Određivanje kinematičkog modela bagerskog uređaja sa dubinskom kašikom

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U ovom radu predstavljen je metod izrade kinematičkog modela bagerskog uređaja sa dubinskom kašikom. Primenjeni metod se koristi u mehanizmima industrijskih robota i koristi Denavit-Hartenberg konvenciju.

Ključne reči: kinematika; kašika bagera; Denavit-Hartenberg

1. UVOD

Jedan od razvojnih puteva u robotici i automatizaciji je automatizacija i robotizacija postojeće opreme [1, 2]. Automatizacija i robotizacija građevinske opreme podrazumeva modeliranje opreme kao i tehnološki process posmatrano iz ugla kinematike i dinamike. Da bi se napravio model postoje dobro poznate metode u stručnoj literaturi pri čemu je svaka ta metoda orijentisana ka nekoj posebnoj tehničkoj oblasti. Kad je reč o građevinskoj mehanizaciji, ona se sastoji od dva dela, šasijsa / okretna platforma / noseći modul i radna oprema. Prvi deo se može modelirati po analogiji sa mobilnim robotima a drugi deo po analogiji sa industrijskim robotima.

Slika 1: Bager

Radna oprema bagera sa dubinskom kašikom, slika 1, predstavlja kinematički lanac otvorenog tipa koji se sastoji od sledećih kinematičkih elemenata: noseći modul, strela, držač i kašika a veze između ovih elemenata su zglobove veze. Na primer, pogonski mehanizam sa hidrauličkim cilindrom između strele i držača je deo zglobove veze koji spaja dva elementa i iz pogonjene veze transformiše ga u pogonski uvezu. Imajući sve ovo u vidu, bagerska oprema za iskopavanje ima četiri stepena kretanja. Ove transformacije se izvode da bi se istražila kinematika ovog mehanizma pomoću metode koja se koristi kod mehanizama industrijskih robota za pozicioniranje i orijentaciju.

Problem kinematičkog modeliranja se može formulisati na sledeći način: ako znamo promenljive pogonskih veza treba da odredimo položaj i orijentaciju krajnjeg efektor. Kinematički model ima dve osnovne upotrebe. Prva je određivanje radnog prostora opreme a druga je određivanje inverznog kinematičkog modela.

2. PREZENTACIJA METODE

U ovoj metodi za svaki kinematički model zakačen je koordinatni sistem prema Denavit-Hartenberg konvenciji [3]. Za kinematički lanac otvorenog tipa koji se sastoji od uzastopnih kinematičkih elemenata $i-2$, $i-1$, i i $i+1$ i kinematičkih veza koji povezuju ove elemente, slika 2, izbor koordinatnih sistema povezanih sa elementima $i-1$ i i izvodi se na sledeći način:

1. pravac ose z_{i-1} koja povezuje element $i-1$ sa i je spojna osa;
2. osa x_{i-1} je spojena sa elementom $i-1$ i izabrana je za zajedničku normalu osa z_{i-1} i z_{i-2} koje su orijentisane od O_{i-2} ka O_{i-1} ;
3. Slično tome, osa x_i je izabrana za zajedničku normalu osa rotacije z_{i-1} i z_i koje su orijentisane od O_{i-1} ka O_i ;
4. Koordinatni počeci se biraju u tačkama preseka između zajedničke normale i ose elementa O_{i-2} and O_{i-1} ;
5. Naziv veze je dat po najvišem redu elemenata koji čine spoj.
6. Ose y su odabrane tako da je koordinatni sistem ortogonalan.

Slika 2: Denavit-Hartenberg konvencija

Geometrijski parametri koji u potpunosti opisuju bilo koje obrtanje u zglobu ili translaciju u kinematičkom lancu su:

θ_i	Ugao zgloba od ose x_{i-1} ka osi x_i , meren u smeru suprotnom od kretanja kazaljke na satu, oko ose z_{i-1}
d_i	Rastojanje od koordinatnog početka O_{i-1} ka preseku ose z_{i-1} sa osom x_i , mereno duž ose z_{i-1}
a_i	Rastojanje od preseka ose z_{i-1} sa x_i ka koordinatnom početku O_i , mereno duž ose x_i (ili najmanje rastojanje između osa z_{i-1} i z_i)
α_i	Ugao između ose z_{i-1} i ose z_i , koji se dobija okretanjem oko ose x_i u smeru suprotnom od smera kretanja kazaljke na satu.

