Temperature Loading of a Thin Metallic Plate Subjected Transversal to Law-Frequency Electromagnetic Field

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Since many devices (such as magnetic circuits of motors, generators, inductors, transformers) work under the influence of the electromagnetic fields, obtaining Joule's heat as a thermal loading of the thin metallic plate subjected transversal in homogenius, time-varying electromagnetic field is presented in this paper. The direction of the field propagation is normal to the surfaces of the plate. Plate thickness is small compared to the depth of penetration of the magnetic field. Time-varying electromagnetic field is the reason of the conducting currents appearance in the material. The problem is solved in analytical form as the interior Dirichlet boundary problem. The intensity of thermal loading of the plate is obtained in dynamic form using the integral-transformation technique (Double Fourier finite-sine transformation and Laplace transformation). It depends on the plate thickness, electric conductivity, magnetic permeability, frequency and magnetic intensity of the external electromagnetic field, impulse's cycle... The influence of the plate thickness, field frequency and characteristic times of an impulse on the dynamic thermal loading are considered. Thermal loading is the entrance for the further calculation of the behavior of the plate, which is usually done by FEM.

Keywords: *depth of penetration, thin metallic plate, electromagnetic field, magnetic induction, heat power.*

1. INTRODUCTION

As a special scientific field, electro-magneto-thermoelasticity has started to develop at the end of the fifties. The first applications were in geophysics, detection of flaws in ferrous metals, optical acoustics, levitation by superconductors and magnetic fusion. A propagation of an elastic field in the presence of magnetic field was considered by Knopoff [1], Dunkin and Eringen [2]. W.F. Brown developed a rigorous phenomenological theory for ferromagnetic materials on the basis of the large deformation theory and the classical theory of ferromagnetism [3]. H.F. Tiersten [4] developed an analogous theory based on a microscopic model. Since the general nonlinear theory is complicated, Pao and Yeh derived a set of linear equations and boundary conditions for soft ferromagnetic elastic materials [5]. They applied linear theory to investigate magnetoelastic buckling of an isotropic plate. The same problem was treated in an other way by Moon and Pao [6]. This theory was applied by Shindo [7] to define the intensification factors of cracks in ferromagnetic elastic solids. Roychoudhuri and Banerjee (Chattopadhyay) [4] considered the influence of the magnetic fields in a rotating media.

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Basic general pieces of information about the theory of magneto-thermoelasticity were presented in monographs by Parkus [9]. A great contribution of a research in this scientific field was given by Ambarcumian et al. [10] and Krakowski [11]. A mathematical model for the temperature field developed during high frequency induction heating was established by Shen et al. [12]. Sharma and Pal investigated the propagation of magnetic-thermoelastic plane wave in homogeneous isotropic conducting plate under uniform static magnetic field [13]. The two-dimensional problem of electromagneto-thermo-elasticity for perfectly conducting thick plate subjected to a time dependent heat source was studied by Allam et al. [14]. A model calculation of a high temperature superconducting microstrip trans-mission lines was performed by Krakovskii [15].

The subject of this paper is obtaining of Joule's heat as a thermal loading of a thin elastic, isotropic, ferromagnetic plate. The plate is subjected transversally to the homogeneous, time-varying magnetic field. Plate thickness is small compared to the depth of penetration of the magnetic field. The problem is described with three systems of differential equations: Maxwell's equations, equations governing temperature field and equations describing deformation and stress fields. Time-varying electromagnetic field is the reason of the conducting currents appearance in the material which provides Joule's heat. Dynamic impulsive electromagnetic field is mathematicaly defined as a sum of Heaviside functions. The problem is solved in an analytical form as the interior Dirichlet boundary problem. The intensity of thermal loading of the plate is obtained in a dynamic form using the integral-transform technique.

2. BASIC EQUATIONS

Let the metallic rectangular plate dimensions $a \times b \times h$ (Fig. 1) be subjected tranversally to the external electromagnetic field induction $\vec{B}_0(t)$. It is assumed that the plate material is elastic, isotropic, soft ferromagnetic, and has good electric conductivity. Many nickel-iron alloys used for the magnetic circuits of motors, generators, inductors, transformers are of this type.

