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Application of quasi-f-power increasing sequence in absolute $\phi - |C, \alpha, \beta; \delta; l|$ of infinite series**

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ABSTRACT. An increasing quasi-f-power sequence of a wider class has been used to establish a universal theorem on a least set of conditions, which is sufficient for an infinite series to be generalized $\phi - |C, \alpha, \beta; \delta; l|_k$ summable. Further, a set of new and well-known arbitrary results have been obtained by using the main theorem. Considering suitable conditions a previous result has been obtained, which validates the current findings. In this way, Bounded Input Bounded Output(BIBO) stability of impulse has been improved by finding a minimal set of sufficient condition for absolute summability because absolute summable is the necessary and sufficient conditions for BIBO stability.

1. INTRODUCTION

Let $\sum a_n$ be an infinite sequence of partial sums, $\{s_n\}$ and n^{th} mean of the sequence $\{s_n\}$ is given by u_n, s.t.,

$$u_n = \sum_{k=0}^{\infty} u_{nk} s_k.$$

An infinite series $\sum a_n$ is absolute summable, if

$$\lim_{n\to\infty} u_n = s,$$

$$\sum_{n=1}^{\infty} |u_n - u_{n-1}| < \infty.$$

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Let t_n represents the n^{th} $(C, 1)$ means of the sequence $\{na_n\}$, then the series $\sum a_n$ is s.t.b. $|C, 1|_k$ summable for $k \leq 1$, [9] if

$$\sum_{n=1}^{\infty} \frac{1}{n} |t_n|^k < \infty.$$

The n^{th} Cesàro means of order $(\alpha,\ \beta)$, with $\alpha + \beta >$ -1, of the sequence $\{na_n\}$ is denoted by $t_n^{\alpha,\beta}$, [1], i.e.

$$t_n^{\alpha,\beta} = \frac{1}{A_n^{\alpha+\beta}} \sum_{v=1}^{n} A_{n-v}^{\alpha-1} A_v^{\beta} v a_v,$$

where

$$A_n^{\alpha+\beta} = \begin{cases} 0, & n < 0, \\ 1, & n = 0, \\ O(n^{\alpha+\beta}), & n > 0. \end{cases}$$

If the sequence $t_n^{\alpha,\beta}$ satisfies

$$\sum_{n=1}^{\infty} \frac{\phi_n^{k-1}}{n^k} |t_n^{\alpha,\beta}|^k < \infty,$$

then the series $\sum a_n$ is said to be summable $\phi - |C, \alpha, \beta|_k$, for $k \geq 1$.

If the mean $t_n^{\alpha,\beta}$ satisfies

$$\sum_{n=1}^{\infty} \frac{\phi_n^{(k-1)l}}{n^{l(k-k\delta)}} |t_n^{\alpha,\beta}|^k < \infty,$$

then the infinite series is said to be summable $\phi - |C, \alpha, \beta; \delta; l|_k$, for $k \geq 1$, $\delta \geq 0$ and l is a real number.

Bor [2–6] gave numerous theorems on absolute summability and Cesàro summability. In 2008, Bor [4] used almost increasing sequence for establishing a theorem on $|C, \alpha, \gamma, \beta|_k$ summable factor. Özarslan [12] generalized the result on $\phi - |C, 1|_k$ by a more general absolute summability $\phi - |C, \alpha|_k$. Özarslan [11,13] has proved some results on absolute summability factors and on generalized Cesàro summability. Sonker and Munjal [14, 15] determined theorems on generalized absolute summability with the sufficient conditions for infinite series. Also Mishra et. al. [17–19] gave results on trignometric approximation. In [20, 21] Srivastava and Singh used the summability to approximate the trigonometric Fourier periodic functions and conjugate functions in Lipschitz class & weighted class. In this paper an advanced study has been carried out to further generalize the result of Sonker and Munjal [15, 16].

2. Known-result

A quasi-f-power increasing sequence is a positive sequence $B = B_n$ with a constant $K = K(B, f) \geq 1$ for all $1 \leq m \leq n$ such that

$$K f_n B_n \geq f_m B_m, \tag{1}$$

$$f = [f_n(\zeta, \eta)] = [n^\zeta (\log n)^\eta,\ \ 0 < \zeta < 1,\ \eta \geq 0]. \tag{2}$$

If we take $\zeta = 0$, then we get a quasi-η-power increasing sequence.

