ANALYSIS OF SHANNON CAPACITY FOR SC AND MRC DIVERSITY SYSTEMS IN $\alpha - \kappa - \mu$ FADING CHANNEL

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ABSTRACT

In this paper, the analysis of Shannon capacity for selection combining (SC) and maximal ratio combining (MRC) diversity systems in the generalized $\alpha - \kappa - \mu$ fading channel is presented. Closed-form expressions for probability density function (PDF) at the output of SC and MRC diversity systems are given. Also, closed-form expressions for Shannon capacity for cases of SC diversity with independent and identically distributed branches, MRC diversity with independent and identically distributed branches and for case of no diversity are derived. The obtained results are numerically calculated and graphically presented for different combinations of fading parameters $\alpha$, $\kappa$ and $\mu$.

Keywords: $\alpha - \kappa - \mu$ distribution, Selection Combiner (SC), Maximal Ratio Combiner (MRC), Shannon capacity, Diversity system.

INTRODUCTION

During transmission, the signal transmitted between the transmitter and the receiver is exposed to various effects. These effects such as shadowing and multipath, adversely affect the signal being transmitted and diminish the performance of the system (Simon & Alouini, 2005a). There are several ways to reduce the impact of the presented effects on system performance. One of the most commonly used ways to reduce the impact of multipath fading is to use the diversity concept such as space diversity (Freeman, 2005). This diversity concept effectively reduce the impact of multipath fading and increase the performance of the system by using multiple transmit or receiver antennas (Stüber, 2002). These antennas are spaced sufficiently far apart so as to obtain signals that fade independently (Ibnkahla, 2005). Diversity combining techniques that are often used are selection combining technique (SC), equal gain combining technique (EGC), maximal ratio combining technique (MRC).

Performance analysis over different channel fading model are considered in many paper. Probability density function (PDF) and cumulative distribution function (CDF) over Rayleigh fading channel for EGC diversity system are given in (Talha et al., 2010). Bit error rate under $\alpha - \mu, \kappa - \mu$ and $\eta - \mu$ fading channel for SC diversity system is presented in (Mitrović et al., 2009; Subadar et al., 2010). Also, bit error rate under $\kappa - \mu$ and $\eta - \mu$ fading channel for MRC diversity system is presented in (Dixit & Sahu, 2012). System performances such as probability output ($P_{out}$) and symbol error rate (SER) over Weibull and $\alpha - \mu$ fading channel for SC diversity are shown in (Katiyar, 2015; Sagias et al., 2003). Same system performances in $\alpha - \mu, \kappa - \mu$ and $\eta - \mu$ fading channel for MRC diversity are given in (Aldalgouni et al., 2013; Subadar et al., 2012; Milišić et al., 2009). Shannon capacity over $\kappa$ and Gamma fading channel for MRC and SC diversity systems is analyzed in (Yilmaz, 2012; Subadar & Das, 2017). Diversity techniques are not the only method to reduce the impact of fading and improve the characteristics of the system. A widely used technique is the technique of adaptive transmission. By combining adaptive transmission techniques and diversity techniques, better performance of the system is achieved. Analysis of the capacities of such systems by using adaptive transmission algorithms and diversity techniques over $\alpha - \mu, \kappa - \mu$, Nakagami-$m$, Weibull and Rayleigh fading channels are investigated in (Mohamed et al., 2013; Subadar & Sahu, 2010; Panić et al., 2013; Subadar et al., 2010; Bessate & El, 2017; Simon & Alouini, 2005b).

In this paper, we investigate Shannon capacity over generalized $\alpha - \kappa - \mu$ fading channel. We consider cases for the proposed fading model when the system does not have a diversity, but also when the system has SC diversity and MRC diversity. Closed-form expressions of PDF and Shannon capacity for these systems will be presented.