Prelaz sa koordinatnog sistema " $i-1$ ", dodatog elementu $i-1$, ka koordinatnom sistemu " i ", dodatom elementu i , izvodi se na sledeći način:

1. okretanje oko ose z_{i-1} za ugao θ_i da bi se osa x_{i-1} preklopila sa x_i ;
2. translatorno kretanje duž ose z_{i-1} sa rastojanjem d_i , što dovodi koordinatni početak O_{i-1} u međupoložaj H_{i-1} (sl. 2);
3. nova translacija duž ose x_{i-1} za rastojanje a_i , pa se koordinatni početak O_{i-1} preklapa sa O_i ;

$${}_{i-1}T^i = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) \cdot \cos(\alpha_i) & \sin(\theta_i) \cdot \sin(\alpha_i) & a_i \cdot \cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \cdot \cos(\alpha_i) & -\cos(\theta_i) \cdot \sin(\alpha_i) & a_i \cdot \sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (3)$$

Pomenuta matrica pomeranja (3) određuje položaj i orijentaciju koordinatnog sistema " i ", dodatog elementu i u odnosu na koordinatni sistem " $i-1$ ", dodatog elementu $i-1$.

3. DOBIJANJE KINEMATIČKOG MODELA BAGERSKOG UREĐAJA SA DUBINSKOM KAŠIKOM

3.1 Dodavanje koordinatnih sistema

Kinematička šema bagerske opreme za iskopavanje je data na slici 3. Na slici su prikazani sledeći

4. Konačno, drugo obrtanje oko ose x_i za ugao

α_i da bi se osa z_{i-1} preklopila sa z_i .

Sva četiri pomeranja se mogu izraziti pomoću elementarne matrice za rotaciono ili translatorno kretanje pa je rezultujuće pomeranje proizvod sledećih matrica:

$${}_{i-1}T^i = Rot(z_{i-1}, \theta_i) \cdot Trans(z_{i-1}, d_i) \cdot Trans(x_{i-1}, a_i) \cdot Rot(x_i, \alpha_i) \quad (1)$$

Množenjem elementarnih matrica dobija se:

$$\begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & 0 \\ \sin(\theta_i) & \cos(\theta_i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_i) & -\sin(\alpha_i) & 0 \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

Matrica pomeranja od koordinatnog sistema " $i-1$ ", dodatog elementu $i-1$, ka koordinatnom sistemu " i ", dodatog elementu i , dobija se na sledeći način:

elementi: 1 – gornji stroj, 2 – strela, 3 – držač, 4 – kašika. Ovi elementi su međusobno povezani spojevima B, C i D, a gornji stroj je takođe povezan sa donjim strojem pomoću spoja A. Prema napred navedenom, uzimajući u obzir da je donjim strojem element „ $i-2 \equiv 0$ ” osa $O_0 z_0$ je vertikalna i njen pozitivni deo ide gore kroz tačku A. Radi uprošćavanja proračuna koordinatni početak O_0 je uzet za tačku A.

Slika 3: Kinematička šema bagera sa dubinskom kašikom

Ose O_1z_1 , O_2z_2 , O_3z_3 i O_4z_4 su dodate na gornji stroj, strelu, držač i kašiku i prolaze kroz tačke A, B, C i D i normalne su na ravan crteža i smer ide od posmatrača ka crtežu. Osa O_1x_1 ima svoj pravac i orijentaciju $\overline{AB}$, osa O_2x_2 ima svoj pravac i orijentaciju $\overline{BC}$, osa O_3x_3 ima svoj pravac i orijentaciju $\overline{CD}$, O_4x_4 ima svoj pravac i orijentaciju $\overline{DE}$. Počeci koordinatnih sistema se nalaze u preseku osa z i x što znači da se O_1 nalazi u tački B, O_2 u C, O_3 u D i O_4 u tački E. Osa O_0x_0 ima svoj pravac u početnom položaju ose O_1x_1 .

3.2. Određivanje matrica prelaza i matrica transformacije

Geometrijski parametri koji opisuju kinematički lanac su predstavljeni u sledećoj tabeli.

Tabela 1: Geometrijski parametri

Zglob	θ_i	d_i	a_i	α_i	Promenljiva q_i
A (0-1)	θ_1	0	$l_1 = AB$	90°	θ_1
B (1-2)	θ_2	0	$l_2 = BC$	0	θ_2
C (2-3)	θ_3	0	$l_3 = CD$	0	θ_3
D (3-4)	θ_4	0	$l_4 = DE$	0	θ_4

