Zoran D. Perović

ACCURACY OF NUMERICAL METHODS FOR ASSESSMENT OF FATIGUE CRACK GROWTH IN WELDED JOINTS

TAČNOST NUMERIČKIH METODA ZA PROCJENU RASTA ZAMORNE PUKOTINE U ZAVARENIM SPOJEVIMA

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Rezime


1. INTRODUCTION

The big part of fatigue life is spent in fatigue crack propagation. Driving force of this process is stress intensity factor (SIF). Various numerical methods are developed in order to determine SIF. Because of its efficiency finite element method (FEM) is most frequently used e.g. virtual crack extension technique, the J-integral method, special elements with introduced $1/\sqrt{r}$ stress singularity by employing

$$ K = M_k F \sigma \sqrt{\pi a} $$

$$ F = \left[ M_1 + M_2 \left( \frac{a}{l} \right)^2 + M_3 \left( \frac{a}{l} \right)^4 \right] f_0 f_w g \sigma / Q $$

2. NUMERICAL ANALYSIS

In this paper SIF is determined by using Raju & Newman solution for surface semielliptical crack in flat plate [1] subjected to the remote uniform tensile stress $\sigma$ and Albrecht’s solution for geometry correction factor for fillet welded joints [2]:

$$ F = \left[ M_1 + M_2 \left( \frac{a}{l} \right)^2 + M_3 \left( \frac{a}{l} \right)^4 \right] f_0 f_w g \sigma / Q $$

Abstract

Fatigue crack could be detected in various structures, subjected to cyclic loading, during the inspection. In order to predict remaining fatigue life, various numerical methods can be used. At the very beginning, stress intensity factor (SIF) describing stress field in vicinity of the crack tip, should be determined. There a few methods (and formulas in some cases) for SIF determination. Then equation for crack growth rate should be chosen and solved by using one of the numerical methods. The difference in final results of the assessment as a consequence of different applied methods and formulas in this procedure for welded joints is considered in this paper.
where:

\[
M_1 = 1.13 - 0.09 \left( \frac{a}{c} \right) ; \quad M_2 = -0.54 + \frac{0.89}{0.2 + \frac{a}{c}} ; \quad M_3 = 0.5 - \frac{1.0}{0.65 + \left( \frac{a}{c} \right)} + 14 \left( 1 - \frac{a}{c} \right)^{24}
\]

\[
f_\varphi = \left[ \left( \frac{a}{c} \right)^2 \cos^2 \varphi + \sin^2 \varphi \right]^{0.25} ; \quad f_w = \left[ \sec \left( \frac{\pi c}{w} \sqrt{\frac{a}{t}} \right) \right]^{0.5} ; \quad g = 1 + \left[ 0.1 + 0.35 \left( \frac{a}{t} \right)^2 \right] \left( 1 - \sin \varphi \right)^2
\]

\[
Q = \left[ 1 + 1.464 \left( \frac{a}{c} \right)^{1.65} \right]^{0.5}
\]

where \(a, c\) = length of the minor and major semi-axis of the elliptical crack, respectively; \(t, w\) = thickness and width of the main plate of the welded joint, respectively; \(\varphi\) = angle that describes the location at the crack front with respect to the major axis of the ellipse. The geometry correction factor \(M_k\), is given by

\[
M_k = \frac{2}{\pi} \sum_{i=1}^{n} \frac{\sigma b_i}{\sigma} \left( \arcsin \frac{b_{i+1}}{a} - \arcsin \frac{b_i}{a} \right)
\]

(3)

where \(\sigma = \) nominal stress in critical section of uncracked joint, \(\sigma_{bi} = \) normal stress in a finite element between the distance \(b_i\) and \(b_{i+1}\) (Fig.1b). It accounts for the effect on \(K\) of a stress concentration produced by a structural detail. This formula was obtained from Eq.4, the solution [3] for a central crack of length \(2a\) in an infinite plate with two equal pairs of splitting forces \(P\), applied at \(x = \pm b\) (Fig.1a).

\[
K = \frac{2P}{\sqrt{\pi a} \sqrt{a^2 - b^2}}
\]

(4)

Fig.1. Crack in infinite plate subjected to: a) two pairs of equal splitting forces; b) pairs of discrete stresses
Verreman et al. [4] used this method for determination of the \( M_k \) factor of a cruciform-welded joint and compared it with the accurate solution obtained by Smith [5] using high-order crack tip elements with an inverse square root singularity. They reported differences smaller than 6%, so this method can be considered accurate for engineering purposes. The advantage of Albrecht’s method is that only one stress analysis needs to be made for each joint geometry, i.e. the stress analysis of an uncracked joint. The computer program was made based on Eq.(1-3) in order to determine SIF for welded joint shown in Fig.2.

