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# Primena GWO algoritma za generisanje zatvorene putanje u optimalnoj sintezi ravnih mehanizama

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*U radu je razmatran problem optimalne sinteze zglobnog četvorougaonog mehanizma i podesivog krivajno klipnog mehanizma za generisanje zatvorene putanje. Razmatrana su dva slučaja. U prvom slučaju cilj je izvršiti optimizaciju putanje zadatu skupom unapred definisanih tačaka. U drugom slučaju razmatran je višekriterijumski optimizacioni problem tj. izvršena je optimizacija putanje i hoda klizača. U procesu optimalne sinteze primenjen je algoritam sivog vuka.*
*Predloženi algoritam je testiran na odgovarajućim numeričkim primerima iz literature čime je pokazana njegova efikasnost.*

**Ključne reči: optimalna sinteza, krivajno klipni mehanizam, zglobni četvorougao, algoritam sivog vuka**

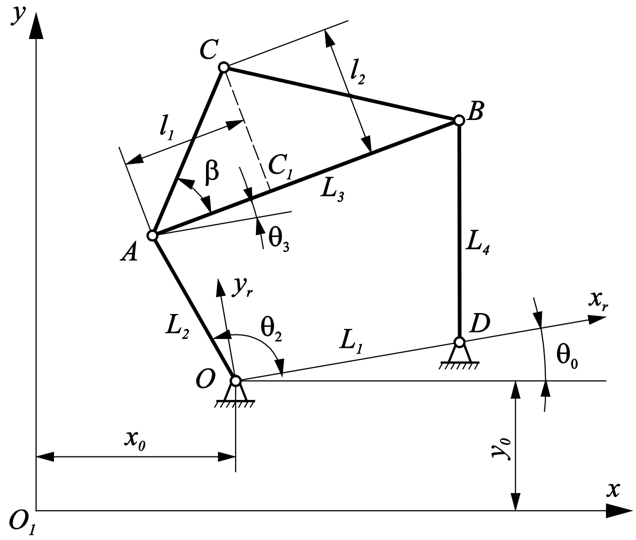
## 1. UVOD

Proučavanje mehanizama vrši se kroz dve etape. Prva etapa obuhvata postupak analize, dok druga druga etapa podrazumeva proces sinteze ili projektovanja mehanizama. Optimalna sinteza podrazumeva projektovanje mehanizma primenom postupka optimizacije [1]. Drugim rečima, optimalna sinteza jeste generisanje najboljeg mehanizma kroz ponovljeni postupak analize [2]. U cilju optimalne sinteze mehanizma potrebno je najpre izvršiti detaljnu analizu istog kako bi se definisale projektne promenljive, funkcija cilja i ograničenja. U nastavku rada biće razmatran problem optimalne sinteze podesivih ravnih mehanizama kao generatora putanje. Ovaj problem razmatran je u referencama [3,4,5].

## 2. FORMULACIJA PROBLEMA SINTEZE ZGLOBNOG ČETVOROUGAONOG MEHANIZMA

### 2.1. Poziciona analiza

Predmet analize je podesivi četvorougaoni mehanizam čiji su parametri prikazani na slici 1.




*Slika 1: Geometrija zglobnog četvorougaonog mehanizma*

Dužine članova mehanizma označene su sa $L_i$. Za potrebe dalje analize uvedena su dva koordinatna sistema – globalni koordinatni sistem $xO_1y$ i lokalni (relativni) koordinatni sistem $x_rOy_r$. Tačka C označava tačku spojke mehanizma koja treba da prođe kroz unapred zadate tačke na putanji.

Analiza zglobnog četvorougaonog mehanizma sprovodi se korišćenjem jednačina koje su poznate u literaturi. Tako se na osnovu Freudenstein-ove jednačine određuju uglovi $\theta_2$ i $\theta_3$, dok je položaj tačke C u odnosu na globalni koordinatni sistem $xO_1y$ definisan jednačinom (1):

$$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} C_{xr} \\ C_{yr} \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \tag{1}$$

gde je:

$$C_{xr} = L_2 \cdot \cos\theta_2 + l_1 \cdot \cos\theta_3 - l_2 \cdot \sin\theta_3$$

$$C_{yr} = L_2 \cdot \sin\theta_2 + l_1 \cdot \sin\theta_3 + l_2 \cdot \cos\theta_3$$

$x_0, y_0$ - koordinate tačke O u odnosu na globalni koordinatni sistem $xO_1y$.

### 2.2. Projektni parametri

U primerima sinteze razmatranog mehanizma kao generatora putanje sa propisanim vremenom optimizuje se devet projektnih promenljivih: $L_1, L_2, L_3, L_4, l_1, l_2, x_0, y_0$ i $\theta_0$. U slučaju sinteze mehanizma kao generatora putanje bez propisanog vremena optimizuju se još i ulazni uglovi krivaje $\theta_2^i \ (i = 1,...,N)$ koji odgovaraju unapred definisanim tačkama na putanji [6]. U opštem slučaju, vektor projektnih promenljivih može se definisati na sledeći način:

$$\mathbf{X} = \left[ L_1, L_2, L_3, L_4, l_1, l_2, x_0, y_0, \theta_0, \theta_2^1, \theta_2^2, ..., \theta_2^N \right] \tag{2}$$

gde je *N* broj zadatih tačaka. Za svaku projektnu promenljivu $x_j$ potrebno je definisati donju $x_j^{lb}$ i gornju $x_j^{ub}$ granicu, dok *NP* označava broj projektnih promenljivih.

$$x_j \in \left[ x_j^{lb}, x_j^{ub} \right], \forall x_j \in \mathbf{X}, j = 1,..., NP \tag{3}$$

### 2.3. Funkcija cilja i ograničenja

Funkcija cilja ima dva dela. Prvi deo funkcije cilja definiše grešku odstupanja sume kvadrata između skupa zadatih i skupa stvarnih tačaka koje u toku kretanja opisuje tačka C spojke. Drugi deo funkcije cilja uzima u obzir ograničenja. Prilikom definisanja optimizacionog problema

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zadaju se dva ograničenja u kojima su sadržane kaznene funkcije, tako da je:

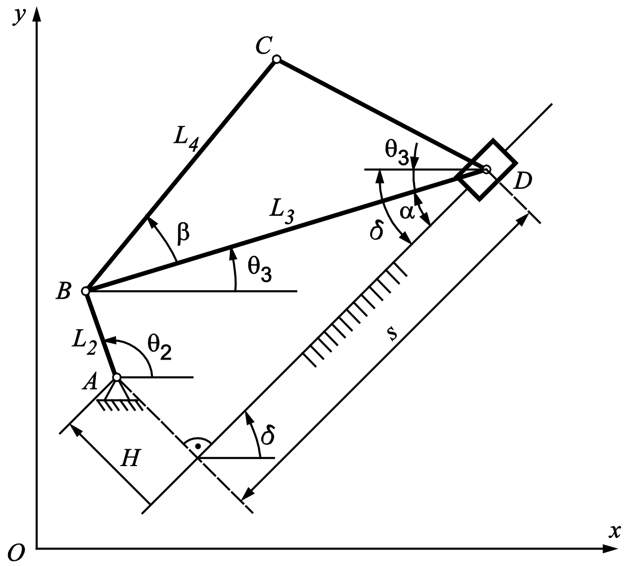
$$\min\left\{\sum_{i=1}^{N}\left[\left(C_{xd}^{i}-C_{x}^{i}\right)^{2}+\left(C_{yd}^{i}-C_{y}^{i}\right)^{2}\right]+M_1h_1(\mathbf{X})+M_2h_2(\mathbf{X})\right\}$$

gde je:
N – broj zahtevanih napadnih tačaka,
$\left(C_{xd}^{i},C_{yd}^{i}\right)$ - koordinate napadnih (zadatih) tačaka u odnosu na globalni koordinatni sistem,
$\left(C_{x}^{i},C_{y}^{i}\right)$ - koordinate koje generiše tačka C spojke (stvarne tačke),
$h_1(\mathbf{X})$ - odnosi se na uslove Grashof-a,
$h_2(\mathbf{X})$ - odnosi se na ulazni ugao krivaje $\theta_2^i\ (i=1,...,N)$
$M_1, M_2$ - kaznene funkcije kojima se kažnjava funkcija cilja kada ograničenja nisu zadovoljena.

## 3. FORMULACIJA PROBLEMA SINTEZE PODESIVOG KRIVAJNO KLIPNOG MEHANIZMA

### 3.1. Poziciona analiza

Predmet analize je podesivi (krivajno klipni) mehanizam čiji su parametri prikazani na slici 2. Dužine članova mehanizma označene su sa $L_i\ (i=2,3,4)$, dok su sa $\theta_i\ (i=2,3,4)$ označeni uglovi kojima se definiše položaj odgovarajućeg člana u odnosu na $x$-osu. Razmatrani mehanizam smešten je u ravni $xOy$.