Zamenom parametara iz tabele 1 u jednačinu (3) dobijamo sledeće matrice:

$${}_0T^3 = {}_0T^1 \cdot {}_1T^2 \cdot {}_2T^3 = \begin{pmatrix} \cos(\theta_1) \cdot \cos(\theta_2 + \theta_3) & -\cos(\theta_1) \cdot \sin(\theta_2 + \theta_3) & \sin(\theta_1) & \cos(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3)) \\ \sin(\theta_1) \cdot \cos(\theta_2 + \theta_3) & -\sin(\theta_1) \cdot \sin(\theta_2 + \theta_3) & -\cos(\theta_1) & \sin(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3)) \\ \sin(\theta_2 + \theta_3) & \cos(\theta_2 + \theta_3) & 0 & l_2 \sin(\theta_2) + l_3 \sin(\theta_2 + \theta_3) \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (8)$$

Transformaciona matrica mehanizma je određena jednačinom (7).

$$\begin{pmatrix} \cos(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) & -\cos(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) & \sin(\theta_1) & \cos(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ \sin(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) & -\sin(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) & -\cos(\theta_1) & \sin(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ \sin(\theta_2 + \theta_3 + \theta_4) & \cos(\theta_2 + \theta_3 + \theta_4) & 0 & l_2 \sin(\theta_2) + l_3 \sin(\theta_2 + \theta_3) + l_4 \sin(\theta_2 + \theta_3 + \theta_4) \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Transformaciona matrica za ceo mehanizam se može dobiti množenjem transformacionih matrica za pozicioniranje i orijentisanje mehanizama i ona glasi:

$$\begin{aligned} n_x &= \cos(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) \\ n_y &= \sin(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) \\ n_z &= \sin(\theta_2 + \theta_3 + \theta_4) \\ o_x &= -\cos(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) \\ o_y &= -\sin(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) \\ o_z &= \cos(\theta_2 + \theta_3 + \theta_4) \\ a_x &= \sin(\theta_1) \\ a_y &= -\cos(\theta_1) \\ a_z &= 0 \\ p_x &= \cos(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \end{aligned} \quad (10)$$

gde je:

$${}_0T^1 = \begin{pmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & l_1 \cdot \cos(\theta_1) \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & l_1 \cdot \sin(\theta_1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

$${}_1T^2 = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_2 \cdot \cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & l_2 \cdot \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

$${}_2T^3 = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & l_3 \cdot \cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & l_3 \cdot \sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (6)$$

$${}_3T^4 = \begin{pmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & l_4 \cdot \cos(\theta_4) \\ \sin(\theta_4) & \cos(\theta_4) & 0 & l_4 \cdot \sin(\theta_4) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (7)$$

Po analogiji sa mehanizmima robota, gornji stroj, strela i držač bagera zajedno sa zglobovima A, B i C čine mehanizam za pozicioniranje koji ima tri stepena pokretljivosti a kašika i spoj D čine mehanizam za orijentisanje koji ima samo jedan stepen pokretljivosti.

Uzevši sve ovo u obzir, transformaciona matrica mehanizma za pozicioniranje glasi:

$$\begin{aligned} p_y &= \sin(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ p_z &= l_2 \sin(\theta_2) + l_3 \sin(\theta_2 + \theta_3) + l_4 \sin(\theta_2 + \theta_3 + \theta_4) \end{aligned}$$

Jednačine (10) predstavljaju kinematički model bagerske opreme za iskopavanje pomoću kojih položaj i orijentacija kašike $n_x, n_y, n_z, o_x, o_y, o_z, a_x, a_y, a_z, p_x, p_y, p_z$ su date kao funkcija promenljivih koje odgovaraju stepenima kretanja mehanizma $\theta_1, \theta_2, \theta_3, \theta_4$.

Položaj bilo koje tačke na kašici u odnosu na fiksni koordinatni sistem se određuje množenjem transformacione matrice (9) sa vektorom kolone čime se dobijaju koordinate tačke u odnosu na pokretni koordinatni sistem. Koordinate vrha zuba kašike su:

$$\vec{r}_0 = {}_0T^4 \cdot \vec{r}_4 \quad (11)$$

$$\begin{pmatrix} \cos(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) & -\cos(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) & \sin(\theta_1) & \cos(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ \sin(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) & -\sin(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) & -\cos(\theta_1) & \sin(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ \sin(\theta_2 + \theta_3 + \theta_4) & \cos(\theta_2 + \theta_3 + \theta_4) & 0 & l_2 \sin(\theta_2) + l_3 \sin(\theta_2 + \theta_3) + l_4 \sin(\theta_2 + \theta_3 + \theta_4) \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\begin{aligned}
x_E &= \cos(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) \\
&+ l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\
y_E &= \sin(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) \\
&+ l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\
z_E &= l_2 \sin(\theta_2) + l_3 \sin(\theta_2 + \theta_3) \\
&+ l_4 \sin(\theta_2 + \theta_3 + \theta_4)
\end{aligned} \quad (12)$$

3.3. Odnos između hoda hidrauličkog cilindra i promenljivih kod kinematičkih zglobova

Zglobovi B i C kod mehanizma opreme se pokreću direktno pomoću hidrauličkih cilindara FG i HK a spoj D pokreće hidraulički cilindar LM pomoću mehanizma NPQD.

