)$$

gde su:

- $\mathbf{M}_{uu}$ - matrica masa sistema u pravcu nepoznatih pomeranja, kg
- $\mathbf{K}_{uu}$ - matrica krutosti sistema u pravcu nepoznatih pomeranja, N/m
- $\ddot{\mathbf{U}}_{uu}$ - vektor nepoznatih ubrzanja, m/s²
- $\mathbf{U}_{uu}$ - vektor nepoznatih pomeranja, m

Sopstvene frekvencije noseće konstrukcije se dobijaju rešavanjem algebarske jednačine:

$$\det(\mathbf{K}_{uu} - \omega^2 \mathbf{M}_{uu}) = 0 \quad (2)$$

Na osnovu sopstvenih frekvencija dobijaju se periodi oscilovanja preko izraza:

$$T_i = \frac{2\pi}{\omega_i} \quad (3)$$

Frekvencije noseće konstrukcije dobijaju se na osnovu izraza:

$$f_i = \frac{\omega_i}{2\pi} = \frac{1}{T_i} \quad (4)$$

3.1. Modalna analiza portala - tip "H"

Korišćenjem metoda sa konzistentnim masama je formiran i rešen 3D konačno-elementni model portala tipa "H" portalne dizalice. Konačno-elementni model portala "H" je prikazan na slici 3. Diskretizacija portala tipa "H" dizalice izvršena je na 18 konačnih grednih elemenata povezanih sa čvorovima. Ceo sistem ima 82 stepena slobode kretanja. Noge i kosi stubovi su modelirani kao jedan konačni element. Kružni prsten je podeljen na 4 konačna elementa. Ram - tipa "H" je podeljen na 6 konačnih elemenata. Konačno-elementni model portala - "H" oslanja se na četiri elastična oslonca.

Slika 3: Konačno-elementni model portala - tip "H"

Karakteristike poprečnih preseka konačnih elemenata portala - "H" jednake su: $A_n=31500 \times 10^{-6} \text{ m}^2$, $I_n=6 \text{ m}$, $I_{yn}=3.117 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.391 \times 10^{-3} \text{ m}^4$, $I_{xn}=2.776 \times 10^{-3} \text{ m}^4$ ($n=1 \dots 4$); $A_n=31500 \times 10^{-6} \text{ m}^2$, $I_n=4.243 \text{ m}$, $I_{yn}=3.117 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.391 \times 10^{-3} \text{ m}^4$, $I_{xn}=2.776 \times 10^{-3} \text{ m}^4$ ($n=5 \dots 8$); $A_n=26500 \times 10^{-6} \text{ m}^2$, $I_n=3 \text{ m}$, $I_{yn}=1.303 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.066 \times 10^{-3} \text{ m}^4$, $I_{xn}=1.630 \times 10^{-3} \text{ m}^4$ ($n=9 \dots 12$); $A_n=25200 \times 10^{-6} \text{ m}^2$, $I_n=3 \text{ m}$, $I_{yn}=1.115 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.119 \times 10^{-3} \text{ m}^4$, $I_{xn}=1.611 \times 10^{-3} \text{ m}^4$ ($n=13 \dots 18$).

Krutost elastičnih oslonaca jednaka je $k=10^7 \text{ N/m}$.

Matrica krutosti konačno-elementnog modela portala tipa "H" u pravcu nepoznatih pomeranja glasi:

$$\mathbf{K}_{uu} = [\mathbf{K}]_{82 \times 82} \quad (5)$$

Matrica masa konačno-elementnog modela portala tipa "H" u pravcu nepoznatih pomeranja glasi:

$$\mathbf{M}_{uu} = [\mathbf{M}]_{82 \times 82} \quad (6)$$

Matematička formulacija jednačina (5) i (6) detaljno je data u [3].

Na osnovu jednačina (1)-(6) i definisanih polaznih podataka korišćenjem Mathematica® [15,16], napisan je kompjuterski program "RV.ModAnP-H" za proračun perioda oscilovanja, kružnih frekvencija i frekvencija. Dobijene su njihove vrednosti za sve stepene slobode ($i=82$).

Iz funkcija frekventnog odgovora portala "H" su izdvojene odgovarajuće računске sopstvene frekvencije pojedinih modova i prikazane u tabeli 1.

Tabela 1: Sopstvene frekvencije portala - tip "H"

Mod	Br.	Oblik	Frekvencija f_H [Hz]		Relativna razlika [%]
			Računski	SAP2000	
1.	Hor.-pod.		2.65	2.65	0.00
2.	Hor.-pop.		2.99	3.01	-0.66
3.	Tor.		5.86	5.84	0.34
4.	Ver.		7.85	7.85	0.00
5.	Hor.-ver.		12.70	12.77	-0.55
6.	Hor.-ver.		17.57	18.02	-2.50

Da bi proverili teorijske rezultate, u konačno-elementnom softveru SAP2000® [17] je formiran i rešen konačno-elementni model portala tipa "H" portalne dizalice. Konačno-elementni model portala tipa "H" je prikazan na slici 4. Prikazani KE model je prostorni model.