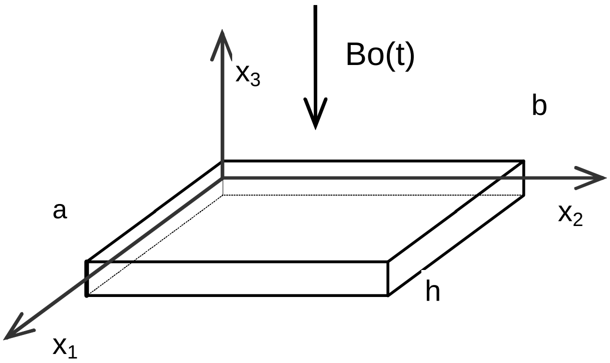

Figure 1. Rectangular plate

Dynamic impulsive electromagnetic field presented in Figure 2 can be mathematicaly defined as a sum of Heaviside functions [16]

$$B_0(t) = B_0 \sum_{i=1}^{k} \sin \omega t \left[H\left(t - t_{oi}\right) - H\left(t - t_{1i}\right) \right], \quad (1)$$

where t_{oi} is the moment of field occurrence, t_{1i} is the moment of field disappearance and ω is the appropriate angular frequency.

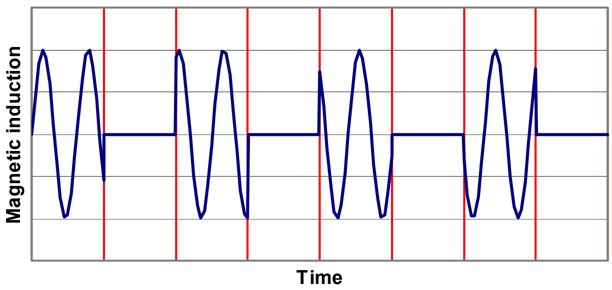

Figure 2. Impulsive electromagnetic field

In the case of the plane electromagnetic wave, field amplitudes decrease according to an exponential law along the trajectory of wave propagation (axis x_3). The penetration constant is in accordance with the decay of one Neper (0.368) and its value is

$$\delta = \frac{1}{\sqrt{\sigma \mu \pi f}}, \quad \omega = 2\pi f, \quad (2)$$

where μ is magnetic permeability, σ is electric conductivity and f is wave frequency. The depth of penetration δ decreases with the increase of frequency, conductivity and permeability.

Figure 3 shows variation of the depth of penetration as a function of wave frequency and relative magnetic permeability μ^* for a soft magnetic material. Electric conductivity of steel is $\sigma = 7.7 \cdot 10^6$ S/m and magnetic permeability for vaccum is $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.

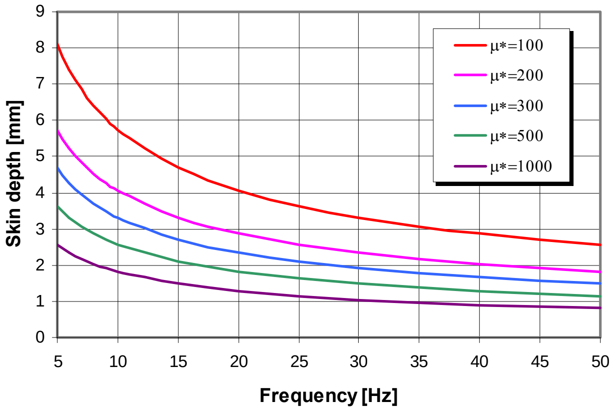

Figure 3. Depth of penetration as a function of wave frequency

As a result of time the changing electromagnetic field conducting currents appear in electric conductors. Let us assume that the change of the electromagnetic field under the influence of the induced conducting currents is small enough that we can prove that the value of magnetic permeability μ is nearly constant.

In the case of the stationary magnetic field conducting currents do not appear in the plate material. So, with the boundary condition

$$B_{\mathrm{p}} = B_0 \Rightarrow \mu H_{\mathrm{p}} = \mu_0 H_0, \quad (3a)$$

magnetic field in the plate H_{p} is

$$H_{\mathrm{p}} = \frac{\mu_0}{\mu} H_0, \quad (3b)$$

Inducted currents form the secondary magnetic field intensity H_1.

This type of a problem is generally mathematically described by a system of Maxwell's equations for slowly moving electrically neutral media and modified Ohm's law [9]

$$\operatorname{rot} \vec{H} = \vec{J} + \frac{\partial \vec{D}}{\partial t}, \ \operatorname{rot} \vec{K} = -\frac{\partial \vec{B}}{\partial t}, \ \operatorname{div} \vec{D} = 0,$$

$$\operatorname{div} \vec{B} = 0, \ \vec{D} = \varepsilon_0 \left(\vec{K} + \dot{\vec{u}} \times \vec{B} \right), \ \vec{B} = \mu_0 \left(\vec{H} - \dot{\vec{u}} \times \vec{D} \right),$$

$$\vec{J} = \sigma \left(\vec{K} + \dot{\vec{u}} \times \vec{B} \right), \quad (4a)$$

where the following notation is applied: H – intensity of the magnetic field, K – intensity of the electric field, B – magnetic flux density (magnetic induction), D – electric induction, J – current density, u – deflection, μ_0 – permeability of vaccum, σ – electric conductivity, ε_0 – dielectric constant of vaccum and t – time.

