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MOMENTI SAVIJANJA I NAPONSKO STANJE U ZAŠTITNOJ STROPNOJ POLICI EKSPLOATACIONOG POLJA, KOJA SE OSTAVLJA IZMEĐU RAZLIČITIH TEHNOLOGIJA EKSPLOATACIJE***

Izvod

U radu se analizira zaštitna stropna polica kao kontinualno opterećen gredni nosač pravougaonog poprečnog preseka. U ovom slučaju statički neodređene veličine su momenti u osloncima. Oslonački momenti su određeni primenom jednačine „triju momenata“ po Klaperjonu. Kontrola napona u grednom nosaču je sprovedena polazeći od diferencijalne jednačine elastične linije koja ima oblik po drugom izvodu.

Ključne reči: *gredni nosač, moment savijanja, reakcija u osloncima, Klaperjon, kritični presek i maksimalni moment savijanja*

1. UVOD

Zaštitna stropna polica se ostavlja u stenskom masivu kada se želi preći sa jedne na drugu tehnologiju eksploatacije mineralne sirovine. U rudniku kamene soli „Tušanj“ u Tuzli, BiH, u jednom vremenskom intervalu korišćene su istovremeno dve tehnologije eksploatacije. Do -250 horizonta korišćena je komorno-stubna metoda eksploatacije sa klasičnim miniranjem u komorama, a ispod -250 horizonta korišćena je tehnologija tzv. kontrolisanog izluživanja soli bušotinama sa -250 horizonta.

Da bi se sprečili deformacioni uticaji stenskog masiva izazvani jednom tehnologijom eksploatacije na deo stenskog masiva koji se eksploatiše drugom tehnologijom, nad celim eksploatacionim poljem mora se ostaviti zaštitna sigurnosna stropna polica. Taj princip važi i za prelazak površinske eksploatacije na pod-zemnu eksploataciju istog rudnog tela.

2. PRORAČUN ZAŠTITNE POLICE EKSPLOATACIONOG POLJA

Zaštitnu policu eksploatacionog polja možemo smatrati i tretirati kao kontinuirano opterećen gredni nosač pravougaonog poprečnog preseka, sl.1. U ovom slučaju sta-

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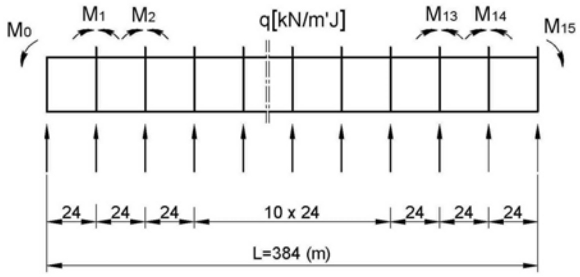
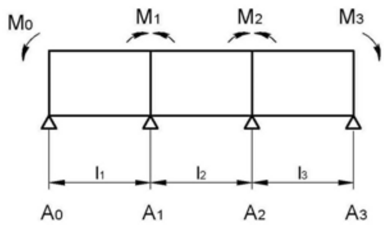
tički neodređene veličine su momenti u osloncima.



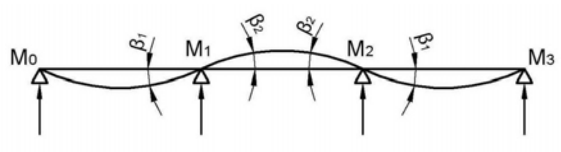
Sl. 1. *Statička šema stropnog nosača eksploatacionog polja*

Pretpostavimo kontinualni nosač sa (*n*) oslonaca, tada je statički neodređen (*n*-2) puta uz pretpostavku da su u krajnjim osloncima pokretni zglobovi. Jedan od načina da se odrede oslonački momenti jeste primena jednačine „triju momenata" po Klaperjon-u. Obzirom da se radi o nosaču kontinuirano opterećenom i jednakim rasponima, to je očigledno da će momenti M_1 do M_{14} imati istu vrednost. Prema tome, zadatak se može svesti na sledeće jednostavno rešenje, razmatrajući kontinuirani nosač sa četiri oslonca, sl.2, koji će biti u ravnoteži ako se u krajnjim presecima isključe transverzalne sile (*dT*) i (*lT*) i oslonački momenti.