Slika 4: SAP2000® KE model portala - tipa "H"

Modalnom analizom u FEA softveru (SAP2000®), dobijene su vrednosti frekvencija oscilovanja i oblici moda prostornog sistema.

U tabeli 1 dato je prvih 6 frekvencija dobijenih prema FEA softveru SAP2000®.

Na slikama 5-7 prezentovana su tri glavna proračunska oblika moda oscilovanja portala tipa "H" portalne.

Poređenjem vrednosti frekvencija dobijenih korišćenjem matematičkog modela sa konačno-elementnim pristupom (metod sa konzistentnim masama) i čisto numeričkim metodom (FEA software SAP2000®), opaža se odlično poklapanje za prvu frekvenciju sa relativnom razlikom 0.00 %. Ostale frekvencije se dobro poklapaju. Tako, na primer, za sledećih pet frekvencija maksimalna relativna razlika je 2.50 %, što je vrlo dobro za prostorni konačno-elementni model.

Slika 5: 1 oblik moda portala - tip "H"

Slika 6: 2 oblik moda portala - tip "H"

Slika 7: 3 oblik moda portala - tip "H"

3.2. Modalna analiza portala - tip "X"

Analogno, kao kod modalne analize portala "H", korišćenjem metoda sa konzistentnim masama je formiran i rešen 3D konačno-elementni model portala tipa "X" portalne dizalice. Konačno-elementni model portala tipa "X" je prikazan na slici 8. Diskretizacija portala tipa "X" dizalice izvršena je na 16 konačnih grednih elemenata povezanih sa čvorovima. Ceo sistem ima 70 stepeni slobode kretanja. Noge i kosi stubovi su modelirani kao jedan konačni element. Kružni prsten je podeljen na 4 konačan elementa. Ram - tipa "X" je podeljen na 4 konačnih elemenata. Konačno-elementni model portala - "X" oslanja se na četiri elastična oslonca.

Slika 8: KE model portala - tip "X"

Karakteristike poprečnih preseka konačnih elemenata portala - "X" jednake su: $A_n=31500 \times 10^{-6} \text{ m}^2$, $l_n=6 \text{ m}$, $I_{yn}=3.117 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.391 \times 10^{-3} \text{ m}^4$, $I_{xn}=2.776 \times 10^{-3} \text{ m}^4$ ($n=1 \dots 4$); $A_n=31500 \times 10^{-6} \text{ m}^2$, $l_n=4.243 \text{ m}$, $I_{yn}=3.117 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.391 \times 10^{-3} \text{ m}^4$, $I_{xn}=2.776 \times 10^{-3} \text{ m}^4$ ($n=5 \dots 8$); $A_n=26500 \times 10^{-6} \text{ m}^2$, $l_n=3 \text{ m}$, $I_{yn}=1.303 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.066 \times 10^{-3} \text{ m}^4$, $I_{xn}=1.630 \times 10^{-3} \text{ m}^4$ ($n=9 \dots 12$); $A_n=25200 \times 10^{-6} \text{ m}^2$, $l_n=4.243 \text{ m}$, $I_{yn}=1.115 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.119 \times 10^{-3} \text{ m}^4$, $I_{xn}=1.611 \times 10^{-3} \text{ m}^4$ ($n=13 \dots 16$).

Krutost elastičnih oslonaca jednaka je $k=10^7 \text{ N/m}$.

Matrica krutosti konačno-elementnog modela portala tipa "X" u pravcu nepoznatih pomeranja glasi:

$$\mathbf{K}_{uu} = [\mathbf{K}]_{70 \times 70} \quad (7)$$

Matrica masa konačno-elementnog modela portala tipa "X" u pravcu nepoznatih pomeranja glasi:

$$\mathbf{M}_{uu} = [\mathbf{M}]_{70 \times 70} \quad (8)$$

Na osnovu jednačina (1)-(4), (7), (8) i definisanih polaznih podataka korišćenjem Mathematica® [15,16], napisan je kompjuterski program "RV.ModAnP-X" za proračun perioda oscilovanja, kružnih frekvencija i frekvencija. Dobijene su njihove vrednosti za sve stepene slobode ($i=70$).

Iz funkcija frekventnog odgovora portala "X" su izdvojene odgovarajuće računске sopstvene frekvencije pojedinih modova i prikazane u tabeli 2.

Tabela 2: Sopstvene frekvencije portala - tip "X"

Mod		Frekvencija f_x [Hz]		Relativna razlika [%]
Br.	Oblik	Računski	SAP2000	
1.	Hor.-pop.	2.87	2.91	-1.37
2.	Hor.-pod.	2.89	2.91	-0.69
3.	Tor.	4.92	4.90	0.41
4.	Ver.	7.92	7.91	0.13
5.	Hor.-ver.	15.31	15.64	-2.11
6.	Hor.-ver.	15.37	15.64	-1.73

Da bi proverili teorijske rezultate, u konačno-elementnom softveru SAP2000® [17] je formiran i rešen konačno-elementni model portala tipa "X" portalne dizalice. Konačno-elementni model portala tipa "X" je prikazan na slici 9. Prikazani KE model je prostorni model.