*Slika 2: Podesivi krivajno klipni mehanizam*

Na početku, potrebno je definisati sledeće veličine [3]:

$$R_{\max}=L_2+L_4 \tag{5}$$

$$R_{\min}=\left|L_2-L_4\right| \tag{6}$$

gde $R_{\max}$ označava najduže rastojanje od tačke A do tačke C spojke, a $R_{\min}$ najkraće rastojanje između ove dve tačke (kada su pogonski član AB i spojka BC kolinearni).
Položaj tačke C u odnosu na koordinatni sistem xOy definiše se na sledeći način:

$$x_C=x_A+L_2\cos\theta_2+L_4\cos(\theta_3+\beta) \tag{7}$$

$$y_C=y_A+L_2\sin\theta_2+L_4\sin(\theta_3+\beta) \tag{8}$$

Ugao $\theta_2$ definiše položaj pogonskog člana $(0\le\theta_2\le 2\pi)$, dok se ugao $\theta_3$ određuje na osnovu relacije:

$$\theta_3=\delta+\arcsin\left[-\frac{H}{L_3}-\frac{L_2}{L_3}\sin(\theta_2-\delta)\right] \tag{9}$$

U opštem slučaju, veličina H (videti sliku 4.2) definiše se relacijom:

$$H=-L_2\sin(\theta_2-\delta)-L_3\sin(\theta_3-\delta) \tag{10}$$

Hod klizača C određen je veličinom s na sledeći način:

$$s=\sqrt{L_2^2+L_3^2-H^2+2L_2L_3\cos(\theta_2-\theta_3)} \tag{11}$$

### 3.2. Definisanje projektnih parametara i funkcije cilja

*Slučaj 1 – Optimizacija putanje*

U ovom slučaju optimizuje se devet projektnih promenljivih, pa se vektor projektnih promenljivih $\mathbf{X}$ definiše na sledeći način:

$$\mathbf{X}=\{L_2,L_3,L_4,\beta,\delta,H,x_A,y_A,\theta_2\} \tag{12}$$

Za svaku projektnu promenljivu $x_j\ (j=1,...,9)$ potrebno je definisati donju $x_j^{lb}$ i gornju $x_j^{ub}$ granicu:

$$x_j\in\left[x_j^{lb},x_j^{ub}\right],\ \forall x_j\in\mathbf{X},\ j=1,...,9 \tag{13}$$

Funkcija cilja definisana je sledećom relacijom:

$$\min\left\{\sum_{i=1}^{N}\left[\left(C_{xd}^{i}-C_{x}^{i}\right)^{2}+\left(C_{yd}^{i}-C_{y}^{i}\right)^{2}\right]+M_1h_1(\mathbf{X})+M_2h_2(\mathbf{X})\right\} \tag{14}$$

Prethodno definisana funkcija cilja podleže ograničenju koje je zadato u obliku nejednakosti:

$$g_1=\left|H+L_2\sin(\theta_2-\delta)\right|-L_3\sin\alpha\le 0 \tag{15}$$

Gde je $\alpha$ ugao kojim je definisan položaj člana BC podesivog mehanizma u odnosu na pravac kretanja klizača C (videti sliku 2).

$$\alpha=\delta-\theta_3 \tag{16}$$

*Slučaj 2 – Optimizacija putanje i hoda klizača*

U ovom slučaju primenom postupka višekriterijumske optimizacije izvršiće se istovremena minimizacija dve funkcije cilja. Optimizacioni problem definisan je na sledeći način:

$$\min\{f_1(\mathbf{X}),f_2(\mathbf{X})\} \tag{17}$$

Optimizacija putanje postiže se zadavanjem funkcije cilja $f_1(\mathbf{X})$ na način kao u nastavku:

$$\min\left\{\sum_{i=1}^{N}\left[\left(C_{xd}^{i}-C_{x}^{i}\right)^{2}+\left(C_{yd}^{i}-C_{y}^{i}\right)^{2}\right]+M_{1}h_{1}\left(\mathbf{X}\right)+M_{2}h_{2}\left(\mathbf{X}\right)\right\} \tag{18}$$

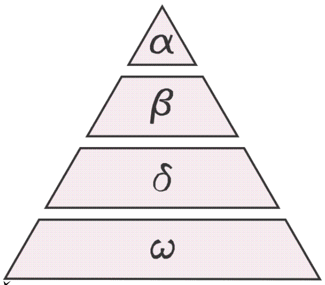
Optimizacija hoda klizača ostvaruje se zadavanjem funkcije cilja $f_2(\mathbf{X})$ kao što sledi:

$$f_2(\mathbf{X}) = \min(\Delta s) \tag{19}$$

gde je $\Delta s = abs(s_{\max} - s_{\min})$. Vektor projektnih promenljivih definiše se relacijom (12), odnosno na isti način kao u *Slučaju 1*.

## 4. ALGORITAM SIVOG VUKA

Algoritam sivog vuka pripada klasi biološki inspirisanih algoritama koji je kreirao Mirjalili [6]. Algoritam oponaša život ove vrste vukova u prirodi, tačnije principe po kojima funkcionišu, a to su stroga hijerarhija i grupni lov. Pošto pripadaju porodici zveri, razmatraju se kao predatori koji se nalaze na vrhu lanca ishrane. Sivi vukovi žive u čoporu koji u proseku broji 5-12 jedinki. U čoporu je zastupljena stroga društvena hijerarhija. Uloga vođe čopora pripada alfa jedinkama koje mogu biti kako mužjaci tako i ženke. Alfa donosi sve bitne odluke i komanduje čoporu. Drugim rečima, alfa jedinka predstavlja prvi nivo u hijerarhiji sivih vukova. Sledeći nivo su beta vukovi koji imaju zadatak da pruže pomoć alfa jedinki pri donošenju odluka, ali i da brinu o disciplini u čoporu. Delta vukovi predstavljaju treći nivo u hijerarhiji sivih vukova tj. podređeni su alfa i beta jedinkama. Delta vukovi su čuvari, izviđači, staratelji, ali i stari vukovi. Najniže rangirani sivi vukovi nazivaju se omega vukovi. Oni su podređeni svim prethodno nabrojanim kategorijama vukova. Iako često izgleda da omega jedinke nisu od posebne važnosti, imaju bitnu ulogu u održavanju strukture dominacije, a neretko preuzimaju brigu o mladima.




*Slika 3: Šematski prikaz hijerarhije sivih vukova*

Sledeća bitna osobina sivih vukova je njihovo ponašanje u lovu, odnosno mehanizam lova. Postoje tri osnovne strategije koje ovi predatori koriste u lovu [6]:

1. Praćenje, gonjenje i približavanje plenu
2. Opkoljavanje i uznemiravanje plena dok se ne smiri
3. Napad na plen

Primena algoritma sivog vuka (GWO) za rešavanje različitih optimizacionih problema može se videti u referencama [7-9].

### 4.1. Matematički model

Da bi se matematički modeliralo ponašanje sivih vukova, potrebno je najpre početnu populaciju u GWO algoritmu podeliti u četiri grupe: $\alpha,\beta,\delta$ i $\omega$. U GWO algoritmu lov tj. potragu za optimalnim rešenjem predvode prva tri najbolja rešenja koja se razmatraju kao $\alpha,\beta$ i $\delta$ vukovi, dok ih $\omega$ vukovi slede u tome.

Glavna faza u grupnom lovu je okruživanje plena i ova strategija u lovu može se modelirati sledećim jednačinama:

$$\mathbf{D} = \left|\mathbf{C}\cdot\mathbf{X_p}(t) - \mathbf{X}(t)\right| \tag{20}$$

$$\mathbf{X}(t+1) = \mathbf{X_p}(t) - \mathbf{A}\cdot\mathbf{D} \tag{21}$$

gde $t$ označava trenutnu iteraciju, $\mathbf{X_p}$ položaj plena, $\mathbf{X}$ je vektor položaja sivog vuka, dok se vektori $\mathbf{A}$ i $\mathbf{C}$ mogu izračunati korišćenjem sledećih izraza:

$$\mathbf{A} = 2\mathbf{a}\cdot\mathbf{r}_1 - \mathbf{a} \tag{22}$$

$$\mathbf{C} = 2\cdot\mathbf{r}_2 \tag{23}$$

gde su $\mathbf{r}_1,\mathbf{r}_2$ nasumični vektori iz opsega $[0,1]$, dok vektor $\mathbf{a}$ linearno opada od 2 do 0 tokom iteracija.

Naime, GWO algoritam polazi od pretpostavke da položaji $\alpha,\beta$ i $\delta$ vukova određuju položaj plena. Prva tri najbolja rešenja (položaja) razmatraju se kao položaji $\alpha,\beta$ i $\delta$ vukova, dok ostali agenti u pretrazi (omega vukovi) menjaju svoje položaje u odnosu na $\alpha,\beta$ i $\delta$ vukove.

Promena položaja omega vukova može se predstaviti sledećim jednačinama:

$$\begin{aligned}\mathbf{D}_\alpha &= \left|\mathbf{C}_1\cdot\mathbf{X}_\alpha - \mathbf{X}\right|;\\ \mathbf{D}_\beta &= \left|\mathbf{C}_2\cdot\mathbf{X}_\beta - \mathbf{X}\right|;\\ \mathbf{D}_\delta &= \left|\mathbf{C}_3\cdot\mathbf{X}_\delta - \mathbf{X}\right|;\end{aligned} \tag{24}$$

$$\begin{aligned}\mathbf{X}_1 &= \left|\mathbf{X}_\alpha - \mathbf{A}_1\cdot\mathbf{D}_\alpha\right|;\\ \mathbf{X}_2 &= \left|\mathbf{X}_\beta - \mathbf{A}_2\cdot\mathbf{D}_\beta\right|;\\ \mathbf{X}_3 &= \left|\mathbf{X}_\delta - \mathbf{A}_3\cdot\mathbf{D}_\delta\right|;\end{aligned} \tag{25}$$

$$\mathbf{X}(t+1) = \frac{\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3}{3} \tag{26}$$

### 4.2. Pseudo kod GWO algoritma

Proces pretrage u GWO algoritmu započinje generisanjem početne populacije, odnosno formiranjem čopora. U daljem iterativnom postupku $\alpha,\beta$ i $\delta$ vukovi procenjuju položaj plena, pri čemu svako potencijalno rešenje podrazumeva približavanje plenu. U tom smislu, kada je $|\mathbf{A}| \succ 1$ potencijalna rešenja teže udaljavanju od plena, odnosno za $|\mathbf{A}| \prec 1$ potencijalna rešenja se približavaju plenu.

U nastavku je dat pseudo kod GWO algoritma.

