SUPER MEAN LABELING OF SOME SUBDIVISION GRAPHS

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ABSTRACT. Let G be a graph and $f : V(G) \rightarrow \{1, 2, 3, \dots, p + q\}$ be an injection. For each edge $e = uv$, the induced edge labeling f^* is defined as follows:

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2}, & \text{if } f(u) + f(v) \text{ is even,} \\ \frac{f(u)+f(v)+1}{2}, & \text{if } f(u) + f(v) \text{ is odd.} \end{cases}$$

Then f is called super mean labeling if $f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, p + q\}$. A graph that admits a super mean labeling is called super mean graph. In this paper, we have studied the super meanness property of the subdivision of the H -graph H_n , $H_n \odot K_1$, $H_n \odot S_2$, slanting ladder, $T_n \odot K_1$, $C_n \odot K_1$ and $C_n @ C_m$.

1. INTRODUCTION

Throughout this paper, by a graph we mean a finite, undirected and simple graph. Let $G(V, E)$ be a graph with p vertices and q edges. For notations and terminology we follow [2].

The path on n vertices is denoted by P_n and a cycle on n vertices is denoted by C_n . A triangular snake is obtained from a path by identifying each edge of the path with an edge of the cycle C_3 . The graph $C_m @ C_n$ is obtained by identifying an edge of C_m with an edge of C_n . The slanting ladder SL_n is a graph obtained from two paths $u_1 u_2 \dots u_n$ and $v_1 v_2 \dots v_n$ by joining each u_i with v_{i+1} , $1 \leq i \leq n - 1$. The H -graph of a path P_n , denoted by H_n is the graph obtained from two copies of P_n with vertices $v_1, v_2, \dots, v_n$ and $u_1, u_2, \dots, u_n$ by joining the vertices $v_{\frac{n+1}{2}}$ and $u_{\frac{n+1}{2}}$ if n is odd and the vertices $v_{\frac{n}{2}+1}$ and $u_{\frac{n}{2}}$ if n is even. The corona of a graph G on p vertices $v_1, v_2, \dots, v_p$ is the graph obtained from G by adding p new vertices $u_1, u_2, \dots, u_p$

Key words and phrases. Super mean graph, super mean labeling.

2010 Mathematics Subject Classification. Primary: 05C78.

Received: November 27, 2014.

Accepted: September 9, 2016.

and the new edges $u_i v_i$ for $1 \leq i \leq p$. The corona of G is denoted by $G \odot K_1$. The 2-corona of a graph G , denoted by $G \odot S_2$ is a graph obtained from G by identifying the center vertex of the star S_2 at each vertex of G . A graph which can be obtained from a given graph by breaking up each edge into one or more segments by inserting intermediate vertices between its two ends. If each edge of a graph G is broken into two by exactly one vertex, then the resultant graph is taken as $S(G)$.

A vertex labeling of G is an assignment $f : V(G) \rightarrow \{1, 2, \dots, p+q\}$ be an injection. For a vertex labeling f , the induced edge labeling $f^*(e = uv)$ is defined by

$$f^*(e) = \begin{cases} \frac{f(u)+f(v)}{2}, & \text{if } f(u) + f(v) \text{ is even,} \\ \frac{f(u)+f(v)+1}{2}, & \text{if } f(u) + f(v) \text{ is odd.} \end{cases}$$

Then f is called super mean labeling if

$$f(V(G)) \cup \{f^*(e) : e \in E(G)\} = \{1, 2, 3, \dots, p+q\}.$$

Clearly f^* is injective. A graph that admits a super mean labeling is called super mean graph.

A super mean labeling of the graph P_7^2 is shown in Figure 1.

FIGURE 1

The concept of mean labeling was introduced and studied by S. Somasundaram and R. Ponraj [5]. Some new families of mean graphs are discussed in [10, 11].

The concept of super mean labeling was introduced and studied by D. Ramya et al. [4]. Further some more results on super mean graphs are discussed in [1, 3, 6–9].

In this paper, we have studied the super meanness of the subdivision of the graphs H -graph H_n , $H_n \odot K_1$, $H_n \odot S_2$, slanting ladder, $T_n \odot K_1$, $C_n \odot K_1$ and $C_n @ C_m$.

2. SUPER MEAN GRAPHS

Theorem 2.1. *The graph $S(H_n)$ is a super mean graph, for $n \geq 3$.*

Proof. Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ be the vertices of the paths of length $n-1$. Each edge $u_i u_{i+1}$ is subdivided by a vertex x_i , $1 \leq i \leq n-1$ and each edge $v_i v_{i+1}$ is subdivided by a vertex y_i , $1 \leq i \leq n-1$. The edge $u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}$ is divided by a vertex z when n is odd. The edge $u_{\frac{n+2}{2}} v_{\frac{n}{2}}$ is divided by a vertex z when n is even. The graph $S(H_n)$ has $4n-1$ vertices and $4n-2$ edges.

Define $f : V(S(H_n)) \rightarrow \{1, 2, 3, \dots, p+q = 8n-3\}$ as follows:

$$f(u_i) = 4i - 3, \quad 1 \leq i \leq n,$$

$$\begin{aligned}
f(v_i) &= \begin{cases} 4(n+i) - 5, & 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 4(n+i) - 3, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n, \end{cases} \\
f(x_i) &= 4i - 1, \quad 1 \leq i \leq n-1, \\
f(y_i) &= \begin{cases} 4(n+i) - 3, & 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 4(n+i) - 1, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n-1, \end{cases} \\
\text{and } f(z) &= \begin{cases} 6n - 4, & \text{if } n \text{ is odd,} \\ 6n - 6, & \text{if } n \text{ is even.} \end{cases}
\end{aligned}$$