For the considered problem the system of equations (4a) has simple form

$$\operatorname{rot} \vec{H} = \vec{J}, \ \operatorname{rot} \vec{K} = -\frac{\partial \vec{B}}{\partial t}, \ \operatorname{div} \vec{B} = 0, \ \vec{J} = \sigma \vec{K}. \quad (4b)$$

Magnetic induction in the plate material after conducting current appearance B is

$$\vec{B} = \mu \vec{H} = \mu\left(\vec{H}_{\mathrm{p}} + \vec{H}_1\right) = \vec{B}_0 + \mu \vec{H}_1 , \qquad (5a)$$

and the system of Maxwell's equations can be presented as

$$\mathrm{rot}\, \vec{H}_1 = \vec{J} \,,\ \mathrm{rot}\, \vec{J} = -\sigma\left(\mu \frac{\partial \vec{H}_1}{\partial t} + \frac{\partial \vec{B}_0}{\partial t}\right). \qquad (5b)$$

3. DIFFERENTIAL EQUATION OF THE PROBLEM

Take the assumption that the plate thickness h is small compared to the other two dimensions a and b and that the middle surface of the plate (x_1, x_2, x_3 = 0) has enclosed contour line C.

Current density in the plate material can be presented as [11]

$$\vec{J} = \frac{1}{h}\mathrm{rot}\left[u\left(x_1, x_2, t\right)\vec{k}\right], \qquad (6)$$

where $u(x_1,x_2,t)$ is scalar function which is equal to zero on the contour C. From the condition (5b) we have

$$\mathrm{rot}\left[\vec{H}_1 - \frac{1}{h}u\left(x_1, x_2, t\right)\vec{k}\right] = 0 . \qquad (7)$$

If the plate is thin and if the magnetic field frequency is low, depth of penetration is longer than the plate thickness and we can assume that the current-induced field H_1 is nearly constant along the x_3 axis. Thus, for the intensity of the current-induced magnetic field we have

$$\vec{H}_1 = \frac{1}{h}u\left(x_1, x_2, t\right)\vec{k} \,. \qquad (8)$$

Equations (5b) and (8) give partial differential equation of the considered problem as

$$\frac{\partial^2 u}{\partial x_1^2} + \frac{\partial^2 u}{\partial x_2^2} - \sigma\mu\frac{\partial u}{\partial t} = \sigma h \frac{\partial B_0(t)}{\partial t} , \qquad (9)$$

which is valid inside the contour C. On the defined contour, function $u(x_1,x_2,t)$ is zero.

4. CONDUCTING CURRENTS

Using the (1) which gives magnetic induction of the external magnetic field, the (9) takes the following form

$$\nabla_1^2 u - \sigma\mu\partial_t u =$$
$$= \sigma h \sum_{i=1}^{k}\left\{B_0 \omega \cos\omega t\left[H\left(t - t_{\mathrm{o}i}\right) - H\left(t - t_{1i}\right)\right] + ,\right.$$
$$\left. B_0 \sin\omega t\left[\delta\left(t - t_{\mathrm{o}i}\right) - \delta\left(t - t_{1i}\right)\right]\right\} , \qquad (10)$$

where ∇_1^2 is the two-dimension Laplace operator, ∂_t is the time derivative and δ is Dirac delta-function. Using the following notation

$$\alpha_m = \frac{m\pi}{a} ,\ \alpha_n = \frac{n\pi}{b} ,\ c_{mn} = \frac{\alpha_n^2 + \alpha_m^2}{\sigma\mu} , \qquad (11a)$$

and applying the double Fourier finite-sine transformation marked as mn and the Laplace transformation marked as *, $t \to p$ we arrive at the transformation function of the function u as (11b).

The inverse Laplace transformation and the inverse double Fourier finite-sine transformation give the final solution for the function $u(x_1,x_2,t)$ in the form (12).