SYSTEM AND CHANNEL MODEL

No diversity

In this case, the system we are discussing consists of a transmitter and receiver without a diversity combiner. Also, transmission through the slowly changing $\alpha - \kappa - \mu$ fading channel is considered. For this system, probability density function for received signal-to-noise ratio is given in (Huang & Yuan, 2018) as:
where parameter $\alpha$ represents nonlinearity of propagation environment, parameter $\kappa$ is Rice factor, parameter $\mu$ represents number of clusters in propagation environment, $\tilde{g}$ represents average SNR and $I_{\alpha}(\cdot)$ denotes modified Bessel function (Huang & Yuan, 2018).

By transforming Bessel function using equation (8.445) from (Gradshteyn & Ryzhik, 2000) and replacing in equation (1) we obtain:

$$f_{g}(g) = \frac{\alpha \kappa (1 + \kappa)^{\frac{\mu}{2}}}{2 \kappa^{2} e^{\mu \kappa}} g^{rac{\mu + 2}{2}} e^{-\frac{\mu \kappa}{2} g^2} \sum_{j=0}^{\infty} \frac{\mu \sqrt{\kappa (1 + \kappa)}}{j! \Gamma (\mu + j) \tilde{g}^{rac{\alpha + j}{2}}}$$ (2)

SC diversity with $L$ branches

In this case, we consider a SC diversity receiver with $L$ branches, operating over $\alpha - \kappa - \mu$ fading channel. All branches are identically and independently distributed. Selective combining (SC) is combining technique where the strongest signal is chosen among $L$ branches of diversity system. PDF of the SNR at the output of the SC receiver with $L$ branches can be calculated by using expression (4) from (Mitrović et al., 2009):

$$f_{g}^{SC}(g) = L f_{g}(g) (F_{g}(g))^{L-1}$$ (3)

where $F_{g}(g)$ represents a cumulative distribution function and $f_{g}(g)$ represent a probability density function for $\alpha - \kappa - \mu$ fading given in equation (2). Cumulative distribution function can be calculated as:

$$F_{g}(g) = \int_{0}^{g} f_{g}(g) dg$$ (4)

Substituting equation (2) in equation (4) and applying equation (3.381/1) from (Gradshteyn & Ryzhik, 2000), cumulative distribution function becomes:

$$F_{g}(g) = \frac{\mu (1 + \kappa)^{\frac{\mu}{2}}}{\kappa^{\frac{\mu + 2}{2}} e^{\mu \kappa}} \sum_{j=0}^{\infty} \frac{\mu \sqrt{\kappa (1 + \kappa)}}{j! \Gamma (\mu + j) \tilde{g}^{rac{\alpha + j}{2}}}$$ (5)

MRC diversity with $L$ branches

In this case, we consider a MRC diversity receiver with $L$ branches, operating over $\alpha - \kappa - \mu$ fading channel. All branches are identically and independently distributed. Maximal Ratio Combining (MRC) is combining technique where received signals from all diversity branches are co-phased, proportionally weighted, and combined to maximize the output SNR (Subadar et al., 2010). PDF of the SNR at the output of the MRC receiver with $L$ branches can be calculated by using relation (12) from (Milišić et al., 2009) where $a_{MRC} = \alpha; b_{MRC} = \kappa; \mu_{MRC} = \mu; \tilde{g}_{MRC} = \tilde{L} g$. Applying this relation in equation (2), PDF of the SNR at the output of the MRC receiver with $L$ branches becomes:

$$f_{g}^{MRC}(g) = \frac{\alpha \mu \kappa (1 + \kappa)^{\frac{\mu}{2}}}{2 \kappa^{2} e^{\mu \kappa}} e^{-\frac{\mu \kappa}{2} \tilde{g}^2} \sum_{j=0}^{\infty} \frac{\mu \sqrt{\kappa (1 + \kappa)}}{j! \Gamma (\mu + j) (\tilde{L} g)^{\frac{\alpha + j}{2}}}$$ (6)

where

$$K_{pdf} = \frac{\mu (1 + \kappa)^{\frac{\mu}{2}}}{\kappa^{\frac{\mu + 2}{2}} e^{\mu \kappa}} \sum_{j=0}^{\infty} \frac{\mu \kappa (1 + \kappa)^{\frac{\mu + 2}{2} - 1}}{j! \Gamma (\mu + j + p + 1) \tilde{L}^2 g^{\frac{\alpha + j}{2}}}$$ (7)

and

$$K_{cdf} = \frac{\mu (1 + \kappa)^{\frac{\mu}{2}}}{\kappa^{\frac{\mu + 2}{2}} e^{\mu \kappa}} \sum_{j=0}^{\infty} \frac{\mu \kappa (1 + \kappa)^{\frac{\mu + 2}{2} - 1}}{j! \Gamma (\mu + j + p + 1) \tilde{L}^2 g^{\frac{\alpha + j}{2}}}$$ (8)