For the vertex labeling f , the induced edge labeling is given as follows:

$$\begin{aligned}
f^*(u_i x_i) &= 4i - 2, \quad 1 \leq i \leq n-1, \\
f^*(x_i u_{i+1}) &= 4i, \quad 1 \leq i \leq n-1, \\
f^*(v_i y_i) &= \begin{cases} 4(n+i) - 4, & 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 4(n+i) - 2, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n-1, \end{cases} \\
f^*(y_i v_{i+1}) &= \begin{cases} 4(n+i) - 2, & 1 \leq i \leq \lfloor \frac{n-3}{2} \rfloor, \\ 6n - 3, & i = \frac{n-1}{2} \text{ and } n \text{ is odd,} \\ 6n - 5, & i = \frac{n-2}{2} \text{ and } n \text{ is even,} \\ 4(n+i), & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n-1, \end{cases} \\
f^*\left(u_{\frac{n+1}{2}} z\right) &= 4n - 2, \quad \text{if } n \text{ is odd,} \\
f^*\left(z v_{\frac{n+1}{2}}\right) &= 6n - 2, \quad \text{if } n \text{ is odd,} \\
f^*\left(u_{\frac{n+2}{2}} z\right) &= 4n - 2, \quad \text{if } n \text{ is even,} \\
\text{and } f^*\left(z v_{\frac{n}{2}}\right) &= 6n - 4, \quad \text{if } n \text{ is even.}
\end{aligned}$$

Thus, f is a super mean labeling and hence $S(H_n)$ is a super mean graph.

For example, a super mean labeling of $S(H_7)$ and $S(H_8)$ are shown in Figure 2. $\square$

Theorem 2.2. *The graph $S(H_n \odot K_1)$ is a super mean graph, for $n \geq 3$.*

Proof. Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ be the vertices of the paths of length $n-1$. Let $a_{1,i}a_{2,i}u_i$ be the path attached at each $u_i, 1 \leq i \leq n$ and $b_{1,i}b_{2,i}v_i$ be the path attached at each $v_i, 1 \leq i \leq n$. Each edge $u_i u_{i+1}$ is subdivided by a vertex $x_i, 1 \leq i \leq n-1$ and each edge $v_i v_{i+1}$ is subdivided by a vertex $y_i, 1 \leq i \leq n-1$. The edge $u_{\frac{n+1}{2}} v_{\frac{n+1}{2}}$ is divided by a vertex z when n is odd. The edge $u_{\frac{n+2}{2}} v_{\frac{n}{2}}$ is divided by a vertex z when n is even. The graph $S(H_n \odot K_1)$ has $8n-1$ vertices and $8n-2$ edges.

FIGURE 2

Define $f : V(S(H_n \odot K_1)) \rightarrow \{1, 2, 3, \dots, p + q = 16n - 3\}$ as follows:

$$\begin{aligned}
 f(u_i) &= \begin{cases} 5, & i = 1, \\ 8i - 7, & 2 \leq i \leq n, \end{cases} \\
 f(v_i) &= \begin{cases} 8n + 3, & i = 1, \\ 8(n + i) - 9, & 2 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 8(n + i) - 7, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n, \end{cases} \\
 f(a_{1,i}) &= \begin{cases} 1, & i = 1, \\ 8i - 2, & 2 \leq i \leq n, \end{cases} \\
 f(a_{2,i}) &= 8i - 5, \quad 1 \leq i \leq n.
 \end{aligned}$$

$$\begin{aligned}
f(b_{1,i}) &= \begin{cases} 8n-1, & i=1, \\ 8(n+i)-4, & 2 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 8(n+i)-2, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n-1, \\ 16n-3, & i=n, \end{cases} \\
f(b_{2,i}) &= \begin{cases} 8(n+i)-7, & 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 8(n+i)-5, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n, \end{cases} \\
f(x_i) &= 8i-1, \quad 1 \leq i \leq n-1, \\
f(y_i) &= \begin{cases} 8(n+i)-3, & 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 8(n+i)-1, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n-1, \end{cases} \\
\text{and } f(z) &= \begin{cases} 12n-6, & \text{if } n \text{ is odd,} \\ 12n-10, & \text{if } n \text{ is even.} \end{cases}
\end{aligned}$$

The induced edge labeling is obtained as follows:

$$\begin{aligned}
f^*(u_i x_i) &= \begin{cases} 6, & i=1, \\ 8i-4, & 2 \leq i \leq n-1, \end{cases} \\
f^*(x_i u_{i+1}) &= 8i, \quad 1 \leq i \leq n-1, \\
f^*(v_i y_i) &= \begin{cases} 8n+4, & i=1, \\ 8(n+i)-6, & 2 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 8(n+i)-4, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n-1, \end{cases} \\
f^*(y_i v_{i+1}) &= \begin{cases} 8(n+i)-2, & 1 \leq i \leq \lfloor \frac{n-3}{2} \rfloor, \\ 12n-5, & i = \frac{n-1}{2} \text{ and } n \text{ is odd,} \\ 12n-9, & i = \frac{n-2}{2} \text{ and } n \text{ is even,} \\ 8(n+i), & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n-1, \end{cases} \\
f^*(a_{1,i} a_{2,i}) &= \begin{cases} 2, & i=1, \\ 8i-3, & 2 \leq i \leq n, \end{cases} \\
f^*(a_{2,i} u_i) &= \begin{cases} 4, & i=1, \\ 8i-6, & 2 \leq i \leq n, \end{cases} \\
f^*(b_{1,i} b_{2,i}) &= \begin{cases} 8n, & i=1 \\ 8(n+i)-5, & 2 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 8(n+i)-3, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n-1, \\ 16n-4, & i=n, \end{cases}
\end{aligned}$$

$$\begin{aligned}
f^*(b_{2,i}v_i) &= \begin{cases} 8n+2, & i=1, \\ 8(n+i)-8, & 2 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 8(n+i)-6, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n, \end{cases} \\
f^*\left(u_{\frac{n+1}{2}}z\right) &= 8n-4, \quad \text{if } n \text{ is odd,} \\
f^*\left(zv_{\frac{n+1}{2}}\right) &= 12n-4, \quad \text{if } n \text{ is odd.} \\
f^*\left(u_{\frac{n+2}{2}}z\right) &= 8n-4, \quad \text{if } n \text{ is even,} \\
f^*\left(zv_{\frac{n}{2}}\right) &= 12n-8, \quad \text{if } n \text{ is even.}
\end{aligned}$$

Thus, f is a super mean labeling and hence $S(H_n \odot K_1)$ is a super mean graph.

