REPEATING PATTERN ACTIVITIES AT PRESCHOOL AGE

Abstract: Activities with patterns are very common and very important in the period of the early education of children. Patterns are present in different forms and contexts in kindergarten, where preschool children often encounter situations in which they work on noticing or predicting certain regularities, whether they are everyday life situations like arranging in a column (boy-girl-boy-girl) or drawing zebra patterns (black-white-black-white), etc. New research on early mathematics education increases this content’s (positive) influence during the development of numerous mathematical competencies, not only those related to algebraic abilities, measurement and spatial reasoning. The aim of this work is to present the development of competences related to the concept of pattern in children of preschool age. In the first part of the paper, the theoretical foundations of mathematical patterns, their categorization and types, their importance at preschool age, as well as specific activities and operations that can be encountered while working with preschool children are presented. The paper also shows the peculiarities of a special category of patterns - patterns of repetition, which are the focus of this paper. Furthermore, the paper presents some of the previous research aimed at children's activities when identifying the structure of the pattern or units of initiation.

Keywords: repeating patterns, preschool children, unit of repeat.

INTRODUCTION

Many scientists believe that pattern is a crucial concept in mathematics education. For example, Steen (1990, p. 611) states that “mathematics is the science of patterns”. Schoenfeld (1992, p. 334) believes that mathematics is “a living subject that seeks to understand the patterns that permeate both
the world around us and the mind within us”, while English (2004) advocates that the development of mathematical reasoning in young children depends on their ability to identify, expand and generalize the pattern. Many documents related to mathematics education include working with patterns as one of the basic algebraic topics (National Council of Teachers of Mathematics, 2000). The ability to notice and analyze a mathematical pattern is characterized by recognizing a repeating unit and identifying how it repeats in that pattern (Mulligan et al., 2020). Studies have shown that children who successfully solve problems related to patterns later have more success in mathematics (Papic, 2007). Introducing preschoolers to patterns is supported by many mathematics education researchers (Sarama & Clements, 2009). Some of the importance of working with patterns at that age is in developing the ability to abstract and generalize (Ibid). Deductive reasoning abilities can also be developed simultaneously with the development of equivalent patterns using different materials and teaching children to predict what is next in an existing pattern (Greenes et al., 2004). In addition to the statement that patterns support the development of specific skills, activities with patterns such as extension, are important component of children’s intellectual development since they provide children with the opportunity to observe and verbalize generalizations and record them symbolically (English & Warren, 1998). Understanding the unit of repetition as a structural part of the pattern is of particular importance because it appears during the development of many mathematical contents, such as measurement (when repeating an identical spatial unit) and multiplication (repeating an identical numerical unit). In each pattern, the various elements are organized in some regular order. For example, in the increasing order of square numbers 0, 1, 4, 9..., the numbers increase by 1, 3, 5 ..., respectively, for a series of odd numbers. In a circle, all points on the circle are the same distance from the center. The way a mathematical pattern is organized is called the structure of that pattern. Finding the structure of patterns is often considered pre-algebraic thinking because the spontaneous development of number concepts begins with the development of algebraic thinking through patterns (Rittle-Johnson et al., 2014).

**MATHEMATICAL PATTERNS AT PRESCHOOL AGE**

Mathematical pattern refers to any predictable regularity that expresses numerical, spatial, or logical relationships among the elements of that pattern (Mulligan & Mitchelmore, 2009). There are different types of patterns that preschool children encounter in early childhood. In the literature, we most often came across the division of patterns according to the way elements are structured within it. Some authors (Papic & Mulligan, 2005; 2007) believe that children of preschool age most often encounter three types of patterns:

1) **repeating patterns**, which have a “cyclical structure that can be generated by repeating smaller parts of that pattern” (Liljedahl, 2004, p. 26-27) (eg. ABABAB...), where the smaller repeating part (AB) is called unit of repetition (Threlfall, 1999);

2) **spatial-structural patterns**, which refer to the invariant relations between different properties of geometric shapes (Papic et al., 2011).
3) **increasing patterns**, consisting of sequences of elements that increase or decrease systematically (e.g., 2, 4, 6, 8, ...) (Papic et al., 2011).

In the educational practice in Serbia, these types of patterns are often found in symbiosis. For example, we can encounter *increasing repetitive patterns* that are repeating patterns and increasing patterns integrated into the same activity, so that the unit of repetition is a particular segment of ascending order. For example, children can make a fence from constructors by placing one constructor, next to it two constructors, then three, and then again one, two, three... or they will represent an ascending pattern where the next element is incremented by 1, but also a repeating pattern with the repeat unit ABC.