A quasi-f-power increasing sequence converted to quasi-ζ-power increasing sequence [10], if η has certain value as η=0 in the condition (2). With the help of Cesàro summability of order α, Bor [7] has proved the following theorem.

Theorem 1. *Let B_n be a quasi-f-power sequence for some ξ $(0 < \xi < 1)$. Also suppose there exists a sequence of numbers $\{D_n\}$ such that it is ξ-quasi-monotone satisfies the following:*

$$\sum n \xi_n B_n = O(1), \tag{3}$$

$$\triangle D_n \leq \xi_n, \tag{4}$$

$$| \triangle \lambda_n| \leq |D_n|, \tag{5}$$

$$\sum D_n B_n \text{ is convergent for all } n. \tag{6}$$

If the conditions

$$|\lambda_n| B_n = O(1), \quad \text{as } n \to \infty, \tag{7}$$

$$\sum_{n=1}^{m} \frac{(w_n^\alpha)^k}{n} = O(B_m), \quad \text{as } m \to \infty, \tag{8}$$

are satisfied, then the series $\sum a_n \lambda_n$ is $|C, \alpha|_k$ summable for $0 < \alpha \leq 1$ and $k \geq 1$.

3. Main result

Theorem 2. *Let B_n be a quasi-f-power increasing sequence for some ζ $(0 < \zeta < 1)$ and $\{D_n\}$ be a ξ-quasi-monotone sequence of numbers, s.t.,*

$$\sum n \xi_n B_n = O(1), \tag{9}$$

$$\triangle D_n \leq \xi_n, \tag{10}$$

$$| \triangle \lambda_n| \leq |D_n|, \tag{11}$$

$$\sum D_n B_n < \infty, \quad \text{for all } n. \tag{12}$$

If the conditions

$$|\lambda_n| \leq |D_n|, \tag{13}$$

$$\sum_{n=v}^{m} \frac{\phi_n^{l(k-1)}}{n^{(\alpha+\beta-l\delta+l)k}} = O\left(\frac{\phi_v^{l(k-1)}}{v^{(\alpha+\beta-l\delta+l)k-1}}\right), \tag{14}$$

$$\sum_{n=1}^{m} \frac{\phi_n^{l(k-1)}}{n^{l(k-\delta k)}} (w_n^{\alpha,\beta})^k = O(B_m), \quad \text{as } m \to \infty, \tag{15}$$

are satisfied, then the series $\sum a_n\lambda_n$ *is* $\phi - |C, \alpha, \beta; \delta; l|_k$ *for* $k \geq 1$, $\delta \geq 0$, $0 < \alpha \leq 1$, $\beta > -1$, $\alpha + \beta > 0$ *and* l *is a real number and* $w_n^{\alpha,\beta}$ *is given by*

$$w_n^{\alpha,\beta} = \begin{cases} \max\limits_{1\leq v\leq n} |t_v^{\alpha,\beta}|, & \beta > -1,\ 0 < \alpha < 1, \\ |t_n^{\alpha,\beta}|, & \beta > -1,\ \alpha = 1. \end{cases} \tag{16}$$

4. LEMMAS

We need the following lemmas for the proof of our main theorem.

Lemma 1. *If* $0 < \alpha \leq 1$, $\beta > -1$ *and* $1 \leq v \leq n$, *then*

$$\left|\sum_{p=0}^{v} A_{n-p}^{\alpha-1} A_p^{\beta} v a_p\right| = \max_{1\leq m\leq v} \left|\sum_{p=0}^{m} A_{m-p}^{\alpha-1} A_p^{\beta} v a_p\right|.$$

Proof. See [8]. □

Lemma 2. *Let* B_n *be a quasi-f-power increasing sequence for some* ζ $(0 < \zeta < 1)$ *and* $\{D_n\}$ *be a* ξ *quasi-monotone sequence of numbers s.t.*

$$\sum n\xi_n B_n < \infty,$$

$$\triangle D_n \leq \xi_n,$$

$$\sum_{n=1}^{\infty} n\xi_n |B_n| < \infty.$$

Then

$$\sum n\xi_n B_n = O(1), \quad \text{as } n \to \infty.$$

Proof. See [6]. □

5. Proof of the main theorem

The series $\sum a_n\lambda_n$ will be $\phi-|C,\alpha,\beta;\delta;l|_k$ summable, if the n^{th} mean $T_n^{\alpha,\beta}$ of order $\alpha+\beta$ of the sequence $\{na_n\lambda_n\}$ satisfies

$$\sum_{n=1}^{\infty}\frac{\phi_n^{l(k-1)}}{n^{l(k-\delta k)}}|T_n^{\alpha,\beta}|^k<\infty. \tag{17}$$