For example, a super mean labeling of $S(H_9 \odot K_1)$ and $S(H_{10} \odot K_1)$ are shown in Figure 3. $\square$

Theorem 2.3. *The graph $S(H_n \odot S_2)$ is a super mean graph, for $n \geq 3$.*

Proof. Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ be the vertices of the paths of length $n-1$. Let $a_{1,i}a_{2,i}u_i$ and $a_{3,i}a_{4,i}u_i$ be the paths attached at each u_i , $1 \leq i \leq n$ and $b_{1,i}b_{2,i}v_i$ and $b_{3,i}b_{4,i}v_i$ be the paths attached at each v_i , $1 \leq i \leq n$. Each edge u_iu_{i+1} is subdivided by a vertex x_i , $1 \leq i \leq n-1$ and each edge v_iv_{i+1} is subdivided by a vertex y_i , $1 \leq i \leq n-1$. The edge $u_{\frac{n+1}{2}}v_{\frac{n+1}{2}}$ is divided by a vertex z when n is odd. The edge $u_{\frac{n+2}{2}}v_{\frac{n}{2}}$ is divided by a vertex z when n is even. The graph $S(H_n \odot S_2)$ has $12n-1$ vertices and $12n-2$ edges.

Define $f : V(S(H_n \odot S_2)) \rightarrow \{1, 2, 3, \dots, p+q = 24n-3\}$ as follows:

$$\begin{aligned}
f(u_i) &= 12i-7, \quad 1 \leq i \leq n, \\
f(v_i) &= \begin{cases} 12(n+i)-9, & 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 12(n+i)-7, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n, \end{cases} \\
f(a_{1,i}) &= \begin{cases} 1, & i=1, \\ 12i-13, & 2 \leq i \leq n, \end{cases} \\
f(a_{2,i}) &= \begin{cases} 3, & i=1, \\ 12i-11, & 2 \leq i \leq n, \end{cases} \\
f(a_{3,i}) &= 12i-3, \quad 1 \leq i \leq n, \\
f(a_{4,i}) &= 12i-5, \quad 1 \leq i \leq n, \\
f(x_i) &= 12i+2, \quad 1 \leq i \leq n-1,
\end{aligned}$$

FIGURE 3

$$f(b_{1,i}) = \begin{cases} 12n - 1, & i = 1, \\ 12(n + i) - 15, & 2 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 18n - 8, & i = \frac{n+1}{2} \text{ and } n \text{ is odd,} \\ 18n - 14, & i = \frac{n}{2} \text{ and } n \text{ is even,} \\ 12(n + i) - 13, & \lfloor \frac{n+3}{2} \rfloor \leq i \leq n, \end{cases}$$

$$\begin{aligned}
f(b_{2,i}) &= \begin{cases} 12n+1, & i=1, \\ 12(n+i)-13, & 2 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 18n-6, & i = \frac{n+1}{2} \text{ and } n \text{ is odd}, \\ 18n-12, & i = \frac{n}{2} \text{ and } n \text{ is even}, \\ 12(n+i)-11, & \lfloor \frac{n+3}{2} \rfloor \leq i \leq n, \end{cases} \\
f(b_{3,i}) &= \begin{cases} 12(n+i)-5, & 1 \leq i \leq \lfloor \frac{n-3}{2} \rfloor, \\ 18n-10, & i = \frac{n-1}{2} \text{ and } n \text{ is odd}, \\ 18n-16, & i = \frac{n-2}{2} \text{ and } n \text{ is even}, \\ 12(n+i)-3, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n, \end{cases} \\
f(b_{4,i}) &= \begin{cases} 12(n+i)-7, & 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 12(n+i)-5, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n, \end{cases} \\
f(z) &= \begin{cases} 18n-4, & \text{if } n \text{ is odd}, \\ 18n-10, & \text{if } n \text{ is even}, \end{cases} \\
\text{and } f(y_i) &= \begin{cases} 12(n+i), & 1 \leq i \leq \lfloor \frac{n-3}{2} \rfloor, \\ 18n-9, & i = \frac{n-1}{2} \text{ and } n \text{ is odd}, \\ 18n-15, & i = \frac{n-2}{2} \text{ and } n \text{ is even}, \\ 12(n+i)+2, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n-1. \end{cases}
\end{aligned}$$

For the vertex labeling f , the induced edge labels are obtained as follows:

$$\begin{aligned}
f^*(a_{1,i}a_{2,i}) &= \begin{cases} 2, & i=1, \\ 12(i-1), & 2 \leq i \leq n, \end{cases} \\
f^*(a_{2,i}u_i) &= \begin{cases} 4, & i=1, \\ 12i-9, & 2 \leq i \leq n, \end{cases} \\
f^*(a_{3,i}a_{4,i}) &= 12i-4, \quad 1 \leq i \leq n, \\
f^*(a_{4,i}u_i) &= 12i-6, \quad 1 \leq i \leq n, \\
f^*(u_i x_i) &= 12i-2, \quad 1 \leq i \leq n-1, \\
f^*(x_i u_{i+1}) &= 12i+4, \quad 1 \leq i \leq n-1, \\
f^*(b_{1,i}b_{2,i}) &= \begin{cases} 12n, & i=1, \\ 12(n+i)-14, & 2 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 18n-7, & i = \frac{n+1}{2} \text{ and } n \text{ is odd}, \\ 18n-13, & i = \frac{n}{2} \text{ and } n \text{ is even}, \\ 12(n+i)-12, & \lfloor \frac{n+3}{2} \rfloor \leq i \leq n, \end{cases}
\end{aligned}$$