Apart from the mentioned categorization, some of the authors (Mulligan & Mitchelmore, 2013) distinguish three forms of structuring according to the type of content present in them: a) numerical, b) spatial-measuring, and c) modal patterns.

### Repeating patterns in preschool children

Repeating patterns are considered as the most accessible type of pattern for young children, possibly because of their very frequent and natural application in everyday activities (Ginsburg et al., 1999; Ginsburg et al., 2003; NCTM, 2006). This type of pattern refers to patterns that have a recognizable, linear unit of repetition (Zazkis & Liljedahl, 2002; Rittle-Johnson et al., 2014). For example, the pattern ABBCABBC has a repeat unit of length of four elements. Considering the importance of researching this type of pattern, it is necessary to support and encourage all activities that support the observation of structures. At preschool age, children form patterns in everyday situations and from elements from their environment. It can be drawing lines (Fox, 2005), making necklaces (Waters, 2004) and various games with models and applications (Bäckman, 2016). The abilities to work on these types of patterns become even more sophisticated around the age of six and seven. In order to create their patterns, children are usually offered tasks and games in which they, usually with help, should continue (expand, extend) a certain pattern. Children first encounter simpler patterns such as those whose unit of repetition is e.g. AB, to make them more difficult by adding repetition units. Many children can successfully solve this type of task which does not necessarily mean recognizing the structure of that pattern (Papic et al., 2011). Certain types of activities can support the creation of this type of capability. Those activities are called “abstract tasks”, and they refer to e.g. identifying the smallest repeat unit by rounding or capping (Papic et al., 2011), creating a particular structure with fewer repeat units but the same pattern structure (Rittle-Johnson et al., 2013). In addition, some of the tasks can be to create or draw the same type of pattern using different materials (Ibid). For example, the pattern ABBC can be represented as circles of different colors: red, yellow, yellow, blue. Children can be required to (re)construct the same pattern with the help of geometric shapes of circle, square and triangle (circle, square, square, triangle). The ability to create the same pattern using different materials is a more complex process than simply copying, expanding and “fixing” the pattern (Sarama & Clements, 2009). By working on activities of this type, children develop abilities and strategies that can later be seen when working with other mathematical content. In other words, exposing preschool children to tasks related to repetition patterns is very important for the development of mathematical
thinking, and it is necessary to increase the awareness of educators and provide them with support in planning activities of this type.

*Operations and activities with (repeating) patterns*

Working with patterns implies certain operations and activities that children can do with them. The most common operations with patterns include: exploring, identifying, expanding, reproducing, presenting and describing patterns (Van de Walle, 2004; Radatz et al., 1998). We will describe the characteristics of each of them below.

1. **Exploring** patterns implies their presentation in a certain context, for which it is necessary to analyze the structure of the patterns beforehand. During the exploration, it is important to find its basic elements (Vogel, 2005).

2. **Identifying** patterns is initially an intuitive process and relates to the structure as a whole. To identify a pattern is to distinguish random occurrences from those that are structured. Identification of patterns is possible only in structured phenomena, in contrast to random ones where this is not possible (Vogel, 2005).

3. **Expanding (continuing)** a certain pattern formation implies mental activities in which the individual is aware of that pattern. Essential to this process is the identification of the characteristics of the pattern - its basic element and its regularities. The expansion of patterns is most often done with the guidance of adults. Pattern extension tasks begin with showing a pattern to the child, and then asking the child to continue the same pattern. Many children succeed in extending the pattern without necessarily recognizing the unit of repetition (Papic et al., 2011). Economopolus (1998) believes that in order for children to be able to generalize and predict patterns, they must mentally separate themselves from simply looking at the structure of patterns, that is, see what the unit of repetition consists of.

4. **Reproduction** of patterns implies the re-creation of an already existing pattern (Rittle-Johnson et al., 2015). When it comes to pattern reproduction, Papić et al. (2011) distinguish two types of reproduction: duplication and copying. *Duplication* implies creating the same pattern using the same materials, while *copying* implies abstracting the unit of repetition and using it while creating a pattern with different materials. For example, children may be presented with a necklace in which the beads alternate in the pattern ABBCABBC, and asked to make several necklaces containing the same bead pattern (duplication). Or, using the same repeat unit, children can make a sash for a dress out of colored flowers that follow the ABBCABBC pattern. Reproduction of patterns requires very precise and consistent analysis. Recreating and comparing patterns is an important requirement for discovering regularities (Vogel, 2005).

5. **Representing** and **describing** patterns involves determining the rules that characterize its structure according to appropriate descriptive features (Stern et al., 2003). These descriptions enable the reconstruction of patterns.