Using Abel's transformation, the n^{th} mean $T_n^{\alpha,\beta}$ of the sequence $\{na_n\lambda_n\}$ is given by

$$\begin{aligned}T_n^{\alpha,\beta}&=\frac{1}{A_n^{\alpha+\beta}}\sum_{v=1}^{n}A_{n-v}^{\alpha-1}A_v^{\beta}va_v\lambda_v\\&=\frac{1}{A_n^{\alpha+\beta}}\sum_{v=1}^{n-1}\Delta\lambda_v\sum_{p=1}^{v}A_{n-p}^{\alpha-1}A_p^{\beta}pa_p\\&\quad+\frac{\lambda_n}{A_n^{\alpha+\beta}}\sum_{v=1}^{n}A_{n-v}^{\alpha-1}A_v^{\beta}va_v,\end{aligned}$$

$$\begin{aligned}|T_n^{\alpha,\beta}|&\le\frac{1}{A_n^{\alpha+\beta}}\sum_{v=1}^{n-1}|\Delta\lambda_v|\left|\sum_{p=1}^{v}A_{n-p}^{\alpha-1}A_p^{\beta}pa_p\right|\\&\quad+\frac{|\lambda_n|}{A_n^{\alpha+\beta}}\left|\sum_{v=1}^{n}A_{n-v}^{\alpha-1}A_v^{\beta}va_v\right|\\&\le\frac{1}{A_n^{\alpha+\beta}}\sum_{v=1}^{n-1}|\Delta\lambda_v|A_v^{\alpha+\beta}w_v^{\alpha,\beta}+|\lambda_n|w_n^{\alpha,\beta}\\&=T_{n,1}^{\alpha,\beta}+T_{n,2}^{\alpha,\beta}\quad(say).\end{aligned} \tag{18}$$

Using Minkowski's inequality,

$$|T_n^{\alpha,\beta}|^k=|T_{n,1}^{\alpha,\beta}|+|T_{n,2}^{\alpha,\beta}|\le 2^k\left(|T_{n,1}^{\alpha,\beta}|^k+|T_{n,2}^{\alpha,\beta}|^k\right). \tag{19}$$

In order to complete the proof of the theorem, it is sufficient to show that

$$\sum_{n=1}^{\infty}\frac{\phi_n^{l(k-1)}}{n^{l(k-\delta k)}}|T_{n,r}^{\alpha,\beta}|^k<\infty,\ \ \text{for } r=1,2. \tag{20}$$

By using Hölder's inequality, Abel's transformation and the conditions of Lemma 1 and Lemma 2, we have

$$\begin{aligned}
&\sum_{n=2}^{m+1}\frac{\phi_n^{l(k-1)}}{n^{l(k-\delta k)}}|T_{n,1}^{\alpha,\beta}|^k\\
&\leq \sum_{n=2}^{m+1}\frac{\phi_n^{l(k-1)}}{n^{l(k-\delta k)}}\frac{1}{A_n^{\alpha+\beta}}\left(\sum_{v=1}^{n-1}|\Delta\lambda_v|A_v^{\alpha+\beta}w_v^{\alpha,\beta}\right)^k\\
&\leq \sum_{n=2}^{m+1}\frac{\phi_n^{l(k-1)}}{n^{(\alpha+\beta-l\delta+l)k}}\sum_{v=1}^{n-1}v^{(\alpha+\beta)k}|D_v|(w_v^{\alpha,\beta})^k\times\left(\sum_{v=1}^{n-1}|D_v|\right)^{k-1}\\
&=O(1)\sum_{v=1}^{m}v^{(\alpha+\beta)k}|D_v|(w_v^{\alpha,\beta})^k\sum_{v+1}^{m+1}\frac{\phi_n^{l(k-1)}}{n^{(\alpha+\beta-l\delta+l)k}}\\
&=O(1)\sum_{v=1}^{m}v^{(\alpha+\beta)k}|D_v|(w_v^{\alpha,\beta})^k\frac{\phi_v^{l(k-1)}}{v^{(\alpha+\beta-l\delta+l)k-1}}\\
&=O(1)\sum_{v=1}^{m}v|D_v|(w_v^{\alpha,\beta})^k\frac{\phi_v^{l(k-1)}}{v^{l(k-\delta k)}}\\
&=O(1)\sum_{v=1}^{m-1}\Delta(v|D_v|)\sum_{r=1}^{v}(w_r^{\alpha,\beta})^k\frac{\phi_r^{k-1}}{r^{k-\delta k}}\\
&\quad+O(1)m|D_m|\sum_{v=1}^{m}(w_v^{\alpha,\beta})^k\frac{\phi_v^{k-1}}{v^{k-\delta k}}\\
&=O(1)\sum_{v=1}^{m-1}|(v+1)\Delta|D_v|-|D_v||B_v+O(1)m|D_m|B_m\\
&=O(1)\sum_{v=1}^{m-1}v|\Delta D_v|B_v+O(1)\sum_{v=1}^{m-1}|D_v||B_v|+O(1)m|D_m|B_m\\
&=O(1)\sum_{v=1}^{m-1}v\xi_vB_v+O(1)\sum_{v=1}^{m-1}|D_v||B_v|+O(1)m|D_m|B_m\\
&=O(1),\ \text{ as } m\to\infty, \qquad (21)
\end{aligned}$$