$$\begin{aligned}
f^*(b_{2,i}v_i) &= \begin{cases} 12n + 2, & i = 1, \\ 12(n + i) - 11, & 2 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 12(n + i) - 9, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n, \end{cases} \\
f^*(b_{3,i}b_{4,i}) &= \begin{cases} 12(n + i) - 6, & 1 \leq i \leq \lfloor \frac{n-3}{2} \rfloor, \\ 18n - 11, & i = \frac{n-1}{2} \text{ and } n \text{ is odd,} \\ 18n - 17, & i = \frac{n-2}{2} \text{ and } n \text{ is even,} \\ 12(n + i) - 4, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n, \end{cases} \\
f^*(b_{4,i}v_i) &= \begin{cases} 12(n + i) - 8, & 1 \leq i \leq \lfloor \frac{n-1}{2} \rfloor, \\ 12(n + i) - 6, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n, \end{cases} \\
f^*(v_iy_i) &= \begin{cases} 12(n + i) - 4, & 1 \leq i \leq \lfloor \frac{n-3}{2} \rfloor, \\ 18n - 12, & i = \frac{n-1}{2} \text{ and } n \text{ is odd,} \\ 18n - 18, & i = \frac{n-2}{2} \text{ and } n \text{ is even,} \\ 12(n + i) - 2, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n - 1, \end{cases} \\
f^*(y_iv_{i+1}) &= \begin{cases} 12(n + i) + 2, & 1 \leq i \leq \lfloor \frac{n-3}{2} \rfloor, \\ 18n - 5, & i = \frac{n-1}{2} \text{ and } n \text{ is odd,} \\ 18n - 11, & i = \frac{n-2}{2} \text{ and } n \text{ is even,} \\ 12(n + i) + 4, & \lfloor \frac{n+1}{2} \rfloor \leq i \leq n - 1, \end{cases} \\
f^*\left(u_{\frac{n+1}{2}}z\right) &= 12n - 2 \text{ if } n \text{ is odd,} \\
f^*\left(zv_{\frac{n+1}{2}}\right) &= 18n - 2 \text{ if } n \text{ is even,} \\
f^*\left(u_{\frac{n+2}{2}}z\right) &= 12n - 2 \text{ if } n \text{ is odd,} \\
\text{and } f^*\left(zv_{\frac{n}{2}}\right) &= 18n - 8 \text{ if } n \text{ is even.}
\end{aligned}$$

Thus, f is a super mean labeling and hence $S(H_n \odot S_2)$ is a super mean graph.

For example, a super mean labeling of $S(H_7 \odot S_2)$ and $S(H_8 \odot S_2)$ are shown in Figure 4. $\square$

Theorem 2.4. *The graph $S(SL_n)$ is a super mean graph, for $n \geq 2$.*

Proof. Let $u_1, u_2, \dots, u_n$ and $v_1, v_2, \dots, v_n$ be the vertices on the paths of length $n - 1$. Let x_i, y_i and z_i be the vertices subdivided the edges u_iu_{i+1}, v_iv_{i+1} and v_iu_{i+1} respectively for each i , $1 \leq i \leq n - 1$. The graph $S(SL_n)$ has $5n - 3$ vertices and $6n - 6$ edges.

Case (i): n is odd.

FIGURE 4

Define $f : V(S(SL_n)) \rightarrow \{1, 2, \dots, p + q = 11n - 9\}$ as follows:

$$f(u_i) = \begin{cases} 1, & i = 1, \\ 5, & i = 2, \\ 13, & i = 3, \\ 11i - 13, & 4 \leq i \leq n \text{ and } i \text{ is even,} \\ 11i - 19, & 4 \leq i \leq n \text{ and } i \text{ is odd,} \end{cases}$$

$$f(v_i) = \begin{cases} 11, & i = 1, \\ 11i - 2, & 2 \leq i \leq n - 1 \text{ and } i \text{ is even,} \\ 11i - 8, & 2 \leq i \leq n - 1 \text{ and } i \text{ is odd,} \\ 11n - 9, & i = n, \end{cases}$$

$$\begin{aligned}
f(x_i) &= \begin{cases} 3, & i = 1, \\ 10, & i = 2, \\ 11i - 5, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd,} \\ 11i - 10, & 3 \leq i \leq n-1 \text{ and } i \text{ is even,} \end{cases} \\
f(y_i) &= \begin{cases} 11i + 6, & 1 \leq i \leq n-2 \text{ and } i \text{ is odd,} \\ 11i + 1, & 1 \leq i \leq n-2 \text{ and } i \text{ is even,} \end{cases} \\
f(y_{n-1}) &= 11(n-1), \\
f(z_i) &= \begin{cases} 7, & i = 1, \\ 11i - 6, & 2 \leq i \leq n-1. \end{cases}
\end{aligned}$$

For the vertex labeling f , the induced edge labeling f^* is given follows:

$$\begin{aligned}
f^*(u_i x_i) &= \begin{cases} 2, & i = 1, \\ 8, & i = 2, \\ 11i - 12, & 3 \leq i \leq n-2 \text{ and } i \text{ is odd,} \\ 11i - 11, & 3 \leq i \leq n-2 \text{ and } i \text{ is even,} \end{cases} \\
f^*(x_i u_{i+1}) &= \begin{cases} 4, & i = 1, \\ 12, & i = 2, \\ 11i - 3, & 3 \leq i \leq n-1 \text{ and } i \text{ is odd,} \\ 11i - 9, & 3 \leq i \leq n-1 \text{ and } i \text{ is even,} \end{cases} \\
f^*(v_i y_i) &= \begin{cases} 14, & i = 1, \\ 11i, & 2 \leq i \leq n-2 \text{ and } i \text{ is even,} \\ 11i - 1, & 2 \leq i \leq n-2 \text{ and } i \text{ is odd,} \\ 11n - 12, & i = n-1, \end{cases} \\
f^*(y_i v_{i+1}) &= \begin{cases} 11i + 8, & 1 \leq i \leq n-2 \text{ and } i \text{ is odd,} \\ 11i + 2, & 1 \leq i \leq n-2 \text{ and } i \text{ is even,} \\ 11n - 10, & i = n-1, \end{cases} \\
f^*(v_i z_i) &= \begin{cases} 9, & i = 1, \\ 11i - 4, & 2 \leq i \leq n-1 \text{ and } i \text{ is even,} \\ 11i - 6, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd,} \end{cases} \\
f^*(z_i u_{i+1}) &= \begin{cases} 6, & i = 1, \\ 11i - 7, & 2 \leq i \leq n-1 \text{ and } i \text{ is even,} \\ 11i - 4, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd.} \end{cases}
\end{aligned}$$

Case (ii): n is even, $n \geq 4$.