The intensity of the magnetic field in the plate material is defined as

$$H_{x_3} = H_{\mathrm{p}} + H_1 = \frac{\mu_0}{\mu}H_0 + \frac{1}{h}u . \qquad (13a)$$

Boundary condition (3b) gives the intensity of the external magnetic field after conducting current appearance in the form

$$H_0^{'} = H_0 + \frac{\mu}{\mu_0}\frac{u}{h} . \qquad (13b)$$

5. JOULE'S HEAT

Joule's heat P, as eddy-current losses, can be calculated from the expression

$$P = \frac{h}{\sigma}\int_S \left\| J \right\|^2 dS , \qquad (14a)$$

$$u_{mn}^{*} = -\frac{4hB_0}{\mu\alpha_m\alpha_n}\sum_{i=1}^{k}\left[\frac{\omega p\left(e^{-pt_{\mathrm{o}i}} - e^{-pt_{1i}}\right)}{\left(p^2 + \omega^2\right)\left(p + c_{mn}\right)} + \frac{\sin\omega t_{\mathrm{o}i} e^{-pt_{\mathrm{o}i}} - \sin\omega t_{1i} e^{-pt_{1i}}}{p + c_{mn}}\right]. \qquad (11b)$$

$$u\left(x_1, x_2, t\right) = -\frac{16hB_0}{\mu ab}\sum_{m=1,3,\ldots}^{\infty}\sum_{n=1,3,\ldots}^{\infty}\frac{\sin\alpha_m x_1 \sin\alpha_n x_2}{\alpha_m\alpha_n} \cdot$$
$$\cdot\sum_{i=1}^{k}\left\{\left[\sin\omega t_{\mathrm{o}i} e^{-c_{mn}\left(t - t_{\mathrm{o}i}\right)} + \omega\frac{c_{mn}\cos\omega\left(t - t_{\mathrm{o}i}\right) + \omega\sin\omega\left(t - t_{\mathrm{o}i}\right) - c_{mn}e^{-c_{mn}\left(t - t_{\mathrm{o}i}\right)}}{c_{mn}^2 + \omega^2}\right]H\left(t - t_{\mathrm{o}i}\right) - \right.$$
$$\left. - \left[\sin\omega t_{1i} e^{-c_{mn}\left(t - t_{1i}\right)} + \omega\frac{c_{mn}\cos\omega\left(t - t_{1i}\right) + \omega\sin\omega\left(t - t_{1i}\right) - c_{mn}e^{-c_{mn}\left(t - t_{1i}\right)}}{c_{mn}^2 + \omega^2}\right]H\left(t - t_{1i}\right)\right\} . \qquad (12)$$

$$J_{x_1}(x_1,x_2,t) = -\frac{16B_0}{\mu ab}\sum_{m=1,3,...}^{\infty}\sum_{n=1,3,...}^{\infty}\frac{\sin\alpha_m x_1\cos\alpha_n x_2}{\alpha_m}\cdot$$

$$\cdot\sum_{i=1}^{k}\left\{\left[\sin\omega t_{oi}e^{-c_{mn}(t-t_{oi})}+\omega\frac{c_{mn}\cos\omega(t-t_{oi})+\omega\sin\omega(t-t_{oi})-c_{mn}e^{-c_{mn}(t-t_{oi})}}{c_{mn}^2+\omega^2}\right]H(t-t_{oi})-\right.$$

$$\left.-\left[\sin\omega t_{1i}e^{-c_{mn}(t-t_{1i})}+\omega\frac{c_{mn}\cos\omega(t-t_{1i})+\omega\sin\omega(t-t_{1i})-c_{mn}e^{-c_{mn}(t-t_{1i})}}{c_{mn}^2+\omega^2}\right]H(t-t_{1i})\right\},$$

$$J_{x_2}(x_1,x_2,t) = -\frac{16B_0}{\mu ab}\sum_{m=1,3,...}^{\infty}\sum_{n=1,3,...}^{\infty}\frac{\cos\alpha_m x_1\sin\alpha_n x_2}{\alpha_n}\cdot$$

$$\cdot\sum_{i=1}^{k}\left\{\left[\sin\omega t_{oi}e^{-c_{mn}(t-t_{oi})}+\omega\frac{c_{mn}\cos\omega(t-t_{oi})+\omega\sin\omega(t-t_{oi})-c_{mn}e^{-c_{mn}(t-t_{oi})}}{c_{mn}^2+\omega^2}\right]H(t-t_{oi})-\right.$$

$$\left.-\left[\sin\omega t_{1i}e^{-c_{mn}(t-t_{1i})}+\omega\frac{c_{mn}\cos\omega(t-t_{1i})+\omega\sin\omega(t-t_{1i})-c_{mn}e^{-c_{mn}(t-t_{1i})}}{c_{mn}^2+\omega^2}\right]H(t-t_{1i})\right\}. \quad (15)$$

$$\|J\| = \sqrt{\|J_{x_1}\|^2+\|J_{x_2}\|^2} = \frac{1}{h}\sqrt{\left|\frac{\partial u}{\partial x_1}\right|^2+\left|\frac{\partial u}{\partial x_2}\right|^2}, \quad (14b)$$

where S is an appropriate area of the middle surface of the plate.