Calculated values of SIF for one arbitrarily chosen stress level are shown in Fig.3. These results are compared with those given in literature. The standard BS 7910:2005 [6] gives formula for SIF for cracks in welded joints, but it is valid for weld profile with sharp radii (<0.1t). The solution were obtained by curve fitting to individual finite element analyses [7]. One can see from ref. [7] that mentioned formula was obtained for \( r/t = 0 \) and recommended for as-welded joints \( (r=\text{weld toe radius}) \). In ref. [7] was performed also second analysis for \( r/t = 0.1 \) and obtained formula was recommended for welded joint with improved profile (TIG dressed or ground joints). That formula has the same shape like formulas (1) and (2) with magnification factor \( M_k \) (accounts for the effect on \( K \) of a stress concentration produced by a structural detail) given by:

\[
M_k = f_1\left(\frac{a}{t}, \frac{c}{t}\right) + f_2\left(\frac{a}{t}, \frac{L}{t}\right) + f_3(\theta)
\]

where

\[
 f_1\left(\frac{a}{t}, \frac{c}{t}\right) = A_1\left(\frac{a}{t}\right)^{A_3} + A_3\left(1 - \frac{a}{t}\right)^{A_4} + A_5\left(\frac{a}{t}\right) + A_6
\]

\[
 f_2\left(\frac{a}{t}, \frac{L}{t}\right) = A_7\left(\frac{a}{t}\right)^{A_8} + A_9\left(1 - \frac{a}{t}\right)^{A_{10}}
\]

\[
A_1 = -3.2172(a/c)^2 + 8.9931(a/c) - 7.7356
\]

\[
A_2 = -0.22457(a/c)^2 - 0.41009(a/c) + 0.86071
\]

\[
A_3 = 0.65009(a/c)^2 - 0.76603(a/c) + 1.0351
\]

\[
A_4 = 0.10745(a/c)^2 - 11.039(a/c) + 30.557
\]

\[
A_5 = 1.2494(a/c)^2 - 7.1510(a/c) + 9.4916
\]

\[
A_6 = 0.33693(a/c)^2 + 0.23884(a/c) + 2.3341
\]

\[
A_7 = -0.0021981(L/t)^2 + 0.0066388(L/t) + 0.23244
\]

\[
A_8 = 0.098096(L/t)^2 - 0.22280(L/t) + 0.19344
\]

\[
A_9 = 0.015584(L/t)^2 + 0.026458(L/t) + 0.31065
\]

\[
A_{10} = -0.29651(L/t)^2 + 1.2995(L/t) + 1.0362
\]

\[
f_3(\theta) = 1.0
\]

where \( L = \text{attachment footprint width (thickness of attachment plus both welds).} \)
3. RESULTS AND DISCUSSION

Weld toe radius effect.

These two methods (eq.1,2,3 and eq.1,2.5) are applied on welded joint (Fig. 2) in order to determine SIF for surface semielliptical crack. Results of the calculation are shown and compared in Table. 1 and figure 3.

<table>
<thead>
<tr>
<th>$a$, mm</th>
<th>$K$, MPa(m)$^{0.5}$ $r/t = 0.1$</th>
<th>$\Delta$, %</th>
<th>$K$, MPa(m)$^{0.5}$ $r/t = 0.2$</th>
<th>$\Delta$, %</th>
<th>$K$, MPa(m)$^{0.5}$ $r/t = 0.4$</th>
<th>$\Delta$, %</th>
<th>$K$, MPa(m)$^{0.5}$ Eq.(1,2,5)</th>
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<tr>
<td>0.1</td>
<td>2.417</td>
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<td>2.240</td>
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<td>1.957</td>
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<td>-9.3</td>
<td>2.727</td>
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<td>2.523</td>
<td>-26.1</td>
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<td>-16.2</td>
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<td>5.343</td>
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<td>-1.4</td>
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</tr>
<tr>
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<td>9.137</td>
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<td>+7.2</td>
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<tr>
<td>5.9</td>
<td>14.221</td>
<td>+1.9</td>
<td>14.762</td>
<td>+5.8</td>
<td>14.496</td>
<td>+3.9</td>
<td>13.956</td>
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<tr>
<td>7.0</td>
<td>16.423</td>
<td>-4.0</td>
<td>16.849</td>
<td>-1.4</td>
<td>16.712</td>
<td>-2.2</td>
<td>17.078</td>
</tr>
</tbody>
</table>