Define $f : V(S(SL_n)) \rightarrow \{1, 2, \dots, p + q = 11n - 9\}$ as follows:

$$\begin{aligned}
 f(u_i) &= \begin{cases} 1, & i = 1, \\ 5, & i = 2, \\ 13, & i = 3, \\ 11i - 13, & 4 \leq i \leq n - 1 \text{ and } i \text{ is even,} \\ 11i - 19, & 4 \leq i \leq n - 1 \text{ and } i \text{ is odd,} \\ 11n - 11, & i = n, \end{cases} \\
 f(v_i) &= \begin{cases} 11, & i = 1, \\ 11i - 2, & 2 \leq i \leq n - 1 \text{ and } i \text{ is even,} \\ 11i - 8, & 2 \leq i \leq n - 1 \text{ and } i \text{ is odd,} \\ 11n - 9, & i = n, \end{cases} \\
 f(x_i) &= \begin{cases} 3, & i = 1, \\ 10, & i = 2, \\ 11i - 5, & 3 \leq i \leq n \text{ and } i \text{ is odd,} \\ 11i - 10, & 3 \leq i \leq n \text{ and } i \text{ is even,} \end{cases} \\
 f(y_i) &= \begin{cases} 11i + 6, & 1 \leq i \leq n - 2 \text{ and } i \text{ is odd,} \\ 11i + 1, & 1 \leq i \leq n - 2 \text{ and } i \text{ is even,} \\ 11n - 12, & i = n - 1, \end{cases} \\
 f(z_i) &= \begin{cases} 7, & i = 1, \\ 11i - 6, & 2 \leq i \leq n - 1. \end{cases}
 \end{aligned}$$

The induced edge labeling is obtained as follows:

$$\begin{aligned}
 f^*(u_i x_i) &= \begin{cases} 2, & i = 1, \\ 8, & i = 2, \\ 11i - 12, & 3 \leq i \leq n - 1 \text{ and } i \text{ is odd,} \\ 11i - 10, & 3 \leq i \leq n - 1 \text{ and } i \text{ is even,} \end{cases} \\
 f^*(x_i u_{i+1}) &= \begin{cases} 4, & i = 1, \\ 12, & i = 2, \\ 11i - 3, & 3 \leq i \leq n - 2 \text{ and } i \text{ is odd,} \\ 11i - 9, & 3 \leq i \leq n - 2 \text{ and } i \text{ is even,} \\ 11n - 13, & i = n - 1, \end{cases} \\
 f^*(v_i y_i) &= \begin{cases} 14, & i = 1, \\ 11i, & 2 \leq i \leq n - 2 \text{ and } i \text{ is even,} \\ 11i - 1, & 2 \leq i \leq n - 2 \text{ and } i \text{ is odd,} \\ 11n - 15, & i = n - 1, \end{cases}
 \end{aligned}$$

$$f^*(y_i v_{i+1}) = \begin{cases} 11i + 8, & 1 \leq i \leq n-2 \text{ and } i \text{ is odd,} \\ 11i + 2, & 1 \leq i \leq n-2 \text{ and } i \text{ is even,} \\ 11n - 10, & i = n-1, \end{cases}$$

$$f^*(v_i z_i) = \begin{cases} 9, & i = 1, \\ 11i - 4, & 2 \leq i \leq n-1 \text{ and } i \text{ is even,} \\ 11i - 7, & 2 \leq i \leq n-1 \text{ and } i \text{ is odd,} \end{cases}$$

$$f^*(z_i u_{i+1}) = \begin{cases} 6, & i = 1, \\ 11i - 7, & 2 \leq i \leq n-2 \text{ and } i \text{ is even,} \\ 11i + 7, & 2 \leq i \leq n-2 \text{ and } i \text{ is odd,} \\ 11n - 14, & i = n-1. \end{cases}$$

Thus, f is a super mean labeling of $S(SL_n)$ and hence $S(SL_n)$ is a super mean graph.

For example, a super mean labeling of $S(SL_7)$ and $S(SL_8)$ are shown in Figure 5.

FIGURE 5

When $n = 2$, a super mean labeling of the graph is shown in Figure 6. □

Theorem 2.5. *The graph $S(T_n \odot K_1)$ is a super mean graph for any n .*

Proof. Let $u_1, u_2, \dots, u_n, u_{n+1}$ be the vertices on the path of length n in T_n and let v_i , $1 \leq i \leq n$ be the vertices of T_n in which v_i is adjacent to u_i and u_{i+1} . Let $v'_i a_i v_i$ be the path attached at each v_i , $1 \leq i \leq n$ and $u'_i b_i u_i$ be the path attached at each u_i , $1 \leq i \leq n+1$. Let x_i, y_i and z_i be the vertices which subdivided the edges $u_i u_{i+1}, u_i v_i$

FIGURE 6

and $v_i u_{i+1}$ respectively for each i , $1 \leq i \leq n$. The graph $S(T_n \odot K_1)$ has $9n + 3$ vertices and $10n + 2$ edges.