---

*Definisanje broja agenata (vukova) N i maksimalnog broja iteracija* $\max iter$

*Inicijalizacija početne populacije* $X_i\ (i = 1,2,...,n)$

*Inicijalizacija vektora* $\mathbf{a},\mathbf{A},\mathbf{C}$

*Izračunavanje fitnes vrednosti svakog agenta*

$X_\alpha =$ *najbolji agent*

$X_\beta =$ *drugi najbolji agent*

$X_\delta =$ *treći najbolji agent*

***while*** $(t < \max iter)$

    ***for*** *%% za svakog agenta pretrage*

*ažuriraj položaj trenutnog agenta pretrage na osnovu jednačine (26)*
***end for***
*Izračunavanje vektora* $\mathbf{a}, \mathbf{A}, \mathbf{C}$
*Izračunavanje fitnes vrednosti svakog agenta*
*Pronalaženje novih* $X_\alpha, X_\beta, X_\delta$
$t = t + 1$
*Sortiranje populacije na bazi fitnes vrednosti*
***for*** $i := (n/2) + 1, n$
*Ažuriraj položaj i-tog vuka na osnovu jednačine (27)*
***end for***
***end while***
*u suprotnom ispisati* $X_\alpha$
*Postprocesiranje rezultata*

## 5. NUMERIČKI PRIMER

### 5.1. Primer 1 - Optimalna sinteza zglobnog četvoročlanog mehanizma

Razmatra se problem sinteze zglobnog četvoročlanog mehanizma kao generatora putanje bez propisanog vremena. Tačka $S$ spojke treba da prođe kroz skup od osamnaest prethodno definisanih tačaka.

Na osnovu relacije (2), projektne promenljive za razmatrani problem definišu se na sledeći način:

$$\mathbf{X} = \left[ L_1, L_2, L_3, L_4, l_1, l_2, x_o, y_o, \theta_0, \theta_2^1 \right] \tag{27}$$

Koordinate željenih (unapred definisanih) tačaka na putanji su:

$$\left\{ C_d^i \right\} = \left\{ \begin{array}{llllll} (0.5,1.1); & (0.4,1.1); & (0.3,1.1); & (0.2,1.0); & (0.1,0.9); & (0.05,0.75); \\ (0.02,0.6); & (0,0.5); & (0,0.4); & (0.03,0.3); & (0.1;0.25); & (0.15,0.2); \\ (0.2,0.3); & (0.3,0.4); & (0.4,0.5); & (0.5,0.7); & (0.6,0.9); & (0.6,1.0) \end{array} \right\} \tag{28}$$

Ulazni ugao krivaje određuje se korišćenjem sledeće relacije:

$$\left\{ \theta_2^i \right\} = \left\{ \theta_2^1, \theta_2^1 + 20 \cdot i \right\},\ i = 1,...,17 \tag{29}$$

Za svaku projektnu promenljivu definisane su granice:

$$\begin{gathered} 0 \le L_1, L_2, L_3, L_4 \le 50\ ; \\ -50 \le l_1, l_2, x_o, y_o \le 50\ ; \\ 0 \le \theta_0,\ \theta_2^1 \le 2\pi \end{gathered} \tag{30}$$

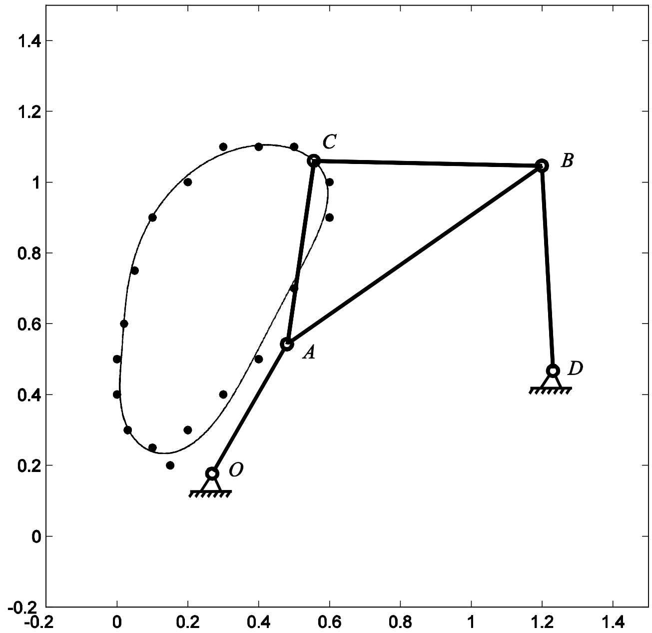
Parametri algoritma GWO koji se primenjuje u postupku optimizacije su: $\max iter = 50$ (maksimalan broj iteracija), dok broj agenata iznosi 30 (SearchAgents_no=30).

Korišćenjem algoritma sivog vuka u optimizacionom postupku dobija se mehanizam čiji su projektni parametri dati u Tabeli 1. Poređenja radi, u pomenutoj tabeli prikazani su i rezultati drugih autora [10, 11, 12, 13, 14] koji su isti problem rešavali primenom različitih optimizacionih algoritama. Dakle, cilj je da se na primeru zglobnog četvoročlanog mehanizma, koji je razmatran u literaturi, pokaže efikasnost GWO algoritma.

*Tabela 1: Uporedni prikaz projektnih parametara dobijenih primenom različitih optimizacionih algoritama*

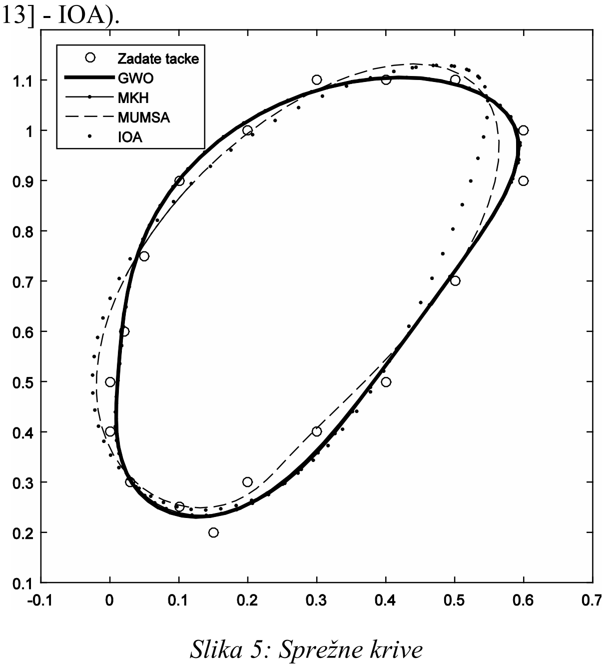
| | Kunjur, Krishnamurty [10] (GA) | Ortiz i dr. [13] (IOA) | Cabrera i dr. [11] (GA) | Cabrera i dr. [12] (MUMSA) | Bulatović i dr.[14] (MKH) | GWO |
|---|---|---|---|---|---|---|
| | 0.274853 | 0.245216 | 0.237803 | 0.297057 | 0.42180 | **0.41970** |
| | 1.180253 | 6.38294 | 4.828954 | 3.913095 | 0.87821 | **0.98857** |
| | 2.138209 | 2.620532 | 2.056456 | 0.849372 | 0.58013 | **0.58240** |
| | 1.879660 | 4.040435 | 3.057878 | 4.453772 | 1.00429 | **1.10427** |
| | -0.833592 | 1.139106 | 0.767038 | 1.6610626 | 0.35907 | **0.40047** |
| | -0.378770 | 1.866109 | 1.850828 | 2.7387359 | 0.38081 | **0.44529** |
| | 1.132062 | 1.891805 | 1.776808 | 2.806964 | 0.26886 | **0.28691** |
| | 0.663433 | -0.761339 | -0.641991 | 4.853543 | 0.17715 | **0.09855** |
| | 4.354224 | 1.187751 | 1.002168 | -1.309243 | 0.29294 | **0.33948** |
| | 2.558625 | 0.000000 | 0.226186 | 4.853543 | 0.88595 | **0.84827** |
| greška | 0.043 | 0.0349 | 0.0337 | 0.0196 | 0.00911 | **0.00908** |

Na slici 4 prikazan je najbolji mehanizam dobijen primenom algoritma sivog vuka (GWO) kao i putanja koju on generiše.




*Slika 4: Najbolji mehanizam u primeru 1*

Na slici 5 uporedno su prikazane putanje koje opisuje tačka C datog mehanizma, a koje su dobijene primenom različitih optimizacionih algoritama (Bulatović i dr.[14] – MKH, Cabrera i dr. [12] – MUMSA algorithm, Ortiz i dr. [13] - IOA).




*Slika 5: Sprežne krive*

5.2 Primer 2 – Optimalna sinteza podesivog (krivajno klipnog) mehanizma

Kao što je prethodno rečeno, u primeru optimalne sinteze krivajno klipnog mehanizma biće razmatrana dva slučaja. U prvom slučaju cilj je optimizacija putanje tj. sinteza krivajno klipnog mehanizma kao generatora putanje. U drugom slučaju, cilj je izvršiti istovremenu (simultanu) optimizaciju putanje i hoda klizača.

<u>*Slučaj 1 - Optimizacija putanje*</u>

Na početku, potrebno je definisati vektor projektnih promenljivih:

$$\mathbf{X} = \{ L_2, L_3, L_4, \beta, \delta, H, x_A, y_A, \theta_2 \} \qquad (31)$$

Koordinate željenih (unapred zadatih) tačaka na putanji iste su kao u Primeru 1, tj. definisane su relacijom (4.28). Naime, cilj je izvršiti sintezu različitih tipova ravnih mehanizama koji generišu istu putanju. Ovde je generator putanje (koja je zadata istim skupom tačaka kao u *Primeru 1* podesivi mehanizam prikazan na sl. 2.

Za svaku projektnu promenljivu definisane su granice:

$$\begin{gathered} 0 \le L_2, L_3, L_4 \le 50\,; \\ -50 \le H, x_A, y_A \le 50\,; \\ 0 \le \theta_2, \beta, \delta \le 2\pi \end{gathered} \qquad (32)$$

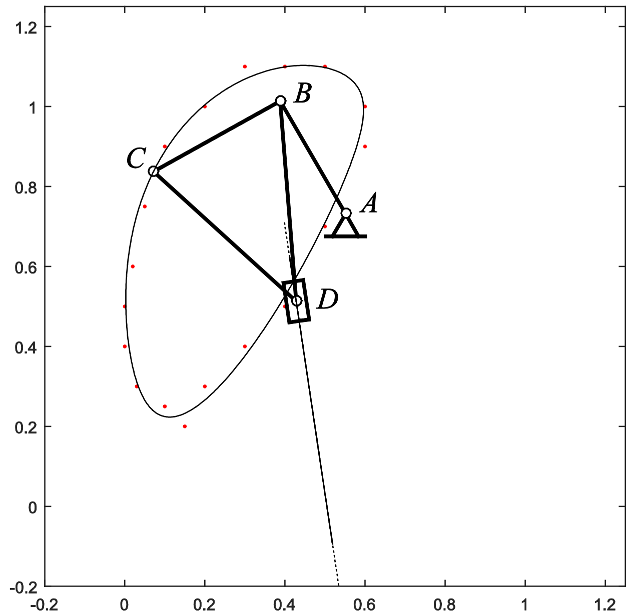
Parametri algoritma GWO koji se primenjuje u postupku optimizacije isti su kao u prethodnom slučaju. Primenom algoritma sivog vuka dobija se podesivi mehanizam čiji su projektni parametri prikazani u Tabeli 2.

