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A Novel Method for the Inclusion of Pipe Roughness in the Hazen-Williams Equation

Accurate estimation of friction losses in pipes is an important engineering task. Due to their simplicity, empirical equations are often used for determining pressure drops in pipes. One of the most widely used empirical equations for calculation of pressure drops in straight pipes is Hazen-Williams equation. In this paper, the authors have established a simple method of the inclusion of pipe roughness in Hazen-Williams equation by comparison with a widely accepted Darcy-Weisbach method coupled with Colebrook friction factor formula for developed turbulent flow.

Keywords: *Hazen-Williams equation, pressure drop, pipe roughness, turbulent flow.*

1. INTRODUCTION

Pressure drop calculation is one of the essential tasks in plant engineering. Pumps, fans, blowers and other machinery are dimensioned based on pressure drop calculations. If these energy losses are underestimated, the plant might not work properly (i.e. not enough fluid will reach the end user). On the other hand, if these losses are overestimated, they will result in an economically unjustifiable plant operation due to high energy consumption (motors will be grossly oversized). Therefore, accurate estimation of pressure losses is important, even in the starting phases of any project.

Calculation of pressure losses is usually not a simple task, but quite the contrary. In the early phases of the project there are a lot of unknowns which render the task of determining the pressure losses all the more complicated. To make the matter even more complex, the formulae which are usually used for pressure drop estimation, such as Colebrook equation coupled with Darcy-Weisbach equation, are not always explicit. Consequently, these equations more often than not need to be solved by employing some more or less complex mathematical apparatus.

In order to overcome the mathematical complexities, roughness estimates and Reynolds number dependencies, simplified equations are often used. These are mainly based on empirical data and provide engineers with directly and easily obtained solutions. However, utilization of such calculation procedures inherently has its pitfalls, mainly due to the number of limitations and approximations set forth by using a simplified approach. In some literature sources users are even advised against using this type of equations [1]. Two such empirical approaches commonly met in fluid mechanics are Hazen-Williams [2] and Manning equations [3].

It needs to be stressed that the discussed topic is very elaborate and vast, and that Hazen-Williams equation is merely one of the available equations for calculation of pressure drop in water filled pipelines. Other equations, such as previously mentioned Colebrook equation or equations published in various literature sources (e.g. [4], [5], [6], etc.) may provide more accurate results, but are also requiring more detailed input data and more calculation effort. A comprehensive overview of most commonly used equations for calculation friction factor is given in [7].

2. HAZEN-WILLIAMS EQUATION AND ITS LIMITATIONS

When pressure drop estimations in water-carrying pipes are observed, one of the most widely used empirical equations for pressure drop is Hazen-Williams equation, which is written in the form [8]

$$\Delta h_f = \frac{\Delta p_f}{\rho \cdot g} = \frac{10.67 \cdot L \cdot Q^{1,852}}{C^{1,852} \cdot D^{4,87}} \tag{1}$$

While this equation is widely used, and even referenced in some codes [9], being an empirical equation it has some obvious drawbacks. Its main limitation is that it does not depend directly on pipe

Received: March 2014, Accepted: May 2014
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doi:10.5937/fmet1501040S
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roughness, but Hazen-Williams parameter C is defined as a constant value, depending only on pipe material, as shown in Table 1.

However, it is a well documented fact that roughness of any one material can vary significantly over the course of its lifetime. For instance, according to [10], absolute roughness of seamless carbon steel pipes can vary from 0.04 to more than 1.5mm, depending on several parameters, but mainly on the conveyed fluid and age of piping (for water lines even values in excess of 5mm are mentioned). The same is true for many other materials referenced in Table 1.

If influence of all other variables, such as dependency of water viscosity on its temperature (note that equation (1) is completely independent on fluid viscosity), is disregarded, it makes sense to investigate in some manner the dependency of Hazen-Williams constant C on pipe roughness. An apparent way for this is to compare the results obtained by Hazen-Williams equation with the ones obtained by utilizing the well-established Darcy-Weisbach equation coupled with Colebrook friction factor, as explained in further text.