Define $f : V(S(T_n \odot K_1)) \rightarrow \{1, 2, \dots, p + q = 19n + 5\}$ as follows:

$$\begin{aligned}
 f(u_i) &= 19i - 14, & 1 \leq i \leq n + 1, \\
 f(v_i) &= 19i - 8, & 1 \leq i \leq n, \\
 f(v'_i) &= 19i - 4, & 1 \leq i \leq n, \\
 f(a_i) &= 19i - 6, & 1 \leq i \leq n, \\
 f(u'_i) &= \begin{cases} 1, & i = 1, \\ 19i - 20, & 2 \leq i \leq n + 1, \end{cases} \\
 f(b_i) &= \begin{cases} 3, & i = 1, \\ 19i - 18, & 2 \leq i \leq n + 1, \end{cases} \\
 f(x_i) &= 19i - 9, & 1 \leq i \leq n, \\
 f(y_i) &= 19i - 12, & 1 \leq i \leq n, \\
 f(z_i) &= 19i + 2, & 1 \leq i \leq n.
 \end{aligned}$$

The induced edge labeling is defined as follows:

$$\begin{aligned}
 f^*(u_i x_i) &= 19i - 11, & 1 \leq i \leq n, \\
 f^*(x_i u_{i+1}) &= 19i - 2, & 1 \leq i \leq n, \\
 f^*(u_i y_i) &= 19i - 13, & 1 \leq i \leq n, \\
 f^*(y_i v_i) &= 19i - 10, & 1 \leq i \leq n, \\
 f^*(v_i z_i) &= 19i - 3, & 1 \leq i \leq n, \\
 f^*(z_i u_{i+1}) &= 19i + 4, & 1 \leq i \leq n, \\
 f^*(v_i a_i) &= 19i - 7, & 1 \leq i \leq n, \\
 f^*(a_i v'_i) &= 19i - 5, & 1 \leq i \leq n, \\
 f^*(u_i b_i) &= \begin{cases} 4, & i = 1, \\ 19i - 16, & 2 \leq i \leq n + 1, \end{cases}
 \end{aligned}$$

$$f^*(b_i u'_i) = \begin{cases} 2, & i = 1, \\ 19(i - 1), & 2 \leq i \leq n + 1. \end{cases}$$

Thus, f is a super mean labeling of $S(T_n \odot K_1)$. $\square$

For example, a super mean labeling of $S(T_6 \odot K_1)$ is shown in Figure 7.

FIGURE 7. $S(T_6 \odot K_1)$

Theorem 2.6. *The graph $S(C_n \odot K_1)$ is a super mean graph, for $n \geq 3$.*

Proof. Let $u_1, u_2, \dots, u_n$ be the vertices of the cycle C_n . Let $v_i y_i u_i$ be the path attached at each u_i , $1 \leq i \leq n$. Each edge $u_i u_{i+1}$ is subdivided by a vertex x_i , $1 \leq i \leq n - 1$ and the edge $u_n u_1$ is subdivided by a vertex x_n .

Case(i): n is odd.

Define $f : V(S(C_n \odot K_1)) \rightarrow \{1, 2, \dots, 8n\}$ as follows:

$$f(u_i) = \begin{cases} 5, & i = 1, \\ 16i - 21, & 2 \leq i \leq \frac{n+1}{2}, \\ 8n, & i = \frac{n+3}{2}, \\ 16(n - i) + 22, & \frac{n+5}{2} \leq i \leq n, \end{cases}$$

$$f(v_i) = \begin{cases} 1, & i = 1, \\ 16i - 17, & 2 \leq i \leq \frac{n+1}{2}, \\ 16(n - i) + 18, & \frac{n+3}{2} \leq i \leq n, \end{cases}$$

$$f(x_i) = \begin{cases} 16i - 9, & 1 \leq i \leq \frac{n-1}{2}, \\ 8n - 3, & i = \frac{n+1}{2}, \\ 16(n - i) + 10, & \frac{n+3}{2} \leq i \leq n, \end{cases}$$

$$f(y_i) = \begin{cases} 3, & i = 1, \\ 16i - 19, & 2 \leq i \leq \frac{n+1}{2}, \\ 16(n-i) + 20, & \frac{n+3}{2} \leq i \leq n. \end{cases}$$

The induced edge labeling is defined as follows:

$$\begin{aligned} f^*(u_i x_i) &= \begin{cases} 6, & i = 1, \\ 16i - 15, & 2 \leq i \leq \frac{n-1}{2}, \\ 8(n-1), & i = \frac{n+1}{2}, \\ 8n - 7, & i = \frac{n+3}{2}, \\ 16(n-i) + 16, & \frac{n+5}{2} \leq i \leq n, \end{cases} \\ f^*(x_i u_{i+1}) &= \begin{cases} 16i - 7, & 1 \leq i \leq \frac{n-1}{2}, \\ 8n - 1, & i = \frac{n+1}{2}, \\ 16(n-i) + 8, & \frac{n+3}{2} \leq i \leq n-1, \end{cases} \\ f^*(x_n u_1) &= 8, \\ f^*(v_i y_i) &= \begin{cases} 2, & i = 1, \\ 16i - 18, & 2 \leq i \leq \frac{n+1}{2}, \\ 16(n-i) + 19, & \frac{n+3}{2} \leq i \leq n, \end{cases} \\ \text{and } f^*(y_i u_i) &= \begin{cases} 4, & i = 1, \\ 16i - 20, & 2 \leq i \leq \frac{n+1}{2}, \\ 8n - 2, & i = \frac{n+3}{2}, \\ 16(n-i) + 21, & \frac{n+5}{2} \leq i \leq n. \end{cases} \end{aligned}$$

Case (ii): n is even.