Sl. 2. *Nosač sa četiri oslonca*

Veza u osloncima (1) i (2) je kruta, tj. uslovi pre i posle deformacija ostaju nepromenjeni. U osloncima (1) i (2) moraju se javiti momenti (M_1) i (M_2), a elastična linija nosača mora zadovoljiti uslov ($\beta_1+\beta_2=0$), sl. 3, drugim rečima, uglovni koeficijenti tangente na elastičnu liniju u tri raspona su isti sa različitim predznakom.



Sl. 3. *Oblik elastične linije*

Iz navedenog uslova, a pretpostavljajući da je moment inercije kao i modul elastičnosti isti, to se po Klaperjon-u dolazi do izraza

$$M_0 \cdot l_1 + 2M_1(l_1 + l_2) + M_2 \cdot l_3 + 6R_{01} = 0 \quad (1)$$

$$M_1 \cdot l_1 + 2M_2(l_2 + l_3) + M_3 \cdot l_3 + 6R_{02} = 0 \quad (2)$$

R_{01} i R_{02} - celokupna reakcija u osloncima (1) i (2)

Jednačine (1) i (2) važe za bilo koja dva susedna polja.

Kako je $M_0 = M_3 = 0$, dolazimo do konačnog oblika:

$$2M_1(l_1 + l_2) + M_2 \cdot l_3 + 6R_{01} = 0 \qquad (3)$$

$$M_1 \cdot l_1 + 2M_2(l_2 + l_3) + 6R_{02} = 0 \qquad (4)$$

Obzirom na jednake raspone l = const., tada je vrednost apsolutnog člana celokupne reakcije

$$R_{01} = R_{02}$$

$$6(R_{01} - R_{02}) = = \frac{1}{2}\left(q_n \cdot l_n^3 + q_{(n+1)}\right) \cdot l_{(n+1)}^3 \qquad (5)$$

q – kontinuirano opterećenje

Za q = const. i l = const. izraz (5) poprima sledeći oblik:

$$6(R_{01} - R_{02}) = \frac{1}{2} q l^3 \qquad (6)$$

Reakcija u osloncu (0) je:

$$A_0 = \frac{q_1 l_1}{2} - \frac{M_1}{l_1}$$

Izraz za transverzalne sile u tački (1) je:

$$lT_1 = q_i \cdot l_1 - A_0$$

$$dT_1 = \frac{q_i \cdot l_1}{2} + \frac{M_1 - M_2}{l}$$

Reakcija u osloncu (1) je

$$A_1 = lT_1 + dT_1$$

Transverzalne sile koje se odnose na oslonac (2), imaju vrednost:

$$lT_2 = q_i \cdot l_2 - dT_1$$

$$dT_2 = \frac{q_2 \cdot l_2}{2} + \frac{M_2}{l_2}$$

Reakcija u osloncu (2) je

$$A_2 = lT_2 + dT_2$$

Obzirom na poznatu vrednost desne transverzalne sile u tački oslonca (2), reakcija u osloncu (3) će biti:

$$A_2 = q_3 l_3 - dT_2$$

Vrednost sveukupnih reakcija u osloncima je:

$$\sum q = A_0 + A_1 + A_2 + A_3$$

Kritični presek i maksimalni moment savijanja u I polju određuje se iz sledeće jednačine:

$$x_1 = \frac{A_0}{q_1}; \quad M_{Imax} = \frac{A_0^2}{2q_1}$$

Kritični presek i maksimalni moment savijanja u II polju:

$$x_{II} = \frac{dT_1}{q_2}; \quad M_{I\,Imax} = \frac{dT^2}{2q_2} - M_1$$

Na osnovu do sada iznetog, stropna polica je kontinuirano opterećen nosač sa 16 oslonaca sa konstantnim rasponom, pri čemu su momenti savijanja M_0 i $M_{15} = 0$, M_1 do M_{14} su jednaki.