Modalnom analizom u FEA softveru (SAP2000®), dobijene su vrednosti frekvencija oscilovanja i oblici moda prostornog sistema.

U tabeli 2 dato je prvih 6 frekvencija dobijenih prema FEA softveru SAP2000®.

Na slikama 9-12 prezentovana su tri glavna proračunska oblika moda oscilovanja portala tipa "X" dizalice.

Poređenjem vrednosti frekvencija dobijenih korišćenjem metoda sa konzistentnim masama i FEA softvera SAP2000®, opaža se odlično poklapanje za prvu frekvenciju sa relativnom razlikom 1.37 %. Ostale frekvencije se dobro poklapaju. Tako, na primer, za sledećih pet frekvencija maksimalna relativna razlika je 2.11 %, što je vrlo dobro za 3D model.

Slika 9: SAP2000® KE model portala - tip "X"

Slika 10: 1 oblik moda portala - tip "X"

Slika 11: 2 oblik moda portala - tip "X"

Slika 12: 3 oblik moda portala - tip "X"

4. UPOREDNA ANALIZA REZULTATA

Tokom modalne analize dva tipa prostornih portala računski odnosno metodom sa konzistentnim masama su određeni njihovi modalni parametri (frekvencije i modovi oscilovanja) i međusobno upoređeni s odgovarajućim veličinama dobijenim pomoću FE programa SAP2000®. Parametri dobijeni metodom konzistentnih masa i pomoću softvera SAP2000® upoređuju se radi verifikacije proračunskih modela i kvaliteta izvedbe. Analizom spomenutih proračunskih i SAP2000® parametara utvrđena je relativno visoka podudarnost za portale tipa "H" i "X", čime su verifikirani proračunski modeli i njihova izvedba.

Jedan od glavnih ciljeva ovog rada je međusobno poređenje modalnih parametara (sopstvenih frekvencija i modova oscilovanja) dva tipa portala.

Iako oba portala imaju iste raspone i visine, postoje razlike u modalnim parametrima. Ove razlike potiču od razlike njihovih horizontalnih ramova u srenjem nivou. U tabeli 3 date su uporedne računске frekvencije portala "H" i "X" i njihove razlike. Takođe, na slici 13 je grafička ilustracija uporednih frekvencija razmatranih tipskih portala.

Tabela 3: Poređenje frekvencija portala ("H" i "X")

Mode No	Calculate Frequency f [Hz]		Relative difference Δ [%]
	Type "H"	Type "X"	
1.	2.65	2.87	-7.66
2.	2.99	2.89	3.34
3.	5.86	4.92	16.04
4.	7.85	7.92	-0.88
5.	12.70	15.31	-7.03
6.	17.57	15.37	12.52

Slika 13: Grafičko poređenje frekvencija oscilovanja dva tipa portala ("H" i "X")

Na osnovu tabele 3 i slike 13 zaključuje se da se prvih 6 frekvencija tipskih portala relativno malo razlikuju. Za konstrukciju su najopasnija rezonantna stanja u prva dva-tri glavna oblika oscilovanja, jer su tada amplitude najveće i brzo dovode do loma konstrukcije [7,18]. U tom smislu pri poređenju portala "H" i "X" dat je akcenat na prva tri moda oscilovanja.

Prvi mod oscilovanja konstrukcije predstavlja dominantni mod oscilovanja. Ovaj mod oscilovanja kod oba razmatrana tipska portala karakteriše horizontalni oblik oscilovanja. Kako je imperativ nosećih konstrukcija transportnih, građevinskih i rudarskih mašina je mala masa, odnosno vitkost, uobičajeno je da prva-najniža (osnovna) frekvencija oscilovanja bude u opsegu od 0,5-3,5 Hz [18]. Prema ovom aspektu kod oba tipa portala prva frekvencija se nalazi u preporučenom opsegu. Portal tipa "H" ima za 7.66 % nižu prvu frekvenciju oscilovanja od portala tipa "X", što znači da je portal - "H" elastičniji sa aspekta prvog moda oscilovanja.

Drugi mod oscilovanja kod oba razmatrana tipska portala karakteriše horizontalni oblik oscilovanja. Portal tipa "H" ima za 3.34 % višu drugu frekvenciju oscilovanja od portala tipa "X". Treći mod oscilovanja kod oba razmatrana tipska portala karakteriše torzioni oblik oscilovanja. Portal tipa "H" ima za 16.04 % višu treću frekvenciju oscilovanja od portala tipa "X".

Dinamičko ponašanje konstrukcije se ocenjuje prvim modom oscilovanja. Valjano dinamičko ponašanje konstrukcije zahteva visoku prvu frekvenciju. Prema ovom aspektu portal - "H" karakteriše nepovoljnija dinamička stabilnost u odnosu na portal - "X".