$$\begin{aligned} f(u_i) &= \begin{cases} 5, & i = 1, \\ 16i - 21, & 2 \leq i \leq \frac{n}{2}, \\ 8n - 4, & i = \frac{n+2}{2}, \\ 16(n-i) + 22, & \frac{n+4}{2} \leq i \leq n, \end{cases} \\ f(v_i) &= \begin{cases} 1, & i = 1, \\ 16i - 17, & 2 \leq i \leq \frac{n}{2}, \\ 8n, & i = \frac{n+2}{2}, \\ 16(n-i) + 18, & \frac{n+4}{2} \leq i \leq n, \end{cases} \\ f(x_i) &= \begin{cases} 16i - 9, & 1 \leq i \leq \frac{n}{2}, \\ 8n - 7, & i = \frac{n+2}{2}, \\ 16(n-i) + 10, & \frac{n+4}{2} \leq i \leq n, \end{cases} \end{aligned}$$

$$f(y_i) = \begin{cases} 3, & i = 1, \\ 16i - 19, & 2 \leq i \leq \frac{n}{2}, \\ 8n - 2, & i = \frac{n+2}{2}, \\ 16(n - i) + 20, & \frac{n+4}{2} \leq i \leq n. \end{cases}$$

For the vertex labeling f , the induced edge labeling f^* is given as follows:

$$\begin{aligned} f^*(u_i x_i) &= \begin{cases} 6, & i = 1, \\ 16i - 15, & 2 \leq i \leq \frac{n}{2}, \\ 8n - 5, & i = \frac{n+2}{2}, \\ 16(n - i + 1), & \frac{n+4}{2} \leq i \leq n, \end{cases} \\ f^*(x_i u_{i+1}) &= \begin{cases} 16i - 7, & 1 \leq i \leq \frac{n-2}{2}, \\ 8n - 6, & i = \frac{n}{2}, \\ 16(n - i) + 8, & \frac{n+2}{2} \leq i \leq n - 1, \end{cases} \\ f^*(x_n u_1) &= 8, \\ f^*(v_i y_i) &= \begin{cases} 2, & i = 1, \\ 16i - 18, & 2 \leq i \leq \frac{n}{2}, \\ 8n - 1, & i = \frac{n+2}{2}, \\ 16(n - i) + 19, & \frac{n+4}{2} \leq i \leq n, \end{cases} \\ \text{and } f^*(y_i u_i) &= \begin{cases} 4, & i = 1, \\ 16i - 20, & 2 \leq i \leq \frac{n}{2}, \\ 8n - 3, & i = \frac{n+2}{2}, \\ 16(n - i) + 21, & \frac{n+4}{2} \leq i \leq n. \end{cases} \end{aligned}$$

Thus, f is a super mean labeling and hence $S(C_n \odot K_1)$ is a super mean graph. $\square$

For example, a super mean labeling of $S(C_{11} \odot K_1)$ and $S(C_{12} \odot K_1)$ are shown in Figure 8.

Theorem 2.7. *The graph $S(C_m @ C_n)$ is a super mean graph for $m, n \geq 3$.*

Proof. $C_m @ C_n$ is a graph obtained by identifying an edge of two cycles C_m and C_n . $C_m @ C_n$ has $m + n - 2$ vertices and $m + n - 1$ edges. In $S(C_m @ C_n)$, $2(m + n - 2)$ vertices lies on the circle and one vertex lies on a chord. Then, the graph $S(C_m @ C_n)$ has $2m + 2n - 3$ vertices and $2(m + n - 1)$ edges.

Let us assume that $m \leq n$.

FIGURE 8

Case (i): m is odd and n is odd.

Let $m = 2k + 1$, $k \geq 1$ and $n = 2l + 1$, $l \geq 1$. We denote the vertices of $S(C_m @ C_n)$ is shown in Figure 9.

FIGURE 9

Define $f : V(S(C_m @ C_n)) \rightarrow \{1, 2, 3, \dots, p + q = 4(m + n) - 5\}$ as follows:

$$f(u_i) = \begin{cases} 1, & i = 1, \\ 8i - 9, & 2 \leq i \leq k, \\ 4m - 6, & i = k + 1, \\ 8i, & k + 2 \leq i \leq k + l, \\ 8(m + n - i) - 9, & k + l + 1 \leq i \leq k + 2l - 1, \\ 4m + 5, & i = k + 2l, \\ 4m, & i = k + 2l + 1, \\ 8(m + n - i) - 6, & k + 2l + 2 \leq i \leq 2k + 2l, \end{cases}$$

$$f(x_i) = \begin{cases} 8i - 5, & 1 \leq i \leq k, \\ 8i + 4, & k + 1 \leq i \leq k + l - 1, \\ 8(m + n - i) - 13, & k + l \leq i \leq k + 2l - 1, \\ 4m + 3, & i = k + 2l, \\ 4m - 5, & i = k + 2l + 1, \\ 8(m + n - i) - 10, & k + 2l + 2 \leq i \leq 2k + 2l, \end{cases}$$

and $f(z) = 4m - 3$.

The induced edge labeling f^* is obtained as follows:

$$f^*(u_i x_i) = \begin{cases} 2, & i = 1, \\ 8i - 7, & 2 \leq i \leq k, \\ 4m + 1, & i = k + 1, \\ 8i + 2, & k + 2 \leq i \leq k + l, \\ 8(m + n - i) - 11, & k + l + 1 \leq i \leq k + 2l - 1, \\ 4m + 4, & i = k + 2l, \\ 4m - 2, & i = k + 2l + 1, \\ 8(m + n - 1 - i), & k + 2l + 2 \leq i \leq 2k + 2l - 2, \end{cases}$$

$$f^*(x_i u_{i+1}) = \begin{cases} 8i - 3, & 1 \leq i \leq k, \\ 8i + 6, & k + 1 \leq i \leq k + l - 1, \\ 8(m + n - i) - 15, & k + l \leq i \leq k + 2l - 2, \\ 4m + 6, & i = k + 2l - 1, \\ 4m + 2, & i = k + 2l, \\ 8(m + n - i) - 12, & k + 2l + 1 \leq i \leq 2k + 2l - 1, \end{cases}$$

$$f^*(x_{2k+2l} u_1) = 4,$$

$$f^*(u_{k+1} z) = 4m - 4,$$

$$\text{and } f^*(z u_{k+2l+1}) = 4m - 1.$$

Thus, f is a super mean labeling. A super mean labeling of $S(C_7 @ C_9)$ is shown in Figure 10.