*Tabela 2 Optimalne vrednosti projektnih parametara u Slučaju 1*

| Projektni parametri | **Optimalne vrednosti GWO** |
|---|---|
| $L_2$ | 0.32563 |
| $L_3$ | 0.50213 |
| $L_4$ | 0.36236 |
| $\beta$ | -1.14240 |
| $\delta$ | -1.42252 |
| $H$ | 0.15411 |
| $x_A$ | 0.55131 |
| $y_A$ | 0.73173 |
| $\theta_2$ | 0.53607 |
| $\Delta s$ | 0.52426 |
| greška | 0.00994 |

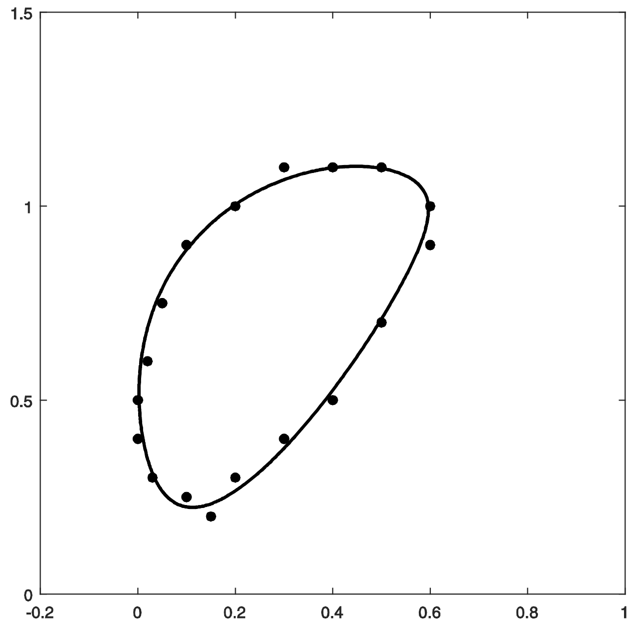
Kako u dostupnoj literaturi ne postoje reference u kojima se razmatra optimizacija putanje ovog tipa ravnih mehanizama, to nije moguće dati uporedne rezultate.

Na slici 6 prikazan je najbolji mehanizam dobijen primenom algoritma sivog vuka (GWO) kao i putanja koju on generiše.




*Slika 6: Najbolji mehanizam u primeru 2 – Slučaj 1*

Na slici 7 prikazana je putanja koje opisuje tačka M razmatranog podesivog mehanizma, a ista je dobijena primenom algoritma sivog vuka.



*Slika 7: Sprežna kriva*

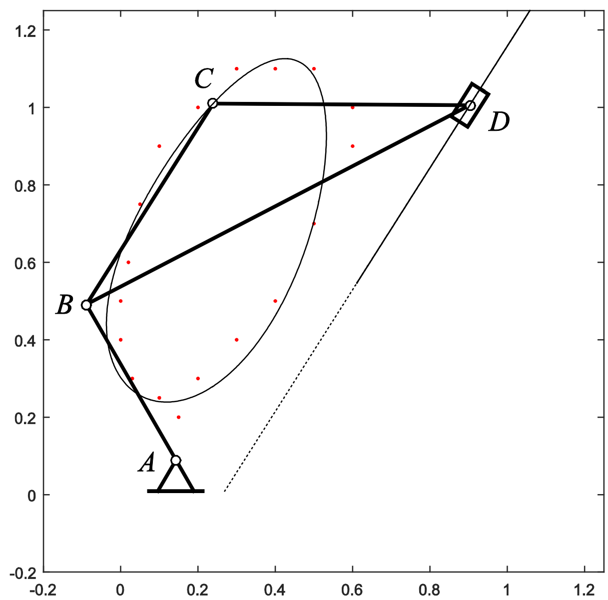
*Slučaj 2 - Optimizacija putanje i hoda klizača*

Vektor projektnih promenljivih u ovom slučaju definiše se na isti način kao u *Slučaju 1*, odnosno relacijom (31). Koordinate željenih tačaka na putanji iste su kao u prethodnim primerima, jer je cilj generisati istu putanju korišćenjem dva tipa ravnih mehanizama. Za razliku od *Slučaja 1* gde se razmatra samo optimizacija putanje, ovde se vrši istovremena optimizacija putanje i hoda klizača. Korišćenjem algoritma sivog vuka u postupku višekriterijumske optimizacije generiše se veći broj podesivih mehanizama (rešenja) sa različitim vrednostima projektnih parametara. U Tabeli 3 prikazani su projektni parametri za četiri najbolja rešenja (mehanizma) koja su dobijena u postupku optimizacije.

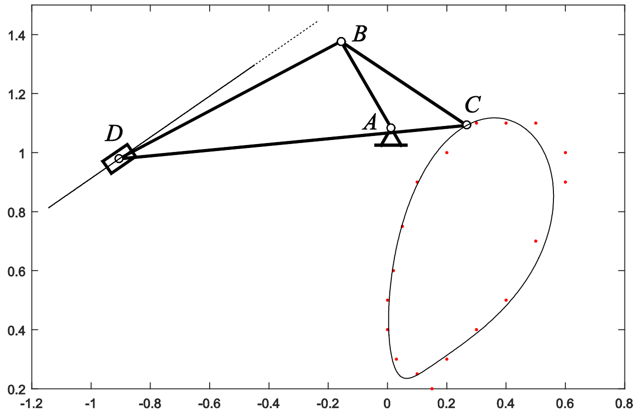
*Tabela 3: Optimalne vrednosti projektnih parametara za četiri najbolja rešenja u Slučaju 2*

| Proj. promenljive | Opt. vred. (Primer 1) | Opt. vred. (Primer 2) | Opt. vred. (Primer 3) | Opt. vred. (Primer 4) |
|---|---|---|---|---|
| $L_2$ | 0.46357 | 0.33887 | 0.46319 | 0.30143 |
| $L_3$ | 1.11772 | 0.84714 | 1.03291 | 1.50697 |
| $L_4$ | 0.61417 | 0.51169 | 0.56197 | 1.02869 |
| $\beta$ | 6.81167 | 8.34521 | 0.53718 | 2.39888 |
| $\delta$ | -5.27894 | -2.53315 | 1.00504 | 3.61733 |
| $H$ | 0.14973 | 0.43825 | 0.15378 | 0.90908 |
| $x_A$ | 0.14271 | 0.01185 | 0.14909 | -0.36041 |
| $y_A$ | 0.08941 | 1.08347 | 0.14390 | 1.47563 |
| $\theta_2$ | -5.22872 | 1.27817 | 1.04741 | 1.29291 |
| $f_{1\min}$ | 0.03292 | 0.03782 | 0.03305 | 0.03602 |
| $f_{2\min}$ | 0.48965 | 0.48626 | 0.49073 | 0.48052 |

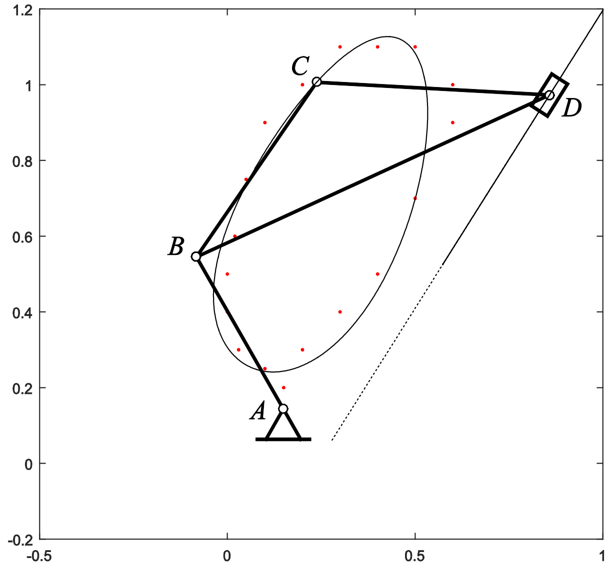
Na slikama 8 - 11 prikazani su mehanizmi (sa parametrima iz Primera 1, 2, 3 i 4, respektivno) koji su dobijeni postupkom višekriterijumske optimizacije i primenom algoritma sivog vuka (GWO). Na istim slikama prikazane su i putanje koje generišu podesivi mehanizmi dobijeni u Primerima 1, 2, 3 i 4.