5. ZAKLJUČAK

Zaključci uporedne modalne analize dva tipa portala "H" i "X" portalnih dizalica su:

- Omogućeno je razumevanje ponašanja dva tipa portala tokom sopstvenih neprigušenih oscilacija;
- Oba portala karakterišu relativno slični oblici oscilovanja;
- Frekvencije portala tipa "H" i "X" su veoma bliske;
- Prva najniža (osnovna) frekvencija oscilovanja oba portala nalazi se u granicama 0,5-3,5 [Hz], tako da je ispunjen je uslov za malom masom, odnosno njihovom vitkošću;
- The portal type "H" is slightly more elastic than portal type "X" which means it has a slightly unfavourable dynamic stability, as it has a lower first frequency of 7.66 %;
- Sprovedena uporedna modalna analiza u ovom radu ostavlja prostor za dalju uporednu dinamičku analizu portala tipa "H" i "X" portalnih dizalica.

LITERATURA

- [1] A.P. Filippov, "Vibration of deformable systems", Moscow (Russia), (1970)
- [2] Z. Maglaić, G. Simonović, R. Hadžović and N. Ademović, "Određivanje osnovne forme i perioda oscilovanja građevina približnim metodama", Materijali i konstrukcije, Vol. 49, No. 3-4, pp. 72–82, (2006)
- [3] S. Timoshenko and D.H. Young, "Vibration problems in engineering", New York (USA), (1955)
- [4] R. Blevins, "Formulas for natural frequency and mode shape", New York (USA), (1979)
- [5] C.P. Filipich and P.A. Laura, "In-plane vibrations of portal frames with end supports elastically restrained against rotation and translation", Journal of Sound and Vibration, pp. 467-473, (1987)
- [6] V. Gašić, A. Obradović and Z. Petković, "Mathematical modelling of the in-plane vibrations of portal cranes with fem verification", Machine Design, Vol. 1, pp. 121–126, (2009)
- [7] F. Ju, Y.S. Choo and F.S. Cui, "Dynamic response of tower crane induced by the pendulum motion of the payload", International Journal of Solids and Structures, Vol. 43, pp. 376–389, (2006)
- [8] R. Vasiljević, M. Savković and R. Bulatović, "The Approaches to the Mathematical-mechanical Modeling Supporting Construction", IMK-14 - Research & Development in Heavy Machinery, Vol. 19, No. 1, pp. EN29-EN38, (2013)
- [9] R. Vasiljević, "Dynamic Analysis of Mechanical Two Post Lift Using Finite Element (ANSYS) and Analytical Method", IMK-14 - Research & Development in Heavy Machinery, Vol. 24, No. 3, pp. EN85-EN91, (2018)
- [10] J.J. Wu, A.R. Whittaker and M.P. Cartmell, "The use of finite element techniques for calculating the dynamic response of structures to moving load", Computers & Structures, Vol. 78, pp. 789-799, (2000)
- [11] А. Ланг, И. Мазовер и В. Майзель, "Портальные краны – расчет и конструирование", Государственное научно-техническое издательство машиностроительной литературы, Москва-Ленинград (Россия), (1962)
- [12] R. Vasiljević, "Analysis of Free Oscillation of 3D Frames Applying the Method of Consistent Masses", Proceedings of IX International Conference "Heavy Machinery HM 2017", Zlatibor (Serbia), 28 June-1 July 2017, pp. D.51-D.56, (2017)
- [13] R. Vasiljević, M. Gašić and M. Savković, "Parameters Influencing the Dynamic Behaviour of the Carrying Structure of a Type H Portal Crane", Strojniški vestnik - Journal of Mechanical Engineering, Vol. 62, No. 10, pp. 591-602, (2016)
- [14] R. Vasiljević, "Comparative modal analysis of the spatial beam frames of a type "H" and "X", Proceedings of International Conference on Mechanical Engineering Technologies and Application COMETA 2018", Jahorina (B&H), November 27-30, pp. 420-427, (2018)
- [15] E. Magrab, "Engineers guide to mathematica®", John Wiley&Sons, Chichester (United Kingdom), (2014)
- [16] Wolfram, Mathematica®, www.wolfram.com
- [17] University of Berkeley. SAP2000® v14.2.0, Computers and Structures, Inc., (2010)
- [18] Z. Petković, "Metalne konstrukcije u mašingradnji 1, Mašinski fakultet, Beograd (Srbija), (1996)

Comparative Modal Analysis of the Portals of a Type “H” and “X” Portal Cranes

Rade Vasiljević^{1*}

¹Kosančičeva 1A/4, 31109 Užice (Serbia)

In this paper a comparative modal analysis with FEM approach of two portals of type “H” and “X” portal cranes is described. These two types of carrying structures have the same main geometric dimensions. They differ only at the medium level in the horizontal plane. The first type of portal on the middle level has a type “H” frame, while the second type of portal on the middle level has a type “X” frame. The paper compares the corresponding modal parameters of two types of portals obtained during their comparative modal analysis. An first estimation has been done for which of the two types of portals is considered to have a more favourable dynamic stability for overturn.

Keywords: Portal, Type “H”, Type “X”, FEM, Modal analysis

1. INTRODUCTION

In structural engineering, it is relatively frequent case that implemented constructing solutions are not optimal. The vibration problem of the carrying structures is very important in mechanical engineering. It is specify attendant in the crane machinery.