FIGURE 10

Case (ii): m is odd and n is even.

Let $m = 2k + 1$, $k \geq 1$ and $n = 2l$, $l \geq 2$.

Define $f : V(S(C_m @ C_n)) \rightarrow \{1, 2, 3, \dots, p + q = 4(m + n) - 5\}$ as follows:

$$f(u_i) = \begin{cases} 1, & i = 1, \\ 8i - 9, & 2 \leq i \leq k, \\ 4m - 6, & i = k + 1, \\ 8i, & k + 2 \leq i \leq k + l - 1, \\ 8(m + n - i) - 9, & k + l \leq i \leq k + 2l - 2, \\ 4m + 5, & i = k + 2l - 1, \\ 4m, & i = k + 2l, \\ 8(m + n - i) - 6, & k + 2l + 1 \leq i \leq 2k + 2l - 1, \end{cases}$$

$$f(x_i) = \begin{cases} 8i - 5, & 1 \leq i \leq k, \\ 8i + 4, & k + 1 \leq i \leq k + l - 1, \\ 8(m + n - i) - 13, & k + l \leq i \leq k + 2l - 2, \\ 4m + 3, & i = k + 2l - 1, \\ 4m - 5, & i = k + 2l, \\ 8(m + n - i) - 10, & k + 2l + 1 \leq i \leq 2k + 2l - 1, \end{cases}$$

and $f(z) = 4m - 3$.

For the vertex labeling f , the induced edge labeling f^* is given as follows:

$$f^*(u_i x_i) = \begin{cases} 2, & i = 1, \\ 8i - 7, & 2 \leq i \leq k, \\ 4m + 1, & i = k + 1, \\ 8i + 2, & k + 2 \leq i \leq k + l - 1, \\ 8(m + n - i) - 11, & k + l \leq i \leq k + 2l - 2, \\ 4m + 4, & i = k + 2l - 1, \\ 4m - 2, & i = k + 2l, \\ 8(m + n - i) - 8, & k + 2l + 1 \leq i \leq 2k + 2l - 1, \end{cases}$$

$$f^*(x_i u_{i+1}) = \begin{cases} 8i - 3, & 1 \leq i \leq k, \\ 8i + 6, & k + 1 \leq i \leq k + l - 1, \\ 8(m + n - i) - 15, & k + l \leq i \leq k + 2l - 3, \\ 4m + 6, & i = k + 2l - 2, \\ 4m + 2, & i = k + 2l - 1, \\ 8(m + n - i) - 12, & k + 2l \leq i \leq 2k + 2l - 2, \end{cases}$$

$$f^*(x_{2k+2l-1} u_1) = 4,$$

$$f^*(u_{k+1} z) = 4m - 4,$$

$$\text{and } f^*(z u_{k+2l}) = 4m - 1.$$

Thus, f is a super mean labeling. A super mean labeling of $S(C_7 @ C_{10})$ is shown in Figure 11.

FIGURE 11

Case (iii): m is even and n is even.

Let $m = 2k$, $k \geq 2$ and $n = 2l$, $l \geq 2$.

Define $f : V(S(C_m @ C_n)) \rightarrow \{1, 2, 3, \dots, p + q = 4(m + n) - 5\}$ as follows:

$$f(u_i) = \begin{cases} 1, & i = 1, \\ 8i - 9, & 2 \leq i \leq k, \\ 4m, & i = k + 1, \\ 4m + 5, & i = k + 2, \\ 8i - 13, & k + 3 \leq i \leq k + l + 1, \\ 8(m + n - i) + 4, & k + l + 2 \leq i \leq k + 2l - 1, \\ 8(m + n - i) - 6, & k + 2l \leq i \leq 2k + 2l - 2, \end{cases}$$

$$f(x_i) = \begin{cases} 8i - 5, & 1 \leq i \leq k + 1, \\ 8i - 9, & k + 2 \leq i \leq k + l, \\ 8(m + n - i), & k + l + 1 \leq i \leq k + 2l - 1, \\ 8(m + n - i) - 10, & k + 2l \leq i \leq 2k + 2l - 2, \end{cases}$$

and $f(z) = 4m - 3$.

For the vertex labeling f , the induced edge labeling f^* is obtained as follows:

$$f^*(u_i x_i) = \begin{cases} 2, & i = 1, \\ 8i - 7, & 2 \leq i \leq k, \\ 4m + 2, & i = k + 1, \\ 4m + 6, & i = k + 2, \\ 8i - 11, & k + 3 \leq i \leq k + l, \\ 8(m + n - i) + 2, & k + l + 1 \leq i \leq k + 2l - 1, \\ 8(m + n - i) - 8, & k + 2l \leq i \leq 2k + 2l - 2, \end{cases}$$

$$f^*(x_i u_{i+1}) = \begin{cases} 8i - 3, & 1 \leq i \leq k - 1, \\ 4m - 2, & i = k, \\ 4m + 4, & i = k + 1, \\ 8i - 7, & k + 2 \leq i \leq k + l, \\ 8(m + n - i) - 2, & k + l + 1 \leq i \leq k + 2l - 2, \\ 4m + 1, & i = k + 2l - 1, \\ 8(m + n - i) - 12, & k + 2l \leq i \leq 2k + 2l - 3, \end{cases}$$

$$f^*(x_{2k+2l-2} u_1) = 4,$$

$$f^*(u_{k+1} z) = 4m - 1,$$

and $f^*(z u_{k+2l}) = 4m - 4$.

Thus, f is a super mean labeling. A super mean labeling of $S(C_6 @ C_8)$ is shown in Figure 12.

Hence, the graph $S(C_m @ C_n)$ is a super mean graph for $m, n \geq 3$. $\square$

FIGURE 12

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