Prema [4] i zamenama na slici 3, između dužina hidrauličkih cilindara i uglova, dobijaju se sledeće jednačine:

$$\cos(\varphi_2) = \frac{s_2^2 - FB^2 - BG^2}{2 \cdot FB \cdot BG}; \quad s_2 = FG \quad (13)$$

$$\cos(\varphi_3) = \frac{s_3^2 - HC^2 - CK^2}{2 \cdot HC \cdot CK}; \quad s_3 = HK \quad (14)$$

$$\cos(\varphi_4) = \frac{s_4^2 - LN^2 - NM^2}{2 \cdot LN \cdot NM}; \quad s_4 = LM \quad (15)$$

a za mehanizam NPQD, zamenama dobija se:

$$\begin{aligned}
a &= \frac{NP^2 - PQ^2 + DQ^2 + ND^2 - 2 \cdot NP \cdot ND \cdot \cos(\varphi_p)}{2 \cdot NP \cdot DQ \cdot \sin(\varphi_p)} \\
b &= \frac{NP \cdot \cos(\varphi_p) - ND}{NP \cdot \sin(\varphi_p)} \\
\varphi_p &= \varphi_4 - \beta_4 - \delta_4
\end{aligned} \quad (16)$$

pa dobijamo:

$$\psi_4 = \arccos \left(\frac{a \cdot b - \sqrt{1 + b^2 - a^2}}{1 + b^2} \right) \quad (17)$$

U ovakvim uslovima ugaone promenljive pogonskih zglobova B, C i D su:

$$\theta_2 = \pi - \beta_2 - \gamma_2 - \varphi_2 \quad (18)$$

$$\theta_3 = 2\pi - \beta_3 - \gamma_3 - \varphi_3 \quad (19)$$

$$\theta_4 = 2\pi - \gamma_4 + \psi_4 \quad (20)$$

4. ODREĐIVANJE RADNOG PROSTORA. PRIMER

Za bager Volvo EC650, [5] i razmeru modela 1:50 sledeće geometrijske karakteristike su utvrđene:

$$\begin{aligned}
l_1 &= 720 \text{ mm}; \quad l_2 = 6560 \text{ mm}; \quad l_3 = 2750 \text{ mm}; \quad l_4 = 2135 \text{ mm} \\
FB &= 674,38 \text{ mm}; \quad BG = 3189,22 \text{ mm}; \quad HC = 3568,923 \text{ mm}; \\
CK &= 1275 \text{ mm}; \quad LN = 3099,428 \text{ mm}; \quad NM = 780 \text{ mm}; \\
\beta_2 &= 24,39^\circ; \quad \gamma_2 = 38,2^\circ; \quad \beta_3 = 149,18^\circ; \\
\gamma_3 &= 30,46^\circ; \quad NP = 620 \text{ mm}; \quad PQ = 500 \text{ mm}; \\
DQ &= 755,11 \text{ mm}; \quad ND = 250 \text{ mm}; \quad \beta_4 = 20,89^\circ; \\
\delta_4 &= 18,51^\circ; \quad \gamma_4 = 100,6^\circ.
\end{aligned}$$

Dužine hidrauličkih cilindara su:

$$s_2 = 2624 \dots 3714 \text{ mm}; \quad s_3 = 2733 \dots 4623 \text{ mm};$$

$$s_4 = 2468 \dots 3490 \text{ mm}.$$

Sa ovim vrednostima izračunate su jednačine (13)...(17), rezultati su ubačeni u jednačine (18)...(20) a onda je dobijena jednačina (12). Za $\theta_1 = 0^\circ = ct$ dobijeni rezultati su dati na slici 4,a. Radi poređenja na slici 4,b predstavljen je radni prostor iz prospekta.

Slika 4

5. ZAKLJUČAK

- Kinematički model kašike bagera, jednačina (10), je određen po analogiji sa mehanizmima kod industrijskih robota upotrebom Denavit-Hartenberg konvencije.
- Pogonski mehanizmi sa hidrauličkim cilindrima koji odgovaraju osama 2, 3 i 4 su uzeti kao deo

obrtnih zglobova B, C i D, i transformisani su u pogonske spojeve.

- Ovaj model se odnosi samo na položaj kinematičkih elemenata. Brzine, ubrzanja različitih tačaka na elementima se mogu lako odrediti diferenciranjem jednačina sličnih jednačini (11).