The power of the conducting current densities (15) can be calculated using (6) and (12).

The presented procedure is suitable to obtain Joule's heat (eddy-current losses) in a plate subjected transversally to time-changing magnetic field in the case when the depth of penetration is large compared to the plate thickness. The assumption that the magnetic field in the plate is constant through the plate thickness is valid only for the low-frequency external magnetic fields. For high-frequency problems the presented procedure has to be performed with the finite element method.

The power of the conducting currents is presented by one type of volume heat source in the plate. The system of equations describing temperature field in a plate is [17]

$$\left(\nabla^2-\frac{1}{\kappa}\frac{\partial}{\partial t}\right)\theta-\eta\dot{u}_{j,j} = -\frac{W}{\lambda_0},$$

$$W = W_{\mathrm{E}}+W_{\mathrm{H}}+\frac{J^2}{\sigma},\ j=1,2,3 \quad (16)$$

where κ is the coefficient of thermal intensity, η is the coupling between the temperature and the deformation fields, λ_0 is the heat conduction coefficient, ∇^2 is the Laplace operator. The temperature field is presented as θ [°C, K] = $T - T_0$ where T_0 is the temperature of the plate in its natural state.

The quantity of heat generated in a unit volume and unit time (heat source intensity) $W(x_1,x_2,x_3,t)$ consists of three parts: the intensity of external heat source W_{E}, the hysterisis losses W_{H} and the Joule's heat (eddy-current losses).

The presented equation has to be completed with an appropriate set of boundary and initial conditions.

If we take into account plate vibrations, we have to involve finite element analysis along with the analytically obtained solutions for the heat power and the temperature field.

6. NUMERICAL RESULTS

6.1 Joule's heat for a sinusoidal field

The first numerical example is given for a thin steel rectangular plate with the thickness of 1 mm. The plate was subjected to the external sinusoidal magnetic field of a induction B_0 = 2T. It is assumed that all field components vary in time t as sinωt. The properties of steel are: relative magnetic permeability μ_{rel} = 500 and electric conductivity $\sigma = 10^6$ S/m.

According to the solution (14a) and (15), the power of the eddy-current losses was calculated for two field frequencies and for three dimensions of the square plates.

Figure 4 presents time variation of the heat power for the field frequency of 2 Hz in the period of 2 seconds. The presented diagram is obtained for the point coordinates $x_1 = x_2$ = 10 mm.

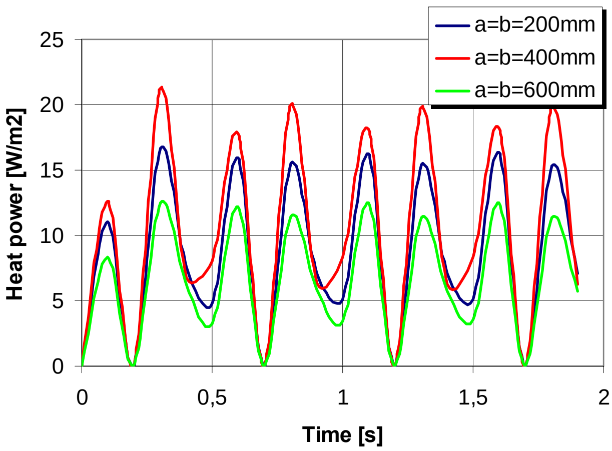

Figure 4. Heat power for the field frequency of 2 Hz at the point $x_1 = x_2$ = 10 mm as a function of time

Dynamic variation of the heat power for the same external field frequency and for $x_1 = x_2 = a/10$ is shown in Figure 5.

The results for the external magnetic field of frequency 20 Hz in the period of 200 ms are presented in Figure 5.

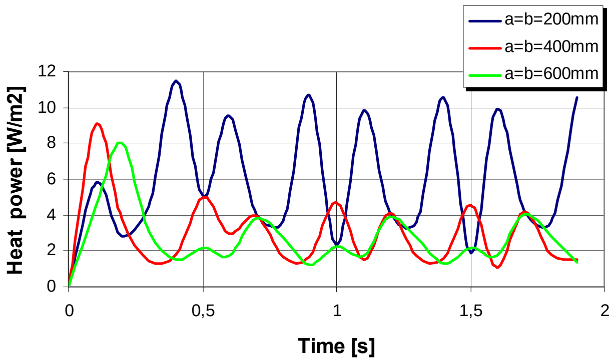

Figure 5. Heat power for the field frequency of 2 Hz and the point *x*1 = *x*2 = *a*/10 as a function of time