Table 1. Values of Hazen-Williams constant from various sources

Material	*C*-factor		
	[8]	[11]	[9]
Asbestos Cement	-	140÷150	140
Brass	-	120÷150	-
Black steel (dry systems)	130	100	100
Black steel (wet systems)	130	120	120
Cast iron - New unlined	130	120÷130	100
Cast iron - 10 years old	100	105÷75	-
Cast iron - 15 years old	100	100÷60	-
Cast iron - 20 years old	80	95÷55	-
Cast iron - 30 years old	80	85÷45	-
Cast iron - 50 years old	80	75÷40	-
Cast iron - Bitumen-lined	-	140	-
Cast iron - Cement-lined	140	140	140
Concrete	120	85÷150	140
Copper	-	120÷150	150
Fibre glass pipe	-	150÷160	-
Fire hose (rubber)	-	135	-
Galvanized steel	-	120	120
Lead	-	130÷150	-
Polyethylene	-	150	-
PVC and plastic pipe	150	150	150
Stainless steel	-	150	150
Steel new and unlined	-	140÷150	-
Steel, welded and seamless	130	100	-
Vitrified clays	-	110	-
Wood	120	-	-
Clay, new riveted steel	110	-	-

2.1 More accurate determination of c factor

Basic idea behind the developed method was to vary pipe diameter, absolute roughness and volumetric flow rate of water in order to calculate the value of C factor predicted by using some of the more sophisticated calculation methods (i.e. Darcy-Weisbach equation with Colebrook friction factor). These values are used to extract a correlation equation between the C factor and relative pipe roughness.

The Darcy-Weisbach equation states that

$$\Delta p_f = f \cdot \frac{L}{D} \cdot \frac{\rho \cdot w^2}{2} \tag{2}$$

where friction factor depends on the Reynolds number and pipe roughness.

In this case, the value of Hazen-Williams coefficient can be expressed by combining equations (1) and (2) as

$$C = \left(\frac{1}{f} \cdot \frac{2}{w^2} \cdot \frac{g \cdot 10.67 \cdot Q^{1,852}}{D^{3,87}} \right)^{\frac{1}{1.852}} \tag{3}$$

or

$$C = \left(129 \cdot \frac{D^{0.13}}{f \cdot Q^{0.148}} \right)^{\frac{1}{1.852}} \tag{4}$$

The Friction factor can be calculated using many well established procedures [7]. In this case, the Colebrook equation will be used for determination of friction factor for fully developed turbulent flow. Formula proposed by Colebrook [12] is broadly accepted design formula for determination of turbulent friction factor in the range of $Re=4000 \div 10^8$ and $\varepsilon=0 \div 0.05$ and is given by equation

$$\frac{1}{\sqrt{f}} = -2 \cdot \log\left(\frac{2.51}{Re \cdot \sqrt{f}} + \frac{\varepsilon}{3.7} \right) \tag{5}$$

where $\varepsilon = \frac{a_r}{D}$ and $Re = \frac{\rho \cdot w \cdot D}{\mu}$.

Correlation equations are derived based on 1300 examined cases. In these test points absolute pipe roughness has varied from 0.01mm to 1mm, pipe diameters ranged from DN80 to DN500, and Reynolds numbers have varied in the range 66500÷680000. These limits are selected based on engineering judgment and authors' experience. In addition, as it will be shown later, the accuracy of Hazen-Williams equation with the constant C factor diminishes with higher Reynolds numbers, which is all the more reason to concentrate on higher Reynolds numbers.

Using equation (3), it can be shown that for aforementioned absolute pipe roughness, diameter and Reynolds number ranges, the Hazen-Williams C coefficient changes roughly within the limits 90÷150. All calculations are performed for thermo-physical properties of water at 4°C (i.e. density of 1000 kg/m^3 and viscosity of 0.00153Pa·s).