Reakcije u osloncima A_1 do A_{14} su jednake $A_0 = A_{15}$.

Transverzalne neparne i parne sile u osloncima su jednake $lT_1 = lT_{13}$, $dT_2 = lT_{14}$, takođe su maksimalni momenti savijanja u sredini raspona jednaki.

3. PROVERA NAPONA U STROPNOJ POLICI EKSPLOATACIONOG POLJA

Polazeći od činjenice da je stropna polica greda prizmatičnog preseka sa kritičnim mestom maksimalnog momenta savijanja, može se napisati $M_{max} = -\frac{ql^2}{8}$, odnosno maksimalni napon savijanja je $\sigma_{max} = \frac{M_{max}}{W}$.

Za bilo koju vrednost x duž raspona stropne grede imamo:

$$M_x = -\frac{ql^2}{8} + \frac{qx^2}{2} \qquad (7)$$

U tom slučaju jednačina elastične linije imaće oblik po drugom izvodu:

$$IEY''=-M_x+\frac{ql^2}{8}-\frac{qx^2}{2} \qquad (8)$$

Odnosno, integralenjem po x – u

$$IEY'=\frac{ql^2}{8}\cdot x-\frac{qx^3}{6}+c_1 \qquad (9)$$

Konstanta c_1 određuje se iz uslova da u tački (0) za x = 0, tangenta na elastičnu liniju je horizontalna tj. y' = 0, što znači da je i c_1 = 0. Ponovnim integralenjem (9) dobije se:

$$IEY=\frac{ql^2x^2}{16}-\frac{qx^4}{24}+c_2 \qquad (10)$$

Prema kritičnom mestu ugiba za $x=\frac{l}{2}$ i izjednačavanjem (10) sa nulom, dobije se vrednost konstante c_2

$$0=\frac{ql^2}{16}\left(\frac{l}{2}\right)^2-\frac{q\cdot\left(\frac{l}{2}\right)^4}{24} \qquad (11)$$

$$c_2=\frac{5ql^4}{384} \qquad (12)$$

Jednačina elastične linije dobiće konačan oblik

$$IEY=\frac{q}{384E}\left(24l^2x^2-16x^4-5l^4\right) \qquad (13)$$

Ugib u sredini raspona za $x=\frac{l}{2}$

$$f_{max}=\frac{5ql^4}{384E} \qquad (14)$$

Moment inercije

$$J=\frac{bh^3}{12}$$

h – visina stropne grede

b = 1,0

Otporni moment preseka:

$$W=\frac{bh_3}{6}$$

Odnos izračunatog napona savijanja i granične vrednosti otpornosti stene na savijanje daje koeficijent sigurnosti

$$n=\frac{\sigma_f}{\sigma_{max\,rač.}}$$

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BENDING MOMENTS AND STRESS STATE IN THE PROTECTIVE ROOF SHELF OF EXPLOITATION FIELD, LEFT BETWEEN DIFFERENT MINING TECHNOLOGIES***

Abstract

This paper analyzes the protective roof shelf as a continuous loaded carrier beam of rectangular cross section. In this case, the statically undetermined sizes are moments in the supports. Support moments are determined using the equation of "three moments" by Clapeyron. Stress control in the carrier beam is carried out starting from the differential equation of elastic line that has a form per the second statement.

Keywords: *carrier beam, bending moment, reaction in supports, Claperyon, critical section, maximum bending moment*

1. INTRODUCTION

Protective roof shelf is left in the rock massif in the case of transfer from one technology to another technology of mining the mineral raw materials. In the rock salt mine "Tušanj" in Tuzla, B&H, two mining technologies were simultaneously used in a time. The chamber-pillar mining method was used up to -250 horizon with the classic blasting in chambers, and the technology of so called controlled leaching of salt was use below -250 horizon with drill holes from -250 horizon.

The protective roof shelf must be left over the entire exploitation field to prevent the deformation effects of rock massive, caused by one mining technology of the rock mass that is exploited by the other technology. This principle is also applied to transfer the open pit mining into the underground mining the same ore body.