Interaction with patterns does not imply strict following of this order and largely depends on the context in which the patterns are used, the tasks that are used as motivation for their use (Vogel, 2005; Tsamir et al., 2017).
In addition to the mentioned operations, some of the authors highlight the most common activities of preschool children when encountering patterns, such as: creating, abstracting and verbalizing patterns (Tsamir et al., 2017).

The creation of patterns initially takes place spontaneously, most often from materials and elements in the environment: painting different lines, making necklaces from play materials, arranging geometric shapes according to a certain order, etc. (Fox, 2005; Waters, 2004; Bäckman, 2016;)

Abstracting patterns implies that children can be asked to make a new pattern modeled on a previous one using different elements but the same unit of repetition (Rittle-Johnson et al., 2013). Being able to translate patterns into new media represents a more advanced stage than being able to copy, expand, or correct a pattern (Sarama & Clements, 2009). The abstraction of patterns occurs as one of the activities of the reproduction operation, which was discussed by Papić et al. (2011).

Some tasks focus on verbalization. For example, a child may consider a pattern such as “blue, yellow, yellow, blue, yellow, yellow” to be the same as “curved, pointed, pointed, curved, pointed, pointed” patterns and describe it in words (Papic et al., 2011).

Previously, the most common activities that we encountered in the literature were presented. It is important to emphasize that in addition to these, activities such as correcting, completing, describing the elements of the patterns and defining the rules of the forms (in a child-specific way) can be done with children.

In many studies (Zazkis & Liljedahl, 2002; Hutchinson & Prounara, 2011; Papic et al., 2011; Rittle-Johnson et al., 2014; Tsamir et al., 2017) it has been proven that children of preschool age, specifically aged 4- 6-year-olds have a wide range of abilities when it comes to repeating patterns. Children of this age are able to successfully copy, expand, and even abstract a repeating pattern. However, the level of success of the aforementioned abilities depends on the complexity of the form itself, i.e. its structure, as well as on the requirements addressed to the children. More precisely, children of preschool age show a high rate of success in those operations and activities that do not necessarily require recognition of the structure of the form, or units of repetition of the proposed pattern. Therefore, activities such as creating one’s own pattern or generalizing it, most children of preschool age are not able to do.

For example, the results of research by Hutchinson and Prounara (2011) show that most children aged 4.5-5.5 years successfully copy (75%) and extend the pattern (87.5%), while a smaller number of them (50%) are able to create their own pattern, with success varying depending on the complexity of the pattern structure and the number of elements. Many of the children's answers indicate that the children did not understand the structure of the pattern, because when asked to explain their pattern, they counted each element of the pattern individually. Only one child out of eight examined was able to recognize that he used the unit of repetition 7 times. When it came to completing the form, which involved looking at the elements to the left and right of the missing field, most children did it successfully. Although there were children who were more successful than others on individual tasks, thus showing better competence, it is not visible that they understand the structure of the pattern, but that they create the pattern according to the principle of “what goes next”. As a key step toward
understanding pattern structure, the authors cite “what goes here” reasoning, as well as the ability to identify the unit of repetition. The authors believe that at this age, work on patterns of this type should be increased, as well as their integration with other contents in the curriculum.

Rittle-Johnson et al. (2014) focused their research on understanding the pattern structure of children aged 4-5 years, relying on construction maps that indicate the level of mastery of certain operations and activities within the pattern. The construction map of this research contains four levels of abilities (less sophisticated abilities are at the bottom of the map and more sophisticated at the top), which differ according to the level of abstraction required in the task and is based on the learning trajectory of patterns and structures proposed by Clements and Sarama (2009). More specifically, Level 1 involves copying patterns, Level 2 expanding patterns, Level 3 abstracting the hidden structure of the pattern using another material, and Level 4 involves explicitly recognizing the smallest unit of the pattern. The research results show that children of this age have mastered Levels 2 and 3, that is, they are successful in pattern expansion tasks, which is also confirmed by the research of Hutchinson and Prournara (2011) and their abstraction (colors and shapes). Contrary to previously mentioned research the authors showed that children were less successful in copying task and explicit recognition of repetition units. Research shows that more exposure to activities with repetition patterns, as well as feedback that follows the activity, contributes to better understanding and better success of children on tasks of this type. For example, during the first research session, 59 out of 66 children made multiple errors on 8 tasks, while the other 7 children were correct on all 8 tasks. The number of errors that children made varied from 2 to 5. More than 65% of children made partial errors, and close to half of the children randomized objects, collated objects, or made a pattern with a simple repeating unit (AB). The number of errors decreased significantly after the children received feedback. In addition to more frequent exposure to the tasks, success was also influenced by the number of elements within the forms. For example, children made fewer mistakes with patterns that had three elements compared to those that had 4 elements.

