Analiza kretanja vibracionog valjka

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U izgradnji cesta, oprema za zbijanje poput valjka s jednostrukim ili tandemskim vibracionim valjkom koristi se i za tlo ili za smeše. U ovom je radu opisan numerički i dinamički model sistema valjak-tlo. Model uzima u obzir najvažnije parametre posebno za vibro valjak, odnosno za materijal ceste. Tlo se smatra elastičnim medijem. Program u Matlab / Simulink 7 razvijen je za rešavanje sistema diferencijalnih jednačina koje opisuju kretanje vibracionog valjka. U radu su napravljene analize stabilnog kretanja valjka tokom procesa sabijanja pomoću povezanosti tehnoloških parametara mašine i parametara tla. Rezultati simulacije prikazani su u obliku dijagrama pomeranja i brzine vibracionog valjka.

Keywords: vibracioni valjak, zemljište, interakcija, dinamička analiza.

1. UVODNA RAZMATRANJA

Zbijanje tla je složen postupak zavistan od uticaja tehnoloških parametara opreme za zbijanje i svojstava slojeva puta. Povezanost svih ovih faktora dovodi do postizanja optimalnog stepena zbijanja nakon minimalnog broja prolaza valjka. Analiza procesa sabijanja i implicitnog kontakta između valjka i tla [1] zahteva:

- poznavanje matematičkih modela koji opisuju ponašanje sistema mašina-tlo [2], [4];
- pisanje i rešavanje diferencijalne jednačine kretanja glavnih podsklopova (valjak, šasijsa);
- validaciju rezultata dobijenih numeričkim postupkom simulacije na bazi eksperimentalnih ispitivanja.

Pored toga, efikasnost sabijanja je zadovoljavajuća ako popznajemo vrednosti vertikalnih ubrzanja šasijsa I valjka u toku procesa sabijanja.

2. MATEMATIČKI MODEL INTERAKCIJE VALJAK-TLO

Vibracioni valjak može se na pojednostavljen način opisati sistemom sa dva stepena slobode (slika 1). Kretanje je definisano pomeranjem vibracionog valjka (k_2) i pomeranjem šasijsa (k_1). Glavni parametri sistema valjak-tlo prikazani su u Tabeli 1.

Vibracioni system smešten u vibracionom valjku proizvodi oscilatorne sile sa amplitudom

$$F_0 = m_0 e \omega^2 \quad (1)$$

dde $\omega = 2\pi f$ predstavlja frekvenciju oscilovanja.

Slika1: Dinamički model interakcije valjak-tlo

Tabela 1. Spisak parametara

Sistem	Parametri	Oznaka
sistem valjak-tlo	masa šasijsa	m_1
	masa valjka	m_2
	ekscentrične mase	m_0
vibracioni sistem	ekscentričnost	e
	učestanost	f
tlo	koeficijent krutosti	k_1
	koeficijent prigušenja	c_1
sistem vešanja	koeficijent krutosti	k_2
	koeficijent prigušenja	c_2

Diferencijalne jednačine kretanja za dve mase (m_1 i m_2) usled dejstva dinamičke sile date su izrazom [2]

$$\begin{cases} m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 - c_1 \dot{x}_2 - k_1 x_2 = 0 \\ m_2 \ddot{x}_2 + (c_1 + c_2) \dot{x}_2 + (k_1 + k_2) x_2 - \\ - c_1 \dot{x}_1 - k_1 x_1 = m_0 e \omega^2 \sin \omega t \end{cases} \quad (2)$$

ili u matricnom obliku

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} m_0 e \omega^2 \sin \omega t \quad (3)$$

Uvođenjem Laplasove transformacije u jednačinu (3) za početne uslove dobija se:

$$[A] \begin{Bmatrix} X_1(s) \\ X_2(s) \end{Bmatrix} = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} F(s) \quad (4)$$

gde je

$$[A] = \begin{bmatrix} m_1 s^2 + c_1 s + k_1 & -c_1 s - k_1 \\ -c_1 s - k_1 & m_2 s^2 + (c_1 + c_2) s + k_1 + k_2 \end{bmatrix} \quad (5)$$

Zavisnosti će biti prikazane preko pomeranja dve mase dinamičkog sistema

$$\begin{Bmatrix} X_1(s) \\ X_2(s) \end{Bmatrix} = [A]^{-1} \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} F(s) \quad (6)$$

gde je determinant matrice A potrebna za određivanje $[A]^{-1}$:

$$\begin{aligned} \det(A) &= \\ &= (m_1 s^2 + c_1 s + k_1) [m_2 s^2 + (c_1 + c_2) s + k_1 + k_2] - \\ &- (c_1 s + k_1)^2 \end{aligned} \quad (7)$$

Ukoliko pretpostavimo da $\det(A)$ mora biti nula, moguće je analizirati karakterističnu jednačinu (7).