The following equation provides the best fit for the entire range of aforementioned 1300 test points

$$C = 50 - 10 \cdot \ln(\varepsilon) = 50 - 10 \cdot \ln\left(\frac{a_r}{D}\right) \tag{6}$$

However, although equation (6) provides the best fit for all data points, in some cases it may yield somewhat underestimated pressure losses compared to those obtained by using the Colebrook equation. For this reason, the following equation has been extracted, which yields conservative results for almost all examined points.

$$C = 45 - 10 \cdot \ln(\varepsilon) = 45 - 10 \cdot \ln\left(\frac{a_r}{D}\right) \tag{7}$$

2.2 Statistical parameters of evaluation

Statistical parameters used in deriving and examining equations (6) and (7) are as follows

a. correlation ratio

$$\theta = \sqrt{1 - \frac{\sum_{i=1}^{n}\left(y_i - y_i^c\right)^2}{\sum_{i=1}^{n}\left(y_i - y_{av}\right)^2}} \tag{8}$$

b. standard deviation

$$\Delta_{av} = \sqrt{\frac{\sum_{i=1}^{n}\left(\frac{y_i - y_i^c}{y_i}\right)^2}{n}} \tag{9}$$

where average value of variable y_i for complete set of input data is calculated as

$$y_{av} = \frac{\sum_{i=1}^{n} y_i}{n} \tag{10}$$

In this case, the variable y_i corresponds to the value of *C* factor calculated by using equation (3), while y_i^c corresponds to the value of *C* factor calculated by using equations (6) or (7). The value of *n* is the total number of data points which have been examined, which is in this case 1300.

Equation (6) yields maximum error of 10.46%, standard deviation of 3.52% and correlation ratio of 0.9349. This equation is presented alongside all examined points in Figure 1.

Figure 1 also shows the equation (7) which yields conservative results for almost entire data set. For this equation standard deviation for the entire data set is 6.67%, while maximum error is 14.32%.

Results of the proposed equation are compared to various values of the Hazen-Williams constant shown in Table 2. Values presented in Table 2 show in how many of the total 1300 cases the *C* factor values calculated by equation (6) yields smaller relative error than the relevant constant value when compared to the *C* factor values calculated by equation (3).

Based on Table 2 it can be concluded that no matter which *C* factor constant value is selected, equation (6) yields better results in the majority of cases. Based on this data, and Figure 1 it can also be concluded that various constant values of the *C* factor can yield non-conservative (underestimated) pressure drop values, thus potentially creating operating problems in a plant.

Table 2. Comparison of results obtained by proposed equation and constant *C* values

	Eq (6)	*C*=90	*C*=100	*C*=110
Max error, %	10.5	39.4	32.6	25.9
No of cases in which equation (6) yields better results than constant	n/a	1283 (98.7%)	1217 (93.6%)	1056 (81.2%)
	C=120	*C*=130	*C*=140	*C*=150
Max error, %	34	45.1	56.3	67.5
No of cases in which equation (6) yields better results than constant	1053 (81%)	1108 (85.2%)	1193 (91.8%)	1290 (99.2%)

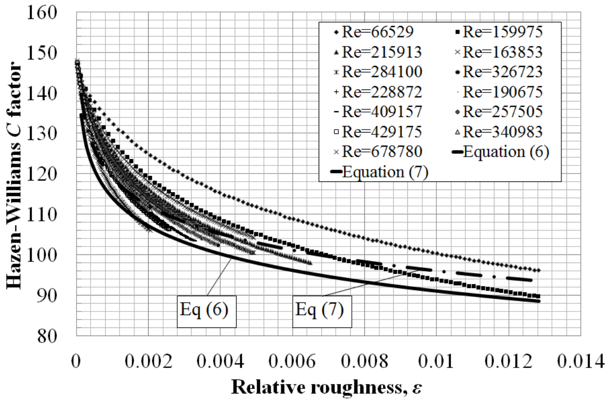

Figure 1. Test points with correlation equation

2.3 Example

Let us consider the flow of 360m^3/h of water in 100m long DN250 (ø273x9.27mm) steel pipe. Absolute roughness of pipe is assumed to equal 0.5mm, which is reasonable assumption after couple of years of service. Water density is adopted as 1000 kg/m^3 and dynamic viscosity as 0.00153Pa·s.

According to the Colebrook equation (5), the friction factor in this example equals 0.0239. This yields the pressure drop of 18226Pa.

If the Hazen-Williams equation is used with constant *C* factor of 130 (refer to Table 1), the calculated pressure drop according to equation (1) would be 14043Pa. This means an underestimate of pressure drop by ~4180Pa, or ~23% compared to the Colebrook equation.