In the dynamic analysis of the carrying structures is very important for improving the performances of the cranes. Modal analysis of conceptual designs of cranes design is the first and most essential element of dynamic analysis for estimation of their dynamic stability. The process of determining the eigenvalues in complex systems (systems with a large number of degrees of freedom) is the most expensive phase in the dynamic analysis. Approximate expressions for determining the natural frequencies of portal frame can be obtained by Reyleigh method (Ref. [1]). Some examples of the approximate methods are shown in [2]. Also, the problem modal analysis is considered in the references [3,4,5,6]. FEM software is widely used for modal analysis of various constructions [7,8,9]. A FE technique for calculating the dynamic response of structures was given in [10].

According to the actuality of this problematic, in this paper is a given comparative modal analysis of the portals of type “H” and “X” portal cranes. Modal analysis of the types of portals was conducted numerically by using the method with consistent masses. The finite element models of the typical portals are formed and solved. The theoretical results of numerical analyses are verified by using the simulation software in program package SAP2000®.

2. DESCRIPTION OF THE PORTALS

In this subtitle, a brief description two type portals (“H” and “X”) portal cranes is given as the basis for the modal analysis. More details about these portals are given in [11].

The main mechanical characteristics of the both portals are equal:

- Modulus of elasticity, $E=2.1 \times 10^{11} \text{ N/m}^2$;
- Modulus of shear, $G=0.8 \times 10^{11} \text{ N/m}^2$; and
- Mass density of beam, $\rho=7850 \text{ kg/m}^3$.

2.1. Description of the portal - type “H”

The discrete model of the portal of type “H” of the portal crane is shown in Fig. 1. The portal is a rigid spatial frame. This portal has four legs with height $H=6 \text{ m}$, which are connected with “H” type frame in the medium level in the horizontal plane. Its base has dimensions $L \times B=6 \times 6 \text{ m}$. The main structural parts of the portal are legs, slanted columns, “H” type frame and circular ring with the diameter $D=3 \text{ m}$. The legs are identical and stand at the same level. The slanted columns are identical and they are connected to the legs, the frame of type “H” and the circular ring. The distance between the frame of type “H” and the circular ring is equal to $H_s=3.675 \text{ m}$. The circular ring has the function of the upper support of the boom slewing. The frame of type “H” has the function of the lower support of the boom slewing.

Figure 1: Discrete model of portal - type “H”

2.2. Description of the portal - type “X”

The discrete model of the portal of type “X” of the portal crane is shown in Fig. 2. This portal has four legs with height $H=6 \text{ m}$, which are connected with “X” type frame in the medium level in the horizontal plane. Others details of the description of the “X” type portal are the same as in the portal of type “H”.

Figure 2: Discrete model of portal - type "X"

3. MODAL ANALYSIS OF THE PORTAL

In this paper, for modal analysis of the portals (description in previous subtitle) by using the method of consistent masses. Analysis of 3D girder finite element using the method of consistent masses is given in Ref. [12]. Also, mathematical formulation of the modal analysis of the 3D girder frames is detail given in the Ref. [12]. Equation of the free undamped oscillation of the system read [12,13,14]:

$$\mathbf{M}_{uu} \ddot{\mathbf{U}}_{uu} + \mathbf{K}_{uu} \mathbf{U}_{uu} = \mathbf{0} \quad (1)$$

where:

- $\mathbf{M}_{uu}$ - mass matrix of the system in the direction of unknown displacements, kg
- $\mathbf{K}_{uu}$ - stiffness matrix of the system in the direction of unknown displacements, N/m
- $\ddot{\mathbf{U}}_{uu}$ - vector of unknown accelerations, m/s²
- $\mathbf{U}_{uu}$ - vector of unknown displacements, m

Eigenfrequencies of the carrying structure are obtained by solving algebraic equation:

$$\det(\mathbf{K}_{uu} - \omega^2 \mathbf{M}_{uu}) = 0 \quad (2)$$

Based on the circular frequencies obtained by the vibration periods via expression:

$$T_i = \frac{2\pi}{\omega_i} \quad (3)$$

Frequencies of the carrying structure are obtained based on expression:

$$f_i = \frac{\omega_i}{2\pi} = \frac{1}{T_i} \quad (4)$$

3.1. Modal analysis of the portal - type "H"

By using the method of consistent masses, 3D finite element model of the portal type "H" of the portal crane was formed and solved. The finite element model of the portal "H" is shown in Fig. 3. Discretization of the portal - type "H" of the crane was performed on 18 finite beam elements connected by nodes. The whole system has 82 degrees of freedom of motion. The legs and slanted columns were modelled as one finite element. The circular ring was divided into 4 finite elements. The frame - type "H" was divided into 6 finite elements. The FE model of the portal - "H" recline on four elastic supports.

