Book review

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LINEAR PROGRAMMING AND ITS APPLICATIONS

Eiselt, H. A., & Sandblom, C.-L. (2007). Berlin Heidelberg: Springer-Verlag,
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In everyday life, as in many scientific fields, there is often a need for finding an optimal (minimum or maximum) solution to a problem, where certain conditions are met. Due to the simplex method, linear programming is one of the most effective approaches to formulating and solving complex problems of decision making. It is therefore increasingly applied in modern society in the planning of economic development and the control of various activities that are limited in the amount of available resources (labor force, materials, a budget, time, etc.).

The book entitled: *Linear Programming and its Applications* (2007), by H. A. Eiselt and C.-L. Sandblom, presents a unique approach to the problem of linear programming, which aims to place an emphasis on models and applications without omitting mathematical accuracy and correctness. Accordingly, the process of solving a problem is illustrated, from setting the model to analyzing the optimal values. The authors' approach, the type and the selection of different applications as well as their limitation to

a small number of sub-themes in the field (which is actually one of its advantages) suggest that the book is primarily intended for advanced students and researchers engaged in operations research and their application to economic problems.

H. A. Eiselt and C.-L. Sandblom note that the book is the last part of the trilogy. The previous two volumes are *Integer Programming and Network Models* (2000) and *Decision Analysis, Location Models, and Scheduling Problems* (2004). Although the order which the books were published in is not entirely clear, all the three volumes are similar in style, emphasize models, applications, formulations/reformulations and provide the numerical examples of the described algorithms.

The book begins with the two (A and B) introductory chapters describing the mathematical apparatus necessary for understanding the continuation of the book. Chapter A (pp. 1-30) provides a clear and concise introduction to linear algebra and highlights concepts important for the field of linear programming. Complicated mathematical proofs are excluded in order to maintain conciseness in interpretation, and readers interested in a deeper analysis are referred to the appropriate literature. Chapter B (pp. 31-44)

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presents a brief account of computational complexity, providing the reader with minimum – however quite enough – knowledge in this field that a book devoted to optimization should offer. Although readers with little prior knowledge in mathematics are unlikely to fully understand the topic briefly described and not so simple, that was not a goal pursued by the authors.

Chapter 1 (pp. 45-66) provides an introduction to the non-classical optimization method known as mathematical programming, which also includes linear programming. The relationship between mathematical programming and modeling is specifically described, for the purpose of presenting an example from real life situations with appropriate variables and parameters using mathematical model. Section 1.5, entitled: *Solving the Model and Interpreting the Printout*, illustrates the theory described with the specific examples. Chapter 2 (pp. 67-128) contains a good selection of the optimization problems pointing to important skills in modeling, noting the difference between a theoretically optimal and practical solution in optimization. The authors clearly describe the mathematical models of the objective function, a set of constraints and the conditions of non-negativity, which are actually the three essential parts of the linear program. Chapter 3 (pp. 129-166) is entirely devoted to the simplex method for solving linear programming problems. The graphical method is first described in two parts and subsequently the algebraic method is described as well. Wherever appropriate, graphic illustrations enhance the effect of the algebraic results. In Chapter 4 (pp. 167-202), there is a formal description of the theory of duality in the first place, only to be followed, for the ease of understanding, by the exemplified demonstration of the relationship between a primal and a dual problem. Because of the principally rigorous theoretical approach, readers with less prior knowledge in mathematics will have more difficulties to understand this chapter than the previous ones. As the dual problem has its own clear and important economic significance, except for calculating the solution of the primal one, the last part of this chapter is devoted to the economic interpretation of duality. Chapter 5 (pp. 203-224) is an upgrading of Chapter 3 in terms of adding the revised simplex

method. It describes the method for introducing the upper bound of constraints and for the generating of the columns of variables when necessary. The chapter contains several examples of the dual-simplex method with a brief and clear explanation. In general, this chapter provides a reader with the main reasons for a possible modification of the simplex method, but does not present enough information necessary for the pursuance of the idea.

Chapter 6 (pp. 225-260) focuses on the post-optimal analysis through the formal presentation of the sensitivity of the optimal solution to changes in initial assumptions. Using a graphical analysis, the consequences of the changes in the free parameters, the objective function coefficients, the adding/deleting of the variables or constraints are described, and generally, the answer to the question: “What if...” is given. The last part explains the economic analysis of optimal solutions with a good numerical example.

Chapter 7 (pp. 261-294) describes alternative non-simplex methods for solving linear programming problems. This part is concise and mainly focused on the interior point method, with a reference to the other methods such as: the traversal method, the external pivoting method, the gravitational method, the bounce and ellipsoid method. The chapter could be expanded with a reference to the relevant literature analyzing the application of these methods in practice. (I also believe it would be good to critically analyze the interior point method, which has increasingly been used in practice in recent decades, and to compare it with the simplex method.)