Table 1. Comparison of SIF values obtained using Eq.(1,2,3) for various $r$ values and those ones obtained by using Eq.(1,2,5)

$\Delta$=Difference among SIF determined by Eq.(1,2,3) and that one determined by Eq.(1,2,5)

Fig.3. Stress intensity factor values obtained using Bowness and Albrecht solution ($\sigma = 100$ MPa)
Weld toe radius effect on SIF magnitude exist until the crack depth is < 0.1t and differences determined in this work for this region are 10% for \( r/t = 0.1 \), 16% for \( r/t = 0.2 \), and 26% for \( r/t = 0.4 \). It can be concluded that Eq. 5 gives more conservative results for bigger weld toe radii in the vicinity of weld toe. Differences of SIF values, for bigger crack depth, are smaller than 7% as a consequence of accuracy of these methods. It is interesting to determine how much these differences in SIF values affect the determination of the crack propagation life. The crack propagation life \( N_p \) is obtained from the equation:

\[
N_p = \frac{1}{4.9 \times 10^{-12} (\Delta \sigma)^3} \int a_f \left[ F(a) \right]^{\frac{1}{3}} da
\]

This equation was solved by the 32-point Gaussian quadrature method. The obtained results are shown in Table 2 and Figure 4.

<table>
<thead>
<tr>
<th>( a/t )</th>
<th>( N_p ) Eq.(1,2,3,6)</th>
<th>( \Delta, % )</th>
<th>( N_p ) Eq.(1,2,3)</th>
<th>( \Delta, % )</th>
<th>( N_p ) Eq.(1,2,5,6)</th>
<th>( \Delta, % )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>1323845</td>
<td>+9.3</td>
<td>126072</td>
<td>+11.2</td>
<td>129425</td>
<td>+14.3</td>
</tr>
<tr>
<td>0.1</td>
<td>80191</td>
<td>+3.3</td>
<td>79164</td>
<td>+2.0</td>
<td>79013</td>
<td>+1.8</td>
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<tr>
<td>0.2</td>
<td>39516</td>
<td>-5.4</td>
<td>38705</td>
<td>-7.6</td>
<td>38218</td>
<td>-9.0</td>
</tr>
</tbody>
</table>

Table 2. Comparison of crack propagation lives \( N_p \) obtained using Albrecht’s method and Bowness method for various \( r \) values (\( \Delta \sigma = 300 \) MPa)

\( \Delta = \) Difference among \( N_p \) determined by Eq.(1,2,3,6) and that one determined by Eq.(1,2,5,6)

Fig. 4 Effect of weld toe radius on fatigue life
Crack shape effect

Fatigue cracks emanating from stress concentrators (weld toe in this case) have semielliptical shape. Cracks change their shape (aspect ratio \( a/2c \)) during their growth. At this place is presented analysis how much crack shape affect on stress intensity factor and crack propagation life. Results of this analysis is shown in Fig.5. and Fig.6.

![Fig.5 Crack shape effect on stress intensity factor; \((\sigma = 300 \text{ MPa})\)](image)

![Fig.6 Crack shape effect on fatigue crack propagation life; \((\sigma = 300 \text{ MPa})\)](image)
4. CONCLUSIONS

Accuracy of various methods for SIF determination is considered in this paper. Stress intensity factor for T-welded joint is obtained by using two methods: Albrecht’s method and Bowness formula. The results show good agreement—difference is smaller than 7% for relative crack depth bigger than a/t = 0.1. The difference in corresponding crack propagation lives is from +3% to -9%. For cracks smaller than 0.1t difference increases with increase of the weld toe radius: 10% for r/t=0.1, 16% for r/t=0.2 and 26% for r/t=0.4. The difference in corresponding crack propagation lives is from +9% to 14%. Bowness formula gives conservative solution for TIG dressed and ground welded joints. The influence of crack shape on SIF and crack propagation life is considered as well. For instance, decrease of aspect ratio a/2c from 0.5 to 0.1 increases SIF 59% for a/t=0.0125 and 179% for a/t=0.875. Corresponding crack propagation lives decrease 5 times (at Δσ=300 MPa, a/t=0.05, a/2c decreases from 0.5 to 0.1) and 6.4 times (at Δσ=300 MPa, a/t=0.2, a/2c decreases from 0.5 to 0.1). These examples illustrate big influence of crack shape on stress intensity factors and crack propagation lives.

REFERENCES