Differential subordination and superordination results for generalized “Srivastava–Attiya” fractional integral operator

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Abstract. In this paper, we derive some subordination and superordination results for the generalized “Srivastava-Attiya” fractional integral operator. Some interesting corollaries for this operator is also obtained.

1. Introduction and preliminaries

Let $H(U)$ denote the class of analytic functions in the open unit disk $U = \{ z \in \mathbb{C} : |z| < 1 \}$ and $S(U)$ denote the subclass of $H(U)$ consisting of functions which are also univalent in $U$. Further let $H[a, p]$ be the subclass of $H(U)$ consisting of function of the form

$$f(z) = a + a_p z^p + a_{p+1} z^{p+1} + \ldots, \quad (a \in \mathbb{C}, \ p \in \mathbb{N} = \{1, 2, 3, \ldots \}).$$

Let $A_p$ denote the class of all analytic functions of the form

$$f(z) = z^p + \sum_{k=p+1}^{\infty} a_k z^k, \quad (p \in \mathbb{N}). \tag{1}$$

For simplicity, we write $A_1 := A$.

Given two functions $f \in H(U)$ and $g \in H(U)$, we say that $f$ is subordinate to $g$ or $g$ is superordinate to $f$ in $U$ and write $f \prec g$, if there exists a Schwarz function $w$, analytic in $U$, with $w(0) = 0$ and $|w(z)| < 1$, $z \in U$, such that $f(z) = g(w(z))$ in $U$. In particular, if $g(z)$ is univalent in $U$, we have the following equivalence:

$$f(z) \prec g(z), \quad (z \in U) \iff [f(0) = g(0) \text{ and } f(U) \subset g(U)].$$

Supposing that $h$ and $k$ are two analytic functions in $U$, let $\phi(r, s, t; z) : \mathbb{C}^3 \times U \to \mathbb{C}$. If $h$ and $\phi(h(z), zh'(z), z^2 h''(z); z)$ are univalent and if $h$ and
\( \phi(h(z), zh'(z), z^2 h''(z); z) \) are univalent functions in \( U \) and \( h \) satisfies the second-order superordination

\[
(2) \quad k(z) \prec \phi(h(z), zh'(z), z^2 h''(z); z),
\]

then \( k(z) \) is said to be a solution of the differential superordination (2). A function \( q \in \mathbb{U} \) is called a subordinant of (2), if \( q(z) \prec h(z) \) for all the functions \( h \) satisfying (2). A univalent subordinant that satisfies \( q(z) \prec \tilde{q}(z) \) for all of the subordinants \( q \) of (2), is said to be the best subordinant. Recently, Miller and Mocanu [6] obtained the sufficient conditions on the functions \( k, q \) and \( \phi \) for which the following implication holds:

\[
k(z) \prec \phi(h(z), zh'(z), z^2 h''(z); z) \Rightarrow q(z) \prec h(z).
\]


\[
q_1(z) \prec \frac{zf'(z)}{f(z)} \prec q_2(z),
\]

where \( q_1 \) and \( q_2 \) are given univalent function in \( \mathbb{U} \). Also, Shanmugam et al. [10] obtained sufficient conditions for a normalized analytic \( f(z) \) to satisfy

\[
q_1(z) \prec \frac{f(z)}{zf'(z)} \prec q_2(z),
\]

\[
q_1(z) \prec \frac{z^2 f'(z)}{(f(z))^2} \prec q_2(z),
\]

where \( q_1 \) and \( q_2 \) are given univalent function in \( \mathbb{U} \) with \( q_1(0) = 1 \) and \( q_2(0) = 1 \). Further subordination results can be found in [7, 8, 11–13].

The fractional integral operator (see [20]) of order \( \lambda (\lambda > 0) \) is defined for a function \( f \) by

\[
D_{z}^{-\lambda} f(z) = \frac{1}{\Gamma(\lambda)} \int_{0}^{z} \frac{f(t)}{(z-t)^{1-\lambda}} dt,
\]

where \( f \) is analytic function in a simply-connected region of \( z \)-plane containing the origin and the multiplicity of \( (z-t)^{1-\lambda} \) is removed by requiring \( \log(z-t) \) to be real, when \( \Re(z-t) > 0 \).