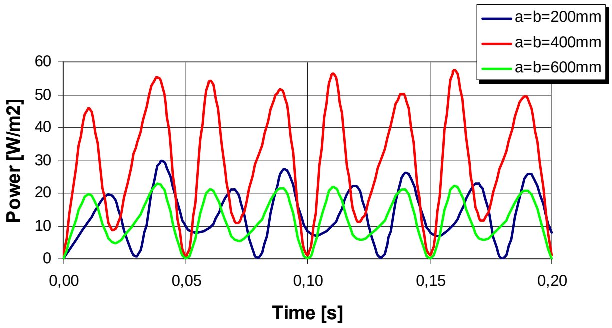

Figure 6. Heat power for the field frequency of 20 Hz at the point $x_1 = x_2 = 10$ mm as a function of time

Distribution of the eddy-current power (Joule's heat) across the middle surface of the plate is presented in Figures 7 and 8. The power was calculated for the field frequency of 2 Hz and at the moment $t = 1.125$ s.

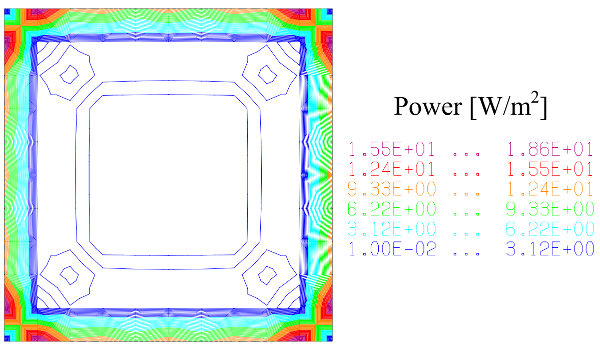

Figure 7. Distribution of the heat power across middle surface of the plate dimensions 200 × 200 × 1 mm

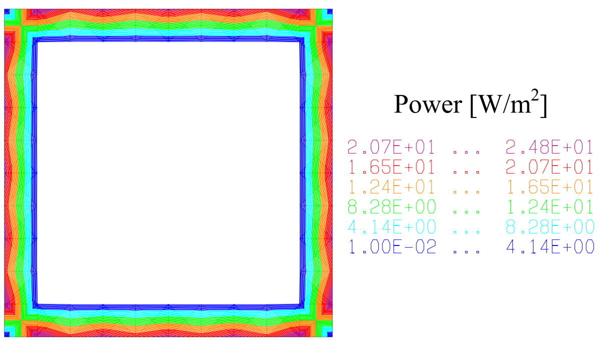

Figure 8. Distribution of the heat power across middle surface of the plate dimensions 400 × 400 × 1 mm

As noticeable, conducting currents appear only near the edges. Heat power is concetrated only in the zone width of about a/10. The center of the plate has not thermal loading.

6.2 Joule's heat for an impulsive magnetic field

Let the square steel plate dimensions 200 × 200 × 1 mm be exposed to impulsive external strong magnetic field of maximum induction of 2*T*.

In the first example the frequency of the field was 1 Hz, while the pulse lasts 0.5 s and the time between two neighboring pulses (relaxation time) was 0.5 s, too. The appropriate diagram is presented in Figure 9.

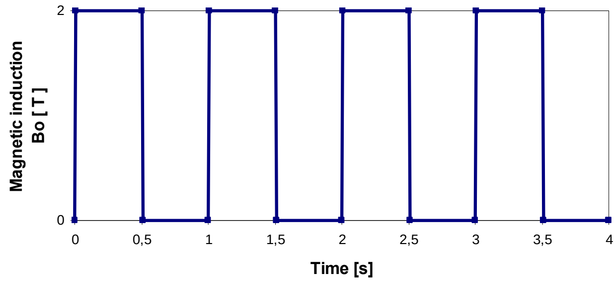

Figure 9. External magnetic induction as a function of time (impulsive magnetic field frequency 1 Hz)

Dynamic variation of the heat power for the impulsive external magnetic field presented in Figure 9 for the points coordinates of $x_1 = x_2 = a/10$ is shown in Figure 10.

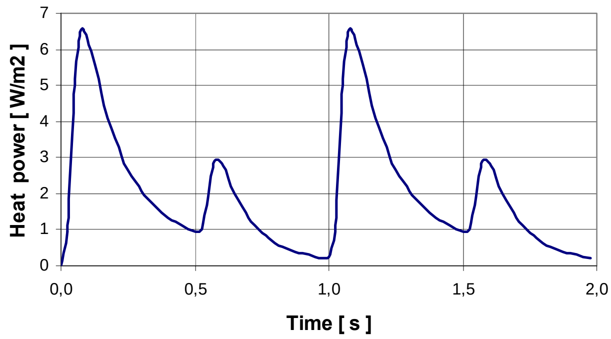

Figure 10. Heat power for the impulsive field of frequency 1 Hz at the point $x_1 = x_2 = a/10$ as a function of time

The power of the conducting currents rapidly increases during the field appearance and disappearance. Duration time and relaxation time of the pulse are long enough that eddy-currents vanish at the end of each circle.