LITERATURA

- [1] “*Robots and automated machines in construction*”, IAARC Catalogue, 1998, <http://www.iaarc.org>
- [2] Carlos Balaguer, Mohamed Abderrahim, “*Robotics and Automation in Construction*”, InTech Europe, 2008, ISBN 978-953-7619-13-8
- [3] Richard S. Hartenberg, Jacques Denavit, “*Kinematic synthesis of linkages*”, McGraw Hill, 1964
- [4] Raicu, A., Bruja, A., “*Mecanisme*”, UTCB, 1996
- [5] Brochure “*Volvo Excavators EC140D*”

Determining the Kinematics Model of a Bucket Excavator's Digging Equipment

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This paper presents a method to build the kinematics model of a bucket excavator's digging equipment. The applied method is the one used for the industrial robots' mechanisms and uses Denavit-Hartenberg convention

Keywords: kinematics; bucket excavator; Denavit-Hartenberg

1. INTRODUCTION

One of the development paths in construction robotics and automation is to automate and robotize existing equipment [1, 2]. Automating and robotizing construction equipment assumes the modelling of the equipment and also its technological process from the kinematic and dynamic point of view. In order to create the model, there are in the specialty literature well-known methods, each of which is oriented to some particular technical field. For construction machinery it's useful to be considered like being composed of 2 parts, the chassis / mobile platform / carrying module and the work equipment, respectively. The first part can be modelled by analogy with mobile robots and the second part by analogy with industrial robots.

Figure 1: Excavator

The work equipment of the bucket excavator, figure 1, can be considered as an open loop kinematic chain composed of the following kinematic elements: undercarriage, boom, stick and bucket and the joints between these elements can be considered driving joints. For example, the driving mechanism with a hydraulic cylinder between boom and stick is considered as being part of the driving joint that connects the 2 elements, transforming it from driven joint to driving joint. Taking these into account, the bucket excavator's digging equipment has 4 degrees of mobility. These transformations are carried out in order to study the kinematics of this mechanism with the method used for positioning and orientation mechanisms of the industrial robots.

The problem of kinematic modelling can be formulated as follows: knowing the variables of the

driving joints we need to find out the position and orientation of the end-effector. The kinematics model has 2 major uses. First one is for determining the workspace of the equipment, and the second one is for determining the inverse kinematic model.

2. METHOD PRESENTATION

In this method to every kinematic element is attached a reference frame according to the *Denavit-Hartenberg* [3] convention. For the open loop kinematic chain composed of consecutive kinematic elements $i-2$, $i-1$, i and $i+1$ and the kinematic joints linking the elements, figure 2, the choosing of the reference frames linked to the elements $i-1$ and i is carried out as follows:

1. direction of the z_{i-1} axis that link the element $i-1$ to i is the joints axis;
2. x_{i-1} axis is linked to the element $i-1$ and is chosen to the common normal of the z_{i-1} and z_{i-2} axes oriented from O_{i-2} to O_{i-1} ;
3. Similarly the axis x_i is chosen as common normal of the rotation axes z_{i-1} and z_i oriented from O_{i-1} to O_i ;
4. The origins of the reference frames are chosen in the intersection points between the common normal and the axis of the revolute joint, respectively O_{i-2} and O_{i-1} ;
5. The name of the joint is given by the highest order of the elements that complete the joint.
6. The y axes are chosen so that the reference frame is orthogonal.

Figure 2: The Denavit-Hartenberg convention

The geometrical parameters which entirely describe any revolute or translational joint in the kinematic chain are as follows:

θ_i	Is the joints angle from axis x_{i-1} to axis x_i , measured counter clockwise, around axis z_{i-1}
d_i	Is the distance from the origin O_{i-1} to the intersection of the axis z_{i-1} with axis x_i , measured along the axis z_{i-1}
a_i	Is the distance from the intersection of the axis z_{i-1} with x_i to the origin O_i , measured along the axis x_i (or the least distance between z_{i-1} and z_i axes)
α_i	Is the angle between the z_{i-1} axis with the axis z_i , by rotating it around x_i axis counter clockwise.

The transition from reference frame "i-1", attached to the element i-1, to the reference frame "i", attached to the element i, is carried out as follows:

1. rotation around z_{i-1} axis by angle θ_i , in order to overlap the x_{i-1} axis with x_i ;
2. a translation along the axis z_{i-1} with the distance d_i , which brings the origin O_{i-1} in the intermediate position H_{i-1} (fig.2);
3. a new translation along the x_{i-1} axis by the a_i distance, thus overlapping the origin O_{i-1} to O_i ;

$${}_{i-1}T^i = \begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) \cdot \cos(\alpha_i) & \sin(\theta_i) \cdot \sin(\alpha_i) & a_i \cdot \cos(\theta_i) \\ \sin(\theta_i) & \cos(\theta_i) \cdot \cos(\alpha_i) & -\cos(\theta_i) \cdot \sin(\alpha_i) & a_i \cdot \sin(\theta_i) \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (3)$$

The said transition matrix (3) sets the position and orientation of the reference frame "i", attached to the element i in relation to the reference frame "i-1", attached to the element i-1.