Channel capacity is one of the most important performance measures of the system. The Shannon capacity of a channel defines its theoretical upper bound for the maximum rate of data transmission at an arbitrarily small bit error probability, without any delay or complexity constraints (Ibnkahla, 2005). It can be expressed as:

$$C = B \int_{0}^{\infty} \log_{2} (1 + g) f_{g}(g) dg$$ (11)
Shannon capacity - no diversity

Shannon capacity for system without diversity over $\alpha - \kappa - \mu$ fading channel can be calculated by replacing equation (2) in equation (12). After replacing equation (2) in equation (12) and after some mathematical manipulation, we obtain:

$$\frac{C_{ND}}{B} = K_{ND} \int_{0}^{\infty} g^{-\frac{\mu p}{2}} \left( \frac{\mu (1 + \kappa)}{2} e^{\frac{p g^2}{1}} \right) \ln (1 + g) \, dg$$

(13)

where

$$K_{ND} = \frac{\alpha \mu (1 + \kappa) \nu}{2 \ln (2) \kappa^{\nu + 1} e^{\mu p}} \sum_{j=0}^{\infty} \left( \frac{\mu \sqrt{\kappa (1 + \kappa)}}{j! \Gamma (\mu + j + 1/2)} \right)$$

(14)

By using equations (8.4.6/5) where is $\ln (1 + x) = G^{1,2}_{2,2} \left[ \frac{1,1}{1,0} | \frac{(1,1)}{1,0} \right]$ and (8.4.3/1) where is $e^{-x} = G^{1,0}_{1,0} \left[ \frac{-1}{0} | x \right]$ from (Prudnikov & Brychkov, 2003), equation (13) becomes:

$$\frac{C_{ND}}{B} = K_{ND} \int_{0}^{\infty} g^{-\frac{\mu p}{2}} \left( \frac{\mu (1 + \kappa)}{2} e^{\frac{p g^2}{1}} \right) \ln (1 + g) \, dg$$

(15)

where $G^{\nu,p}_{\mu,q} \left[ \frac{a_1, \ldots, a_p, a_{p+1}, \ldots, a_q}{b_1, \ldots, b_m, b_{m+1}, \ldots, b_q} \right]$ represents MeijerG function. Integral from equation (15) can be solved by using equation (2.24.1/1) from (Prudnikov & Brychkov, 2003). Applying this equation, Shannon capacity for system without diversity is:

$$\frac{C_{ND}}{B} = K_{ND} \int_{0}^{\infty} g^{-\frac{\mu p}{2}} \left( \frac{\mu (1 + \kappa)}{2} e^{\frac{p g^2}{1}} \right) \ln (1 + g) \, dg$$

(16)

where

$$\Lambda_{ND} = \left( \Delta \alpha, - \left( \frac{\alpha (\mu + j)}{2} \right), \Delta \alpha, 1 - \frac{\alpha (\mu + j)}{2} \right)$$

(17)

and

$$\Psi_{ND} = (\Delta k, b_1, \Delta \alpha, - \left( \frac{\alpha (\mu + j)}{2} \right), \Delta \alpha, - \left( \frac{\alpha (\mu + j)}{2} \right)$$

(18)

Equation (16) represent closed-form expression of Shannon capacity for system without diversity over $\alpha - \kappa - \mu$ fading channel. This expression is valid only for integer values of fading parameter $\alpha$. Also, this expression is general, and Shannon capacity for system without diversity over other fading models such as $\alpha - \mu$, $\kappa - \mu$, Nakagami - $m$ and Rayleigh models can be obtained from it.