Figure 3: FE model of portal - type "H"

The properties of the cross-sections of the finite elements of the portal - "H" are equal: $A_n=31500 \times 10^{-6} \text{ m}^2$, $I_n=6 \text{ m}$, $I_{yn}=3.117 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.391 \times 10^{-3} \text{ m}^4$, $I_{xn}=2.776 \times 10^{-3} \text{ m}^4$ ($n=1 \dots 4$); $A_n=31500 \times 10^{-6} \text{ m}^2$, $I_n=4.243 \text{ m}$, $I_{yn}=3.117 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.391 \times 10^{-3} \text{ m}^4$, $I_{xn}=2.776 \times 10^{-3} \text{ m}^4$ ($n=5 \dots 8$); $A_n=26500 \times 10^{-6} \text{ m}^2$, $I_n=3 \text{ m}$, $I_{yn}=1.303 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.066 \times 10^{-3} \text{ m}^4$, $I_{xn}=1.630 \times 10^{-3} \text{ m}^4$ ($n=9 \dots 12$); $A_n=25200 \times 10^{-6} \text{ m}^2$, $I_n=3 \text{ m}$, $I_{yn}=1.115 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.119 \times 10^{-3} \text{ m}^4$, $I_{xn}=1.611 \times 10^{-3} \text{ m}^4$ ($n=13 \dots 18$).

Stiffness of the elastic supports is equal $k=10^7 \text{ N/m}$.

The stiffness matrix of the FE model of portal of type "H" in the direction of unknown displacements read:

$$\mathbf{K}_{uu} = [\mathbf{K}]_{82 \times 82} \quad (5)$$

Mass matrix of the FE model of portal of type "H" in the direction of unknown displacements read:

$$\mathbf{M}_{uu} = [\mathbf{M}]_{82 \times 82} \quad (6)$$

In [4], in detail is given mathematical formulation of the Eqs. (5) and (6).

Based on Eqs. (1)-(6) and defined starting data by using Mathematica® [15,16], was written computer code "RV.ModAnP-H" for calculating the vibration periods, circular frequencies and the frequencies. Obtained are their value for all degrees of freedom ($i=82$).

From the frequency response functions of the "H" - portal, the corresponding calculation frequencies of the individual modes are separated and shown in the Table 1.

Table 1: Frequencies of portal - type "H"

Mode		Frequency f_H [Hz]		Relative difference [%]
No	Shape	Calculate	SAP2000	
1.	Hor.-long.	2.65	2.65	0.00
2.	Hor.-lat.	2.99	3.01	-0.66
3.	Tor.	5.86	5.84	0.34
4.	Ver.	7.85	7.85	0.00
5.	Hor.-ver.	12.70	12.77	-0.55
6.	Hor.-ver.	17.57	18.02	-2.50

In order to verification the theoretical results, the FE software SAP2000® [17] is formed and solved finite element model of the portal of type “H” of the portal crane. The FE model of the portal of type “H” is shown in Fig. 4. Presented FE model is a spatial model.

Figure 4: SAP2000® FE model of portal - type “H”

With modal analysis in FEA software (SAP2000®), are obtained values of vibration frequencies and mode shapes in spatial system.

Table 1 gives first 6 frequencies obtained from FEA software SAP2000®.

On Figs. 5-7 there are presented three main calculated mode shapes of vibration of the portal of type “H” crane.

By comparing the values of frequencies obtained by using the mathematical model with the finite element approach (method of consistent masses) and the purely numerical method (FEA software SAP2000®), excellent coinciding of the first frequency with relative difference 0.00 % is noticed. Other frequencies good coincide. Thus, for example, for the next five frequencies the maximum relative difference is 2.50 %, which is very good for the spatial FE model.

Figure 5: 1st mode shape of portal - type “H”

Figure 6: 2nd mode shape of portal - type “H”

Figure 7: 3rd mode shape of portal - type “H”

3.2. Modal analysis of portal - type “X”

Analogically, as with the modal analysis of the portal - “H”, by using the method of consistent masses, 3D finite element model of the portal type “X” of the portal crane was formed and solved. The finite element model of the portal “X” is shown in Fig. 8. Discretization of the portal - type “X” of the crane was performed on 16 finite beam elements connected by nodes. The whole system has 70 degrees of freedom of motion. The legs and slanted columns were divided into 1 finite element. The upper beam (the circular ring with the diameter D) was divided into 4 finite elements. The lower beam (frame - type “X”) was divided into 4 finite elements. The FE model of the portal - “X” recline on four elastic supports.

Figure 8: FE model of portal - type "X"

The properties of the cross-sections of the finite elements of the portal - "X", are equal: $A_n=31500 \times 10^{-6} \text{ m}^2$, $l_n=6 \text{ m}$, $I_{yn}=3.117 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.391 \times 10^{-3} \text{ m}^4$, $I_{xn}=2.776 \times 10^{-3} \text{ m}^4$ ($n=1 \dots 4$); $A_n=31500 \times 10^{-6} \text{ m}^2$, $l_n=4.243 \text{ m}$, $I_{yn}=3.117 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.391 \times 10^{-3} \text{ m}^4$, $I_{xn}=2.776 \times 10^{-3} \text{ m}^4$ ($n=5 \dots 8$); $A_n=26500 \times 10^{-6} \text{ m}^2$, $l_n=3 \text{ m}$, $I_{yn}=1.303 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.066 \times 10^{-3} \text{ m}^4$, $I_{xn}=1.630 \times 10^{-3} \text{ m}^4$ ($n=9 \dots 12$); $A_n=25200 \times 10^{-6} \text{ m}^2$, $l_n=4.243 \text{ m}$, $I_{yn}=1.115 \times 10^{-3} \text{ m}^4$, $I_{zn}=1.119 \times 10^{-3} \text{ m}^4$, $I_{xn}=1.611 \times 10^{-3} \text{ m}^4$ ($n=13 \dots 16$).