Recently, Srivastava and Attiya [21] introduced and investigated the linear operator: Now for \( f \in \mathcal{A}, b \in \mathbb{C} \setminus \mathbb{Z}_0^− \) and \( s \in \mathbb{C} \), we define the function \( G_{s, b}(z) \) by

\[
G_{s, b}(z) := (1 + b)^s \left[ \Phi(z, s, b) - b^{-s} \right], \quad (z \in \mathbb{U}).
\]

We also denote by

\[
J_{s, b}(f) : \mathcal{A} \rightarrow \mathcal{A}
\]
the linear operator defined by
\[
J_{s,b}(f)(z) := G_{s,b}(z) \ast f(z), \quad (z \in \mathbb{U}; f \in \mathcal{A}; b \in \mathbb{C} \setminus \mathbb{Z}_0^-; s \in \mathbb{C})
\]
in terms of the Hadamard product (or convolution).

We note that
\[
J_{s,b}f(z) = z + \sum_{k=2}^{\infty} \left( \frac{1+b}{k+b} \right)^s a_k z^k, \quad (z \in \mathbb{U}; b \in \mathbb{C} \setminus \mathbb{Z}_0^-; s \in \mathbb{C}; f \in \mathcal{A}).
\]

**Remark 1.** It follows from (5) and (6) that one can define the operator 
\( J_{s,b}(f) \) for \( b \in \mathbb{C} \setminus \mathbb{Z}_0^- \). Therefore, we may use the following limit relationship:
\[
J_{s,0}f(z) := \lim_{b \to 0} \{ J_{s,b}(f)(z) \}.
\]

Motivated essentially by the above-mentioned “Srivastava-Attiya” operator, Wang [22] introduced the operator for the class \( \mathcal{A}_p \).
\[
J_{s,b}^{\alpha,p}(f) : \mathcal{A}_p \to \mathcal{A}_p,
\]
which is defined as
\[
J_{s,b}^{\alpha,p}f(z) = z^p + \sum_{k=1}^{\infty} \left( \frac{\alpha + p}{k} \right) \left( \frac{p+b}{p+k+b} \right)^s a_{p+k} z^{p+k}, \quad (z \in \mathbb{U}),
\]
where \((\nu)_k\) is the Pochhammer symbol defined by
\[
(\nu)_k := \begin{cases} 
1, & k = 0, \\
\nu(\nu+1) \cdots (\nu+k-1), & k \in \mathbb{N}.
\end{cases}
\]

Recently q-extension of “Srivastava-Attiya” operator have been studied in [19], the mathematical applications of q-calculus, fractional q-calculus and the fractional q-derivative operators can be seen in [15]. Srivastava et al. [18] also reconnoiter the not-yet-widely-known fact that the so-called \((p,q)\) variation of classical q-calculus is a rather trivial and inconsequential variation of classical q-calculus. For more detail and related works one can see in ([9,14,16,17]).

Unless otherwise mentioned, we assume throughout this paper that the parameter \( s, b, p \) and \( \alpha \) are constrained as follows:
\[
(\nu)_k := \begin{cases} 
1, & k = 0, \\
\nu(\nu+1) \cdots (\nu+k-1), & k \in \mathbb{N}.
\end{cases}
\]

\[ s \in \mathbb{C}; \ b \in \mathbb{C} \setminus \mathbb{Z}_0^-; \ p \in \mathbb{N} \text{ and } \alpha > -p. \]

From (3) and (9), we get the fractional integral operator \( D_z^{-\lambda} J_{s,b}^{\alpha,p} f(z) \) defined as
\[
D_z^{-\lambda} J_{s,b}^{\alpha,p} f(z) = \frac{\Gamma(p+1)}{\Gamma(\lambda + p + 1)} z^{\lambda+p} + \sum_{k=1}^{\infty} \frac{(\alpha + p)_k}{k!} \frac{\Gamma(p + k + 1)}{\Gamma(\lambda + p + k + 1)} \left( \frac{p+b}{p+k+b} \right)^s a_{p+k} z^{p+k+\lambda}
\]
for \((\lambda + p + 1 > 0, \alpha + p > 0)\). Also, it is easily verified from (12) that
\[
(13) \quad z \left( \mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z) \right)' = (\lambda - \alpha) \mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z) + (\alpha + p) \mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha+1,p} f(z).
\]

**Definition 1** (Miller and Mocanu [6]). Denote by \(Q\) the set of all functions \(f(z)\) that are analytic and injective on \(\mathbb{U}\setminus E(f)\), where
\[
E(f) = \{\eta \in \partial \mathbb{U} : \lim_{z \to \eta} f(z) = \infty\},
\]
and are such that \(f'(\eta) \neq 0\) for \(\eta \in \partial \mathbb{U}\setminus E(f)\).