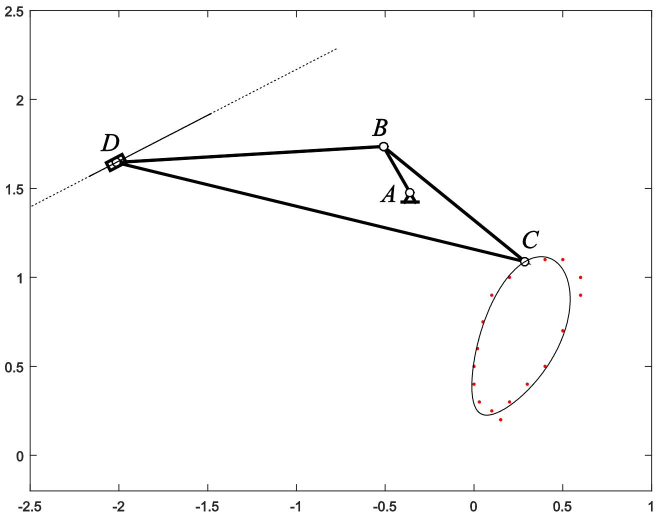

*Slika 8: Mehanizam i njegova putanja – Primer 1*




*Slika 9: Mehanizam i njegova putanja – Primer 2*




*Slika 10: Mehanizam i njegova putanja – Primer 3*




*Slika 11: Mehanizam i njegova putanja – Primer 4*

## 6. ZAKLJUČAK

U ovom radu razmatran je problem optimalne sinteze podesivih ravnih mehanizama kao generatora putanje. U rešavanju problema optimalne sinteze primenjen je algoritam sivog vuka. U dostupnoj literaturi ne postoje istraživanja u kojima je problem optimalne sinteze mehanizama rešavan primenom GWO algoritma. Testiranje efikasnosti algoritma sivog vuka izvršeno je na primeru optimalne sinteze četvoročlanog mehanizma kao generatora putanje (Primer 1). Rezultati dobijeni pri-menom GWO algoritma bolji su od rezultata u [10,11,12,13], odnosno približni rezultatima u [14] (videti Tabelu 1). Zatim su razmatrana dva slučaja optimalne sinteze podesivog krivajno klipnog mehanizma (Primer 2). Najpre, u *Slučaju 1*, izvršena je optimalna sinteza putanje koja je definisana istim skupom tačaka kao u Primeru 1. Dobijeni rezultati su odlični, tj. putanja je gotovo identična onoj koju generiše četvoročlani mehanizam (Slika 7). Odstupanja stvarne od željene putanje su minimalna. U *Slučaju 2* izvršena je višekriterijumska optimizacija putanje i hoda klizača. Međutim, rezultati dobijeni u ovom slučaju nisu zadovoljavajući. Naime, veličina $\Delta s$ je neznatno smanjena u odnosu na *Slučaj 1* (jednokriterijumska optimi-zacija), dok postoji drastično odstupanje stvarne od željene (zadate) putanje (videti slike 8 - 11).

Na osnovu prethodnog, nameće se zaključak da algoritam sivog vuka daje odlične rezultate u slučaju jednokriterijumske optimizacije. Međutim, u postupku višekri-terijumske optimizacije primena GWO algoritma nije dala očekivano dobre rezultate. U tom smislu, potrebno je izvršiti određene modifikacije tj. poboljšanja standardnog GWO algoritma kako bi se poboljšala njegova efikasnost u postupku MOO.

Na kraju, treba istaći da u dostupnoj literaturi ne postoje reference u kojima je razmatran problem višekriterijumske optimizacije putanje i hoda klizača podesivog krivajno klipnog mehanizma. Ovim je dat izvestan doprinos proučavanju problema optimalne sinteze podesivih ravanskih mehanizama.

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# Application of GWO Algorithm for Closed Path Generation in Optimal Synthesis of Planar Mechanisms

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*The problem of optimal synthesis of four-bar linkage and adjustable slider crank mechanism for generating a closed path was considered in this paper. Two cases were considered. In the first case, the goal is to optimize the path given by a set of predefined points. In the second case, a multi-criteria optimization problem is considered, ie. the path and adjustable length of slider were optimized. The grey wolf algorithm was applied in the process of optimal synthesis. The proposed algorithm has been tested on appropriate numerical examples from the literature to demonstrate its efficiency.*

**Key words: Optimal synthesis, Adjustable slider crank mechanism, Four-bar linkage, Grey wolf optimizer**

## 1. INTRODUCTION

Mechanisms are studied through two stages. The first stage involves the process of analysis, while the second stage involves the process of synthesis or design of mechanisms. Optimal synthesis involves the design of the mechanism using the optimization process [1]. In other words, optimal synthesis means the generation of the best mechanism through the repeated procedure of analysis [2]. For the purpose of optimal synthesis of the mechanism, it is first necessary to perform a detailed analysis of the mechanism in order to define project variables, objective function and constraints. Further more, the problem of optimal synthesis of adjustable planar mechanisms as a path generator will be discussed. This problem has been discussed in references [3,4,5].

## 2. FORMULATION OF THE PROBLEM OF SYNTHESIS OF THE FOUR-BAR LINKAGE

### 2.1. Position analysis

The subject of analysis is an adjustable four-bar linkage whose parameters are shown in Figure 1.

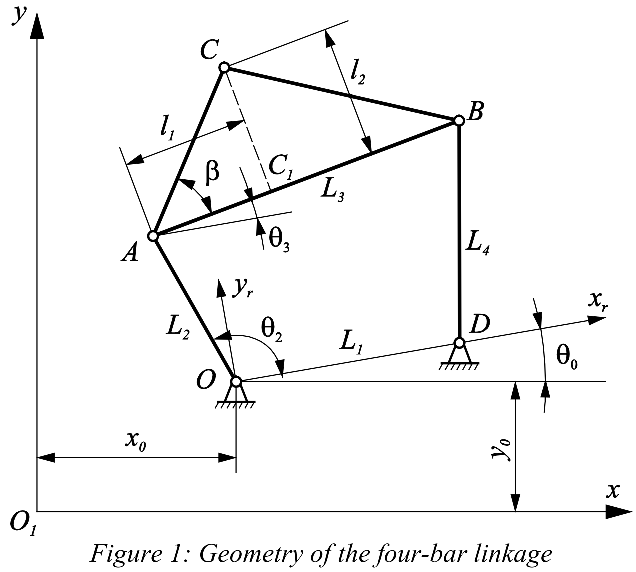


*Figure 1: Geometry of the four-bar linkage*

The lengths of the mechanism links are indicated by $L_i$. For the purpose of further analysis, two coordinate systems were introduced - the global coordinate system $xO_1y$ and the local (relative) coordinate system $x_rOy_r$. Point $C$ indicates the point of the coupler that must pass through the preset points on the path.

The analysis of the four-bar linkage is performed using equations known in the literature. Thus, on the basis of Freudenstein equation, the angles $\theta_2$ and $\theta_3$ are determined, while the position of point C with respect to the global coordinate system $xO_1y$ is defined by equation (1):

$$\begin{bmatrix} C_x \\ C_y \end{bmatrix} = \begin{bmatrix} \cos\theta_0 & -\sin\theta_0 \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} C_{xr} \\ C_{yr} \end{bmatrix} + \begin{bmatrix} x_0 \\ y_0 \end{bmatrix} \tag{1}$$

where:

$C_{xr} = L_2 \cdot \cos\theta_2 + l_1 \cdot \cos\theta_3 - l_2 \cdot \sin\theta_3$

$C_{yr} = L_2 \cdot \sin\theta_2 + l_1 \cdot \sin\theta_3 + l_2 \cdot \cos\theta_3$

$x_0, y_0$ - the coordinates of the point $O$ with respect to the global coordinate system $xO_1y$.

### 2.2. Design parameters

In the examples of synthesis of the considered mechanism as a path generator with prescribed time, nine design variables are optimized: $L_1, L_2, L_3, L_4, l_1, l_2, x_0, y_0$ and $\theta_0$. In the case of the synthesis of the mechanism as a path generator without prescribed time, the input angles of the crank $\theta_2^i \ (i = 1,...,N)$ corresponding to predefined points on the path are also optimized [6]. In general, the vector of design variables can be defined as follows:

$$\mathbf{X} = \left[ L_1, L_2, L_3, L_4, l_1, l_2, x_0, y_0, \theta_0, \theta_2^1, \theta_2^2, ..., \theta_2^N \right] \tag{2}$$

where $N$ is the number of given points. For each design variable it is necessary to define the lower $x_j^{lb}$ and upper $x_j^{ub}$ bounds, while the $NP$ indicates the number of design variables.

$$x_j \in \left[ x_j^{lb}, x_j^{ub} \right], \forall x_j \in \mathbf{X}, j = 1,..., NP \tag{3}$$

### 2.3. Objective function and constraints

The objective function has two parts. The first part of the objective function defines the error of the deviation of the sum of the squares between the set of given and the set of real points described by the point $C$ of the coupler during the

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motion. The second part of the objective function considers constraints. When defining an optimization problem, two constraints that contain penal functions are imposed, so that:

$$\min\left\{\sum_{i=1}^{N}\left[\left(C_{xd}^{i}-C_{x}^{i}\right)^{2}+\left(C_{yd}^{i}-C_{y}^{i}\right)^{2}\right]+M_{1}h_{1}(\mathbf{X})+M_{2}h_{2}(\mathbf{X})\right\}$$

where:
$N$ – the number of required set points,
$\left(C_{xd}^{i}, C_{yd}^{i}\right)$ - the coordinates of set points with respect to the global coordinate system,
$\left(C_{x}^{i}, C_{y}^{i}\right)$ - the coordinates generated by the point C of coupler (real points),
$h_1(\mathbf{X})$ - refers to the conditions of a Grashof,
$h_2(\mathbf{X})$ - refers to the input angle of the crank $\theta_2^i\ (i=1,...,N)$,
$M_1, M_2$ - penalty functions that penalize the objective function when constraints are not satisfied.

## 3. FORMULATION OF THE PROBLEM OF SYNTHESIS OF ADJUSTABLE SLIDER CRANK MECHANISM

### 3.1. Position analysis

The subject of analysis is an adjustable (slider crank) mechanism whose parameters are shown in Figure 2. The lengths of the links are indicated by $L_i\ (i=2,3,4)$, while the angles defining the position of the corresponding link with respect to the $x$-axis are indicated by $\theta_i\ (i=2,3,4)$. The mechanism is placed in the $xOy$ plane.