2. CALCULATION THE PROTECTIVE ROOF SHELF OF EXPLOITATION FIELD

The protective shelf of exploitation field can be considered and treated as a continuous loaded beam carrier of a rec-tangular cross-section, Figure 1. In this case, the

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statically undetermined sizes are the moments in supports.

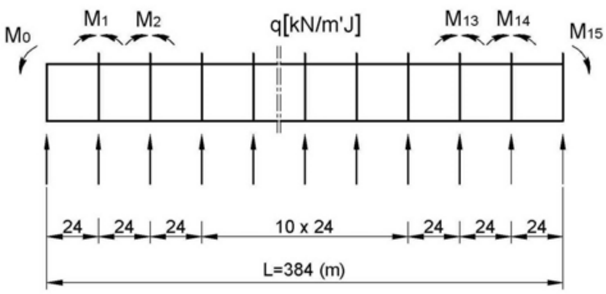

Fig. 1. *Static scheme of roof beam of the exploitation field*

Suppose a continuous carried with *(n)* supports, then it is statically undetermined for (*n*-2) times with an assumption that the movable joints are in the end supports. One way to determine the support moments is to apply the equation of "three moments" by Claperyon. Since it is a carried a continuous loaded carrier and with the same ranges, it is obvious that the moments M_1 to M_{14} will have the same value. Therefore, the task can be reduced to the following simple solution, considering the continuous beam with four supports, Figure 2, which will be in equilibrium if the transverse forces *(dT)* and *(lT)* and support moments are excluded in the final sections.

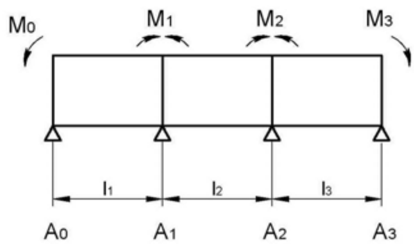

Fig. 2. *Carrier with four supports*

Connection in supports (1) and (2) is rigid, i.e. the conditions before and after deformation remain unchanged. In the supports (1) and (2), the moments (M_1) and (M_2) must appear, and the elastic line of carrier must satisfy the condition (β1 + β2 = 0), Figure 3; in the other words, the angular coefficient of tangent on the elastic line in the three ranges are the same with a different sign.

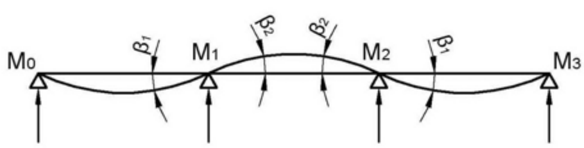

Fig. 3. *Form of the elastic line*

From the specified condition, and assuming that the moments of inertia as well as the elastic modulus are the same, according Claperyon, the expression follows

$$M_0 \cdot l_1 + 2M_1(l_1 + l_2) + M_2 \cdot l_3 + 6R_{01} = 0 \quad (1)$$

$$M_1 \cdot l_1 + 2M_2(l_2 + l_3) + M_3 \cdot l_3 + 6R_{02} = 0 \quad (2)$$

R_{01} and R_{02} – overall reaction in supports (1) and (2)

Equations (1) and (2) are valid for any two adjacent fields.

As $M_0 = M_3 = 0$, it is come to the final form:

$$2M_1(l_1 + l_2) + M_2 \cdot l_3 + 6R_{01} = 0 \quad (3)$$

$$M_1 \cdot l_1 + 2M_2(l_2 + l_3) + 6R_{02} = 0 \quad (4)$$

Regard to the same tanges l = const., then the value of absolute member of the overall reaction

$$R_{01} = R_{02}$$

$$6(R_{01} - R_{02}) = = \frac{1}{2}\left(q_n \cdot l_n^3 + q_{(n+1)}\right) \cdot l_{(n+1)}^3 \quad (5)$$

q – Continuous load

For q = const. and l = const. expression (5) has the following form:

$$6(R_{01} - R_{02}) = \frac{1}{2} q l^3 \quad (6)$$

Reaction in the support (0) is:

$$A_0 = \frac{q_1 l_1}{2} - \frac{M_1}{l_1}$$

Expression for transversal forces at point (1) is

$$lT_1 = q_i \cdot l_1 - A_0$$

$$dT_1 = \frac{q_i \cdot l_1}{2} + \frac{M_1 - M_2}{l}$$

Reaction in the support (1) is $A_1 = lT_1 + dT_1$

Transversal forces relating to the support (2) have value:

$$lT_2 = q_i \cdot l_2 - dT_1$$

$$dT_2 = \frac{q_2 \cdot l_2}{2} + \frac{M_2}{l_2}$$

Reaction in the support (2) is

$$A_2 = lT_2 + dT_2$$

Considering the known value of the right transversal force at the point of support (2), the reaction in support (3) will be:

$$A_2 = q_3 l_3 - dT_2$$

Value of overall reactions in supports is:

$$\sum q = A_0 + A_1 + A_2 + A_3$$

Critical section and maximum bending moment in the field I is determined from the following equation:

$$x_1 = \frac{A_0}{q_1}; \quad M_{Imax} = \frac{A_0^2}{2q_1}$$

Critical section and maximum bending moment in the field II:

$$x_{II} = \frac{dT_1}{q_2}; \quad M_{I\,Imax} = \frac{dT^2}{2q_2} - M_1$$

Based on the above mentioned, the roof shelf is a continuous loaded carrier with 16 supports with a constant range, where the bending moments M_0 and $M_{15} = 0$, M_1 to M_{14} are equal.

Reactions in the supports A_1 to A_{14} are equal $A_0 = A_{15}$.

Unpaired and paired transversal forces in the supports are equal $lT_1 = lT_{13}$, $dT_2 = lT_{14}$, also maximum bending moments in the mid-range are equal.

3. CHECKING THE STRESS IN THE ROOF SHELF OF EXPLOITATION FIELD

Based on the fact that the roof shelf is a beam of prismatic cross-section with critical point of maximum bending moment, the following can be written $M_{max} = -\frac{ql^2}{8}$, i.e. maximum bending stress is $\sigma_{max} = \frac{M_{max}}{W}$.

For any value x along the range of roof beam, the following is:

$$M_x = -\frac{ql^2}{8} + \frac{qx^2}{2} \quad (7)$$

In this case, the equation of elastic line will have the form according to the second statement:

$$IEY'' = -M_x + \frac{ql^2}{8} - \frac{qx^2}{2} \quad (8)$$

That is, using integral per x – u

$$IEY' = \frac{ql^2}{8} \cdot x - \frac{qx^3}{6} + c_1 \quad (9)$$

Constant c_1 is determined from the condition that at point (0) for x = 0, the tangent on elastic line is horizontal, i.e. y' = 0, which means that also $c_1 = 0$ and using integral (9), the following is obtained:

$$IEY = \frac{ql^2 x^2}{16} - \frac{qx^4}{24} + c_2 \quad (10)$$

According to the critical point of deflection for $x = \frac{l}{2}$ and equating (10) to zero, the value of constant c_2 is obtained

$$0 = \frac{ql^2}{16}\left(\frac{l}{2}\right)^2 - \frac{q \cdot \left(\frac{l}{2}\right)^4}{24} \quad (11)$$

$$c_2 = \frac{5ql^4}{384} \quad (12)$$

The equation of elastic line will obtain the final form

$$IEY = \frac{q}{384E}\left(24l^2x^2 - 16x^4 - 5l^4\right) \quad (13)$$

Deflection in the middle of range for $x = \frac{l}{2}$

$$f_{max} = \frac{5ql^4}{384E} \quad (14)$$

Moment of inertia

$$J = \frac{bh^3}{12}$$

h – height of roof beam
b = 1.0

Resistant moment of section

$$W = \frac{bh_3}{6}$$

Ratio of calculated bending stress and limit value of rock strength to bending gives the safety coeficient

$$n = \frac{\sigma_f}{\sigma_{max\,rač.}}$$

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