To prove our results we shall need the following lemmas.

**Lemma 1** (Bulboacă [4]). Let \(q(z)\) be convex univalent in the unit disk \(\mathbb{U}\) and \(\theta\) and \(\psi\) be analytic in a domain \(\mathbb{D}\) containing \(q(\mathbb{U})\). Suppose that
1. \(\Re[\theta'(q(z))/\psi(q(z))] > 0\) for \(z \in \mathbb{U}\),
2. \(zq'(z)\psi(q(z))\) is starlike in \(\mathbb{U}\).

If \(p(z) \in H[q(0),1] \cap Q\) with \(p(\mathbb{U}) \subseteq \mathbb{D}\) and \(\theta(p(z)) + zp'(z)\psi(p(z))\) is univalent in \(\mathbb{U}\) and
\[
(14) \quad \theta(q(z)) + zq'(z)\psi(q(z)) \prec \theta(p(z)) + zp'(z)\psi(p(z)).
\]
then \(q(z) \prec p(z)\) and \(q\) is the best subordinant of (14).

**Lemma 2** (Frasin [5]). Let the function \(p(z)\) and \(q(z)\) be analytic in \(\mathbb{U}\) and suppose that \(q(z) \neq 0\) (\(z \in \mathbb{U}\)) is also univalent in \(\mathbb{U}\) and that \(zq'(z)/q(z)\) is starlike univalent in \(\mathbb{U}\). If \(q(z)\) satisfies
\[
(15) \quad \Re \left( 1 + \frac{c_1}{\beta} q(z) + \frac{2c_2}{\beta} (q(z))^2 + \cdots + \frac{nc_n}{\beta} (q(z))^n - \frac{zq'(z)}{q(z)} + \frac{zq''(z)}{q'(z)} \right) > 0
\]
and
\[
(16) \quad c_0 + c_1 p(z) + c_2 (p(z))^2 + \cdots + c_n (p(z))^n + \beta \frac{zp'(z)}{p(z)} \prec c_0 + c_1 q(z) + c_2 (q(z))^2 + \cdots + c_n (q(z))^n + \beta \frac{zq'(z)}{q(z)},
\]
\((z \in \mathbb{U}; c_0, c_1, c_2, \ldots, c_n, \beta \in \mathbb{C}; \beta \neq 0)\),
then \(p(z) \prec q(z)\) (\(z \in \mathbb{U}\)) and \(q\) is the best dominant.

We now first prove the following subordination result involving the operator \(\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)\).

2. **Subordination results for analytic functions**

**Theorem 1.** Let the function \(q(z)\) be analytic and univalent in \(\mathbb{U}\) such that \(q(z) \neq 0\), (\(z \in \mathbb{U}\)). Suppose that \(zq'(z)/q(z)\) is starlike univalent in \(\mathbb{U}\) and the
inequality (15) holds true. Let
\[
\Omega^m_j(c_0, c_1, c_2, \ldots c_n, \beta, \alpha, \lambda, p, f)
\]

(17)  
\[
= c_0 + c_1 \left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right) + c_2 \left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right)^2
\]

+ \cdots + c_n \left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right)^n + \beta(\alpha + p) \left( \frac{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha+1,p} f(z)}{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)} - 1 \right).
\]

If \( q(z) \) satisfies
\[
\Omega^m_j(c_0, c_1, c_2, \ldots c_n, \beta, \alpha, \lambda, p, f)
\]

(18)  
\[
< c_0 + c_1 q(z) + c_2 (q(z))^2 + \cdots + c_n (q(z))^n + \beta \frac{z q'(z)}{q(z)},
\]

\((z \in \mathbb{U}; c_0, c_1, c_2, \ldots c_n, \beta \in \mathbb{C}; \beta \neq 0)\),
then
\[
\left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right) < q(z), \quad (z \in \mathbb{U}\setminus\{0\}),
\]
and \( q \) is the best dominant.