Tsamir et al. (2017) showed in their study that children aged 5 to 6 can successfully expand patterns by more than one element, but that the success of the expansion depends both on the structure of the pattern and on the proposed continuation, and that children are more successful in extending a pattern that ends with a complete unit of repetition. When it comes to abstraction tasks, with the increase in abstractness and the ability to generalize, children’s success in perceiving the structure of a pattern also decreases, since when comparing two patterns, children were most successful when the patterns were made of the same material, and least successful when they had to recognize different structures on different materials. As part of the research, examples were chosen in which the mentioned units of repetition were complete or incomplete and in which the structure and materials shown to the children varied. The results of the research show that there were children who were successful when asked to extend the pattern, but also that this success inevitably depended on whether the repetition unit was complete or not: 67% of them successfully extended the pattern by choosing the appropriate continuation of the pattern when it ended with a complete unit of repetition, on the contrary, 60% of children chose an adequate continuation of the pattern when the unit of repetition
was not completed, or 15% of the children managed to correctly choose the endings for both patterns, without simultaneously choosing the wrong endings. Although the difference is not small, these data suggest that children are still able to see the complete repetition unit within the patterns. When it comes to understanding pattern structure in abstraction tasks, where children compared pairs of patterns in which the type of material and repetition units were varied, the results show the following:

- Children of preschool age are more successful in recognizing the structure of the pattern if the same material is used in the observed pair.
- When pairs of patterns are made of the same materials, children are equally successful at spotting the structure and non-structure of the pattern;
- When patterns are made of different materials, children perceive patterns with the same rather than different structures more easily.

Based on the previous construction maps from the research of Sarama & Clements (2009) and Rittle-Johnson et al. (2014), we suggested a construction map (Table 1) of which operations and activities preschool children first master, more precisely which are more sophisticated operations and activities at this age.

<table>
<thead>
<tr>
<th>Level</th>
<th>Operation/activity</th>
<th>Researches</th>
</tr>
</thead>
<tbody>
<tr>
<td>Level 6</td>
<td>Identification of unit of repetition</td>
<td>Hutchinson &amp; Prounara, 2011; Rittle-Johnson et al, 2014; Tsamir et al., 2017;</td>
</tr>
<tr>
<td>Level 5</td>
<td>Producing</td>
<td>Hutchinson &amp; Prounara, 2011;</td>
</tr>
<tr>
<td>Level 4</td>
<td>Abstraction</td>
<td>Rittle-Johnson et al, 2014; Tsamir et al., 2017;</td>
</tr>
<tr>
<td>Level 3</td>
<td>Completing</td>
<td>Hutchinson &amp; Prounara, 2011;</td>
</tr>
<tr>
<td>Level 2</td>
<td>Copying</td>
<td>Hutchinson &amp; Prounara, 2011; Rittle-Johnson et al, 2014;</td>
</tr>
<tr>
<td>Level 1</td>
<td>Extending</td>
<td>Hutchinson &amp; Prounara, 2011; Tsamir et al., 2017;</td>
</tr>
</tbody>
</table>

The existing construction map of research by Rittle-Johnson et al. (2014) was supplemented with 2 more levels, i.e. activities of completing and creating patterns. In addition, a change was made in the order of the levels suggested by the mentioned researchers. The aforementioned additions and changes to the construction maps were based on data on the success of children of a certain age in these activities. For example, the copying task was one of the more successful operations in Hutchinson & Prounara (2011), while in Rittle-Johnson et al. (2014) children were less successful during these activities. The reason may be in the differences in the age of the children, that is, in the cognitive abilities of children who at a younger age had a weaker ability to perceive the unit of repetition or failed to develop a sufficiently effective strategy in order to reach the correct solution. When it comes to the completing (replenishment) activity, which until now was not visible in the construction maps, we considered it to be at Level 3. Comparing the results with other operations and activities in the research of Hutchinson & Prounara (2011), the conclusion is reached that the children were less
successful in relation to the extending and copying activities, but still better in relation to the task of creating patterns. When it comes to Level 4 and Level 5 from the table shown, we established the order based on the age of the children and success in these activities, since the activities of abstraction and creating patterns were not found in all three studies. In order to determine the sequence of levels with certainty, it is necessary to rely on more relevant research that deals with these operations and activities. The results of all three studies agree that the most sophisticated level of children’s thinking with repetition patterns is the identification of the unit of repetition.