3. ATTAINING THE KINEMATIC MODEL OF THE BUCKET EXCAVATOR'S DIGGING EQUIPMENT

3.1 Attaching reference frames

The kinematic scheme of the bucket excavator's digging equipment is presented in figure 3. In the figure

Figure 3: The kinematic scheme of the bucket excavator's digging equipment

4. finally a second rotation around x_i axis by an α_i angle, in order to overlap z_{i-1} axis with z_i .

Each one of these 4 movements can be expressed using an elementary rotation or translation matrix and the resulting movement being the product of the matrices:

$${}_{i-1}T^i = Rot(z_{i-1}, \theta_i) \cdot Trans(z_{i-1}, d_i) \cdot Trans(x_{i-1}, a_i) \cdot Rot(x_i, \alpha_i) \quad (1)$$

Doing the multiplication of the elementary matrices:

$$\begin{pmatrix} \cos(\theta_i) & -\sin(\theta_i) & 0 & 0 \\ \sin(\theta_i) & \cos(\theta_i) & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & a_i \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos(\alpha_i) & -\sin(\alpha_i) & 0 \\ 0 & \sin(\alpha_i) & \cos(\alpha_i) & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (2)$$

The transition matrix from the reference frame "i-1", attached to the element i-1, to the reference frame "i", attached to the element i, is obtained:

were noted: 1 – upper carriage, 2 – boom, 3 – stick, 4 – bucket. These elements are linked between them through joints B, C and D, also the upper carriage is linked to the under carriage through Joint A. According to the method presented in the previous chapter, considering the under carriage being the element „i-2≡0” the O_0z_0 axis is vertical with the positive side going up and going through point A. For simplifying the calculus the origin O_0 is chosen in point A.

The axes O_1z_1 , O_2z_2 , O_3z_3 and O_4z_4 are attached to the upper carriage, boom, stick and bucket going through points A, B, C and D and are normal to the drawing plane and the sense going from the viewer to the drawing. The axis O_1x_1 has its direction and orientation by $\overline{AB}$, axis O_2x_2 has its direction and orientation by $\overline{BC}$, axis O_3x_3 has its direction and orientation by $\overline{CD}$, O_4x_4 has its direction and orientation by $\overline{DE}$. The origins of the reference frames are found to the intersection of the z axes with the x axes which means that O_1 find itself in point B, O_2 in C, O_3 in D and O_4 in point E. The axis O_0x_0 has its direction in the initial position of the O_1x_1 axis.

3.2. Determining the transition matrices and transformation matrices

The geometrical parameters that describe the kinematic chain are presented in the following table.

Table 1. The geometrical parameters

Joint	θ_i	d_i	a_i	α_i	Variable q_i
A (0-1)	θ_1	0	$l_1 = AB$	90°	θ_1
B (1-2)	θ_2	0	$l_2 = BC$	0	θ_2
C (2-3)	θ_3	0	$l_3 = CD$	0	θ_3
D (3-4)	θ_4	0	$l_4 = DE$	0	θ_4

Replacing in eq. (3) the parameters from table 1, we obtain the following matrices:

$${}^0T^3 = {}^0T^1 \cdot {}^1T^2 \cdot {}^2T^3 = \begin{pmatrix} \cos(\theta_1) \cdot \cos(\theta_2 + \theta_3) & -\cos(\theta_1) \cdot \sin(\theta_2 + \theta_3) & \sin(\theta_1) & \cos(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3)) \\ \sin(\theta_1) \cdot \cos(\theta_2 + \theta_3) & -\sin(\theta_1) \cdot \sin(\theta_2 + \theta_3) & -\cos(\theta_1) & \sin(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3)) \\ \sin(\theta_2 + \theta_3) & \cos(\theta_2 + \theta_3) & 0 & l_2 \sin(\theta_2) + l_3 \sin(\theta_2 + \theta_3) \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (8)$$

And the transformation matrix of the orientation mechanism is eq. (7).