On the other hand, if equation (6) is used to calculate the C factor of 112.3, then the pressure drop calculated by equation (1) would be 18408Pa, which is in good correlation with the results obtained by using the Colebrook equation.

Finally, if conservative equation (7) is used to determine the *C* factor of 109.3, then the pressure drop determined by equation (1) would be 20028Pa, which is ~9% greater value than the pressure drop calculated by using the Colebrook equation.

For this particular example, the correspondence between the pressure drop calculated according to

Hazen-Williams equation using the C factor of 130 and the pressure drop calculated by using Colebrook equation would be for the absolute roughness of 0.14mm. In other words, for pipe absolute roughness greater than 0.14mm, the Hazen-Williams equation with the C factor of 130 would give erroneous (underestimated) results.

Summary of the results is given in Table 3.

Table 3. Summary of example results

Calculation method	f	C	Δp, Pa	Error, %
Colebrook equation	0.0239	n/a	18226	n/a
Hazen-Williams equation – constant C	n/a	130	14403	-26.5
Hazen-Williams equation – C calculated with eq (6)	n/a	112.3	18408	+1.0
Hazen-Williams equation – C calculated with eq (7)	n/a	109.3	20028	+8.8

3. CONCLUSION

The Hazen-Williams equation is often used in real world problems mainly due to its simplicity. However, as it is shown, it can yield uncertain and, even more importantly, non-conservative results, especially for greater Reynolds numbers and relative roughness values. For example, as shown in Figure 1, the C factor value of 130 (cited for steel pipes), gives relatively conservative results only up to the relative roughness value of $0.8\cdot10^{-3}$, and only for relatively small Reynolds numbers.

Data presented in Figure 1 show that there is dependency of the Hazen-Williams C factor on pipe roughness, while data shown in Table 2 shows that if proposed equation (6) is used for calculation of the Hazen-Williams C factor, in a vast majority of cases it yields better results than a constant value.

In addition, equation (7) is given, which still produces conservative, but more realistic results than the constant, for almost all 1300 examined cases.

It is not implied in the paper that the Hazen-Williams equation is the most accurate or even correct choice for determining the pressure drop in water carrying pipelines. It still has many drawbacks, and frequently falls short in accuracy in comparison to some of the other equations, such as the Colebrook and Darcy-Weisbach equations which have been used as a benchmark in this paper. However, its sheer simplicity renders it very useful when no detailed routing data exists and quick and easy estimations are required. Equations presented in this paper are meant to be used as a supplement of the original equation by including the influence of pipe roughness, thus increasing its accuracy without impeding its simplicity.

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NOMENCLATURE

a_r, m	absolute pipe roughness
C	C-factor (Hazen-Williams constant)
D, m	pipe internal diameter
f	friction factor
g, m/s^2	gravitational acceleration
Δh, m,	head loss
L, m	pipe length
n	Number of examined data points
Δp, Pa	pressure drop
Q, m^3/s	water volumetric flow rate
Re	Reznolds number
w, m/s	fluid velocity
y_i	i-th value of variable

Greek symbols

Δ	standard deviation
ε	relative pipe roughness
μ, Pa·s	dynamic fluid viscosity
ρ, g/m^3	water density
θ	correlation ratio

Superscripts

av	average
f	friction

НОВИ МЕТОД ЗА УКЉУЧИВАЊЕ ЕФЕКАТА ХРАПАВОСТИ ЦЕВИ У ХАЗЕН-ВИЛИЈАМСОВУ ЈЕДНАЧИНУ

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Прецизна процена пада притиска услед трења у цевоводима је веома битан инжењерски задатак. Пре свега због једноставности, емпиријске једначине често налазе примену за прорачуне пада притиска у цевоводима. Једна од најчешће коришћених емпиријских једначина која се примењује у овој области је Хазен-Вилијамсова једначина. У овом раду су аутори извели једноставан приступ који може да се примени за укључивање ефеката храпавости цеви у Хазен-Вилијамсову једначину. Приступ се састоји у поређењу наведене једначине са добро познатом Дарси-Вајсбахом једначином и Колбруковом једначином за прорачун фактора трења за развијено турбулентно струјање.