**IMPLICATIONS FOR THE WORK WITH PRESCHOOL CHILDREN**

At the very beginning, it is important to note that the educational practice in Serbia, as well as none of the previous and current preschool education programs (Pravilnik o opštim osnovama predškolskog programa, 2006; Pravilnik o opštim osnovama predškolskog programa, 2018) do not recognize the pattern as one of mathematical content that must be implemented in everyday work, in contrast to preschool programs in other countries (Australian curriculum, 2018; Statutory framework for the early years foundation stage, 2021).

In order to realize the potential of patterns in early education plans and programs, it is clear that practice in kindergarten is crucial. It is necessary to implement them in both mathematical and other forms of activity with children of this age. Activities with patterns can be developed through a range of contexts, such as those present in literature, music, dance, children’s physical activity, etc. Many authors recommend explicit pattern learning as well as child-initiated play (Garrick et al., 2005; McCluskey et al., 2018). Pattern teaching activities should include as many activities and operations as possible, not only those that involve copying or extending them (Garrick et al., 2005; Miller et al., 2016).

Apart from the indisputable need to implement content that breaks patterns in educational work with children of preschool age, it is also necessary to emphasize the need for training both kindergarten teachers and future kindergarten teachers to work with them. Some of existing research shows that pattern teaching should be supported by knowledge of its developmental progression (Frye et al., 2013) since practitioner understanding of patterns (Houssart, 2000) and identification of pattern types (McGarvey, 2012) vary in their sophistication and complexity. A specific assessment of children’s ability to work on patterns should enable overcoming difficulties through practice.

**CONCLUSION**

As already has been pointed out several times, the importance of patterns in general, and especially repeating patterns at preschool age is very big. Therefore, it is not surprising that there is a large number of research, especially at preschool age, which we encountered during the literature review. Researches show that children of preschool age master operations such as copying, expanding and abstracting patterns, which does not necessarily imply that children are able to recognize structure or the repetition unit of the pattern. More precisely, we can conclude that: 1. children at preschool age are
able to successfully work with repeating pattern operations that do not necessarily imply recognition of its structure, and 2. children's success largely depends on the complexity of the structure, the number of pattern elements and the number of elements within the repetition unit.

Given that most of the research we encountered deals with repeating patterns, some implications for future research may include examining the possibilities and abilities of preschool children in working with other types of patterns, such as growing and spatial patterns. Since most of the existing research is based on operations and activities that have already been examined at these ages, it is especially important to pay attention to other operations and activities that preschool children perform when working with patterns, such as describing, verbalizing, “fixing” the pattern, etc. Some future research should examine children’s success in these activities. Additional systematization of existing research with new data can help expand the already existing construct map (Sarama & Clements, 2009, Rittle-Johnson et al., 2014). Observation of children while working on other activities with patterns or individual interviews with them can contribute to supplementing and elucidating the picture about the strategies used with patterns, but also about in which specific cases they lead children to the correct solution and successful understanding of the repetition unit.

References


29. Pravilnik o osnovama programa predškolskog vaspitanja i obrazovanja (2018). Prosvetni glasnik, br. 16


ПОНАВЉАЊЕ ШАБЛОНСКИХ АКТИВНОСТИ У ПРЕДШКОЛСКОМ УЗРАСТУ

РЕЗИМЕ: Активности са обрасцима су веома честе и веома важне у периоду раног образовања деце. Обрасци су присутни у различитим облицима и контекстима у вртићу, где се деца предшколског узраста често сусрећу са ситуацијама у којима раде на уочавању или предвиђању одређених правилности, било да је реч о свакодневним животним ситуацијама попут ређања у колону (дечак-девојчица-дечак-девојчица) или цртања шара зебри (црно-бело-црно-бело) итд. Нова истраживања раног математичког образовања указују на (позитиван) утицај овог садржаја на развој бројних математичких компетенција, не само оних које се односе на алгебарске способности, мерење и просторно закључивање. Циљ овог рада јесте да прикаже развој компетенција у вези са појмом образца код деце предшколског узраста. У првом делу рада су приказана теоријске основе математичких образаца, њихова категоризација и типови, њихов значај у предшколском узрасту, као и специфичне активности и операције са којима се могу сусрети у раду са децом предшколског узраста. У раду су приказане и особености посебне категорије образаца – образаца понављања, који су у фокусу овог рада. Надаље, у раду су приказана нека од претходних истраживања која су усмерена на дечије активности приликом идентификовања структуре образаца или јединица понављања.

КЛЮЧНЕ РЕЧИ: обрасци понављања, предшколски узраст, јединица понављања.