The transformation matrix for the entire

$$\begin{pmatrix} \cos(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) & -\cos(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) & \sin(\theta_1) & \cos(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ \sin(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) & -\sin(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) & -\cos(\theta_1) & \sin(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ \sin(\theta_2 + \theta_3 + \theta_4) & \cos(\theta_2 + \theta_3 + \theta_4) & 0 & l_2 \sin(\theta_2) + l_3 \sin(\theta_2 + \theta_3) + l_4 \sin(\theta_2 + \theta_3 + \theta_4) \\ 0 & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

which means that the first three columns are the direction cosines of the mobile reference frame, and the last column the coordinates of the mobile reference frame's origin to the end-effector:

$$\begin{aligned} n_x &= \cos(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) \\ n_y &= \sin(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) \\ n_z &= \sin(\theta_2 + \theta_3 + \theta_4) \\ o_x &= -\cos(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) \\ o_y &= -\sin(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) \\ o_z &= \cos(\theta_2 + \theta_3 + \theta_4) \\ a_x &= \sin(\theta_1) \\ a_y &= -\cos(\theta_1) \\ a_z &= 0 \\ p_x &= \cos(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \end{aligned} \quad (10)$$

$${}^0T^1 = \begin{pmatrix} \cos(\theta_1) & 0 & \sin(\theta_1) & l_1 \cdot \cos(\theta_1) \\ \sin(\theta_1) & 0 & -\cos(\theta_1) & l_1 \cdot \sin(\theta_1) \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (4)$$

$${}^1T^2 = \begin{pmatrix} \cos(\theta_2) & -\sin(\theta_2) & 0 & l_2 \cdot \cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2) & 0 & l_2 \cdot \sin(\theta_2) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (5)$$

$${}^2T^3 = \begin{pmatrix} \cos(\theta_3) & -\sin(\theta_3) & 0 & l_3 \cdot \cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3) & 0 & l_3 \cdot \sin(\theta_3) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (6)$$

$${}^3T^4 = \begin{pmatrix} \cos(\theta_4) & -\sin(\theta_4) & 0 & l_4 \cdot \cos(\theta_4) \\ \sin(\theta_4) & \cos(\theta_4) & 0 & l_4 \cdot \sin(\theta_4) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (7)$$

By analogy with the robot's mechanisms we can consider that the upper carriage, boom and stick of the excavator along with joints A, B and C are composing the positioning mechanism, having 3 degrees of mobility and the bucket and joint D compose the orientation mechanism having only 1 degree of mobility.

Taking these into account, the transformation matrix of the positioning mechanisms is:

mechanism can be obtained by multiplying the transformation matrices of the positioning and orientation mechanisms which is:

$$\begin{aligned} p_y &= \sin(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ p_z &= l_2 \sin(\theta_2) + l_3 \sin(\theta_2 + \theta_3) + l_4 \sin(\theta_2 + \theta_3 + \theta_4) \end{aligned}$$

Equations (10) represent the kinematic model of the bucket excavator's digging equipment through which the position and orientation of the end-effector are set $n_x, n_y, n_z, o_x, o_y, o_z, a_x, a_y, a_z, p_x, p_y, p_z$ as a function of the variables corresponding to the degrees of mobility of the mechanism $\theta_1, \theta_2, \theta_3, \theta_4$.

The position of any point on the bucket in relation to the fixed reference frame is determined by multiplying the transformation matrix (9) with the column vector which gives the coordinates of the point in relation to the mobile reference frame. In particular, the coordinates of the bucket's tooth tip are as follows:

$$\vec{r}_0 = {}^0T^4 \cdot \vec{r}_4 \quad (11)$$

$$\begin{pmatrix} \cos(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) & -\cos(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) & \sin(\theta_1) & \cos(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ \sin(\theta_1) \cdot \cos(\theta_2 + \theta_3 + \theta_4) & -\sin(\theta_1) \cdot \sin(\theta_2 + \theta_3 + \theta_4) & -\cos(\theta_1) & \sin(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ \sin(\theta_2 + \theta_3 + \theta_4) & \cos(\theta_2 + \theta_3 + \theta_4) & 0 & l_2 \sin(\theta_2) + l_3 \sin(\theta_2 + \theta_3) + l_4 \sin(\theta_2 + \theta_3 + \theta_4) \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}$$

of which

$$\begin{aligned} x_E &= \cos(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ y_E &= \sin(\theta_1) \cdot (l_1 + l_2 \cos(\theta_2) + l_3 \cos(\theta_2 + \theta_3) + l_4 \cos(\theta_2 + \theta_3 + \theta_4)) \\ z_E &= l_2 \sin(\theta_2) + l_3 \sin(\theta_2 + \theta_3) + l_4 \sin(\theta_2 + \theta_3 + \theta_4) \end{aligned} \quad \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$$

3.3. The relation between the hydraulic cylinders strokes and variables of the driving kinematic joints

The joints B and C of the equipment's mechanism are driven directly by the hydraulic cylinders FG and HK and joint D is driven by the hydraulic cylinder LM through the four-bar mechanism NPQD.