$$\begin{aligned}
\sum_{n=2}^{m+1}\frac{\phi_n^{l(k-1)}}{n^{l(k-\delta k)}}|T_{n,2}^{\alpha,\beta}|^k &= O(1)\sum_{n=1}^{m}\frac{\phi_n^{l(k-1)}}{n^{l(k-\delta k)}}|\lambda_n|(w_n^{\alpha,\beta})^k\\
&= O(1)\sum_{n=1}^{m-1}\Delta|\lambda_n|\sum_{v=1}^{n}\frac{\phi_v^{l(k-1)}}{v^{l(k-\delta k)}}(w_v^{\alpha,\beta})^k\\
&\quad+O(1)|\lambda_m|\sum_{n=1}^{m}\frac{\phi_n^{l(k-1)}}{n^{l(k-\delta k)}}(w_n^{\alpha,\beta})^k
\end{aligned}$$

$$= O(1)\sum_{n=1}^{m-1} \Delta|\lambda_n|B_n + O(1)|\lambda_m|B_m$$

$$= O(1)\sum_{n=1}^{m-1} |D_n|B_n + O(1)|\lambda_m|B_m$$

$$= O(1), \quad \text{as } m \to \infty. \tag{22}$$

Collecting (17)-(22), we have

$$\sum_{n=1}^{\infty} \frac{\phi_n^{l(k-1)}}{n^{l(k-\delta k)}} |T_{n,r}^{\alpha,\beta}|^k < \infty. \tag{23}$$

Hence the proof of the theorem is complete.

6. Corollaries

Corollary 1. *Let B_n be a quasi-f-power increasing sequence for some ζ $(0 < \zeta < 1)$ and $\{D_n\}$ be a ξ quasi-monotone sequence of numbers satisfying* (9)-(13) *and the following conditions:*

$$\sum_{n=v}^{m} \frac{\phi_n^{l(k-1)}}{n^{(\alpha-l\delta+l)k}} = O\left(\frac{\phi_v^{l(k-1)}}{v^{(\alpha-l\delta+l)k-1}}\right), \tag{24}$$

$$\sum_{n=1}^{m} \frac{\phi_n^{l(k-1)}}{n^{l(k-\delta k)}} (w_n^{\alpha})^k = O(B_m), \quad \text{as } m \to \infty. \tag{25}$$

Then the series $\sum a_n\lambda_n$ is $\phi - |C,\alpha;\delta;l|_k$, for $k \geq 1$, $\delta \geq 0$, $0 < \alpha \leq 1$ and l is a real number and w_n^{α} is given by

$$w_n^{\alpha} = \begin{cases} \max\limits_{1\leq v\leq n} |t_v^{\alpha}|, & 0 < \alpha < 1, \\ |t_n^{\alpha}|, & \alpha = 1. \end{cases}$$

Proof. By using $\beta = 0$ in main theorem, we will get (24) and (25). We omit the details of the proof as it is similar to that of the main theorem 2. □