U nedostatku spoljne sile i prigušenja, jednačine kretanja masa m_1 i m_2 postaju:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (8)$$

Rešenja jednačine (8) su

$$\begin{aligned} x_1 &= A_1 \sin(\omega t + \varphi_1) \\ x_2 &= A_2 \sin(\omega t + \varphi_2) \end{aligned} \quad (9)$$

tako da jednačina (8) postaje

$$[A^*] \begin{Bmatrix} A_1 \\ A_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (10)$$

gde je

$$[A^*] = \begin{bmatrix} m_1 \omega^2 + k_1 & -k_1 \\ -k_1 & m_2 \omega^2 + k_1 + k_2 \end{bmatrix} \quad (11)$$

Ukoliko pretpostavimo da $\det(A^*)$ mora biti nula, dobijaja se izraz za spostvene vrednosti sistema mašina-tlo

$$m_1 m_2 \omega^4 - \omega^2 (m_1 k_1 + m_2 k_1 + m_1 k_2) + k_1 k_2 = 0 \quad (12)$$

čije je rešenje

$$\omega_{1,2} = \sqrt{\frac{a \pm \sqrt{a^2 - b^2}}{2m_1 m_2}} \quad (13)$$

U jednačini (13) uvedeni parametric su dati u obliku:

$$\begin{aligned} a &= m_1 k_1 + m_2 k_1 + m_1 k_2 \\ b &= (4k_1 k_2 m_1 m_2)^{0.5} \end{aligned} \quad (14)$$

Prikaz spostvenih vrednosti dinamičkog sistema dobijenih analitičkim putem prikazan je na slici 1, i dovodi do određivanja rezonance oscilovanja [5]. Poznavanje ovih vrednosti, zajedno sa setom eksperimentalnih podataka, nudi mogućnost procene koeficijenta prigušivanja za analizirani sistem [6], [3].

3. NUMERIČKA SIMULACIJA

Važan aspekt analize procesa sabijanja je stabilnost funkcije kretanja sistema valjak-zemlja po glavnim parametrima dva podsistema u kontaktu [7], [8]. Na primer, nadalje su predstavljeni rezultati numeričke simulacije za neke slučajeve koji bi trebalo da budu reprezentativni.

Za simulaciju interakcije mašina-tlo korišćeni su sledeći podaci: $m_1=2000$ kg; $m_2=2800$ kg; $f=30$ Hz; $c_1=106$ Ns/m; $k_1=3.5 \times 10^8$ N/m; $c_2=5$ Ns/m; $k_2=4.5 \times 10^6$ N/m. Pretpostavlja se da je material u kontaktu pesak.

Prvo, korišćena je dinamička sila (F_0) sa frekvencijom od 30 Hz i promenljive amplitude sa statičkim momentum ekscentrične mase m_0 . Vreme simulacije je 400 ms.

Na slici 2 prikazan je dijagram promene kontaktne sile između valjaka i peska, a na slici 3 kretanje u faznoj ravni sistema.

4. ZAKLJUČAK

Na osnovu dijagrama sa slike 2 može se doći do zaključaka koji su prisutni iz razloga tehnološki parametri valjka nisu u uzajamnoj vezi sa karakteristikama terena:

Slika 2: Zavisnost kontaktne sile od vremena
 — $M_{st}=0.5 \text{ kgm}^2$; - - - $M_{st}=2.5 \text{ kgm}^2$; - x - $M_{st}=5 \text{ kgm}^2$;

Slika 3: Fazna ravan

a) $M_{st}=0.5 \text{ kgm}^2$; b) $M_{st}=2.5 \text{ kgm}^2$.