Shannon capacity - SC diversity with $L$ branches

Shannon capacity at the output of SC diversity receiver with $L$ branches can be calculated by replacing equation (7) in equation (12). Substituting equation (7) in equation (12), applying relation for transforming logarithm and exponential functions over MeijerG function what is explained in previous section and using equation (2.24.1/1) from (Prudnikov & Brychkov, 2003) we obtain Shannon capacity:

$$\frac{C_{SC}}{B} = L K_{pdf} \left( K_{cdf}^{SC} \right)^{L - 1} G^{2+2a,\nu}_{2a+2,2a} \left( \frac{\mu (1 + \kappa) \nu}{4 \nu} \right) \Psi_{SC}$$

(19)

where

$$K_{pdf} = \frac{\mu (1 + \kappa) \nu}{\sqrt{2} \ln (2) \kappa^{\nu + 1} e^{\mu p}} \sum_{j=0}^{\infty} \left( \frac{\mu \sqrt{\kappa (1 + \kappa)}}{j! \Gamma (\mu + j)} \right)$$

(20)

$$K_{cdf} = \frac{\mu (1 + \kappa) \nu}{\sqrt{2} \ln (2) \kappa^{\nu + 1} e^{\mu p}} \sum_{j=0}^{\infty} \sum_{p=0}^{\infty} \left( \frac{\mu \sqrt{\kappa (1 + \kappa)}}{j! \Gamma (\mu + j + p + 1)} \right)$$

(21)

$$\Lambda_{SC} = \left( \Delta \alpha, - \left( \frac{\alpha (\mu + j + p) - \alpha p)}{2} \right), \Delta \alpha, 1 - \frac{\alpha (\mu + j + p) - \alpha p)}{2} \right)$$

(22)

$$\Psi_{SC} = (\Delta k, b_1, \Delta \alpha, - \left( \frac{\alpha (\mu + j + p) - \alpha p)}{2} \right), \Delta \alpha, - \left( \frac{\alpha (\mu + j + p) - \alpha p)}{2} \right)$$

(23)

Shannon capacity - MRC diversity with $L$ branches

Shannon capacity at the output of SC diversity receiver with $L$ branches can be calculated by replacing equation (10) in equation (12). Substituting equation (10) in equation (12), applying relation for transforming logarithm and exponential functions over MeijerG function what is explained in previous section and using equation (2.24.1/1) from (Prudnikov & Brychkov, 2003) we obtain Shannon capacity:

$$\frac{C_{MRC}}{B} = K_{MRC} G^{2+2a,\nu}_{2a+2,2a} \left( \frac{\mu (1 + \kappa) \nu}{4 (\nu)} \right) \Psi_{MRC}$$

(24)

where

$$K_{MRC} = \frac{2 \sqrt{\ln (2) \kappa^{\nu + 1} e^{\mu p} 2^{2}}}{\ln (2) \kappa^{\nu + 1} e^{\mu p}} \sum_{j=0}^{\infty} \left( \frac{\mu \sqrt{\kappa (1 + \kappa)}}{j! \Gamma (\mu + j)} \right)$$

(25)

$$\Lambda_{MRC} = \left( \Delta \alpha, - \left( \frac{\alpha (\mu + j)}{2} \right), \Delta \alpha, 1 - \left( \frac{\alpha (\mu + j)}{2} \right) \right)$$

(26)
Ψ_{MRC} = \left( \Delta k, b_1, \Delta \alpha, -\left( \frac{\alpha (L \mu + j)}{2} \right) \right).

(27)

NUMERICAL RESULTS

In this section, the analytically obtained results will be numerically calculated and graphically presented.

Figure 1 depicts Shannon capacity over $\alpha - \kappa - \mu$ fading channel for system without diversity. The results are shown for different combinations of parameters $\kappa$ and $\mu$ while the value of the fading parameter $\alpha$ is fixed. From Figure 1 it can be seen that increasing the value of these parameters increases the capacity of the channel. Also, from the Figure 1 it can be seen that parameter $\mu$ more influence on the channel capacity than parameter $\kappa$.