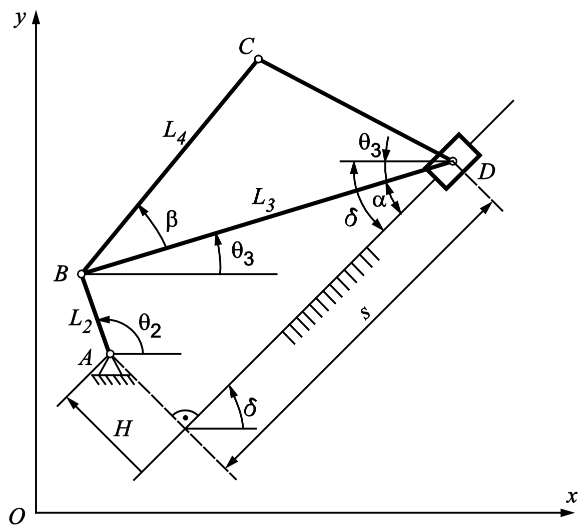


*Figure 2: Adjustable slider crank mechanism*

At the outset, it is necessary to define the following sizes [3]:

$$R_{\max} = L_2 + L_4 \quad (5)$$

$$R_{\min} = \left|L_2 - L_4\right| \quad (6)$$

where $R_{\max}$ indicates the longest distance from point $A$ to point $C$ of the coupler, and $R_{\min}$ is the shortest distance between these two points (when the driving member $AB$ and the coupler $BC$ are collinear).
The position of point $C$ with respect to the coordinate system $xOy$ is defined as follows:

$$x_C = x_A + L_2\cos\theta_2 + L_4\cos(\theta_3+\beta) \quad (7)$$

$$y_C = y_A + L_2\sin\theta_2 + L_4\sin(\theta_3+\beta) \quad (8)$$

The angle $\theta_2$ defines the position of the driving link $(0 \le \theta_2 \le 2\pi)$, while the angle $\theta_3$ is determined by the relation:

$$\theta_3 = \delta + \arcsin\left[-\frac{H}{L_3} - \frac{L_2}{L_3}\sin(\theta_2-\delta)\right] \quad (9)$$

In general, the size $H$ (see Figure 4.2) is defined by the relation:

$$H = -L_2\sin(\theta_2-\delta) - L_3\sin(\theta_3-\delta) \quad (10)$$

Adjustable value $s$ is determined as follows:

$$s = \sqrt{L_2^2 + L_3^2 - H^2 + 2L_2L_3\cos(\theta_2-\theta_3)} \quad (11)$$

### 3.2. Defining the design parameters and objective functions

*Case 1 – Path optimization*

In this case, nine design variables are optimized, so the vector of project variables $\mathbf{X}$ is defined as follows:

$$\mathbf{X} = \{L_2, L_3, L_4, \beta, \delta, H, x_A, y_A, \theta_2\} \quad (12)$$

For each design variable $x_j\ (j=1,...,9)$ the lower $x_j^{lb}$ and upper $x_j^{ub}$ bounds must be defined:

$$x_j \in \left[x_j^{lb}, x_j^{ub}\right],\ \forall x_j \in \mathbf{X},\ j=1,...,9 \quad (13)$$

The objective function is defined by the following relation:

$$\min\left\{\sum_{i=1}^{N}\left[\left(C_{xd}^{i}-C_{x}^{i}\right)^{2}+\left(C_{yd}^{i}-C_{y}^{i}\right)^{2}\right]+M_{1}h_{1}(\mathbf{X})+M_{2}h_{2}(\mathbf{X})\right\} \quad (14)$$

The previously defined objective function is subject to the constraint given in the form of inequality:

$$g_1 = \left|H + L_2\sin(\theta_2-\delta)\right| - L_3\sin\alpha \le 0 \quad (15)$$

where $\alpha$ is the angle that defines the position of the link $BC$ of the adjustable mechanism relative to the direction of movement of slider $C$ (see Figure 2).

$$\alpha = \delta - \theta_3 \quad (16)$$

*Case 2 – Optimization of path and adjustable length s*

In this case, applying the multi-criteria optimization procedure the two objective functions will be simultaneously minimize. The optimization problem is defined as follows:

$$\min\{f_1(\mathbf{X}), f_2(\mathbf{X})\} \quad (17)$$

Path optimization is achieved by defining an objective function $f_1(\mathbf{X})$ as follows:

$$\min\left\{\sum_{i=1}^{N}\left[\left(C_{xd}^{i}-C_{x}^{i}\right)^{2}+\left(C_{yd}^{i}-C_{y}^{i}\right)^{2}\right]+M_1h_1(\mathbf{X})+M_2h_2(\mathbf{X})\right\} \tag{18}$$

Optimization of adjustable lenght is achieved by defining an objective function $f_2(\mathbf{X})$ as follows:

$$f_2(\mathbf{X}) = \min(\Delta s) \tag{19}$$

where $\Delta s = abs(s_{\max} - s_{\min})$. The vector of design variables is defined by relation (12), that is, on the same way as in *Case 1*.

## 4. THE GREY WOLF ALGORITHM

The grey wolf algorithm belongs to the class of biologically inspired algorithms created by Mirjalili [6]. The algorithm mimics the life of these species of wolves in nature, ie. the principles by which they function, namely strict hierarchy and group hunting. Since they belong to the family of the beast, they are considered as predators that are at the top of the food chain. Grey wolves live in a pack of 5-12 individuals on average. There is a strict social hierarchy in the pack. The role of leader belongs to alphas that can be both males and females. Alpha makes all the important decisions and commands the pack. In other words, alpha is the first level in the hierarchy of grey wolves. The next level is the beta wolves who are tasked with assisting alphas in making decisions as well as taking care of discipline in the pack. Delta represent the third level in the grey wolf hierarchy, ie. they are subordinate to alpha and beta wolves. Delta wolves are guardians, scouts, but also old wolves. The lowest ranked grey wolves are called omega wolves. They are subordinate to all the above categories of wolves. Although it often seems that omega wolves are not of particular importance, they have an essential role in maintaining the structure of domination and often take care of the young.

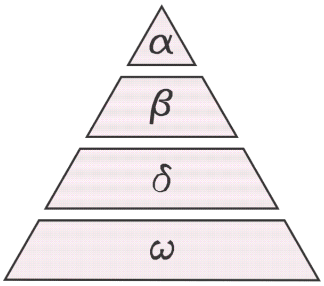


*Figure 3: Schematic representation of the grey wolf hierarchy*

Another important feature of grey wolves is their hunting behavior ie. the hunting mechanism. There are three basic strategies that these predators use in hunting [6]:

1. Tracking, pursuing and approaching prey
2. The encirclement and harassment prey until it calms down
3. Attack on prey

The application of the grey wolf algorithm (GWO) to solve various optimization problems can be seen in references [7-9].

### 4.1. The mathematical model

In order to mathematically model the behavior of grey wolves, it is necessary to divide the initial population in the GWO algorithm into four groups: $\alpha, \beta, \delta$ and $\omega$. In the GWO algorithm hunting ie. the search for the optimal solution is led by the first three best solutions that are considered as $\alpha, \beta$ and $\delta$ wolves, while $\omega$ wolves follow them.

The main stage in group hunting is the surrounding of prey and this hunting strategy can be modeled by the following equations:

$$\mathbf{D} = \left|\mathbf{C}\cdot\mathbf{X_p}(t) - \mathbf{X}(t)\right| \tag{20}$$

$$\mathbf{X}(t+1) = \mathbf{X_p}(t) - \mathbf{A}\cdot\mathbf{D} \tag{21}$$

where $t$ denotes current iteration, $\mathbf{X_p}$ is the position of prey, $\mathbf{X}$ is vector of grey wolf position, while the vectors $\mathbf{A}$ i $\mathbf{C}$ can be calculated using the following expressions:

$$\mathbf{A} = 2\mathbf{a}\cdot\mathbf{r}_1 - \mathbf{a} \tag{22}$$

$$\mathbf{C} = 2\cdot\mathbf{r}_2 \tag{23}$$

where $\mathbf{r}_1, \mathbf{r}_2$ are random vectors from range $[0,1]$, while the vector $\mathbf{a}$ decreases linearly from 2 to 0 during iterations.

Namely, the GWO algorithm starts from the assumption that positions of $\alpha, \beta$ and $\delta$ wolves determine the position of prey. The first three best solutions (positions) are considered as positions of $\alpha, \beta$ and $\delta$ wolves, while other agents in the search (omega wolves) change their positions with respect to $\alpha, \beta$ and $\delta$ wolves.

Changing the position of omega wolves can be represented by the following equations:

$$\begin{aligned}\mathbf{D}_\alpha &= \left|\mathbf{C}_1\cdot\mathbf{X}_\alpha - \mathbf{X}\right|;\\ \mathbf{D}_\beta &= \left|\mathbf{C}_2\cdot\mathbf{X}_\beta - \mathbf{X}\right|;\\ \mathbf{D}_\delta &= \left|\mathbf{C}_3\cdot\mathbf{X}_\delta - \mathbf{X}\right|;\end{aligned} \tag{24}$$

$$\begin{aligned}\mathbf{X}_1 &= \left|\mathbf{X}_\alpha - \mathbf{A}_1\cdot\mathbf{D}_\alpha\right|;\\ \mathbf{X}_2 &= \left|\mathbf{X}_\beta - \mathbf{A}_2\cdot\mathbf{D}_\beta\right|;\\ \mathbf{X}_3 &= \left|\mathbf{X}_\delta - \mathbf{A}_3\cdot\mathbf{D}_\delta\right|;\end{aligned} \tag{25}$$

$$\mathbf{X}(t+1) = \frac{\mathbf{X}_1 + \mathbf{X}_2 + \mathbf{X}_3}{3} \tag{26}$$

The search process in the GWO algorithm begins by generating an initial population, ie. by forming a pack. In a further iterative procedure, $\alpha, \beta$ and $\delta$ wolves also assess the position of the prey, with each potential solution implying approaching the prey. In this sense, when $|\mathbf{A}| \succ 1$ potential solutions tend to stray away from prey, respectively when $|\mathbf{A}| \prec 1$ potential solutions are approaching prey.

The pseudo code of the GWO algorithm is given below.

1: *Defining the number of agents (wolves) N and the maximum number of iterations* $\max iter$
2: *Initialization of initial population* $X_i$ $(i = 1,2,...,n)$
3: *Initialization of vectors* $\mathbf{a}, \mathbf{A}, \mathbf{C}$
4: *Calculating the fitness value of each agent*
5: $X_\alpha =$ *the best agent*
6: $X_\beta =$ *the second best agent*
7: $X_\delta =$ *the third best agent*
8: ***while*** $(t < \max iter)$
9: ***for*** *%% each search agent*

*update position of current search agent based on equation (26)*
***end for***
*Calculating of vectors* $\mathbf{a}, \mathbf{A}, \mathbf{C}$
*Calculating fitnes value of each agent*
*Finding new* $X_\alpha, X_\beta, X_\delta$
$t = t + 1$
*Sorting the population based on fitness value*
***for*** $i := (n/2)+1, n$
*Update position of i-th wolf based on equation (27)*
***end for***
***end while***
*otherwise print* $X_\alpha$
*Postprocessing of results*

## 5. NUMERICAL EXAMPLE

### 5.1. Example 1 - Optimal synthesis of four-bar linkage

The problem of synthesis of four-bar linkage as a path generator without prescribed time is considered. The point S of the coupler should pass through a set of eighteen predefined points.