- Kontaktna sila između valjka i zemljišta ima samo pozitivnu vrednost što znači da je kontakt permanentan (linearan), slika 2 i Slika 3a;

- Kontaktna sila ima kratko vreme trajanja kada je vrednost nula što znači da je kontakt trenutni (nelinearan). Slika 2 i slika 3b.

U poslednjem slučaju, nejednakost dodira dovodi do nestabilnog kretanja valjka pod dinamičkim silama. Zaključno, poznavajući svojstva tla, računarska simulacija omogućava uspostavljanje stabilnih i optimalnih režima za radni proces.

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The Analysis of Vibratory Roller Motion

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In road building, the compaction equipment like rollers with single or tandem vibratory drum are used both for soil or mixture. A numerical and dynamical model of the drum-soil system is described in this paper. The model takes into account the most important parameters specifically for vibratory roller and, respectively for road pavement material. The soil is considered by an elastic medium. A program in Matlab / Simulink 7 was developed to solve the system of differential equations, which describe the vibratory roller movement. In paper the authors make a study of the roller’s stabile motion during the compaction process through correlation of the machine’s technological parameters with soil parameters. The simulation results are presented as diagrams of displacement and velocity of the vibratory drum.

Keywords: vibratory roller, soil, interaction, dynamic analysis.

1. INTRODUCTION

The soil compaction is a complex process influenced by the technological parameters of compaction equipment and the properties of road system layer’s. The correlation of all these factors leads to obtaining an optimal compaction degree after a minimum roller’s passes.

The analysis of compaction process and implicit of the contact between drum and ground [1] supposes:

- knowing of mathematical models, which describe the behavior of machine-ground system [2], [4];
- writing and solving of differential equation of motion for the main subassemblies (drum, chassis);
- validation of results obtained by numerical simulation process based on experimental tests.

In addition, the compaction efficiency it is appreciated if we know the values of chassis and drum vertical accelerations during on compaction process.

2. MATEMATICAL MODEL OF ROLLER-GROUND INTERACTION

The vibratory roller can be describing, in simplified way, by a system with two degree of freedom (Figure 1). The roller’s motion is defining by the vibratory drum displacement (x_2) and chassis’ displacement (x_1). The main parameters of roller-ground system are show in Table 1.

The vibration system maintained into roller drum produces oscillatory force with amplitude

$$F_0 = m_0 e \omega^2 \quad (1)$$

where $\omega = 2\pi f$ represents the force pulsation.

Figure 1: Dynamic model of roller-ground interaction

Table 1. Parameter's identification

Systems	Parameters	Notations
roller-ground system	chassis mass	m_1
	drum mass	m_2
	eccentrically pieces mass	m_0
vibration system	eccentricity	e
	frequency	f
ground system	stiffness coefficient	k_1
	damping coefficient	c_1
suspension system	stiffness coefficient	k_2
	damping coefficient	c_2

The differential equations of motion for the two mass (m_1 and m_2) under dynamic force action are the next forms [2]

$$\begin{cases} m_1 \ddot{x}_1 + c_1 \dot{x}_1 + k_1 x_1 - c_1 \dot{x}_2 - k_1 x_2 = 0 \\ m_2 \ddot{x}_2 + (c_1 + c_2) \dot{x}_2 + (k_1 + k_2) x_2 - \\ - c_1 \dot{x}_1 - k_1 x_1 = m_0 e \omega^2 \sin \omega t \end{cases} \quad (2)$$

or in matrix writing

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} c_1 & -c_1 \\ -c_1 & c_1 + c_2 \end{bmatrix} \begin{Bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ I \end{Bmatrix} m_0 e \omega^2 \sin \omega t \quad (3)$$

Applying Laplace transformation to the Eq. (3) in zero conditions it's obtained:

$$[A] \begin{Bmatrix} X_1(s) \\ X_2(s) \end{Bmatrix} = \begin{Bmatrix} 0 \\ I \end{Bmatrix} F(s) \quad (4)$$

where

$$[A] = \begin{bmatrix} m_1 s^2 + c_1 s + k_1 & -c_1 s - k_1 \\ -c_1 s - k_1 & m_2 s^2 + (c_1 + c_2) s + k_1 + k_2 \end{bmatrix} \quad (5)$$

Relation will be given by the displacements of the two masses of dynamical system is

$$\begin{Bmatrix} X_1(s) \\ X_2(s) \end{Bmatrix} = [A]^{-1} \begin{Bmatrix} 0 \\ I \end{Bmatrix} F(s) \quad (6)$$

where determinant of matrix A useful for evaluation of $[A]^{-1}$ is:

$$\begin{aligned} \det(A) &= \\ &= (m_1 s^2 + c_1 s + k_1) [m_2 s^2 + (c_1 + c_2) s + k_1 + k_2] - \\ &- (c_1 s + k_1)^2 \end{aligned} \quad (7)$$

If we suppose that $\det(A)$ must be null, it is possible the polls evaluation of the characteristic equation (7).