Chapter 8 (pp. 295-324) describes the techniques important for solving the problems that do not fit in the “standard” model of linear programming. This is a very useful chapter because the reader is referred to the methods of overcoming difficulties in the modeling of the given optimization problem. The method of reformulating a problem in the general form of a linear programming problem is demonstrated (by replacing variables, conditions, and objective functions). Although the chapter covers the basics of the field, readers can expect to come across interesting and useful illustrative examples.

Chapter 9 (pp. 325-362), the last one, as a good complement to the rest of the book, gives an introduction to the problem of multi-objective programming. The concept of the vector versus scalar optimization is well explained. It also describes the established methods for solving multi-objective programming, such as: the *weighting method*, *constraint*, *reference point programming*, *fuzzy programming*, *goal programming* and *bilevel programming*. Some of these methods are widely used in economics, which is shown on concrete examples.

In their introduction, H. A. Eiselt and C.-L. Sandblom clearly indicate their desire to ensure the “longevity” of the book by paying more attention to analyzing

the essence of the problem and to describing the appropriate models; however, the book lacks the use of a software package for solving optimization problems. Thus, it would be easier for researchers with insufficient experience in the field of linear programming to identify a high potential of the applications of the field in practice. However, since it is obviously impossible to introduce the whole mathematical apparatus used in linear programming in just one book, the authors have rightly focused on what is mathematically the most basic and economically the most important. Readers who study the book carefully will be able to understand the majority of scientific and professional articles in journals dedicated to this topic.

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Prikaz knjige

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U svakodnevnom životu, kao i u mnogim naučnim oblastima, često se nameće potreba nalaženja optimalnog (minimalnog ili maksimalnog) rešenja problema pri čemu su zadovoljeni određeni uslovi. Zahvaljujući simpleks metodi, linearno programiranje je jedan od najefikasnijih pristupa formulisanju i rešavanju složenih problema donošenja odluka. Zbog toga se sve više primenjuje u modernom društvu prilikom planiranja ekonomskog razvoja i kontrole raznih aktivnosti koje su ograničene količinom raspoloživih resursa (radnom snagom, sirovinama, budžetom, vremenom i sl.).

Knjiga pod naslovom: *Linear Programming and its Applications* (2007), autora H. A. Eiselt-a i C.-L. Sandblom-a, predstavlja jedinstven pristup problematici linearnog programiranja, čiji je cilj da bez izostavljanja matematičke preciznosti i korektnosti akcenat stavi na modele i aplikacije. Shodno tome, ilustrovan je proces rešavanja problema od postavljanja modela do analize dobijenih optimalnih vrednosti. Pristup autora, vrste i izbor različitih aplikacija, kao i

ograničavanje na manji broj podtema ove oblasti (što je upravo jedna od njenih prednosti) ukazuju na to da je knjiga pre svega namenjena naprednim studentima i istraživačima koji se bave operacionim istraživanjima i njihovom primenom u ekonomskim problemima.

H. A. Eiselt i C.-L. Sandblom napominju da je ova knjiga poslednji deo trilogije čija su prethodna dva toma *Integer Programming and Network Models* (2000) i *Decision Analysis, Location Models, and Scheduling Problems* (2004). Iako redosled izdavanja ovih knjiga nije u potpunosti jasan, sva tri toma su slična po stilu, naglašavaju modele, aplikacije, formulacije/reformulacije i daju numeričke primere opisanih algoritama.

Knjiga počinje sa dva uvodna poglavlja A i B, u kojima je opisan matematički aparat neophodan za razumevanje nastavka knjige. Poglavlje A (1-30 str.) daje jasan i sažet uvod u linearnu algebru i ističe pojmove značajne za oblast linearnog programiranja. Izostavljeni su komplikovaniji matematički dokazi zbog održavanja konciznosti u izlaganju, a čitaoci koji su zainteresovani za dublju analizu su upućeni na odgovarajuću literaturu. Poglavlje B (31-44 str.) objašnjava određivanje složenosti algoritama i vremena rešavanja problema, pružajući čitaocima

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minimum znanja iz ove oblasti, ali sasvim dovoljno koliko knjiga posvećena optimizaciji treba da ponudi. Čitaoci koji nemaju odgovarajuće matematičko predznanje verovatno neće u potpunosti razumeti ukratko opisanu ne tako jednostavnu problematiku, ali autorima to i nije bio cilj.