Based on relation (2), the design variables for the considered problem are defined as follows:

$$\mathbf{X} = \left[ L_1, L_2, L_3, L_4, l_1, l_2, x_o, y_o, \theta_0, \theta_2^1 \right] \quad (27)$$

The coordinates of the desired (predefined) points on the path are:

$$\left\{ C_d^i \right\} = \left\{ \begin{array}{llllll} (0.5,1.1); & (0.4,1.1); & (0.3,1.1); & (0.2,1.0); & (0.1,0.9); & (0.05,0.75); \\ (0.02,0.6); & (0,0.5); & (0,0.4); & (0.03,0.3); & (0.1;0.25); & (0.15,0.2); \\ (0.2,0.3); & (0.3,0.4); & (0.4,0.5); & (0.5,0.7); & (0.6,0.9); & (0.6,1.0) \end{array} \right\} \quad (28)$$

The input angle of the curve is determined using the following relation:

$$\left\{ \theta_2^i \right\} = \left\{ \theta_2^1, \theta_2^1 + 20 \cdot i \right\},\ i = 1,...,17 \quad (29)$$

For each design variable, boundaries are defined:

$$0 \le L_1, L_2, L_3, L_4 \le 50\ ;$$
$$-50 \le l_1, l_2, x_o, y_o \le 50\ ; \quad (30)$$
$$0 \le \theta_0,\ \theta_2^1 \le 2\pi$$

The parameters of the GWO algorithm used in the optimization process are: max $iter = 50$ (maximum number of iteration), while the number of the agents is 30 (SearchAgents_no=30).

Using the grey wolf algorithm in the optimization procedure one obtains a mechanism whose design parameters are given in Table 1. For comparison, the table shows the results of other authors [10,11,12,13,14] who solved the same problem by applying different optimization algorithms. Thus, the aim is to demonstrate the effectiveness of the GWO algorithm in the example of the four-bar linkage, discussed in the literature.

*Table 1: Comparative view of design parameters obtained using different optimization algorithms*

| | Kunjur, Krishnamurty [10] (GA) | Ortiz et al. [13] (IOA) | Cabrera et al. [11] (GA) | Cabrera et al. [12] (MUMSA) | Bulatović et al.[14] (MKH) | GWO |
|---|---|---|---|---|---|---|
| | 0.274853 | 0.245216 | 0.237803 | 0.297057 | 0.42180 | **0.41970** |
| | 1.180253 | 6.38294 | 4.828954 | 3.913095 | 0.87821 | **0.98857** |
| | 2.138209 | 2.620532 | 2.056456 | 0.849372 | 0.58013 | **0.58240** |
| | 1.879660 | 4.040435 | 3.057878 | 4.453772 | 1.00429 | **1.10427** |
| | -0.833592 | 1.139106 | 0.767038 | 1.6610626 | 0.35907 | **0.40047** |
| | -0.378770 | 1.866109 | 1.850828 | 2.7387359 | 0.38081 | **0.44529** |
| | 1.132062 | 1.891805 | 1.776808 | 2.806964 | 0.26886 | **0.28691** |
| | 0.663433 | -0.761339 | -0.641991 | 4.853543 | 0.17715 | **0.09855** |
| | 4.354224 | 1.187751 | 1.002168 | -1.309243 | 0.29294 | **0.33948** |
| | 2.558625 | 0.000000 | 0.226186 | 4.853543 | 0.88595 | **0.84827** |
| error | 0.043 | 0.0349 | 0.0337 | 0.0196 | 0.00911 | **0.00908** |

Figure 4 shows the best mechanism obtained by applying the grey wolf algorithm (GWO) as well as the path it generates.

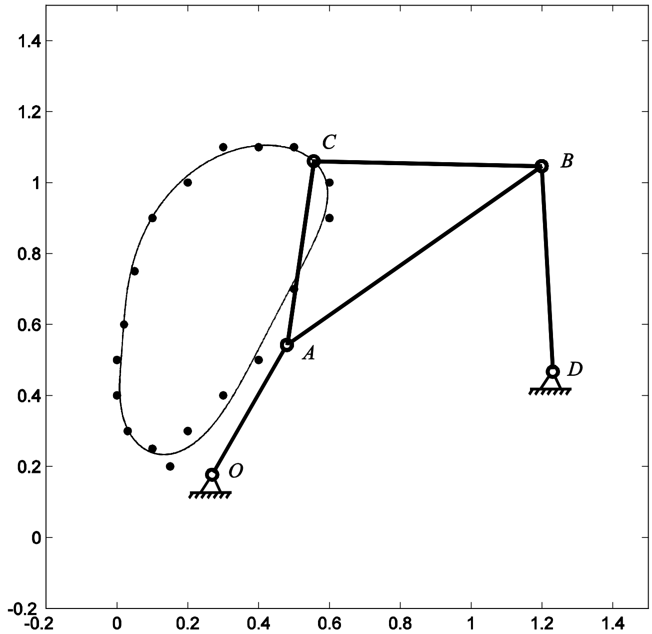


*Figure 4: The best mechanism in Example 1*

Figure 5 shows, in parallel, the paths described by point *C* of a given mechanism, which were obtained using various optimization algorithms (Bulatovic et al. [14] - MKH, Cabrera et al. [12] - MUMSA algorithm, Ortiz et al. [13 ] - IOA).

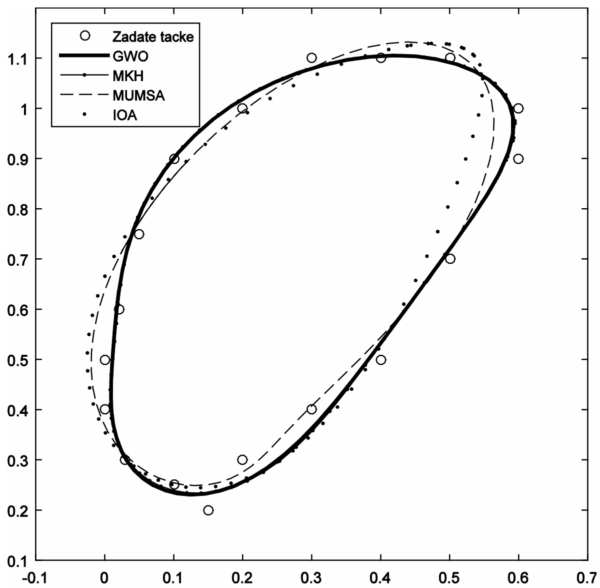


*Figure 5: Coupling curves*

5.2 Example 2 – Optimal synthesis of adjustable (slider crank) mechanism

As previously stated, two cases will be considered in the example of optimal synthesis of the slider crank mechanism. In the first case, the goal is to optimize the path, ie. synthesis of adjustable mechanism as a path generator. In the second case, the goal is to perform simultaneous optimization of the path and adjustable length *s*.

*Case 1 – Path optimization*

Initially, it is necessary to define a vector of design variables:

$$\mathbf{X} = \{ L_2, L_3, L_4, \beta, \delta, H, x_A, y_A, \theta_2 \} \tag{31}$$

The coordinates of the desired (preset) points in the path are the same as in Example 1, ie. they are defined by relation (4.28). Namely, the goal is to perform the synthesis of various types of planar mechanisms which generate the same trajectory. Here, the path generator (which is given by the same set of points as in Example 1) is the adjustable mechanism shown in Fig. 2.

For each design variable, boundaries are defined:

$$\begin{gathered} 0 \le L_2, L_3, L_4 \le 50\,; \\ -50 \le H, x_A, y_A \le 50\,; \\ 0 \le \theta_2, \beta, \delta \le 2\pi \end{gathered} \tag{32}$$

The parameters of the GWO algorithm used in the optimization process are the same as in the previous case. Using the grey wolf algorithm, an adjustable mechanism is obtained and its design parameters are shown in Table 2.

*Table 2 Optimal values of design parameters for Case 1*

| Design parameters | **Optimal values GWO** |
|---|---|
| $L_2$ | 0.32563 |
| $L_3$ | 0.50213 |
| $L_4$ | 0.36236 |
| $\beta$ | -1.14240 |
| $\delta$ | -1.42252 |
| $H$ | 0.15411 |
| $x_A$ | 0.55131 |
| $y_A$ | 0.73173 |
| $\theta_2$ | 0.53607 |
| $\Delta s$ | 0.52426 |
| error | 0.00994 |

As there are no references in the available literature to consider the path optimization of this type of planar mechanisms, it is not possible to provide comparative results.

Figure 6 shows the best mechanism obtained by applying the grey wolf algorithm (GWO) as well as the path it generates.

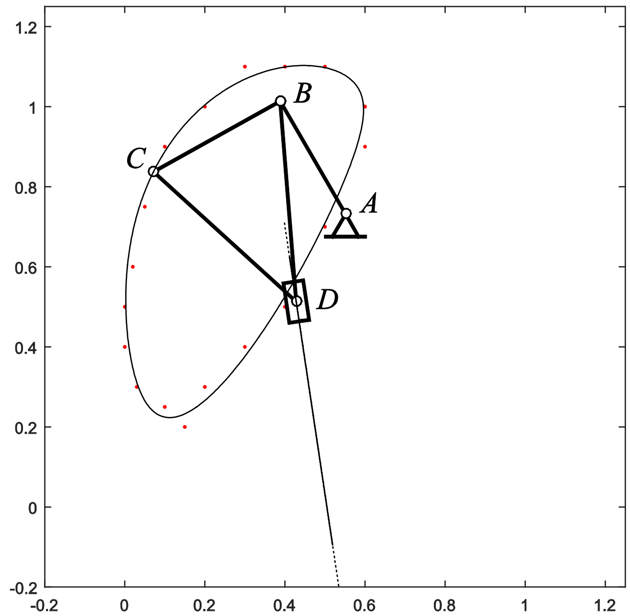


*Figure 6: The best mechanism in Example 2 – Case 1*

Figure 7 shows the path described by point $M$ of the considered adjustable mechanism, and the same is obtained using the grey wolf algorithm.