Corollary 2. *Let B_n be a quasi-f-power increasing sequence for some ζ $(0 < \zeta < 1)$ and $\{D_n\}$ be a ξ quasi-monotone sequence of numbers satisfying* (9)-(13) *and the following conditions:*

$$\sum_{n=v}^{m} \frac{\phi_n^{(k-1)}}{n^{(\alpha+1)k}} = O\left(\frac{\phi_v^{(k-1)}}{v^{(\alpha+1)k-1}}\right), \tag{26}$$

$$\sum_{n=1}^{m} \frac{\phi_n^{(k-1)}}{n^{k}} (w_n^{\alpha})^k = O(B_m), \quad \text{as } m \to \infty. \tag{27}$$

Then the series $\sum a_n\lambda_n$ *is* $\phi - |C,\alpha|_k$*, for* $k \geq 1$*,* $0 < \alpha \leq 1$ *and* w_n^α *is given by*

$$w_n^\alpha = \begin{cases} \max\limits_{1\leq v\leq n} |t_v^\alpha|, & 0<\alpha<1, \\ |t_n^\alpha|, & \alpha = 1. \end{cases} \tag{28}$$

Proof. By using $\beta = 0$, $\delta = 0$ and $l = 1$ in main theorem, we will get (26) and (27). We omit the details of the proof as it is similar to that of the main theorem 2. □

Corollary 3. *Let* B_n *be a quasi-*f*-power increasing sequence for some* ζ $(0 < \zeta < 1)$ *and* $\{D_n\}$ *be a* ξ *quasi-monotone sequence of numbers satisfying* (9)-(13) *and the following condition*

$$\sum_{n=1}^{m} \frac{(w_n^\alpha)^k}{n^k} = O(B_m), \quad as\ m \to \infty. \tag{29}$$

Then the series $\sum a_n\lambda_n$ *is* $|C,\alpha|_k$*, for* $k \geq 1$*,* $0 < \alpha \leq 1$ *and* w_n^α *is given by*

$$w_n^\alpha = \begin{cases} \max\limits_{1\leq v\leq n} |t_v^\alpha|, & 0<\alpha<1, \\ |t_n^\alpha|, & \alpha = 1. \end{cases} \tag{30}$$

Proof. By using $\phi = n$, $\beta = 0$, $\delta = 0$ and $l = 1$ in main theorem, we will get (29). We omit the details of the proof as it is similar to that of the main theorem 2. □

Corollary 4. *Let* B_n *be a quasi-*f*-power increasing sequence for some* ζ $(0 < \zeta < 1)$ *and* $\{D_n\}$ *be a* ξ *quasi-monotone sequence of numbers satisfying* (9)-(13) *and the following conditions*

$$\sum_{n=v}^{m} \frac{\phi_n^{(k-1)}}{n^{(\alpha+\beta+1)k}} = O\left(\frac{\phi_v^{(k-1)}}{v^{(\alpha+\beta+1)k-1}}\right), \tag{31}$$

$$\sum_{n=1}^{m} \frac{\phi_n^{(k-1)}}{n^k}(w_n^{\alpha,\beta})^k = O(B_m), \quad as\ m \to \infty, \tag{32}$$

are satisfied, then the series $\sum a_n\lambda_n$ *is* $\phi - |C,\alpha,\beta|_k$*, for* $k \geq 1$*,* $0 < \alpha \leq 1$*,* $\beta > -1$*,* $\alpha + \beta > 0$ *and* $w_n^{\alpha,\beta}$ *is given by*

$$w_n^{\alpha,\beta} = \begin{cases} \max\limits_{1\leq v\leq n} |t_v^{\alpha,\beta}|, & \beta > -1,\ 0<\alpha<1, \\ |t_n^{\alpha,\beta}|, & \beta > -1,\ \alpha = 1. \end{cases} \tag{33}$$

Proof. By using $\delta = 0$ and $l = 1$ in main theorem, we will get (31) and (32). We omit the details of the proof as it is similar to that of the main theorem 2. □

The idea of summability of infinite series has been applied in almost all application areas of science like rectification of signals in FIR filter (Finite Impulse Response Filter) and IIR filter (Infinite Impulse Response Filter), to speed of the rate of convergence, orthogonal series and approximation theory. Our concept of $\phi - |C, \alpha, \beta; \delta; l|$ is also used to approximate the trigonometric Fourier periodic functions and conjugate functions and help the researchers of other areas of science who are working on the existing results of Cesàro summability.

7. Conclusion

The aim of our paper is to obtain the minimal set of conditions for an infinite series to be absolute Cesàro summable $\phi - |C, \alpha, \beta; \delta; l|_k$. Through the investigation we may conclude that our theorem is a generalized version which can be reduced for several well known summabilities as shown in corollaries.

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