Figure 2 depicts Shannon capacity over $\alpha - \kappa - \mu$ fading channel and other fading models obtained from it for system without diversity. Results of channel capacity are shown for $\alpha - \kappa - \mu$, $\alpha - \mu$, $\kappa - \mu$, Nakagami-$m$ and Rayleigh fading models. Figure 2 is obtained by using values from Table 1 and by using equation (16). Also, Table 1 represents a generality of $\alpha - \kappa - \mu$ fading model.

Figure 3 depicts Shannon capacity over $\alpha - \kappa - \mu$ fading channel for SC diversity receiver with $L$ branches. Results are shown for values given in Table 1 and by using equation (19). The Figure 3 shows that the higher capacity is obtained for SC diversity receiver. Also, by increasing the number of the branches, channel capacity is increased.

Figure 4 depicts Shannon capacity over $\alpha - \kappa - \mu$ fading channel for MRC diversity receiver with $L$ branches. Results are shown for values given in Table 1 and by using equation (24). The Figure 4 shows that the higher capacity is obtained for MRC diversity receiver. Also, by increasing the number of the branches, channel capacity is increased.

Table 1. Generality of $\alpha - \kappa - \mu$ fading model

<table>
<thead>
<tr>
<th>Other fading models obtained from $\alpha - \kappa - \mu$ fading model</th>
<th>$\alpha - \kappa - \mu$</th>
<th>$\alpha - \mu$</th>
<th>$\kappa - \mu$</th>
<th>Nakagami-$m$</th>
<th>Rayleigh</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>1.5</td>
<td>$\kappa \to 0$</td>
<td>1.5</td>
<td>$\kappa \to 0$</td>
<td>$\kappa \to 0$</td>
</tr>
<tr>
<td>$\mu$</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1.5</td>
<td>1</td>
</tr>
</tbody>
</table>

Figure 5 shows the difference in the achieved capacity when we use the system without diversity, SC diversity with $L$ branches and MRC diversity with $L$ branches. The comparison was made in relation to the number of branches that use diversity systems. Thus, it can be seen from the Figure 5 that the maximum capacity is reached when the number of branches is $L = 3$. When we perform the comparison for the same number of branches but dif-
different diversity systems, we see that the MRC diversity system gives a better capacity than the SC diversity system. When we observe a system for $L = 2$ branches, we see that it gives a smaller capacity in relation to a system with $L = 3$ branches, but, more capacity than a system that does not have diversity. For $L = 2$ branches, MRC diversity gives better capacity than SC diversity. Results shown in Figure 5 are obtained from equations (16), (19) and (24) and parameter values given in Table 1.

![Figure 4](image-url)  
**Figure 4.** Shannon capacity for MRC diversity system with $L$ branches.

![Figure 5](image-url)  
**Figure 5.** Comparison of Shannon capacity for SC and MRC diversity systems with $L$ branches.

With the increase in SNR, capacity of the channel is also increased. In this paper, Shannon capacity for lower SNR values on the receiver (from 0 – 10dB) in order to represent the influence of parameters and used diversity techniques on channel capacity is considered.

**CONCLUSION**

In this paper, Shannon capacity analysis for $\alpha - \kappa - \mu$ fading channel by using different system models is presented. Closed-form expression for probability density function and Shannon capacity of system without diversity, SC diversity system with $L$ branches and MRC diversity system with $L$ branches are given.

Numerical results obtained from analytical closed-form expression are calculated and graphically presented for different combination of fading parameters and different system models. At the end, comparison of proposed system models are made.

The higher channel capacity are achieved by using MRC diversity system with $L$ branches. The least channel capacity are achieved for system without diversity. Although it provides less capacity compared to the MRC diversity, SC diversity is often used in practice realization for simpler implementation. Also, diversity systems with more than $L = 3$ branches are not implemented in the practical applications because of complexity. Complexity is reflected in increasing the computational complexity of receiving and transmitting algorithms, as well as the power required to process the signals. A larger number of antennas will also affect the price of the receiver.

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