Stiffness of the elastic supports is equal $k=10^7 \text{ N/m}$.

The stiffness matrix of the FE model portal type "X" in the direction of unknown displacements read:

$$\mathbf{K}_{uu} = [\mathbf{K}]_{70 \times 70} \quad (7)$$

Mass matrix of the FE model portal type "X" in the direction of unknown displacements read:

$$\mathbf{M}_{uu} = [\mathbf{M}]_{70 \times 70} \quad (8)$$

Based on Eqs. (1)-(4), (7), (8) and defined starting data in Mathematica® [15,16], was written computer code "RV. ModAnP-X" for calculating the vibration period, circular frequencies, and the frequency. Obtained are their value for all degrees of freedom ($i=70$).

From the frequency response functions of the "X" - portal, the corresponding calculation frequencies of the individual modes are separated and shown in the Table 2.

Table 2: Frequencies of portal - type "X"

Mode		Frequency f_X [Hz]		Relative difference [%]
No	Shape	Calculate	SAP2000	
1.	Hor.-lat.	2.87	2.91	-1.37
2.	Hor.-long.	2.89	2.91	-0.69
3.	Tor.	4.92	4.90	0.41
4.	Ver.	7.92	7.91	0.13
5.	Hor.-ver.	15.31	15.64	-2.11
6.	Hor.-ver.	15.37	15.64	-1.73

In order to verification the theoretical results, the FE software SAP2000® [17] is formed and solved finite element model of the portal type "X" of the portal crane. The FE model of the portal type "X" is shown in Fig. 9. Presented FE model is a 3D model.

With modal analysis in software SAP2000®, are obtained values of vibration frequencies and mode shapes in spatial system.

Table 2 (4 column) gives first 6 frequencies obtained from software SAP2000®.

On Figs. 10-12 there are presented three main calculated mode shapes of vibration of the portal of type "X" crane.

By comparing the values of frequencies obtained by using method of consistent masses and the FEA software SAP2000®, excellent coinciding of the first frequency with relative difference 1.37 % is noticed. Other frequencies good coincide. Thus, for example, for the next five frequencies the maximum relative difference is 2.11 %, which is very good for the 3D FE model.

Figure 9: SAP2000® FE model of portal - type "X"

Figure 10: 1st mode shape of portal - type "X"

Figure 11: 2nd mode shape of portal - type "X"

Figure 12: 3rd mode shape of portal - type "X"

4. COMPARATIVE ANALYSIS RESULTS

During the modal analysis of two types of spatial portals calculating, i.e. method with consistent masses, its modal parameters (frequencies and vibration modes) are determined and compared with corresponding values obtained by FE software SAP2000[®]. Parameters obtained by method of consistent masses and using software SAP2000[®] are compared to verify calculated models and quality of producing. Analysis of calculating and SAP2000[®] parameters confirmed a relatively high accordance for both portals (type "H" and type "X"), whereby verified calculating models and their producing. The main aim of this paper is the comparison of modal parameters (eigenfrequencies and vibration modes) the two types of portal ("H" and "X").

Although both portals have the same spread and height, there are differences in modal parameters. These differences come from the difference in their frames at the

medium level in the horizontal plane. Table 3 presents comparison of the magnitudes of the first six frequencies of the two type portals of the portal-slewing crane. Also, the relative differences of these parameters are given. Figure 13 shows a graphic illustration of the first six comparative frequencies of the considered portal types ("H" and "X").

Table 3: Comparison of frequencies of portals ("H" and "X")

Mode No	Calculate Frequency f [Hz]		Relative difference Δ [%]
	Type "H"	Type "X"	
1.	2.65	2.87	-7.66
2.	2.99	2.89	3.34
3.	5.86	4.92	16.04
4.	7.85	7.92	-0.88
5.	12.70	15.31	-7.03
6.	17.57	15.37	12.52

Figure 13: Graphic comparison of frequencies of vibration of the two portals type ("H" and "X")

Based on Table 3 and Figure 13 it is concluded that the first 6 frequencies of the portal types are relative slightly different. For the construction the most dangerous are first two or three major forms of vibration, since there are the greatest amplitude and quickly lead to failure of the structure [7,18]. In this regard, when comparing portals "H" and "X", the accent is on the first three modes of vibration.