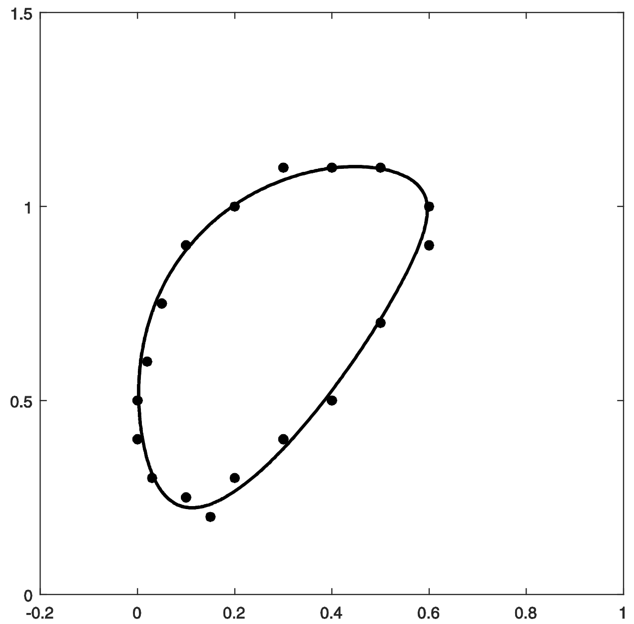

*Figure 7: Copling curve*

## *Case 2 – Optimization of path and adjustable length s*

The vector of design variables in this case is defined in the same way as in Case 1, by using relation (31). The coordinates of the desired points in the path are the same as in the previous examples, since the goal is to generate the same path using two types of planar mechanisms. Unlike Case 1, where only path optimization is considered, simultaneous optimization of the path and adjustable length is performed here. Using the grey wolf algorithm in the multicriteria optimization process, a number of adjustable mechanisms (solutions) with different values of design parameters are generated. Table 4.3 shows the design parameters for the four best solutions (mechanisms) obtained in the optimization process.

*Table 3: Optimal values of design parameters for the four best solutions in Case 2*

| Design variab. | Opt.values (Example1) | Opt. values (Example2) | Opt. values (Example3) | Opt.values (Example4) |
|---|---|---|---|---|
| $L_2$ | 0.46357 | 0.33887 | 0.46319 | 0.30143 |
| $L_3$ | 1.11772 | 0.84714 | 1.03291 | 1.50697 |
| $L_4$ | 0.61417 | 0.51169 | 0.56197 | 1.02869 |
| $\beta$ | 6.81167 | 8.34521 | 0.53718 | 2.39888 |
| $\delta$ | -5.27894 | -2.53315 | 1.00504 | 3.61733 |
| $H$ | 0.14973 | 0.43825 | 0.15378 | 0.90908 |
| $x_A$ | 0.14271 | 0.01185 | 0.14909 | -0.36041 |
| $y_A$ | 0.08941 | 1.08347 | 0.14390 | 1.47563 |
| $\theta_2$ | -5.22872 | 1.27817 | 1.04741 | 1.29291 |
| $f_{1\min}$ | 0.03292 | 0.03782 | 0.03305 | 0.03602 |
| $f_{2\min}$ | 0.48965 | 0.48626 | 0.49073 | 0.48052 |

Figures 8 - 11 show the mechanisms (with the parameters of Examples 1, 2, 3 and 4, respectively) obtained by the multicriteria optimization procedure and the application of the grey wolf algorithm (GWO). The same figures show the paths generated by the adjustable mechanisms obtained in Examples 1, 2, 3 and 4.

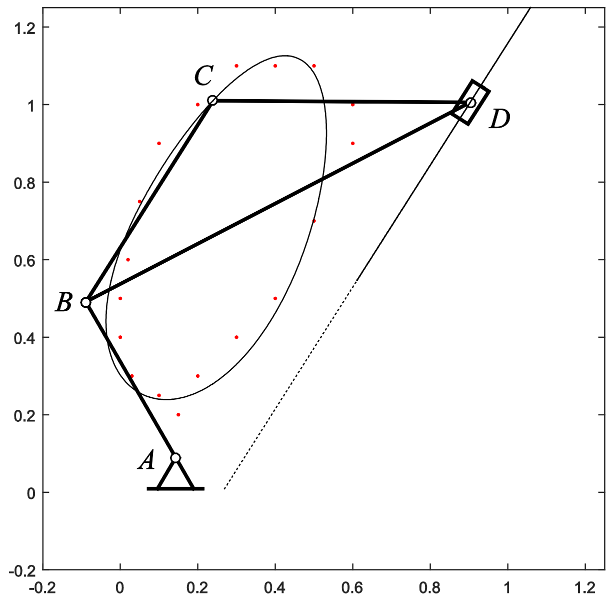


*Figure 8: Mechanism and its path – Example 1*

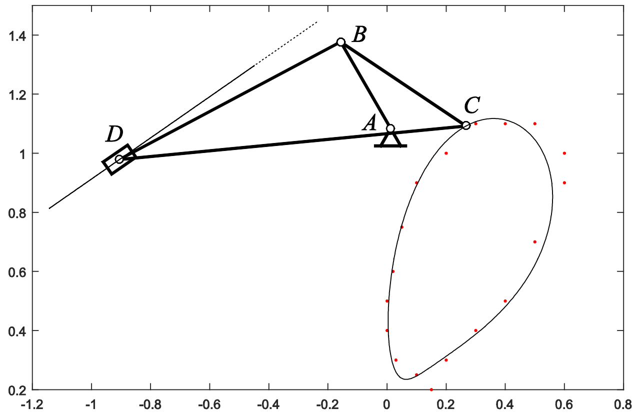


*Figure 9: Mechanism and its path – Example 2*

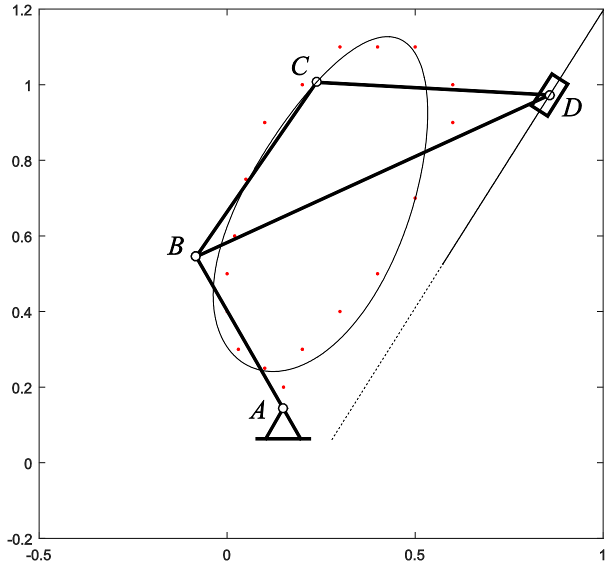


*Figure 10: Mechanism and its path – Example 3*

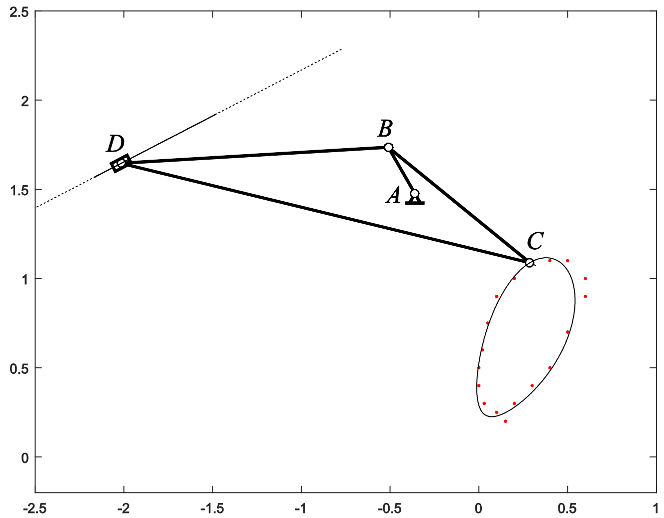


*Figure 11: Mechanism and its path – Example 4*

## 6. CONCLUSION

In this paper, the problem of optimal synthesis of planar mechanisms as a path generator is discussed. To solve the problem of optimal synthesis, the grey wolf algorithm was applied. There is no research available in the literature in which the problem of optimal synthesis of mechanisms was solved by applying the GWO algorithm. Testing the efficiency of the grey wolf algorithm was performed by the example of optimal synthesis of a four-bar linkage as a path generator (Example 1). The results obtained by applying the GWO algorithm are better than the results in [10,11,12,13], while they are approximate to results in [14] (see Table 1). Then, two cases of optimal synthesis of an adjustable slider crank mechanism were considered (Example 2). Firstly, in Case 1, it was performed an optimal synthesis of path which is defined by the same set of points as in Example 1. The obtained results are excellent, the path is almost identical to that one generated by the four-bar linkage (Figure 7). The deviations between actual and desired path are minimal. In Case 2, multi-criteria optimization of the path and adjustable length $s$ was performed. However, the results obtained in this case are not satisfactory. Namely, the magnitude $\Delta s$ is slightly reduced compared to Case 1 (single-criteria optimization), while there is a drastic deviation of the actual from the desired (given) path (see Figures 8 - 11).

Based on the above, the conclusion is that the grey wolf algorithm provides excellent results in the case of single-criteria optimization. However, the application of the GWO algorithm did not give the expected good results in the multi-objective optimization. In this sense, certain modifications should be made to the standard GWO algorithm to improve its efficiency in the MOO process.

Finally, it should be noted that there are no references in the available literature in which the problem of multi-criteria optimization of the trajectory and stroke of the slider of the adjustable curved piston mechanism has been discussed. This has made some contribution to the study of the problem of optimal synthesis of tunable plane mechanisms.

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