According to the analytical solution, in the second example, heat power of the eddy-current losses was calculated for the ten times higher frequency field. Characteristic times of the pulse are described in the Figure 11.

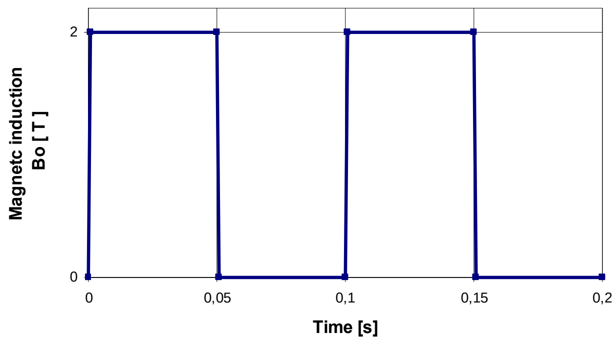

Figure 11. External magnetic induction as a function of time (impulsive magnetic field frequency 10 Hz)

The frequency of the field was 10 Hz, while the pulse lasts 0.05 s. The diagram describing dynamic variation of the heat power is presented in Figure 12.

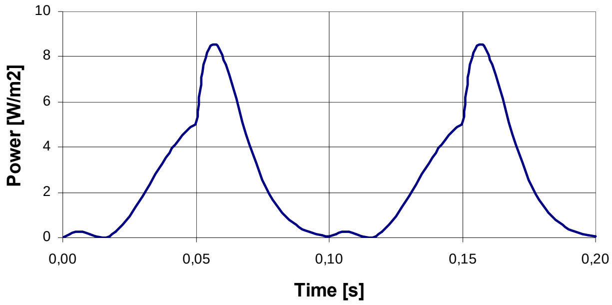

Figure 12. Heat power for the impulsive field frequency 10 Hz in the point $x_1 = x_2 = a/10$ as a function of time

As noticeable from the presented diagram, for the field frequency of 10 Hz the power of the conducting currents rapidly increases during the field disappearance.

The following example presents thermal loading of a thin rectangular plate with dimensions $a = b = 200$ mm and $h = 1$ mm. The distribution of the heat power (Joule's heat) across the middle surface of the plate is presented in Figures 13 and 14.

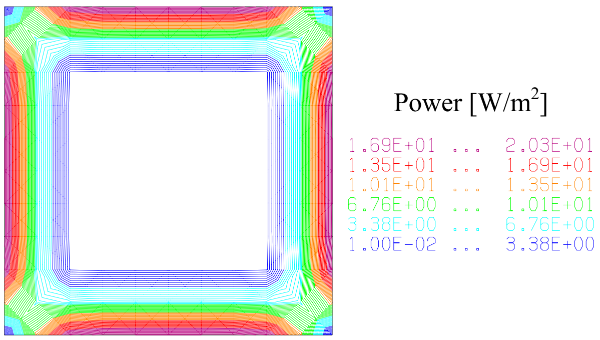

Figure 13. Distribution of the heat power across the middle surface for the plate dimensions 200 × 200 × 1 mm at $t = 0.1$ s

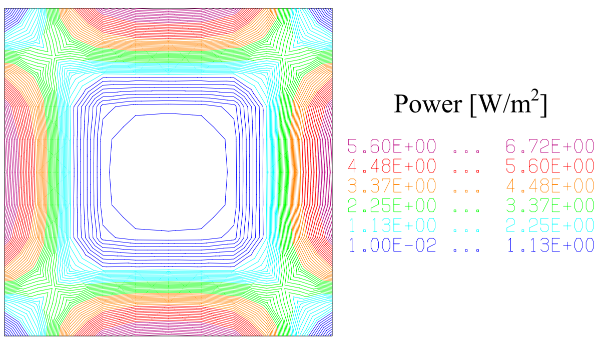

Figure 14. Distribution of the heat power across the middle surface for the plate dimensions 200 × 200 × 1 mm at $t = 0.3$ s

Let the field frequency is 1 Hz according to the diagram in Figure 9.

The material properties are the same as in the previous examples.