Poglavlje 1 (45-66 str.) daje uvod u neklasičnu metodu optimizacije poznatu kao matematičko programiranje koje uključuje i linearno programiranje. Specijalno je opisan odnos između matematičkog programiranja i modeliranja, čija je svrha da pomoću odgovarajućih promenljivih veličina i parametara matematičkim modelom predstavi primer iz realne životne situacije. Odeljak 1.5, pod naslovom: *Solving the Model and Interpreting the Printout*, ilustruje opisanu teoriju na konkretnom primeru. Poglavlje 2 (67-128 str.) sadrži dobar izbor problema optimizacije koji ukazuju na važne veštine prilikom modeliranja navodeći razliku između teorijski optimalnog i praktičnog rešenja pri optimizaciji. Jasno su opisani matematički modeli sa funkcijom cilja, skupom ograničenja i uslovima nenegativnosti koji u stvari i čine tri bitna dela linearnog programa. Poglavlje 3 (129-166 str.) je u potpunosti posvećeno simpleks metodi za rešavanje problema linearnog programiranja. U dva dela je najpre opisana grafička metoda, a zatim i algebarska. Gdegod je prikladno, grafičke ilustracije pojačavaju efekat algebarskih rezultata. U Poglavlju 4 (167-202 str.) je najpre izložen formalan opis teorije dualnosti, a zatim zbog lakšeg razumevanja, na primerima je pokazana veza između primarnog i dualnog problema. Zbog uglavnom strogog teorijskog pristupa, čitaocima sa slabijim matematičkim predznanjem ovo poglavlje će biti teže za razumevanje od prethodnih. Kako dualni problem, osim za izračunavanje rešenja primala, ima i sopstveno jasno i važno ekonomsko značenje, poslednji deo ovog poglavlja je posvećen ekonomskoj interpretaciji dualnosti. Poglavlje 5 (203-224 str.) predstavlja nadogradnju Poglavlja 3 u smislu dodavanja revidirane simpleks metode. Opisuje se način uvođenja gornje granice u ograničenjima i generisanje kolona promenljivih, kada je to potrebno. Poglavlje sadrži i nekoliko primera dual-simpleks metode sa kratkim i jasnim objašnjenjima. U principu, ovo poglavlje pruža čitaocu osnovne razloge moguće modifikacije simpleks metode, ali ne i dovoljno informacija neophodnih za spovođenje te ideje.

Poglavlje 6 (225-260 str.) je fokusirano na post-optimalnu analizu kroz formalno izlaganje o osetljivosti optimalnog rešenja na promene polaznih pretpostavki. Primenom grafičke analize opisane su posledice promena slobodnih parametara, koeficijenta funkcije cilja, dodavanje/brisanje promenljivih ili ograničenja i uopšte, dat je odgovor na pitanje: „Šta ako...?“ . U poslednjem delu je na jednom dobrom numeričkom primeru objašnjena ekonomska analiza optimalnog rešenja.

Poglavlje 7 (261-294 str.) opisuje neke alternativne nesimpleks metode za rešavanje problema linearnog programiranja. Ovaj deo je sažet i fokusiran uglavnom na metodi unutrašnje tačke (*Interior point methods*), uz osvrtanje na druge metode kao što su: *traversal*, *external pivoting method*, *gravitational method*, *bounce* i *ellipsoid method*. Poglavlje bi moglo da se proširi upućivanjem na relevantnu literaturu u kojoj se analizira primena navedenih metoda u praksi. (Takođe, bilo bi dobro metodu unutrašnje tačke, koja se poslednjih decenija sve češće koristi u praksi, kritički analizirati i uporediti sa simpleks metodom).

Poglavlje 8 (295-324 str.) opisuje važne tehnike za rešavanje problema koji se ne uklapaju u „standardni“ model linearnog programiranja. Ovo je veoma korisno poglavlje, jer se čitalac upućuje na načine prevazilaženja teškoća oko modeliranja datog optimizacionog problema. Demonstriran je način preformulisanja datog problema do opšteg oblika problema linearnog programiranja (zamenom promenljivih veličina, uslova i funkcije cilja). Iako Poglavlje pokriva samo osnovu iz ove oblasti, čitaoci mogu očekivati da će naići na zanimljive i korisne ilustrativne primere.

Poslednje, Poglavlje 9 (325-362), daje uvod u problematiku višeciljnog programiranja. Koncept vektorske naspram skalarne optimizacije je dobro objašnjen. Takođe su opisane i utvrđene metode za rešavanje višeciljnog programiranja, kao što su: *weighting method*, *constraint*, *reference point programming*, *fuzzy programming*, *goal programming*, *bilevel programming*. Neke od ovih metoda imaju veliku primenu u ekonomiji, što je pokazano na konkretnim primerima.

H. A. Eiselt i C.-L. Sandblom, u Uvodu jasno ističu svoju želju da knjizi obezbede „dugovečnost“, tako što

će više pažnje posvetiti analiziranju suštine problema i opisivanju odgovarajućeg modela. Međutim, ono što knjizi nedostaje jeste korišćenje nekog od *software* paketa za rešavanje optimizacionih problema. Tako bi istraživači koji nemaju dovoljno iskustva u oblasti linearnog programiranja lakše uočili velike mogućnosti primene ove oblasti u praksi. Međutim,

pošto je očigledno nemoguće samo u jednoj knjizi uvesti ceo matematički aparat koji se upotrebljava u linearnom programiranju, autori su se, sa pravom, usredsredili na ono što je matematički najosnovnije i praktično najvažnije. Čitaoci koji knjigu pažljivo prouče moći će da razumeju većinu naučnih i stručnih članaka u časopisima na ovu temu.

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