Proof. Define the function \( h(z) \) by
\[
h(z) = \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}}, \quad (z \in \mathbb{U}\setminus\{0\}).
\]
Then a computation shows that
\[
\frac{zh'(z)}{h(z)} = \frac{z \mathcal{D}_z^{-\lambda} (J_{s,b}^{\alpha,p} f(z))'}{\mathcal{D}_z^{-\lambda} (J_{s,b}^{\alpha,p} f(z))} - (\lambda + p).
\]
By using the identity (13), we obtain
\[
\frac{zh'(z)}{h(z)} = (\alpha + p) \left( \frac{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha+1,p} f(z)}{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)} - 1 \right),
\]
which, in light of hypothesis (16), yields the following subordination
\[
c_0 + c_1 h(z) + c_2 (h(z))^2 + \cdots + c_n (h(z))^n + \beta \frac{zh'(z)}{h(z)}
\]

\[
< c_0 + c_1 q(z) + c_2 (q(z))^2 + \cdots + c_n (q(z))^n + \beta \frac{z q'(z)}{q(z)},
\]
and Theorem 1 follows by an application of Lemma 2.

For the choices \( q(z) = \frac{1+Az}{1+Bz}, \ -1 \leq B < A \leq 1 \) and \( q(z) = \left( \frac{1+z}{1-z} \right)^\mu, \ 0 \leq \mu \leq 1 \) in Theorem 1, we get Corollaries 1 and 2 below. □
Corollary 1. Assume that (15) holds true. If \( f \in \mathcal{A}_p \) and
\[
\Omega^m_j(c_0, c_1, c_2, \ldots, c_n, \beta, \alpha, \lambda, p, f)
\prec c_0 + c_1 \left( \frac{1 + Az}{1 + Bz} \right) + c_2 \left( \frac{1 + Az}{1 + Bz} \right)^2 + \cdots
+ c_n \left( \frac{1 + Az}{1 + Bz} \right)^n
+ \beta \frac{(A - B)z}{(1 + Az)(1 + Bz)}
(z \in \mathbb{U}; c_0, c_1, c_2, \ldots, c_n, \beta \in \mathbb{C}; \beta \neq 0),
\]
where \( \Omega^m_j(c_0, c_1, c_2, \ldots, c_n, \beta, \alpha, \lambda, p, f) \) is as defined in equation (17), then
\[
\left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right) \prec \frac{1 + Az}{1 + Bz}.
\]
and \( \frac{1 + Az}{1 + Bz} \) is the best dominant.

Corollary 2. Assume that (15) holds true. If \( f \in \mathcal{A}_p \) and
\[
\Omega^m_j(c_0, c_1, c_2, \ldots, c_n, \beta, \alpha, \lambda, p, f)
\prec c_0 + c_1 \left( \frac{1 + z}{1 - z} \right)^\mu + c_2 \left( \frac{1 + z}{1 - z} \right)^{2\mu} + \cdots
+ c_n \left( \frac{1 + z}{1 - z} \right)^{2n\mu}
+ \frac{2\beta \mu z}{1 - z^2},
(z \in \mathbb{U}; c_0, c_1, c_2, \ldots, c_n, \beta \in \mathbb{C}; \beta \neq 0),
\]
where \( \Omega^m_j(c_0, c_1, c_2, \ldots, c_n, \beta, \alpha, \lambda, p, f) \) is as defined in equation (17), then
\[
\left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right) \prec \left( \frac{1 + z}{1 - z} \right)^\mu,
\]
and \( \frac{1 + z}{1 - z} \) is the best dominant.

For \( q(z) = e^{\epsilon Az}, (|\epsilon A| < \pi) \), in Theorem 1, we get the following result.

Corollary 3. Assume that (15) holds true. If \( f \in \mathcal{A}_p \) and
\[
\Omega^m_j(c_0, c_1, c_2, \ldots, c_n, \beta, \alpha, \lambda, p, f) \prec c_0 + c_1 e^{\epsilon Az} + c_2 e^{2\epsilon Az} + c_n e^{n\epsilon Az} + \beta \epsilon Az,
\]
where \( \Omega^m_j(c_0, c_1, c_2, \ldots, c_n, \beta, \alpha, \lambda, p, f) \) is as defined in equation (17), then
\[
\left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{\mathcal{D}_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right) \prec e^{\epsilon Az}, \quad (z \in \mathbb{U}\{0\}),
\]
and \( e^{\epsilon Az} \) is the best dominant.
3. SUPERORDINATION FOR ANALYTIC FUNCTIONS

Next, applying Lemma 1, we obtain the following two theorems.