Under the influence of the field appearance the power of the eddy-currents increases and achieves its maximum after about 0.08 s. Thus, 0.1 s after the field appearance the power of the conducting currents is still concentrated near the edges. During the time, the intensity of the heat power decreases and propagates to the center of the plate.

The distribution of the heat power obtained using analytical solution for $t = 0.1$ s is presented in Figure 13 and for $t = 0.3$ s in Figure 14.

Further calculation (obtaining temperature field, stress and deformation) is usually done using the finite element analysis.

7. CONCLUSION

As many constructions (type magnetic circuits of motors, generators, inductors, transformers) work under the influence of the electromagnetic fields, the subject of this paper is obtaining Joule's heat as a thermal loading of a thin metallic plate. The plate is subjected to the homogeneous, time-changing electro-magnetic field. The direction of the field propagation is normal to the surfaces of the plate. It is assumed that the plate material is elastic, isotropic, soft ferromagnetic which has good electric conductivity. The plate thickness is small compared to the penetration depth of the magnetic field.

The problem of a metallic plate subjected transversally to a strong, homogeneous, time-varying magnetic field can be described through three systems of differential equations: Maxwell's equations, equations governing temperature field and equations describing deformation and stress fields.

Time-varying electromagnetic field is the cause of the conducting currents appearance in the material which provides Joule's heat. Dynamic impulsive electromagnetic field can be mathematicaly defined as a sum of Heaviside functions. If the plate is thin and if the magnetic field frequency is low, the depth of penetration is large compared to the plate thickness and we can assume that the current-induced field is nearly constant along the plate thickness.

The presented problem is analytically solved as the interior Dirichlet boundary problem and the intensity of the thermal loading of the plate is obtained in the dynamic form. A very suitable method for solving the problem, as shown in this paper, is the integral-transformation technique (Double Fourier finite-sine transformation and Laplace transformation). It depends on the plate thickness, electric conductivity, magnetic permeability, frequency and magnetic intensity of the exterior electromagnetic field, impulse cycle... So, the influence of the plate thickness, field frequency and characteristic times of the pulse on the dynamic thermal loading of the plate is considered.

In the case of sinusoidal magnetic field conducting currents appear only near the edges. Heat power is concetrated only in the zone close to edges. The center of the plate has not thermal loading.

When the magnetic field has impulsive character the power of the conducting currents rapidly increases during the field occurrence and disappearance. Under the influence of the field appearance the power of the eddy-currents increases and achieves maximum after about several hundreds of seconds. After field appearance, the power of the conducting currents is concentrated near the edges. During the time the intensity of the heat power decreases and propagates to the center of the plate.

Thermal loading is the entrance for the further calculation of the behavior of the plate, which is usually done by FEM.

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NOMENCLATURE

a, b	middle surface dimensions
h	plate thickness
t	time
B	magnetic induction
H	magnetic field intensity
J	conducting currents density
P	heat power
K	electric field intensity
f	magnetic field frequency
ω	angular frequency
μ	magnetic permeability
μ_0	magnetic permeability of vaccum
σ	electric conductivity
δ	Dirac delta-function
∇_1^2	two-dimension Laplace operator

ТЕМПЕРАТУРСКО ОПТЕРЕЋЕЊЕ ТАНКЕ МЕТАЛНЕ ПЛОЧЕ ПОД ДЕЈСТВОМ МАГНЕТНОГ ПОЉА НИСКЕ ФРЕКВЕНЦИЈЕ

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Пошто многе конструкције типа магнетских кола електромотора, генератора, индуктора или трансформатора раде под утицајем електромагнетских поља, у раду је приказано одређивање Џулове топлоте као термичког оптерећења танке металне плоче. Плоча је постављена трансверзално у односу на хомогено временски променљиво електромагнетско поље. Правац простирања поља је управан на средњу раван плоче. Дебљина плоче је мала у поређењу са дубином продирања магнетног поља. Временски променљиво електромагнетно поље доводи до појаве кондукционих струја у материјалу плоче. Проблем је решен у аналитичком облику као унутрашњи Дирихлеов гранични проблем. Интензитет топлотног оптерећења плоче одређен је у динамичком облику применом технике интегралних трансформација (двоструке коначне синусне Фуријеове трансформације и Лапласове трансформације). Он зависи од дебљине плоче,

електричне проводности материјала, магнетске пермеабилности, фреквенције и индукције спољашњег магнетног поља, импулсног циклуса... Разматран је утицај димензија плоче, фреквенције поља и карактеристичних времена импулса на динамичко термичко оптерећење плоче. Тако дефинисано термичко оптерећење представља улазни податак за одређивање понашања плоче (напон, деформација), које се уобичајено одређује применом методе коначних елемената.