According to [4] and using the substitutions in figure 3, between the lengths of the hydraulic cylinders and position angles of the driven bars the following eq. are written:

$$\cos(\varphi_2) = \frac{s_2^2 - FB^2 - BG^2}{2 \cdot FB \cdot BG}; \quad s_2 = FG \quad (13)$$

$$\cos(\varphi_3) = \frac{s_3^2 - HC^2 - CK^2}{2 \cdot HC \cdot CK}; \quad s_3 = HK \quad (14)$$

$$\cos(\varphi_4) = \frac{s_4^2 - LN^2 - NM^2}{2 \cdot LN \cdot NM}; \quad s_4 = LM \quad (15)$$

and for the four-bar mechanism NPQD, with the substitutions:

$$a = \frac{NP^2 - PQ^2 + DQ^2 + ND^2 - 2 \cdot NP \cdot ND \cdot \cos(\varphi_p)}{2 \cdot NP \cdot DQ \cdot \sin(\varphi_p)}$$

$$b = \frac{NP \cdot \cos(\varphi_p) - ND}{NP \cdot \sin(\varphi_p)} \quad (16)$$

$$\varphi_p = \varphi_4 - \beta_4 - \delta_4$$

we obtain

$$\psi_4 = \arccos\left(\frac{a \cdot b - \sqrt{1 + b^2 - a^2}}{1 + b^2}\right) \quad (17)$$

In these conditions the angular variables of the driving joints B, C and D are as follows:

$$\theta_2 = \pi - \beta_2 - \gamma_2 - \varphi_2 \quad (18)$$

$$\theta_3 = 2\pi - \beta_3 - \gamma_3 - \varphi_3 \quad (19)$$

$$\theta_4 = 2\pi - \gamma_4 + \psi_4 \quad (20)$$

4.DETERMINING THE WORK SPACE.

EXAMPLE

For the excavator Volvo EC650, [5] and a scale model to 1:50, were determined the following geometrical characteristics:

$l_1 = 720 \text{ mm}$; $l_2 = 6560 \text{ mm}$; $l_3 = 2750 \text{ mm}$; $l_4 = 2135 \text{ mm}$
 $FB = 674,38 \text{ mm}$; $BG = 3189,22 \text{ mm}$; $HC = 3568,923 \text{ mm}$;
 $CK = 1275 \text{ mm}$; $LN = 3099,428 \text{ mm}$; $NM = 780 \text{ mm}$;
 $\beta_2 = 24,39^\circ$; $\gamma_2 = 38,2^\circ$; $\beta_3 = 149,18^\circ$;
 $\gamma_3 = 30,46^\circ$; $NP = 620 \text{ mm}$; $PQ = 500 \text{ mm}$;
 $DQ = 755,11 \text{ mm}$; $ND = 250 \text{ mm}$; $\beta_4 = 20,89^\circ$;
 $\delta_4 = 18,51^\circ$; $\gamma_4 = 100,6^\circ$. The lengths of the hydraulic cylinders are as follows:
 $s_2 = 2624 \dots 3714 \text{ mm}$; $s_3 = 2733 \dots 4623 \text{ mm}$;
 $s_4 = 2468 \dots 3490 \text{ mm}$.

With these values the eq. (13)...(17) are computed, the results are pushed into eq. (18)...(20) and further the eq. (12) are computed. For $\theta_1 = 0^\circ = ct$ the obtained results are presented in figure 4,a. For comparison in figure 4,b is presented the work space from the product brochure.

Figure 4: Determining a work space

5. CONCLUSIONS

- The forward kinematic model of the bucket excavator's digging equipment, eq. (10), is determined by analogy with the industrial robot's mechanisms and using the Denavit-Hartenberg convention.
- The driving mechanisms with hydraulic cylinders corresponding to axes 2, 3 and 4 were considered as being part of the revolute joints B, C and D, transforming these from driven joint to driving joints. This fact allows that the mechanism to be studied as an open loop kinematic chain.
- The present model regards only the positions study of the kinematic elements. Velocities, accelerations of different points on the elements are determined easily by differentiating some similar eq. like eq. (11).

REFERENCES

- [1] "*Robots and automated machines in construction*", IAARC Catalogue, 1998, <http://www.iaarc.org>
- [2] Carlos Balaguer, Mohamed Abderrahim, "*Robotics and Automation in Construction*", InTech Europe, 2008, ISBN 978-953-7619-13-8
- [3] Richard S. Hartenberg, Jacques Denavit, "*Kinematic synthesis of linkages*", McGraw Hill, 1964
- [4] Raicu, A., Bruja, A., "*Mecanisme*", UTCB, 1996
- [5] Brochure "*Volvo Excavators EC140D*"