In absence of external force and amortisation, motion equation of m_1 and m_2 becomes:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix} \begin{Bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{Bmatrix} + \begin{bmatrix} k_1 & -k_1 \\ -k_1 & k_1 + k_2 \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \quad (8)$$

	pieces mass	

The solution for the Eq. (8) are

$$\begin{aligned} x_1 &= A_1 \sin(\omega t + \varphi_1) \\ x_2 &= A_2 \sin(\omega t + \varphi_2) \end{aligned} \quad (9)$$

and Eq.(8) becomes

$$[A^*] = \begin{bmatrix} m_1 \omega^2 + k_1 & -k_1 \\ -k_1 & m_2 \omega^2 + k_1 + k_2 \end{bmatrix} \quad (10)$$

where

$$[A^*] = \begin{bmatrix} m_1 \omega^2 + k_1 & -k_1 \\ -k_1 & m_2 \omega^2 + k_1 + k_2 \end{bmatrix}$$

If we suppose that $\det(A^*)$ must be null, then we obtained the expressions for eigenvalues of roller-ground system

$$m_1 m_2 \omega^4 - \omega^2 (m_1 k_1 + m_2 k_1 + m_1 k_2) + k_1 k_2 = 0 \quad (11)$$

with solutions

$$\omega_{1,2} = \sqrt{\frac{a \pm \sqrt{a^2 - b^2}}{2 m_1 m_2}} \quad (12)$$

In Eq. (13) we make the next notations:

$$\begin{aligned} a &= m_1 k_1 + m_2 k_1 + m_1 k_2 \\ b &= (4 k_1 k_2 m_1 m_2)^{0.5} \end{aligned} \quad (13)$$

Evaluation on analytical way of eigenvalues of dynamical system depicted into Figure 1 leads to finding of pulsations resonance [5]. The knowing of these values, in completion with a set of experimental data offers the possibility of damping coefficient evaluation for the analysed system [6], [3].

3. NUMERICAL SIMULATION

An important aspect of compaction process study consists on stability analysis of roller-ground system motion function by main parameters of two subsystems in contact [7], [8]. For example, in the next are presented the results of numerical simulation for some cases supposed to be representative.

For simulation roller-ground interaction, following data are used: $m_1=2000$ kg; $m_2=2800$ kg; $f=30$ Hz; $c_1=106$ Ns/m; $k_1=3.5 \times 10^8$ N/m; $c_2=5$ Ns/m; $k_2=4.5 \times 10^6$ N/m. We supposed that the ground material is sand.

Firstly, it has been used since exciter dynamic force (F_0) with frequency of 30 Hz and variable amplitude function by static torque of eccentric mass m_0 . Simulation time was 400 ms.

In Figure 2 was show the diagram of contact force variation between roller-sand and Figure 3 the motion into phases plane for the system.

4. CONCLUSIONS

Based on diagrams from figure 2, we can observe some aspects which appear because the roller technological parameters are not correlated with terrain characteristics:

In the last case, the no uniformity of contact deals to the instable motion of roller under dynamical forces.

In conclusion, knowing the soil properties, the computer simulation enables to establish the stabile and optimal regimes for working process.

Figure 2: Contact force vs time

— $M_{st}=0.5 \text{ kgm}^2$; ----- $M_{st}=2.5 \text{ kgm}^2$; ---x--- $M_{st}=5 \text{ kgm}^2$;

Figure 3: Phase plane

a) $M_{st}=0.5 \text{ kgm}^2$; b) $M_{st}=2.5 \text{ kgm}^2$.

- the contact force between drum and soil have only positive values that means the contact is permanently (linear), as can be seen in Figure 2 and Figure 3a);
- the contact force have short duration time when his value is null that means the contact is temporary (nonlinear), such as Figure 2 and Figure 3b).

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