The first mode of vibration of the construction represents the dominant vibration mode. This vibration mode for both considered types of portals is characterized by a horizontal shape of vibration. As the imperatives of carrying structures of transport, building and mining machinery, small mass, and their slimness is common to that the first lowest (basic) frequency is in the range of 0.5...3.5 Hz [18]. In this aspect, for both types of portal, the first frequency is in the recommended range. The portal type "H" has a 7.66 % lower first vibration frequency than a portal of type "X", which means that portal "H" is more elastic from the aspect of the first vibration mode.

The second vibration mode in both considered types of portal is characterized by a horizontal shape of vibration.

The portal type “H” has a 3.44 % higher second vibration frequency than a portal type “X”. The third mode of vibration in both types of portal types is characterized by torsional vibration. The type “H” has a 16.04 % higher third vibration frequency than a type “X”.

Dynamic behaviour of construction is evaluated by the first vibration mode. Valid dynamic behaviour of a construction requires a high first vibration frequency. According to this aspect, portal “H” is characterized by unfavourable dynamic stability compared to portal “X”.

5. CONCLUSION

The conclusions of a comparative modal analysis of two types “H” and “X” portal cranes are:

- It is possible to understand the behaviour of two types of portal during its own undamped vibrations;
- Both type portals are characterized by approximately the similar shapes of vibration;
- The vibration frequencies of portals type “H” and type “X” are fairly similar;
- The first lowest (basic) vibration frequency of both portals is within the limits of 0.5-3.5 Hz, so the condition for small mass, or their slimness, is fulfilled;
- The portal type “H” is slightly more elastic than portal type “X” which means it has a slightly unfavourable dynamic stability, as it has a lower first frequency of 7.66 %;
- This paper leaves room for the farther of comparative dynamic analysis of portals type “H” and “X” of portal cranes.

REFERENCES

- [1] A.P. Filippov, “Vibration of deformable systems”, Moscow (Russia), (1970)
- [2] Z. Maglaić, G. Simonović, R. Hadžović and N. Ademović, “Određivanje osnovne forme i perioda oscilovanja građevina približnim metodama”, Materijali i konstrukcije, Vol. 49, No. 3-4, pp. 72–82, (2006)
- [3] S. Timoshenko and D.H. Young, “Vibration problems in engineering”, New York (USA), (1955)
- [4] R. Blevins, “Formulas for natural frequency and mode shape”, New York (USA), (1979)
- [5] C.P. Filipich and P.A. Laura, “In-plane vibrations of portal frames with end supports elastically restrained against rotation and translation”, Journal of Sound and Vibration, pp. 467-473, (1987)
- [6] V. Gašić, A. Obradović and Z. Petković, “Mathematical modelling of the in-plane vibrations of portal cranes with fem verification”, Machine Design, Vol. 1, pp. 121–126, (2009)
- [7] F. Ju, Y.S. Choo and F.S. Cui, “Dynamic response of tower crane induced by the pendulum motion of the payload”, International Journal of Solids and Structures, Vol. 43, pp. 376–389, (2006)
- [8] R. Vasiljević, M. Savković and R. Bulatović, “The Approaches to the Mathematical-mechanical Modeling Supporting Construction”, IMK-14 - Research & Development in Heavy Machinery, Vol. 19, No. 1, pp. EN29-EN38, (2013)
- [9] R. Vasiljević, “Dynamic Analysis of Mechanical Two Post Lift Using Finite Element (ANSYS) and Analytical Method”, IMK-14 - Research & Development in Heavy Machinery, Vol. 24, No. 3, pp. EN85-EN91, (2018)
- [10] J.J. Wu, A.R. Whittaker and M.P. Cartmell, “The use of finite element techniques for calculating the dynamic response of structures to moving load”, Computers & Structures, Vol. 78, pp. 789-799, (2000)
- [11] А. Ланг, И. Мазовер и В. Майзель, “Портальные краны – расчет и конструирование”, Государственное научно-техническое издательство машиностроительной литературы, Москва-Ленинград (Россия), (1962)
- [12] R. Vasiljević, “Analysis of Free Oscillation of 3D Frames Applying the Method of Consistent Masses”, Proceedings of IX International Conference “Heavy Machinery HM 2017”, Zlatibor (Serbia), 28 June-1 July 2017, pp. D.51-D.56, (2017)
- [13] R. Vasiljević, M. Gašić and M. Savković, “Parameters Influencing the Dynamic Behaviour of the Carrying Structure of a Type H Portal Crane”, Strojniški vestnik - Journal of Mechanical Engineering, Vol. 62, No. 10, pp. 591-602, (2016)
- [14] R. Vasiljević, “Comparative modal analysis of the spatial beam frames of a type “H” and “X”, Proceedings of International Conference on Mechanical Engineering Technologies and Application COMETA 2018”, Jahorina (B&H), November 27-30, pp. 420-427, (2018)
- [15] E. Magrab, “Engineers guide to mathematica®”, John Wiley&Sons, Chichester (United Kingdom), (2014)
- [16] Wolfram, Mathematica®, www.wolfram.com
- [17] University of Berkeley. SAP2000® v14.2.0, Computers and Structures, Inc., (2010)
- [18] Z. Petković, “Metalne konstrukcije u mašinogradnji 1, Mašinski fakultet, Beograd (Srbija), (1996)