**Theorem 2.** Let \( q \) be analytic and convex univalent in \( U \) such that \( q(z) \neq 0 \) and \( \frac{zq'(z)}{q(z)} \) is starlike univalent in \( U \). Suppose also that

\[
\Re \left( \frac{c_1}{\beta} q(z) + \frac{2c_2}{\beta} (q(z))^2 + \ldots + \frac{n c_n}{\beta} (q(z))^n \right) > 0, \\
(z \in U; c_0, c_1, c_2, \ldots c_n, \beta \in \mathbb{C}; \beta \neq 0).
\]

If \( f \in A_p \)

\[
\left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{\mathcal{D}_z^{-\lambda} J^\alpha_{s,b} f(z)}{z^{\lambda+p}} \right) \in \mathcal{H}[q(0), 1] \cap Q
\]

and \( \Omega_j^m(c_0, c_1, c_2, \ldots, c_n, \beta, \alpha, \lambda, p, f) \) defined in (17) is univalent in \( U \), then the following superordination:

\[
c_0 + c_1 q(z) + c_2 (q(z))^2 + \cdots + c_n (q(z))^n + \beta \frac{zq'(z)}{q(z)} \prec \Omega_j^m(c_0, c_1, c_2, \ldots, c_n, \beta, \alpha, \lambda, p, f), \\
(z \in U; c_0, c_1, c_2, \ldots c_n, \beta \in \mathbb{C}; \beta \neq 0),
\]

implies that

\[
q(z) \prec \left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{\mathcal{D}_z^{-\lambda} J^\alpha_{s,b} f(z)}{z^{\lambda+p}} \right), \\
(z \in U \setminus \{0\}),
\]

and \( q(z) \) is the best subordinant.

**Proof.** Let

\[
\theta(\omega) = c_0 + c_1 \omega + c_2 \omega^2 + \ldots c_n \omega^n \quad \text{and} \quad \psi(\omega) := \beta \frac{\omega'}{\omega}.
\]

Then, we observe that \( \theta(\omega) \) is analytic in \( \mathbb{C} \), \( \psi(\omega) \) is analytic in \( \mathbb{C}^* = \mathbb{C} \setminus \{0\} \) and that \( \psi(\omega) \neq 0 \) (\( \omega \in \mathbb{C}^* \)). Since \( q \) is a convex univalent in \( U \), it follows that

\[
\Re \left( \frac{\theta'(q(z))}{\psi(q(z))} \right) = \Re \left( \frac{c_1}{\beta} q(z) + \frac{2c_2}{\beta} (q(z))^2 + \ldots + \frac{n c_n}{\beta} (q(z))^n \right) > 0, \\
(z \in U; c_0, c_1, c_2, \ldots, c_n, \beta \in \mathbb{C}; \beta \neq 0).
\]

Theorem 2 follows as an application of Lemma 1. \( \square \)

Combining the results of differential subordination and superordination, we state that the following sandwich result.
Theorem 3. Let $q_1$ be convex univalent and $q_2$ be univalent in $U$ such that $q_1(z) \neq 0$ and $q_2(z) \neq 0$ ($z \in U$). Suppose also that $q_2$ satisfies (19) and $q_1$ satisfies (15). If $f \in A_p$,

$$
\left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{D_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right) \in H[0,1] \cap Q
$$

and

$$
c_0 + c_1 \left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{D_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right) + c_2 \left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{D_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right)^2 + \cdots + c_n \left( \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{D_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \right)^n + \beta(\alpha + p) \left( \frac{D_z^{-\lambda} J_{s,b}^{\alpha+1,p} f(z)}{D_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)} - 1 \right),
$$

(z $\in U$; $c_0, c_1, c_2, \ldots, c_n, \beta \in \mathbb{C}$; $\beta \neq 0$)

is univalent in $U$, then the subordination given by

$$
c_0 + c_1 q_1(z) + c_2(q_1(z))^2 + \cdots + c_n(q_1(z))^n + \beta \frac{z q_1'(z)}{q_1(z)}
\prec \Omega^m_j(c_0, c_1, c_2, \ldots, c_n, \beta, \alpha, \lambda, p, f)
\prec c_0 + c_1 q_2(z) + c_2(q_2(z))^2 + \cdots + c_n(q_2(z))^n + \beta \frac{z q_2'(z)}{q_2(z)},
$$

(z $\in U$; $c_0, c_1, c_2, \ldots, c_n, \beta \in \mathbb{C}$; $\beta \neq 0$),

implies that

$$
q_1(z) \prec \frac{\Gamma(\lambda + p + 1)}{\Gamma(p + 1)} \frac{D_z^{-\lambda} J_{s,b}^{\alpha,p} f(z)}{z^{\lambda+p}} \prec q_2(z),
$$

and $q_1$ and $q_2$ are respectively, the best subordinant and the best dominant of (21).

REFERENCES


Differential subordination and superordination results


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